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► **To cite this version:**

Laurent Krähenbühl, Patrick Dular, Victor Péron, Ronan Perrussel, Clair Poignard, et al.. Asymptotic delta-Parametrization of Surface-Impedance Solutions. IEEE CEFC, May 2014, Annecy, France. hal-00959750

**HAL Id: hal-00959750**

**<https://hal.science/hal-00959750v1>**

Submitted on 7 Mar 2019

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# Asymptotic delta-Parametrization of Surface-Impedance Solutions

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**Abstract**—The surface impedance methods are among the most efficient for solving time-harmonic eddy-current problems with a small penetration depth. When the solution is required for a wide range of frequencies (or material conductivities) the standard approach leads to the solution of a complex-valued problem for each frequency (or conductivity). Hereafter we introduce a close method, parametrized by the skin depth ( $\delta$ ), based on a formal asymptotic expansion. It provides accurate results with a reduced computational cost for a wide range of  $\delta$  values.

**Index Terms**—Surface impedance, asymptotics, parametric solutions.

## I. SURFACE IMPEDANCES

The classical surface impedance method allows to solve approximately and quite accurately a time-harmonic eddy-current problem in a conductor (with a linear magnetic behavior), when the skin depth  $\delta$  is small compared to the characteristic size  $D$  of the conducting parts of the device under study [1]. If the boundary  $\Sigma$  of the conductor is regular enough, one can compute the electromagnetic field in the outer domain  $\Omega$  by imposing a surface impedance condition on  $\Sigma$ :

$$\text{curl } \mathbf{H} = \mathbf{J}_s \text{ in } \Omega, \quad (1)$$

$$\mathbf{n} \times \mathbf{E} = Z_s \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \text{ on } \Sigma, \quad (2)$$

$$Z_s = \frac{1+j}{\sigma\delta}, \quad (3)$$

with  $\mathbf{H}$  the magnetic field,  $\mathbf{J}_s$  the source current density,  $\mathbf{E}$  the electric field,  $\mathbf{n}$  the outward normal,  $j$  the imaginary unit and  $Z_s$  the so-called surface impedance that depends on the electric conductivity  $\sigma$  and  $\delta$ . The finite element solution is straightforward, e.g. in a 2D plane case, the vector potential ( $A$ ) formulation gives ( $A$  and  $J$  with only one component):

$$-\Delta A = \mu_0 J_s \text{ in } \Omega; \quad A = \alpha \delta \partial_n A \text{ on } \Sigma; \quad \alpha = \frac{j-1}{2}. \quad (4)$$

If the frequency (or conductivity) is modified, the solution has to be performed again.

## II. ASYMPTOTIC EXPANSION AND PARAMETRIZATION

The solution to Problem (4) can be expanded in a formal series in power of  $\alpha\delta$  as in [2]:

$$A = \sum_{i \geq 0} (\alpha\delta)^i A_i, \quad (5)$$

where the coefficients are real-valued solutions to elementary problems (6)–(7) independent of  $\delta$ :

$$-\Delta A_0 = \mu_0 J_s \text{ in } \Omega; \quad A_0 = 0 \text{ on } \Sigma. \quad (6)$$

$$\forall i \geq 1, \quad -\Delta A_i = 0 \text{ in } \Omega; \quad A_i = \partial_n A_{i-1} \text{ on } \Sigma. \quad (7)$$

In practice, the computation of only 2, or 3 terms ( $A_0, A_1, A_2$ ) suffices to ensure high accuracy (see Section III). Furthermore, the solution (4) for any new small value  $\delta$  can be simply reconstructed by combining linearly the pre-computed terms as in (5); what amounts to a considerable gain in computational time for sensitivity or parametric studies.

## III. NUMERICAL EXAMPLE

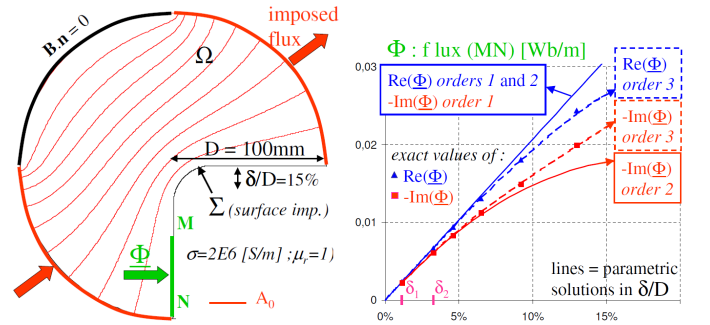


Fig. 1. Domain  $\Omega$  with  $A_0$  on the left. Flux vs.  $\delta/D$  on the right.

A simple test case is represented in Fig. 1. We enforce a flux at part of the boundary  $\Sigma$  of a conducting angle. The first term of (5) is depicted in Fig. 1, left. The flux through segment  $MN$ ,  $\Phi = A(M) - A(N)$ , is shown as a function of  $\delta/D$  and compared to the exact solution in Fig. 1, right. The approximate flux is observed to be accurate for  $\delta/D < 15\%$ .

At the conference, we will detail how we compute the 3 first orders based on 2 solutions with surface impedance for 2 distinct values of  $\delta$ ,  $\delta_1$  and  $\delta_2$ . The 3 first orders seem to provide an “accurate” behavior; see Fig. 1, right. We will also discuss error estimates, the case of a linear magnetic conductor and the 3D formulations.

## REFERENCES

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