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To cite this version:

Sergey Alexandrov, Sibasish Banerjee. Fivebrane instantons in Calabi-Yau compactifications. Physical Review D, 2014, 90, pp.041902(R). 10.1103/PhysRevD.90.041902 . hal-00958486

HAL Id: hal-00958486

https://hal.science/hal-00958486v1

Submitted on 4 Jun 2021

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Fivebrane instantons in Calabi-Yau compactifications

Sergei Alexandrov and Sibasish Banerjee*
Laboratoire Charles Coulomb, CNRS UMR 5221,
Université Montpellier 2, F-34095 Montpellier, France

We provide the last missing piece of the complete non-perturbative description of the low energy effective action emerging from Calabi-Yau compactifications of type II string theory — NS5-brane instanton corrections to the hypermultiplet moduli space $\mathcal{M}_H$. We find them using S-duality symmetry of the type IIB formulation. The result is encoded in a set of holomorphic functions on the twistor space of $\mathcal{M}_H$ and includes all orders of the instanton expansion.

INTRODUCTION

One of the outstanding problems in string theory is to derive the effective theories appearing as a low energy limit of string compactifications. This is an important step in connecting string theory to the real world, which can also shed light on its non-perturbative structure. Eventually, we are interested in compactifications preserving $N = 1$ or no supersymmetry, since these cases are relevant from the phenomenological point of view. However, at present the full low energy description of such compactifications seems to be beyond our reach.

At the same time, compactifications preserving eight supercharges in four dimensions seem to be more tractable, and still involving very non-trivial and rich physics. One type of such compactifications is provided by type II string theory on a Calabi-Yau manifold. During recent years a significant progress has been achieved in getting its non-perturbative description (see [1, 2] for reviews). The corresponding effective theory is completely determined by the metric on its moduli space, which is a direct product of vector and hypermultiplet components, $\mathcal{M}_V$ and $\mathcal{M}_H$. It is in the latter space where all complications are hidden. Being a quaternion-Kähler (QK) manifold [3], $\mathcal{M}_H$ receives instanton corrections coming from branes wrapping non-trivial cycles of the Calabi-Yau [4]. There are two types of such branes which contribute to the non-perturbative metric: D-branes and NS5-branes. Using twistorial techniques [5–7], which provide a very efficient parametrization of quaternionic manifolds, the D-brane instantons have been incorporated in a series of works [8–10]. This result left NS5-brane instantons as the only remaining unknown piece of the full non-perturbative picture.

An attempt to include these corrections has been done in [11]. It was based on the fact that in the type IIB formulation S-duality maps D5-branes into NS5-branes, which makes possible to get the latter once we know the former. However, due to a complicated action of S-duality on the twistor space, where the D-instantons have the most simple formulation, this idea had been realized only in the one-instanton approximation.

In this paper we go beyond this restriction and provide a complete description of fivebrane instantons which includes all orders of the instanton expansion. The clue to such a result is an improved parametrization of contact transformations which encode the geometry of the twistor space of $\mathcal{M}_H$. It allows essentially to linearize the action of S-duality and thus to extract fivebrane instanton corrections to the hypermultiplet metric. In a companion paper [12] we will show the consistency of our result with the U-duality symmetry group of $\mathcal{M}_H$, which requires an improved realization of these symmetries, and extend it to include contributions from interactions between fivebrane and D1-D(-1)-instantons.

TWISTOR APPROACH AND CONTACT GEOMETRY

Darboux coordinates and transition functions

We start with a brief review of twistorial techniques which are necessary tools for describing instanton corrections to the metric on $\mathcal{M}_H$ [1]. The need for these techniques stems from the fact that a generic QK manifold is not even a complex manifold so that the constraints of the QK geometry appear to be highly non-trivial. At the same time, $4n$-dimensional QK spaces $\mathcal{M}$ are in one-to-one correspondence with $(4n+2)$-dimensional Kähler spaces $\mathcal{Z}_\mathcal{M}$ carrying a *holomorphic contact structure*. The latter are known as *twistor spaces* and appear as $\mathbb{C}P^1$ bundles over the original QK manifolds.

The presence of the complex and holomorphic contact structures makes the twistor spaces much easier to work with. In particular, the contact structure can be represented by a set of holomorphic one-forms $\mathcal{X}^{[i]}$ on each of the patches of the covering $\mathcal{Z}_\mathcal{M} = \cup\mathcal{U}_i$. They are defined up to rescalings by nowhere vanishing holomorphic smooth functions, and such that $\mathcal{X}^{[i]} \wedge \left(\mathrm{d}\mathcal{X}^{[i]}\right)^n \neq 0$ is a holomorphic top form. Given these one-forms, in each patch one can introduce a system of Darboux coordinates $(\xi^\Lambda_{[i]}, \tilde{\xi}^{[i]}_\Lambda, \alpha^{[i]})$, $\Lambda = 0, \ldots, n-1$, fixed (non-uniquely) by the requirement that

$$\mathcal{X}^{[i]} = \mathrm{d}\alpha^{[i]} + \xi^\Lambda_{[i]}\mathrm{d}\tilde{\xi}^{[i]}_\Lambda. \tag{1}$$

Then all information about the geometry of $\mathcal{Z}_\mathcal{M}$, and hence $\mathcal{M}$, is contained in the changes of the Darboux coordinates between different patches. They should preserve the contact one-form up to an overall holomorphic factor $\mathcal{X}^{[i]} = \hat{f}^2_{ij}\,\mathcal{X}^{[j]}$. Such contact transformations can

be parametrized by holomorphic functions $H^{[ij]}$ which, by analogy with canonical transformations, are taken to depend on ξ^Λ in patch $\mathcal{U}_i$ and $\tilde{\xi}_\Lambda, \alpha$ in patch $\mathcal{U}_j$. In these terms the gluing conditions read [10]

$$\begin{aligned} \xi^\Lambda_{[j]} &= \xi^\Lambda_{[i]} - \partial_{\tilde{\xi}_\Lambda^{[ij]}} H^{[ij]} + \xi^\Lambda_{[i]} \partial_{\alpha^{[ij]}} H^{[ij]}, \\ \tilde{\xi}^{[ij]}_\Lambda &= \tilde{\xi}^{[i]}_\Lambda + \partial_{\xi^\Lambda_{[i]}} H^{[ij]}, \\ \alpha^{[ij]} &= \alpha^{[i]} + H^{[ij]} - \xi^\Lambda_{[i]} \partial_{\xi^\Lambda_{[i]}} H^{[ij]}, \end{aligned} \tag{2}$$

which results in $\hat{f}^2_{ij} = 1 - \partial_{\alpha^{[ij]}} H^{[ij]}$. Supplemented by proper regularity conditions, these relations can be rewritten as integral equations and solved with respect to Darboux coordinates as functions of coordinates on the QK base and the $\mathbb{C}P^1$ fiber. Starting from these solutions, there is a straightforward, although a bit non-trivial procedure to derive the metric on $\mathcal{M}$ [7].

Thus, the twistorial description encodes the geometry of a QK space into a covering of its twistor space and the associated set of holomorphic functions $H^{[ij]}$. In fact, for the purpose of constructing the *local* metric on $\mathcal{M}$, the recipe can even be simplified: it is sufficient to provide a set of contours ℓ_i on $\mathbb{C}P^1$ and a set of transition functions $H^{[i]}$ attached to each contour. Whereas the closed contours typically correspond to boundaries of open patches $\mathcal{U}_i$, open contours arise as an effective description of transition functions with branch cuts. We refer to [1] for more details.

Applying this approach to the problem of computing the non-perturbative metric on the hypermultiplet moduli space $\mathcal{M}_H$, we see that it reduces to the problem of finding contours and holomorphic functions for each type of quantum corrections contributing to the metric. The perturbative metric was put into this language in [7] and the twistor data for D-instantons have been found in [9, 10]. As a result, a D-instanton of charge $\gamma = (p^\Lambda, q_\Lambda)$ is generated by the data consisting of the contour

$$\ell_\gamma = \{\ell \in \mathbb{C}P^1 : \ Z_\gamma/\ell \in \mathrm{i}\mathbb{R}^-\}, \tag{3}$$

where Z_γ is the central charge of the $N = 1$ supersymmetry algebra preserved by the D-brane, and the transition function

$$H^{[\gamma]} = H_\gamma - \frac{1}{2} q_\Lambda p^\Lambda \left(H'_\gamma\right)^2, \tag{4}$$

where H_γ is given by

$$H_\gamma(\Xi_\gamma) = \frac{\bar{\Omega}(\gamma)}{4\pi^2} \sigma_D(\gamma) \, \mathrm{e}^{-2\pi \mathrm{i} \Xi_\gamma}. \tag{5}$$

Here $\sigma_D(\gamma)$ is the the so-called quadratic refinement, $\bar{\Omega}(\gamma)$ are rational DT invariants [23], and we used the notation $\Xi_\gamma = q_\Lambda \xi^\Lambda - p^\Lambda \tilde{\xi}_\Lambda$.

Contact bracket and improved transition functions

Although the parametrization of the contact transformations (2) using transition functions $H^{[ij]}$ is very explicit, it also has some inconveniences. The most important problem comes from that the arguments of $H^{[ij]}$ belong to different patches. As a result, all symmetries of the twistor space are realized on the transition functions in a very non-trivial way. An example is the symplectic invariance of the D-instanton corrections: whereas the function (5) is clearly symplectic invariant, as (p^Λ, q_Λ) and $(\xi^\Lambda, \tilde{\xi}_\Lambda)$ transform as vectors under symplectic transformations, this is not the case for the transition function (4). This suggests that there should be another way to parametrize the contact transformations where the fundamental role is shifted to the function (5) [24].

To introduce the new parametrization, we need first to define the so-called "contact bracket", which can be viewed as a lift of the Poisson bracket to the realm of contact geometry and was defined previously in [9, Eq.(2.44)]. This can be done in a coordinate independent way as follows. Let us associate with a function h the "contact vector field" X_h determined by the following relations

$$\iota_{X_h} \mathrm{d}\mathcal{X} = -\mathrm{d}h + R(h)\mathcal{X}, \qquad \iota_{X_h} \mathcal{X} = h, \tag{6}$$

where ι_X is contraction of vector X with a differential form, $\mathcal{X}$ is the contact one-form, and R is the Reeb vector field which is the unique element of the kernel of $\mathrm{d}\mathcal{X}$ such that $\mathcal{X}(R) = 1$. Then the contact bracket between two functions h and f is defined as

$$\{h, f\} = X_h(f). \tag{7}$$

In terms of Darboux coordinates, it reads explicitly as

$$\begin{gathered} \{h, \xi^\Lambda\} = -\partial_{\tilde{\xi}_\Lambda} h + \xi^\Lambda \partial_\alpha h, \qquad \{h, \tilde{\xi}_\Lambda\} = \partial_{\xi^\Lambda} h, \\ \{h, \alpha\} = h - \xi^\Lambda \partial_{\xi^\Lambda} h. \end{gathered} \tag{8}$$

Note that the bracket is not antisymmetric, but satisfies

$$\{h, h\} = hR(h) = h\partial_\alpha h, \tag{9}$$

which can be obtained by applying ι_{X_h} to the first equation in (6). Besides, the bracket does not follow the Leibnitz rule in the first argument giving instead

$$\{h_1 h_2, f\} = h_1\{h_2, f\} + h_2\{h_1, f\} - h_1 h_2 \partial_\alpha f. \tag{10}$$

The reason for the different behaviour with respect to the two arguments is that actually they are supposed to be local sections of different bundles, $\mathcal{O}(2)$ and $\mathcal{O}(0)$, respectively, and the contact bracket maps them into an $\mathcal{O}(0)$ section [9].

A crucial property of the contact bracket, which immediately follows from its definition, is that it generates an infinitesimal transformation scaling the contact one-form

$$\mathcal{L}_{X_h} \mathcal{X} = \mathrm{d}\left(\iota_{X_h} \mathcal{X}\right) + \iota_{X_h} \mathrm{d}\mathcal{X} = R(h)\mathcal{X}, \tag{11}$$

i.e. a contact transformation. Conversely, any *finite* contact transformation, for instance, the one which relates Darboux coordinates in different patches, can be

parametrized by a holomorphic function $h^{[ij]}$ and generated by the contact bracket via the following action

$$\Xi^{[j]} = e^{\{h^{[ij]},\cdot\}}\,\Xi^{[i]}, \tag{12}$$

where $\Xi^{[i]}$ denotes the set of Darboux coordinates in the patch $\mathcal{U}_i$. What is important here is that $h^{[ij]}$ is considered as a function of Darboux coordinates in *one* patch. We call it *improved transition function*.

Comparing the gluing conditions (12) and (2), one obtains relations between the two types of transition functions. In particular, this gives the following explicit expression (we dropped the patch indices)

$$H = \left(e^{\{h,\cdot\}} - 1\right)\alpha + \xi^{\Lambda}\left(e^{\{h,\cdot\}} - 1\right)\tilde{\xi}_{\Lambda}. \tag{13}$$

To illustrate the new parametrization, let us apply the relation (13) to the D-instanton case taking the improved transition function to be $h = H_\gamma(\Xi_\gamma)$. Using the fact that α-independent functions commute with themselves (see (9)), one obtains

$$\begin{aligned} \left(e^{\{H_\gamma,\cdot\}} - 1\right)\tilde{\xi}_{\Lambda} &= q_{\Lambda} H'_\gamma, \\ \left(e^{\{H_\gamma,\cdot\}} - 1\right)\alpha &= H_\gamma - q_{\Lambda}\xi^{\Lambda} H'_\gamma - \frac{1}{2}p^{\Lambda} q_{\Lambda}(H'_\gamma)^2. \end{aligned} \tag{14}$$

Substituting this into (13), one reproduces the previous result (4).

S-DUALITY IN TWISTOR SPACE

As we explained in the introduction, we are going to find fivebrane instanton contributions to the metric on $\mathcal{M}_H$ by applying S-duality to the D5-brane corrections. Therefore, it is important to understand how S-duality acts at the level of the twistor space, in particular, on Darboux coordinates and on transition functions.

This question has been already addressed in the previous works [9, 13, 14]. It was found that for an $SL(2,\mathbb{Z})$ transformation $\mathrm{g} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to be an isometry of $\mathcal{M}_H$ lifted to the twistor space the Darboux coordinates should transform as

$$\begin{aligned} \xi^0 &\mapsto \frac{a\xi^0 + b}{c\xi^0 + d}, \qquad \xi^a \mapsto \frac{\xi^a}{c\xi^0 + d}, \\ \tilde{\xi}_a &\mapsto \tilde{\xi}_a + \frac{c\kappa_{abc}\xi^b\xi^c}{2(c\xi^0 + d)} - c_{2,a}\varepsilon(\mathrm{g}), \\ \begin{pmatrix} \tilde{\xi}_0 \\ \alpha \end{pmatrix} &\mapsto \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}\begin{pmatrix} \tilde{\xi}_0 \\ \alpha \end{pmatrix} - \frac{c}{6}\kappa_{abc}\xi^a\xi^b\xi^c \begin{pmatrix} \frac{-c}{c\xi^0+d} \\ \frac{c(a\xi^0+b)+2}{(c\xi^0+d)^2} \end{pmatrix}, \end{aligned} \tag{15}$$

where $a = 1, \ldots, n-1$, κ_{abc} are triple intersection numbers of the Calabi-Yau, $c_{2,a}$ are components of the second Chern class along a basis of 2-forms, and $\varepsilon(\mathrm{g})$ is the multiplier system of the Dedekind eta function.

Combined with the gluing conditions (2), the transformation (15) can be used to get the transformation property of the transition functions $H^{[ij]}$. Although its explicit form was found [14], it is highly non-linear and its direct application is rather involved. We do not give it here since we found a way to proceed which is much more instructive and elegant. The idea is that the passage to the improved transition functions $h^{[ij]}$ will also improve transformation properties under S-duality: one can hope that the non-linearities appearing in the transformation law for $H^{[ij]}$ are of the same origin as those in (4) and will disappear when one works in terms of $h^{[ij]}$.

To show that this is indeed the case, let us note the following property of the contact bracket

$$\{(c\xi^0 + d)\mathrm{g}\cdot h, \mathrm{g}\cdot f\} = \mathrm{g}\cdot\{h, f\}, \tag{16}$$

which can be verified by direct calculation using (8) and (15). With its help it becomes easy to evaluate the operator relating Darboux coordinates in two patches after the $SL(2,\mathbb{Z})$ transformation. One finds

$$\begin{aligned} \mathrm{g}\cdot e^{\{h,\cdot\}}\cdot\mathrm{g}^{-1} &= e^{\mathrm{g}\cdot\{h,\cdot\}\cdot\mathrm{g}^{-1}} \\ &= e^{\{(c\xi^0+d)\mathrm{g}\cdot h,\cdot\}}. \end{aligned} \tag{17}$$

This implies that a QK manifold carries the isometric action of the S-duality group $SL(2,\mathbb{Z})$ only if the improved transition functions on its twistor space are split into $SL(2,\mathbb{Z})$ multiplets and transform *linearly* inside each multiplet with weight -1, e.g. [25]

$$h_{m,n} \mapsto \frac{h_{m',n'}}{c\xi^0 + d}, \quad \begin{pmatrix} m' \\ n' \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix}, \tag{18}$$

where the pair (m, n) labels the elements of the multiplet.

FIVEBRANE INSTANTONS

An important property of type IIB string theory compactified on a Calabi-Yau is that quantum corrections to the hypermultiplet moduli space $\mathcal{M}_H$ are arranged into different sectors invariant under S-duality. This happens according to the following pattern:

α':	perturb.	w.s. inst		
g_s:	1-loop D(-1)	D1	D3	D5 NS5

(19)

Thus, D(-1)-instantons are combined with perturbative α' and g_s-corrections, D1-instantons mix with worldsheet instantons, D3-instantons are S-duality invariant, whereas D5 and NS5-instantons transform as a doublet under $SL(2,\mathbb{Z})$. This splitting allows to study each sector independently of the others. In particular, a twistorial description of the first two sectors, which is manifestly S-duality invariant, has been given in [13, 15]. The sector of D3-branes has been studied in [16], where it was shown that the transition functions (4) restricted to this sector are consistent with S-duality. Here we concentrate on the sector of fivebranes and derive all corresponding instanton corrections from the knowledge of D5-instantons.

In type IIB theory D5-branes, or more precisely D5-D3-D1-D(-1)-bound states, are characterized by the rational valued generalized Mukai vector $\gamma = (p^0, p^a, q_a, q_0)$ with $p^0 \neq 0$ [17]. Below we will also need the so-called invariant charges [11]

$$\begin{aligned} \hat{q}_a &= q_a + \tfrac{1}{2}\kappa_{abc}\tfrac{p^b p^c}{p^0}, \\ \hat{q}_0 &= q_0 + \tfrac{p^a q_a}{p^0} + \tfrac{1}{3}\kappa_{abc}\tfrac{p^a p^b p^c}{(p^0)^2} \end{aligned} \tag{20}$$

and the reduced charge vector $\hat{\gamma} = (p^a, \hat{q}_a, \hat{q}_0)$.

Since fivebrane instantons form a doublet of $SL(2,\mathbb{Z})$, their improved transition functions must follow the transformation law (18). Identifying D5-branes with the component $(m,n) = (0,p^0)$, all of them can be obtained by applying g to the function $h^{[\hat{\gamma}]}_{0,p^0} = H_\gamma$. Taking into account the physical interpretation of the charges, it is convenient to take

$$c = -k/p^0, \qquad d = p/p^0, \qquad p^0 = \gcd(p,k). \tag{21}$$

Then k appears to be precisely the NS5-brane charge. Using (15), it is straightforward to obtain

$$h^{[\hat{\gamma}]}_{k,p} = -\frac{\bar{\Omega}_{k,p}(\hat{\gamma})}{4\pi^2}\frac{k}{p^0}(\xi^0 - n^0)\sigma_D(\gamma)\, e^{2\pi i S_{k,p;\hat{\gamma}}}, \tag{22}$$

where

$$\begin{aligned} S_{k,p;\hat{\gamma}} \;=\; & k\left(\alpha + n^\Lambda \tilde{\xi}_\Lambda + F^{\rm cl}(\xi - n)\right) \\ & + \frac{p^0(k\hat{q}_a(\xi^a - n^a) + p^0\hat{q}_0)}{k^2(\xi^0 - n^0)} + \frac{a}{k}p^0 q_0 - c_{2,a}p^a c(\mathrm{g}), \end{aligned} \tag{23}$$

$F^{\rm cl}(X) = -\kappa_{abc}\frac{X^a X^b X^c}{6X^0}$ is the classical prepotential, $n^a = p^a/k$, $n^0 = p/k$, and $\bar{\Omega}_{k,p}(\hat{\gamma}) \equiv \bar{\Omega}(\gamma; \mathrm{g}\cdot z)$ takes into account the fact that DT invariants are only piecewise constant and generically jump along lines of marginal stability in the moduli space of Kähler structure deformations z^a. It should not be surprising that the resulting function (22), up to the factor ensuring the correct modular weight, coincides with the result found in [11, Eq.(5.30)] in the one-instanton approximation: in this approximation the two types of transition functions are identical and the remarkable feature of (22) is that it is *exact* at linear order.

Furthermore, it is possible to evaluate explicitly the contact transformation generated by the function $h^{[\hat{\gamma}]}_{k,p}$ by applying the operator (12). Referring to [12] for details, here we give the result just for the transition function defined by the relation (13) [26]

$$H^{[\hat{\gamma}]}_{k,p} = h^{[\hat{\gamma}]}_{k,p} - 2\pi^2 (h^{[\hat{\gamma}]}_{k,p})^2 \left[\frac{\hat{q}_0 (p^0)^2}{k(\xi^0 - n^0)} - \frac{2k^2 F^{\rm cl}(\xi - n)}{\left(1 - 2\pi i k h^{[\hat{\gamma}]}_{k,p}\right)^2}\right]. \tag{24}$$

One observes that this function generates an infinite expansion in instantons equivalent to the expansion in DT invariants or in $h^{[\hat{\gamma}]}_{k,p}$. In [12] we also show that it solves the non-linear S-duality constraint found in [14] and is consistent with all discrete symmetries generating the U-duality group of the compactified theory.

CONCLUSIONS

In this paper we found a twistorial description of fivebrane instanton corrections to the hypermultiplet moduli space $\mathcal{M}_H$ of type II string theory on a Calabi-Yau. It is provided by the holomorphic functions (22) and (24). With these results at hand, one can now write integral equations for Darboux coordinates on the twistor space of $\mathcal{M}_H$, whose solution uniquely defines the metric on the moduli space and thereby the non-perturbative low energy effective action of the compactified theory.

This progress has become possible due to a new parametrization (12) of contact transformations with the help of the so-called contact bracket. The improved transition functions $h^{[\hat{\gamma}]}$ entering this parametrization appear to be more fundamental than their ordinary cousins $H^{[\hat{\gamma}]}$. As a result, all transformation laws and results for instantons take a much simpler linear form being reformulated in their terms.

Although our results provide a complete description of the fivebrane sector of quantum corrections to $\mathcal{M}_H$, it still remains to put all quantum effects, shown in (19), into one unifying picture. This problem is addressed in [12], where we provide the twistor space construction including all sectors except the one of D3-branes. The latter represents a challenge since, despite it is captured by the transition functions (4), even in the one-instanton approximation it has not been reformulated yet in an explicitly S-duality invariant way. It is expected that a crucial role in such a reformulation will be played by mock modular forms [16].

Another interesting problem is to put our type IIB construction into the mirror type IIA formulation. In particular, it is interesting to see how the NS5-brane instantons deform the integrable structure of the D-instantons encoded in the Thermodynamic Bethe Ansatz equations [18, 19].

Besides, there is a number of important issues which can be approached once the fivebrane instantons have been incorporated. These include: a resolution of the singularity generated by the one-loop correction on $\mathcal{M}_H$; divergence of the sum over charges due to the exponential growth of DT invariants in supergravity, which was argued to be related to the NS5-brane effects [20]; and consistency of the NS5-brane instantons with wall-crossing.

Finally, one can hope that the results presented here can be useful as well for phenomenological studies. For instance, in [21] it was argued that the fivebrane instantons may be crucial for the derivation of the Starobinsky model [22] of the inflationary cosmology from compactifications of string theory.

Acknowledgments

We are grateful to Sylvain Ribault for careful reading of the manuscript.

* Electronic address:
sakexand@univ-montp2.fr, sitsasishbanerjee@live.in

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[23] The original result has been formulated in terms of the di-logarithm and integer DT invariants, which also makes explicit its relation to Thermodynamic Bethe Ansatz [18, 19]. The two formulations are readily equivalent.

[24] In fact, such a symplectic invariant description has been already proposed in [10]. But it works only if transition functions satisfy a certain integrability condition, which is indeed the case for H_γ, but turns out not to be the case for the NS5-transition functions found below.

[25] It is possible also to have on the r.h.s. some regular contributions, which can always be absorbed into a redefinition of Darboux coordinates not affecting the contact structure.

[26] The function (24) is written in terms of Darboux coordinates in one patch what is not sufficient to calculate its derivatives entering the gluing conditions (2). Their expressions can be found in [12].