Power Allocation Strategies in Energy Harvesting Wireless Cooperative Networks

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***Abstract*—In this paper, a wireless cooperative network is considered, in which multiple source-destination pairs communicate with each other via an *energy harvesting* relay. The focus of this paper is on the relay's strategies to distribute the harvested energy among the multiple users and their impact on the system performance. Specifically, a non-cooperative strategy that uses the energy harvested from the i-th source as the relay transmission power to the i-th destination is considered first, and asymptotic results show that its outage performance decays as $\frac{\log SNR}{SNR}$. A faster decay rate, $\frac{1}{SNR}$, can be achieved by two centralized strategies proposed next, of which a water filling based one can achieve optimal performance with respect to several criteria, at the price of high complexity. An auction based power allocation scheme is also proposed to achieve a better tradeoff between system performance and complexity. Simulation results are provided to confirm the accuracy of the developed analytical results.**

I. INTRODUCTION

Low cost mobile devices have been recognized as crucial components of various wireless networks. Examples include wireless sensor networks which have been developed for a variety of applications, including surveillance, environmental monitoring and health care. Such low cost devices are typically equipped with fixed energy supplies, such as batteries with limited operation life. Replacing batteries for such devices is either impossible or expensive, particularly in the case in which sensors are deployed in hostile environments. Therefore energy harvesting, a technique to collect energy from the surrounding environment, has recently received considerable attention as a sustainable solution to overcoming the bottleneck of energy constrained wireless networks [1].

Conventional energy harvesting techniques rely on external energy sources that are not part of communication networks, such as those based on solar power, wind energy, etc. [1], [2]. Recently a new approach to energy harvesting has been proposed that involves collecting energy from ambient radio frequency signals [3], [4], so that wireless signals can be used as a means for the delivery of information and power simultaneously. In addition, such an approach can also reduce the cost of communication networks, since peripheral equipment to take advantage of external energy sources can be avoided. The concept of simultaneous power and information delivery was first proposed in [3] for flat fading channels, where the fundamental tradeoff between the energy and information rate is characterized for point-to-point communication scenarios. The extension of such a concept to frequency selective channels is considered in [4]. In [5] the authors study energy harvesting for communication scenarios with co-channel interference, where such interference is identified as a potential energy source. The simultaneous transfer of power and information is also studied in multiple-input multiple-output systems with imperfect channel information at the transmitter in [6].

A difficulty that such energy harvesting networks is that practical circuits cannot realize energy harvesting and data detection from wireless signals at the same time. This challenge has motivated a few recent works deviating from the ideal assumption that a receiver can detect signals and harvest energy simultaneously. In [7], the authors introduced a general receiver architecture, in which the circuits for energy harvesting and signal detection are operated in a time sharing or power splitting manner. The impact of power splitting on the tradeoff between the achievable information transfer rate and the harvested energy is characterized in [8], and the performance difference between power splitting and time sharing is studied in broadcasting scenarios in [9]. These approaches are naturally applicable to cooperative networks, and their impact on the outage probability for amplify-and-forward (AF) relaying networks with one source-destination pair is studied in [10].

In this paper, a general wireless cooperative network is considered, in which multiple pairs of sources and destinations communicate through an *energy harvesting* relay. Specifically, multiple sources deliver their information to the relay via orthogonal channels, such as different time slots. The relaying transmissions are powered by the signals sent from the sources. Assuming that the battery of the relay is sufficiently large, the relay can accumulate a significant amount of power for relaying transmissions. The aim of this paper is to study how to efficiently distribute such power among the multiple users and investigate the impact of these power allocation strategies on the system performance.

The contribution of this paper is four-fold. *Firstly*, a non-cooperative individual transmission strategy is developed, in

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which the relaying transmission to the i-th destination is powered by using only the energy harvested from the i-th source. Such a simple power allocation scheme will serve as a benchmark for other more sophisticated strategies developed in the paper. The decode-and-forward (DF) strategy is considered, and an exact expression for the outage probability achieved by such a scheme is obtained. Based on this expression, asymptotic studies are carried out to show that the average outage probability for such a scheme decays with the signal-to-noise ratio (SNR) at a rate of $\frac{\log SNR}{SNR}$. Similar asymptotic results have been recently reported in [11] in the context of conventional energy harvesting networks.

Secondly, the performance of an equal power allocation scheme is investigated, in which the relay distributes the accumulated power harvested from the sources evenly among relaying transmissions. The advantage of such a scheme is that a user pair with poor channel conditions can be helped since more relay transmission power will be allocated to them compared to the individual transmission strategy. Exact expressions for the outage performance achieved by this transmission scheme are obtained. Analytical results show that the equal power allocation scheme can always outperform the individual transmission strategy. For example, the average outage probability achieved by the equal power allocation scheme decays at the rate of $\frac{1}{SNR}$, faster than the individual transmission scheme.

Thirdly, a more opportunistic power allocation strategy based on the sequential water filling principle is studied. The key idea of such a strategy is that the relay will serve a user with a better channel condition first, and help a user with a worse channel condition afterwards if there is any power left at the relay. This sequential water filling scheme can achieve the optimal performance for the user with the best channel conditions, and also maximize the number of successful destinations. Surprisingly, it can also be proved that such a scheme minimizes the worst user outage probability. Several bounds are developed for the average outage probability achieved by such a scheme, and asymptotic studies are carried out to show that such bounds exhibit the same rate of decay at high SNR.

Finally, an auction based power allocation scheme is proposed, and the properties of its equilibrium are discussed. Recall that the sequential water filling scheme can achieve superior performance in terms of reception reliability; however, such a scheme requires that channel state information (CSI) be available at the transmitter, which can consume significant system overhead in a multi-user system. As demonstrated by the simulation results, the auction based distributed scheme can achieve much better performance than the equal power and individual transmission schemes, and in performance close to the water filling strategy.

II. Energy harvesting relaying transmissions

Consider an energy harvesting communication scenario with M source-destination pairs and one relay. Each node is equipped with a single antenna. The sources communicate with their respective destinations via the relay, and through orthogonal channels, such as different time slots. All channels are assumed to be identically and independently quasi-static Rayleigh fading, and large scale path loss will be considered only in Section VI in order to simplify the analytical development.

The basic idea of energy harvesting relaying is that an energy constrained relay recharges its battery by using the energy from its observations[1]. Among the various energy harvesting relaying models, we focus on power splitting [7], [10]. Specifically, the cooperative transmission consists of two time slots of duration $\frac{T}{2}$. At the end of the first phase, the relay splits the observations from the i-th transmitter into two streams, one for energy harvesting and the other for detection. Let θ_i denote the power splitting coefficient for the i-th user pair, i.e. θ_i is the fraction of observations used for energy harvesting. At the end of the first phase, the relay's detection is based on the following observation:

$$y_{r,i} = \sqrt{(1-\theta_i)P_i}h_i s_i + n_{r,i}, \tag{1}$$

where P_i denotes the transmission power at the i-th source, h_i denotes the channel gain between the i-th source and the relay, s_i is the source message with unit power, and $n_{r,i}$ denotes additive white Gaussian noise (AWGN) with unit variance. As discussed in [10], such noise consists of the baseband AWGN as well as the sampled AWGN due to the radio-frequency band to baseband signal conversion. We consider a pessimistic case in which power splitting reduces only the signal power, but not to the noise power, which can provide a lower bound for relaying networks in practice.

The data rate at which the relay can decode the i-th source's signal is

$$R_{r,i} = \frac{1}{2}\log(1 + (1-\theta_i)P_i|h_i|^2), \tag{2}$$

and the parameter θ_i can be set to satisfy the criterion $R_{r,i} = R$, i.e.,

$$\theta_i \triangleq 1 - \frac{2^{2R}-1}{P_i|h_i|^2}, \tag{3}$$

where R is the targeted data rate. Due to the use of the DF strategy, the choice of the power splitting parameter in (3) is optimal. Particularly, if the splitting parameter is set as $\theta_i > \tilde{\theta}_i \triangleq \left(1 - \frac{2^{2R}-1}{P_i|h_i|^2}\right)$, too much power has been directed to the energy harvesting circuit and there is not sufficient signal power for decoding, which leads to a decoding failure. If the power splitting parameter is set as $\theta_i < \tilde{\theta}_i$, too much signal power has been directed to the detection circuit, whereas the circuit needs only $(1-\tilde{\theta}_i)P_i|h_i|^2$ to guarantee correct decoding. Therefore a choice of $\theta_i < \tilde{\theta}_i$ leads to an inefficient use of the incoming signals. Therefore the optimal choice of the power splitting parameter is $\theta_i \triangleq \tilde{\theta}_i$.

[1]It is assumed in this paper that the relay has its own energy source, e.g. a sensor operating with limited battery power. And the use of wireless power transfer is to prolong the battery life of the relay. For example, the proposed energy harvesting method can ensure that relaying transmissions are powered by the energy harvested from the relay observations, instead of the energy from the relay battery. As a result, the energy stored in the relay battery is used only to support the transmit and detection circuits, which is important to increase the lifetime of the relay battery.

At the end of the first phase, the relay harvests the following amount of energy from the i-th source:

$$E_{H,i} = \eta P_i |h_i|^2 \theta_i \frac{T}{2}, \tag{4}$$

where η denotes the energy harvesting efficiency factor. During the second time slot, this energy can be used to power the relay transmissions. However, how to best use such harvested energy is not a trivial problem, since different strategies will have different impacts on the system performance. In the following subsection, we first introduce a non-cooperative individual transmission strategy, which serves as a benchmark for the transmission schemes proposed later.

A non-cooperative individual transmission strategy

A straightforward strategy to use the harvested energy is allocating the energy harvested from the i-th source to the relaying transmission to the i-th destination, i.e., the relaying transmission power for the i-th destination is

$$P_{ri} \triangleq \frac{E_{H,i}}{\frac{T}{2}} = \eta P_i |h_i|^2 \theta_i. \tag{5}$$

During the second time slot, the DF relay forwards the i-th source message if the message is reliably detected at the relay, i.e. $|h_i|^2 > \epsilon$, where $\epsilon = \frac{2^{2R}-1}{P_i}$. Therefore, provided there is a successful detection at the relay, the i-th destination receives the observation, $\sqrt{P_{ri}} g_i s_i + n_{d,i}$, which yields a data rate at the i-th destination of

$$R_{d,i} = \frac{1}{2}\log(1 + P_{ri}|g_i|^2), \tag{6}$$

where g_i denotes the channel between the relay and the i-th destination and $n_{d,i}$ denotes the noise at the destination. For notational simplicity, it is assumed that the noise at the destination has the same variance as that at the relay. The outage probability for the i-th user pair can be expressed as

$$\mathcal{P}_{i,I} = \Pr\left(\frac{1}{2}\log(1 + P_i|h_i|^2) < R\right) + \Pr\left(\frac{1}{2}\log(1 + P_i|h_i|^2) > R, \frac{1}{2}\log(1 + P_{ri}|g_i|^2) < R\right). \tag{7}$$

In addition we also define the worst and best outage performance among the M users as $\max\{\mathcal{P}_{i,I}, 1 \le i \le M\}$ and $\min\{\mathcal{P}_{i,I}, 1 \le i \le M\}$, respectively. The following proposition characterizes the outage of such a strategy.

Proposition 1: The use of the non-cooperative individual transmission strategy yields an outage probability at the i-th destination of

$$\mathcal{P}_{i,I} = 1 - \frac{e^{-\frac{a}{P_i}}}{P_i}\sqrt{\frac{4aP_i}{\eta}}\mathbf{K}_1\left(\sqrt{\frac{4a}{\eta P_i}}\right), \tag{8}$$

where $a = 2^{2R} - 1$ and $\mathbf{K}_n(\cdot)$ denotes the modified Bessel function of the second kind with order n. Under the assumptions that all the channels are identically and independently distributed and the sources use the same transmission power, the worst and best outage performance among the M users are $1 - (1 - \mathcal{P}_{i,I})^M$ and $(\mathcal{P}_{i,I})^M$, respectively.

Proof: The first term on the righthand side of (7) can be calculated as $\left(1 - e^{-\frac{a}{P_i}}\right)$ by using the exponential distribution. On denoting the second probability on the righthand side of (7) by Q_2, we have

$$\begin{aligned} Q_2 &= \Pr\left(\frac{1}{2}\log(1 + P_i|h_i|^2) > R, \frac{1}{2}\log(1 + \eta P_i|h_i|^2\theta_i|g_i|^2) < R\right) \\ &= \Pr\left(\frac{1}{2}\log(1 + P_i|h_i|^2) > R, \frac{1}{2}\log(1 + \eta\left(P_i|h_i|^2 - 2^{2R} + 1\right)|g_i|^2) < R\right). \end{aligned} \tag{9}$$

On setting $z = P_i|h_i|^2 - 2^{2R} + 1$, we can write the density function of z as $f_z(z) = \frac{1}{P_i}e^{-\frac{z+a}{P_i}}$, which yields

$$\begin{aligned} Q_2 &= \int_0^\infty \left(1 - e^{-\frac{a}{\eta z}}\right) f_z(z) dz \\ &= e^{-\frac{a}{P_i}} - \frac{e^{-\frac{a}{P_i}}}{P_i}\sqrt{\frac{4aP_i}{\eta}}\mathbf{K}_1\left(\sqrt{\frac{4a}{\eta P_i}}\right), \end{aligned} \tag{10}$$

where the last equation is obtained by applying Eq. (3.324.1) in [12]. Combining the two probabilities in (7), the first part of the lemma is proved. The worst and best outage performance can be obtained by using the assumption that the channels are identically and independently distributed. ■

The asymptotic high SNR behavior of the outage performance can be used as an benchmark for comparing power allocation strategies. Our intuition is that such a straightforward strategy is most likely inefficient, as illustrated in the following. Suppose that two source nodes with channels $|h_1|^2 >> |h_2|^2 \approx \frac{a}{P_2}$ and $|g_1|^2 >> |g_2|^2$ have information correctly detected at the relay. Based on the individual transmission scheme, there is little energy harvested from the second source transmission, which results in $P_{r2} \to 0$ and therefore a possible detection failure at the second destination. A more efficient solution to such a case is to allow the users to share the harvested power efficiently, which can help the user with a poor connection. This scenario is discussed in the following sections.

III. Centralized mechanisms for power allocation

Recall that each user uses the power splitting fraction $\theta_i = 1 - \frac{2^{2R}-1}{P_i|h_i|^2}$, which implies that total power reserved at the relay at the end of the first phase is[2]

$$P_r = \sum_{i=1}^{N} \frac{E_{H,i}}{\frac{T}{2}} = \sum_{i=1}^{N} \eta P_s |h_i|^2 \theta_i, \tag{11}$$

where N denotes the number of sources whose information can be reliably detected at the relay. Note that N is a random variable whose value depends on the instantaneous source-relay channel realizations. To simplify the analysis, it is assumed that all the source transmission powers are the

[2]Instead of all M sources, we consider only the power harvested from the N sources that can deliver their information to the relay successfully. Or in other words, we consider a pessimistic strategy such that for each source, the relay will first direct the received signals to the detection circuit until the receive SNR is sufficient for successful detection. If there is a failure of detection, all the energy must have already been directed to the detection circuit, and there will be no energy left for energy harvesting.

same $P_i = P_s$. In the following, we study how to distribute such power among the users based on various criteria. Specifically, an equal power allocation strategy is introduced first, and then we will investigate the water filling based strategy which achieves a better outage performance but requires more complexity.

A. Equal power allocation

In this strategy, the relay allocates the same amount of power to each user, i.e., $P_{ri} = \frac{1}{N}\sum_{i=1}^{N}\eta P_s|h_i|^2\theta_i$. The advantage of such a strategy is that there is no need for the relay to know the relay-destination channel information, which can reduce the system overhead significantly, particularly in a multi-user system. The following theorem describes the outage performance achieved by such a power allocation scheme.

Theorem *1:* Based on the equal power allocation, the outage probability for the i-th destination is given by

$$\mathcal{P}_{i,II} = \sum_{n=1}^{M}\frac{1}{(n-1)!}\left((n-1)! - 2\left(\frac{b_n}{P}\right)^{\frac{n}{2}}\mathbf{K}_n\left(2\sqrt{\frac{b_n}{P}}\right)\right) \times\frac{(M-1)!}{(n-1)!(M-n)!}e^{-n\epsilon}\left(1-e^{-\epsilon}\right)^{M-n} + \left(1-e^{-\epsilon}\right),$$

where $b_n = \frac{na}{\eta}$.

Proof: See the appendix. ∎

Note that the theorem is obtained by assuming that all the channel gains are independent and identically complex Gaussian distributed, which is valid for many indoor applications, such as smart home and personal area networks. When large scale path loss is taken into consideration, the outage probability becomes difficult to evaluate, since channels may not be identically distributed. One possible solution is to apply stochastic geometry and assume that the nodes are randomly located within a certain area, which can ensure the channel gains to be identically distributed again, as shown in [13]. Consideration of this scenario is beyond the scope of this paper due to space limitations.

Based on Theorem 1, we also obtain the best outage and worst outage performance among the M users achieved by the equal power allocation scheme as follows.

Proposition 2: Based on the use of the equal power allocation, the outage probability of the user with the best channel conditions among the M users is

$$\mathcal{P}_{best,II} = \sum_{n=1}^{M}\frac{2}{(n-1)!}\sum_{i=0}^{n}\binom{n}{i}(-1)^i\left(\frac{ib_n}{P}\right)^{\frac{n}{2}} \times\mathbf{K}_n\left(2\sqrt{\frac{ib_n}{P}}\right)\frac{M!e^{-n\epsilon}}{n!(M-n)!}\left(1-e^{-\epsilon}\right)^{M-n} + \left(1-e^{-\epsilon}\right)^{M},$$

and the worst outage performance among the M users is

$$\mathcal{P}_{worst,II} = \frac{1}{(M-1)!}\left((M-1)! - 2\left(\frac{Mb_M}{P}\right)^{\frac{M}{2}} \times\mathbf{K}_M\left(2\sqrt{\frac{Mb_M}{P}}\right)\right)e^{-M\epsilon} + 1 - e^{-M\epsilon}.$$

Proof: Suppose that there are N sources whose messages can be reliably received by the relay. Among these N users, order the relay-destination channels as $g_{(1)} \leq \cdots \leq g_{(N)}$, and the outage performance for the best outage performance can be expressed as

$$\begin{aligned}\mathcal{P}_{best,II} &= \sum_{n=1}^{M}\Pr\left(\frac{1}{2}\log\left(1+\frac{P_r}{N}|g_{(N)}|^2\right) < R, N=n\right) \\ &\quad+\Pr\left(N=0\right) \\ &= \sum_{n=1}^{M}\Pr\left(\frac{1}{2}\log\left(1+\frac{P_r}{N}|g_{(N)}|^2\right) < R|N=n\right) \\ &\quad\times\Pr(N=n) + \Pr\left(N=0\right). \end{aligned} \tag{12}$$

By applying the density function of $\sum_{i=1}^{N}|h_{n_i}|^2$ shown in the proof for Theorem 1, the best outage probability can be expressed as

$$\begin{aligned}\mathcal{P}_{best,II} &= \sum_{n=1}^{M}\Pr(N=n)\int_{n\epsilon}^{\infty}\left(1-e^{-\frac{b_n}{Py-na}}\right)^n \\ &\quad\times\frac{1}{(n-1)!}(y-n\epsilon)^{n-1}e^{-(y-\epsilon)}dy + \Pr\left(N=0\right).\end{aligned}$$

By applying (44) from appendix, the best outage probability can be obtained as shown in the proposition. The worst outage probability can be expressed as

$$\begin{aligned}\mathcal{P}_{worst,II} &= \Pr\left(\frac{1}{2}\log\left(1+\frac{P_r}{M}|g_{(1)}|^2\right) < R, N=n\right) \\ &\quad+\sum_{n=0}^{M-1}\Pr\left(N=n\right).\end{aligned} \tag{13}$$

Note that

$$\sum_{n=1}^{M-1}\Pr\left(N=n\right) = 1 - \Pr(N=M) = 1 - e^{-M\epsilon}. \tag{14}$$

Combining the density function of $\sum_{i=1}^{N}|h_{n_i}|^2$ shown in the proof of Theorem 1, and the results in (44) and (14), the probability can be evaluated and the proposition is proved. ∎

B. Sequential water filling based power allocation strategy

Provided that the relay has access to global channel state information, a more efficient strategy that maximizes the number of successful destinations can be designed as follows. First recall that in order to ensure the successful detection at the i-th destination, the relay needs to allocate the relaying transmission power $P_{i,targeted} = \frac{2^{2R}-1}{|g_i|^2}$ to the i-th destination. Suppose that n sources can deliver their information to the relay reliably, and the required relaying transmission power for these n destinations can be ordered as

$$\frac{2^{2R}-1}{|g_{(1)}|^2} \geq \cdots \geq \frac{2^{2R}-1}{|g_{(n)}|^2}.$$

The sequential water filling power allocation strategy is described in the following. The relay first serves the destination with the strongest channel by allocating power $\frac{2^{2R}-1}{|g_{(n)}|^2}$ to it, if the total harvested energy at the relay is larger than or equal to $\frac{2^{2R}-1}{|g_{(n)}|^2}$. And then the relay tries to serve the destination with the second strongest channel with the power $\frac{2^{2R}-1}{|g_{(n-1)}|^2}$, if possible. Such a power allocation strategy continues until either all users are served or there is not enough power left at the relay. If there is any power left, such energy is reserved at the relay, where it is assumed that the capacity of the relay battery is infinite.

The probability of having m successful receivers among n users can be expressed as

$$\Pr\left(\sum_{i=1}^{m}\frac{2^{2R}-1}{|g_{(n-i+1)|^2}}<P_r,\sum_{i=1}^{m+1}\frac{2^{2R}-1}{|g_{(n-i+1)}|^2}>P_r,N=n\right),$$

from which the average number of successful destinations can be calculated by carrying out the summation among all possible choices of m and n. Evaluating the above expression is quite challenging, mainly because of the complexity of the density function of the sum of inverse exponential variables. However, explicit analytical results for such a power allocation scheme can be obtained based on other criteria. Particularly we are interested in the outage performance achieved by the water filling strategy.

Although such a water filling power allocation scheme is designed to maximize the number of successful destinations, it can also minimize the outage probability for the user with the best channel conditions, since such a user is the first to be served and has access to the maximal relaying power. The following proposition provides an explicit expression for such a outage probability.

Proposition 3: With the sequential water filling power allocation strategy, the outage probability for the user with the best channel conditions is

$$\mathcal{P}_{best,III}=\sum_{n=1}^{M}\frac{2}{(n-1)!}\sum_{i=1}^{n}\binom{n}{i}(-1)^i\left(\frac{i\tilde{b}}{P}\right)^{\frac{n}{2}}\mathbf{K}_n\left(2\sqrt{\frac{i\tilde{b}}{P}}\right)$$
$$\times\frac{M!}{n!(M-n)!}e^{-n\epsilon}\left(1-e^{-\epsilon}\right)^{M-n}+\left(1-e^{-\epsilon}\right)^{M},$$

where $\tilde{b}=\frac{a}{\eta}$.

Proof: The outage probability for the user with the best channel channel conditions is

$$\mathcal{P}_{best,III}=\sum_{n=1}^{M}\Pr\left(\sum_{i=1}^{n}\eta\left(P_s|h_i|^2-a\right)<\frac{2^{2R}-1}{|g_{(n)}|^2},\right.\quad(15)$$
$$\left.N=n\right)+\Pr(N=0).$$

Following steps similar to the ones in the proofs for Theorem 1 and Proposition 2, the probability in Proposition 3 can be evaluated, and the details of such algebraic manipulations are consequently omitted. ■

The water filling scheme is not only optimal in terms of maximizing the number of successful destinations, but also optimal in terms of minimizing the outage performance for the destination with the best channel conditions, since this destination is served first and has access to all the harvested energy. However, it is surprising that the performance of the water filling scheme for the user with the worst outage probability is the same as that attained for the worst user with the optimal strategy, as shown in the following lemma.

Lemma *1:* Denote by $\mathcal{P}_i(\mathsf{s})$ the outage probability for the i-th user achieved by a power allocation strategy s, where $\mathsf{s}\in\mathcal{S}$ and $\mathcal{S}$ contains all possible strategies. Define $\mathcal{P}_{worst}(\mathsf{s})\triangleq\max\{\mathcal{P}_\mathsf{i}(\mathsf{s}),\mathsf{i}\in\{1,\cdots,\mathsf{M}\}\}$ and $\mathcal{P}_{worst,III}$ as the worst user performance achieved by the sequential water filling scheme. $\mathcal{P}_{worst,III}=\min\{\mathcal{P}_{worst}(\mathsf{s}),\mathsf{s}\in\mathcal{S}\}$ holds.

Proof: See the appendix. ■

Lemma 1 can also be used to study the fairness among the M user pairs. For example, given one snapshot of the source-relay and relay-destination channels, the problem of achieving fairness among the M users for this snapshot is equivalent to finding a power allocation strategy that can minimize the worst outage performance, i.e. $\min_{\mathbf{s}}\max\{\mathcal{P}_1(\mathbf{s}),\ldots,\mathcal{P}_M(\mathbf{s})\}$. As shown in Lemma 1, the sequential water filling strategy is optimal for minimizing the worst outage performance. Or in other words, the sequential water filling strategy can achieve the optimal fairness among the M user pairs, for one snapshot of the channels. Note that fairness is characterized by using the criterion of outage balancing, similar to SNR balancing in [14]. It is also important to study fairness based on other criteria or even the long term fairness over multiple snapshots of the channels, which is beyond the scope of this paper.

It is quite challenging to find an exact expression for such an outage probability, for the following reason. Denote $z_i=\frac{1}{|g_i|^2}$. Since the channels are Rayleigh faded, the probability density and cumulative distribution functions of z_i can be obtained as follows:

$$f_{z_i}(z)=\frac{1}{z^2}e^{-\frac{1}{z}},F_{z_i}(z)=e^{-\frac{1}{z}}.\qquad(16)$$

Obtaining an exact expression for (51) requires the density function of $\sum_{i=1}^{M}z_i$, which is the sum of inverse exponential variables. The Laplace transform for the density function of an individual z_i is $\mathcal{L}_{z_i}(s)=2\sqrt{s}\mathbf{K}_1(2\sqrt{s})$, so that the Laplace transform for the overall sum is $\mathcal{L}_{\sum_{i=1}^{M}z_i}(s)$ is $2^Ms^{\frac{M}{2}}\mathbf{K}_1^M(2\sqrt{s})$, a form difficult to invert. There are a few existing results regarding to the sum of inverted gamma/chi-square distributed variables [15], [16]; however, the case with 2 degrees of freedom, i.e. inverse exponential variables, is still an open problem, partly due to the fact that its moments are not bounded. The following proposition provides upper and lower bounds of the outage performance of the users with the worst channel conditions.

Proposition 4: The outage probability for the user with the worst channel conditions achieved by the water filling strategy can be upper bounded by

$$\mathcal{P}_{worst,III}<e^{-M\epsilon}\int_0^\infty\left(1-e^{-\frac{M^2}{w}}\right.$$
$$\left.-M\int_{\frac{w}{M}}^{w}\frac{e^{-\frac{(M-1)^2}{w-v}-\frac{1}{v}}}{v^2}dv\right)f_w(w)dw+1-e^{-M\epsilon},\qquad(17)$$

and lower bounded by

$$\mathcal{P}_{worst,III}>\left(1-\frac{2}{(M-1)!}\left(\frac{M\epsilon}{\eta}\right)^{\frac{M}{2}}\mathbf{K}_M\left(2\sqrt{\frac{M\epsilon}{\eta}}\right)\right)\quad(18)$$
$$\times e^{-M\epsilon}+1-e^{-M\epsilon},$$

where $f_w(w)=\frac{1}{(M-1)!}\left(\frac{\epsilon}{\eta}\right)^Mw^{M-1}e^{-\frac{\epsilon}{\eta}w}$.

Proof: See the appendix. ■

While the expression in (17) can be evaluated by numerical methods, it is difficult to carry out asymptotic studies for such an expression with integrals, and the following proposition provides a bound slightly looser than (17) that enables asymptotic analysis.

Proposition 5: The outage probability for the user with the worst channel conditions achieved by the water filling strategy can be upper bounded as follows:

$$\mathcal{P}_{worst,III} < 1 - e^{-M\epsilon}\left(\frac{2}{(M-1)!}\left(\frac{\epsilon M^2}{\eta}\right)^{\frac{M}{2}}\right. \times \mathbf{K}_M\left(2\sqrt{\frac{\epsilon M^2}{\eta}}\right) + \frac{M}{(M-1)!}\left(\frac{\epsilon}{\eta}\right) \times \left.\int_c^{M-1} 2\left(\frac{a(y)\epsilon}{\eta}\right)^{\frac{M-1}{2}}\mathbf{K}_{M-1}\left(2\sqrt{\frac{a(y)\epsilon}{\eta}}\right)dy\right), \quad (19)$$

where $a(y) = (y+1)\left(\frac{(M-1)^2+1}{y}\right)$ and c is a constant to facilitate asymptotic analysis, $c \in [0, M-1]$.

Proof: See the appendix. ■

The upper bound in Proposition 4 is a special case of the one in Proposition 5 by setting $c = 0$ as shown in the appendix. The reason to use the parameter c is to facilitate asymptotic analysis and ensure that the factor $a(y)\epsilon$ approaches zero at high SNR, as illustrated in the next section.

Recall that the two bounds in Proposition 4 were developed based on (53), which is recalled in the following:

$$\mathcal{P}_w\left(z_{(M)} > w\right) < \mathcal{P}_w\left(\sum_{i=1}^{M} z_i > w\right) \leq \mathcal{P}_w\left(z_{(M)} + (M-1)z_{(M-1)} > w\right), \quad (20)$$

where z_i has been ordered as $z_{(1)} \leq \cdots \leq z_{(M)}$ and w is a random variable related to the source-relay channels and transmission power. Intuitively such bounds should be quite loose since the two order statistics, $z_{(M)}$ and $z_{(M-1)}$, are expected to become the same with large M.

However, as shown by the simulation in Section VI, such bounds are surprisingly tight, even for large M. This is because for the addressed scenario the statistical properties of $z_{(M)}$ and $z_{(M-1)}$ are very different. In the following it will be shown that the expectations of $z_{(M-i)}$, $1 \leq i \leq (M-1)$, *are bounded*, but the expectation of $z_{(M)}$ is *not bounded*, for any fixed M. The expectation of $z_{(M-1)}$ is

$$\begin{aligned}\mathcal{E}_{z_{(M-1)}} &= M(M-1)\int_0^\infty \frac{1}{x}e^{-\frac{M-1}{x}}\left(1-e^{-\frac{1}{x}}\right)dx \quad (21)\\ &= M(M-1)\sum_{k=1}^{\infty}\frac{(-1)^k}{k!}\int_0^\infty \frac{1}{x^{k+1}}e^{-\frac{M-1}{x}}dx,\end{aligned}$$

where the first equality follows from the probability density function (pdf) of $z_{(M-1)}$, $f_{z_{(M-1)}}(x) = M(M-1)\frac{e^{-\frac{M-1}{x}}}{x^2}\left(1-e^{-\frac{1}{x}}\right)$, and the second equality follows from the series expansion of exponential functions. Furthermore we have

$$\begin{aligned}\mathcal{E}_{z_{(M-1)}} &= M(M-1)\sum_{k=1}^{\infty}\frac{(-1)^k}{k!}\frac{1}{(M-1)^k} \quad (22)\\ &< M(M-1)\sum_{k=1}^{\infty}\frac{(-1)^k}{(M-1)^k} = (M-1)^2.\end{aligned}$$

Therefore the expectation of $z_{(M-1)}$ is finite, but the expectation of $z_{(M)}$ is not finite since

$$\mathcal{E}_{z_{(M)}} = M\int_0^\infty \frac{1}{x}e^{-\frac{M}{x}}dx = M \cdot \lim_{t\to 0}\mathbf{E}_i(-t) \to \infty. \quad (23)$$

IV. Asymptotic Analysis of the Outage Performance

In the previous sections, exact expressions for the outage performance achieved by the addressed power allocation schemes have been developed. Most of these expressions contain Bessel functions, which makes it difficult to get any insight from the analytical results. In this section, high SNR asymptotic studies for the outage performance are carried out. To do this we need asymptotic expression for $x^n\mathbf{K}_n(x)$, when $x \to 0$. By applying the series representation of Bessel functions, $x^n\mathbf{K}_n(x)$ can be approximated as [12]

$$\begin{aligned}x^n\mathbf{K}_n(x) &= x^n(-1)^{n+1}\mathbf{I}_n(x)\left(\ln\frac{x}{2}+\mathbf{C}\right) \quad (24)\\ &\quad +\frac{1}{2}(-1)^n\sum_{l=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{n+2l}x^n}{l!(n+l)!}\left(\sum_{k=1}^{l}\frac{1}{k}+\sum_{k=1}^{n+l}\frac{1}{k}\right)\\ &\quad +\frac{1}{2}\sum_{l=0}^{n-1}\frac{(-1)^l(n-l-1)!}{l!}\frac{x^{2l}}{2^{2l-n}}\\ &\approx \frac{1}{2}\sum_{l=0}^{n-1}\frac{(-1)^l(n-l-1)!}{l!}\frac{x^{2l}}{2^{2l-n}} + x^{2n}q(\ln x),\end{aligned}$$

for $n \geq 2$, where $\mathbf{I}_n(x) = \sum_{k=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{n+2k}}{k!(n+k)!}$ and $q(\ln x) = (-1)^{n+1}\frac{\ln\frac{x}{2}}{2^n n!}$. For the case $n = 1$, we have

$$\begin{aligned}x\mathbf{K}_1(x) &= 1 + x\mathbf{I}_1(x)\left(\ln\frac{x}{2}+\mathbf{C}\right) \quad (25)\\ &\quad -\frac{1}{2}\sum_{l=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{2l+1}x}{l!(l+2)!}\left(\sum_{k=1}^{l}\frac{1}{k}+\sum_{k=1}^{l+2}\frac{1}{k}\right)\\ &\approx 1 + \frac{x^2}{2}\ln\frac{x}{2}.\end{aligned}$$

These approximations will be used for the following high SNR asymptotic analysis of the outage performance.

A. Average outage performance

According to Theorem 1, the average performance achieved by the equal power allocation strategy can be expressed as

$$\mathcal{P}_{i,II} = \sum_{n=1}^{M} \frac{1}{(n-1)!}\left((n-1)! - \frac{1}{2^{n-1}}\left(2\sqrt{\frac{b_n}{P}}\right)^n \times \mathbf{K}_n\left(2\sqrt{\frac{b_n}{P}}\right)\right)\frac{(M-1)!}{(n-1)!(M-n)!}e^{-n\epsilon} \times\left(1-e^{-\epsilon}\right)^{M-n} + \left(1-e^{-\epsilon}\right) \tag{26}$$
$$\approx \sum_{n=1}^{M}\left(1-\left(1-\frac{1}{(n-1)}\left(\frac{b_n}{P}\right)\right)\right)\frac{(M-1)!}{(n-1)!(M-n)!} \times(1-n\epsilon)\epsilon^{M-n}+\epsilon$$
$$\approx \left(1+\frac{M}{(M-1)\eta}\right)\epsilon,$$

where only the first two factors containing x^0 and x^2 in (24) are used. On the other hand, according to Proposition 1, the average outage performance achieved by the non-cooperative individual strategy is

$$\begin{aligned}\mathcal{P}_{i,I} &\approx 1-(1-\epsilon)\left(1+\frac{2\epsilon}{\eta}\ln\sqrt{\frac{\epsilon}{\eta}}\right) \\ &\approx \epsilon\left(1-\frac{2}{\eta}\ln\sqrt{\frac{\epsilon}{\eta}}\right).\end{aligned} \tag{27}$$

An important observation from (26) and (27) is that the average outage probability for the individual transmission scheme decays as $\frac{\log SNR}{SNR}$, where the equal power allocation scheme can achieve better performance, i.e. a faster rate of decay, $\frac{1}{SNR}$. Another aspect for comparison is to study the normalized difference of the two probabilities. When $\epsilon \to 0$, we can approximate this difference as

$$\frac{\mathcal{P}_{i,I}-\mathcal{P}_{i,II}}{\mathcal{P}_{i,II}} \approx \frac{\frac{1}{\eta}\ln\frac{1}{\epsilon\eta}-\frac{n}{2(n-1)\eta}}{1+\frac{M}{(M-1)\eta}} > 0. \tag{28}$$

This difference can be significant since the factor $\ln\frac{1}{\epsilon}$ approaches infinity as $\epsilon \to 0$. In terms of the average outage performance, the water filling strategy can also achieve performance similar to that of the equal power allocation scheme, i.e., its average outage probability decays as ϵ. Although we cannot obtain an explicit expression for the water filling strategy, the rate of decay of $\frac{1}{SNR}$ can be proved by studying the outage probability for the user with the worst channel conditions as shown in Section IV-C.

B. Best outage performance

Following the previous discussions about the average outage performance, the best outage performance achieved by the individual transmission scheme can be approximated as follows:

$$\mathcal{P}_{best,I} = (\mathcal{P}_{i,I})^M \approx \epsilon^M\left(1-\frac{2}{\eta}\ln\sqrt{\frac{\epsilon}{\eta}}\right)^M. \tag{29}$$

Comparing Proposition 2 and Proposition 3, we can see that the equal power allocation scheme and the water filling scheme achieve similar performance for the user with the best channel conditions. So in the following, we focus only on the equal power allocation scheme. The following corollary provides a high SNR approximation to the best outage performance achieved by equal power allocation.

Proposition 6: With the equal power allocation scheme, the outage probability for the user with the best channel conditions can be approximated at high SNR by

$$\mathcal{P}_{best,II} \approx \epsilon^M(1-c\ln\epsilon),$$

where $c = \sum_{n=1}^{M} 2\left(\frac{n}{\eta}\right)^n \frac{M!}{(n-1)!n!(M-n)!}$ is a constant not depending on ϵ.

Proof: According to Proposition 2, the use of equal power allocation yields the best outage performance among the M users as in (30). Recall that the sum of binomial coefficients has the following properties [12]:

$$\sum_{i=0}^{n}\binom{n}{i}(-1)^i i^l = 0, \tag{31}$$

for $0 \le l < n$, and

$$\sum_{i=0}^{n}\binom{n}{i}(-1)^i i^n = (-1)^n n!. \tag{32}$$

By using such properties, the expression shown in (30) can be simplified significantly. Specifically all the factors of the order of $\left(\frac{ib}{P}\right)^l$, $l < n$, will be completely removed, so we can write

$$\begin{aligned}\mathcal{P}_{best,II} &\approx \sum_{n=1}^{M}\frac{1}{2^{n-1}(n-1)!}\sum_{i=0}^{n}\binom{n}{i}(-1)^i\left(\left(\frac{ib}{P}\right)^n \times 2^{2n}q(\ln x)\right)\frac{M!}{n!(M-n)!}(1-n\epsilon)\epsilon^{M-n}+\epsilon^M \\ &\approx -c\epsilon^M\ln\epsilon+\epsilon^M,\end{aligned}$$

and the proposition is proved. ■

An interesting observation here is that the high SNR approximation to the outage probability achieved by the equal power allocation scheme also includes a term $\ln\epsilon$, similar to the individual transmission scheme. Compared to traditional relaying networks, such a phenomenon is unique, which is due to the fact that, in energy harvesting cases, the relaying transmission power is constrained by the source-relay channel attenuation during the first phase transmissions.

C. Worst outage performance

The worst outage performance achieved by the non-cooperative individual strategy will be

$$\begin{aligned}\mathcal{P}_{worst,I} &\approx 1-(1-M\epsilon)\left(1+\frac{2\epsilon}{\eta}\ln\sqrt{\frac{\epsilon}{\eta}}\right)^M \\ &\approx M\epsilon\left(1-\frac{2}{\eta}\ln\sqrt{\frac{\epsilon}{\eta}}\right),\end{aligned} \tag{33}$$

which still decays as $\frac{\log SNR}{SNR}$. And the worst outage performance achieved by the equal power allocation can be approximated as

$$\begin{aligned}\mathcal{P}_{worst,II} &\approx \frac{M}{M-1}\frac{Mb}{P}e^{-M\epsilon}+1-e^{-M\epsilon} \\ &\approx \epsilon M\left(1+\frac{M}{\eta(M-1)}\right),\end{aligned}$$

$$\mathcal{P}_{best,II} = 2\sum_{n=1}^{M}\frac{1}{(n-1)!}\sum_{i=0}^{n}\binom{n}{i}(-1)^i\left(\frac{ib}{P}\right)^{\frac{n}{2}}\mathbf{K}_n\left(2\sqrt{\frac{ib}{P}}\right)\frac{M!}{n!(M-n)!}e^{-n\epsilon}\left(1-e^{-\epsilon}\right)^{M-n}+\left(1-e^{-\epsilon}\right)^{M}$$
$$\approx \sum_{n=1}^{M}\frac{1}{2^{n-1}(n-1)!}\sum_{i=0}^{n}\binom{n}{i}(-1)^i\left(2^{n-1}\sum_{l=0}^{n-1}\frac{(-1)^l(n-l-1)!}{l!}\left(\frac{ib}{P}\right)^l\right.$$
$$\left.+\left(\frac{ib}{P}\right)^n 2^{2n}q(\ln x)\right)\frac{M!}{n!(M-n)!}(1-n\epsilon)\epsilon^{M-n}+\epsilon^M. \quad (30)$$

which decays as $\frac{1}{SNR}$, and hence performs better than the individual transmission scheme. According to Lemma 1, the water filling strategy can achieve the optimal performance for the user with the worst channel conditions. Upper and lower bounds have been developed in Lemma 4. In the following we show that such bounds converge at high SNR. We first focus on the upper bound which can be rewritten as

$$\mathcal{P}_{worst,III} < 1-e^{-M\epsilon}\left(\left(1-\frac{1}{(M-1)}\frac{\epsilon M^2}{\eta}\right)\right.$$
$$+M\left(\frac{\epsilon}{\eta}\right)\int_c^{M-1}\left(\frac{1}{M-1}\right.$$
$$\left.\left.-\frac{a(y)\epsilon}{\eta}\frac{1}{(M-1)(M-2)}\right)dy\right)$$
$$\approx 1-e^{-M\epsilon}\left(\left(1-\frac{1}{(M-1)}\frac{\epsilon M^2}{\eta}\right)\right.$$
$$\left.+M\left(\frac{\epsilon}{\eta}\right)\int_c^{M-1}\frac{1}{M-1}dy\right)$$
$$\approx \epsilon\left(M+\frac{M(M-1-c)}{(M-1)\eta}+\frac{M^2}{(M-1)\eta}\right),$$

for $M>2$, where the first inequality follows from (24) and c is a constant to ensure $a(y)\epsilon\to 0$ for $y\in[c,M-1]$. As can be observed from the above equation, the upper bound decays as $\frac{1}{SNR}$. The same conclusion can be obtained for the case of $M=2$ by applying the approximation in (25). On the other hand, the lower bound of the worst case probability can be similarly approximated as follows:

$$\mathcal{P}_{worst,III} > \epsilon M\left(1+\frac{1}{\eta(M-1)}\right).$$

Since both upper and lower bounds decay as $\frac{1}{SNR}$, we can conclude that the worst case outage probability achieved by the water filling scheme also decays as $\frac{1}{SNR}$, which also shows the rate of decay of the average outage probability for the water filling scheme.

V. Auction based Power Allocation

In the previous sections, three different strategies to use the harvested energy have been studied, where the water filling strategy can achieve the best performance in inters of several criteria. However, such a centralized method requires that the relay has access to global CSI. For a system with a large number of users, the provision of global CSI consumes significant system overhead, which motivates the study of the following auction based strategy to realize distributed power allocation.

Power auction game

The addressed power allocation problem can be modeled as a game in which the multiple destinations compete with each other for the assistance of the relay. Note that we will need to consider only the destinations whose corresponding source messages can be reliably decoded at the relay. Specifically each destination submits a bid to the relay, and the relay will update the power allocation of the users at the end of each iteration. Each destination knows only its own channel information, and the relay has no access to relay-destination channel information. The described game can be formulated as follows:

- Bids: Each user submits a scalar b_i to the relay;
- Allocation: The relay will allocate the following transmission power to each user:

$$P_{ri}=\frac{b_i}{\sum_{j=1}^{M}b_j+\xi}P_r=\frac{b_i}{\sum_{j=1}^{M}b_j+\xi}\sum_{i=1}^{M}\eta P_s|h_i|^2\theta_i, \quad (34)$$

  where ξ is a factor related to the power reserved at the relay.
- Payments: Upon the allocated transmission power P_{ri}, user i pays the relay $C_i=\pi P_{ri}$.

The rationale for using the strategy in (34) is that the relay does not know the relay-destination CSI, and will allocate the power according to the bids submitted by the destinations [17]. More discussions about the power allocation strategy will be provided at the end of this section. Recall that the data rate the i-th destination can achieve is $R_{d,i}=\frac{1}{2}\log(1+P_{ri}|g_i|^2)$, and therefore it is natural to consider a game in which the i-th user selects b_i to maximize its payoff as follows:

$$U_i(b_i;\mathbf{b}_{-i},\pi) = \frac{1}{2}\log(1+P_{ri}|g_i|^2)-\pi P_{ri}, \quad (35)$$

where $\mathbf{b}_{-i}=(b_1,\cdots,b_{i-1},b_{i+1},b_N)$. The addressed game in strategic form is a triplet $G=(\mathcal{N},(b_i)_{i\in\mathcal{N}},(U_i)_{i\in\mathcal{N}})$, where $\mathcal{N}=\{1,\ldots,N\}$ includes all the destinations whose source messages can be delivered to the relay successfully, and $b_i\in\mathbb{R}^+$, where $\mathbb{R}^+$ denotes the nonnegative real numbers. A desirable outcome of such a game is the following:

Definition 1: The Nash Equilibrium (NE) of the addressed game, G, is a bidding profile $\mathbf{b}^*$ which ensures that no user wants to deviate unilaterally, i.e.,

$$U_i(b_i^*;\mathbf{b}_{-i}^*,\pi)\geq U_i(b_i;\mathbf{b}_{-i}^*,\pi),\forall i\in\mathcal{N},\forall b_i\geq 0. \quad (36)$$

The following proposition provide the best response functions and the uniqueness of the NE for the addressed game.

$$BR_i(\mathbf{b}_{-i},\pi)=\begin{cases}\infty, & \pi<\min\{\pi_{u,i}, i\in\mathcal{N}\}\\ 0, & \pi>\max\left\{\frac{|g_i|^2}{2\ln 2}, i\in\mathcal{N}\right\}\\ \frac{\left(\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}\right)}{P_r-\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}}\left(\sum_{j=1,j\neq i}^{M} b_j+\xi\right), & otherwise\end{cases}, \tag{37}$$

Proposition 7: For the addressed power auction game, there exists a threshold price such that *a unique NE exists* when the price π is larger than such a threshold, otherwise there are infinitely many equilibria. In addition, the unique best response function for each player i can be expressed as in (37), where $\pi_u=\frac{|g_i|^2}{2\ln 2(1+P_r|g_i|^2)}$.

Proof: See the appendix. ■

Some explanation of the choice of the best response function shown in Proposition 7 follows. When the price is too large, a player's payoff function is always negative, and therefore it simply quits the game, i.e. $b_i=0$. Another extreme case is that the price is too small, which motivates a player to compete aggressively with other players by using a large bid. Unlike [17] and [18], the uniqueness of the NE is shown by using the contraction mapping property of the best response functions, which can simplify the following discussion about practical implementation.

The addressed power auction game can be implemented in an iterative way. The relay will first announce the price to all players. During the n-th iteration, each user will update its bid according to the following:

$$BR_i(\mathbf{b}_{-i}(n-1)) = \frac{\left(\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}\right)}{P_r-\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}} \times\left(\sum_{j=1,j\neq i}^{M} b_j^{n-1}+\xi\right), \tag{38}$$

where $BR_i(\mathbf{b}_{-i}(n-1))$ quantifies the best response dynamics determined by the actions from the previous iteration. For simplicity, we consider only the case of $\max\left\{\frac{|g_i|^2}{2\ln 2}, i\in\mathcal{N}\right\}\leq\pi\leq\min\{\pi_{u,i}, i\in\mathcal{N}\}$. As shown in the proof for Proposition 7, the best response function for the addressed power auction game is a contraction mapping, provided that the price is larger than the threshold, which means that this iterative algorithm converges to a unique fixed point, namely the NE of the addressed game. Note that such a convergence property is proved without the need for the nonnegative matrix theory as in [17] and [18].

In practice, the implementation of the iterative steps in (38) requires a challenging assumption that each user knows the other players' actions, and such an assumption can be avoided by using the following equivalent updating function,

$$BR_i(\mathbf{b}_{-i}(n-1))=\frac{\left(\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}\right)}{P_r-\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}}\frac{(P_r-P_{ri}^{n-1})b_j^{n-1}}{P_{ri}^{n-1}}, \tag{39}$$

where we only consider the case $\left(\frac{|g_i|^2}{2\ln 2(1+P_{ri}|g_i|^2)}-\pi_i\right)>0$. Therefore each destination updates its bid only according to its local information, such as its local CSI, its previous allocated power and its previous bid, without needing to know the actions of other users, as shown in (39). The system reaches a fixed point at which no destination wants to deviate from the recommended power allocation strategy.

VI. Numerical results

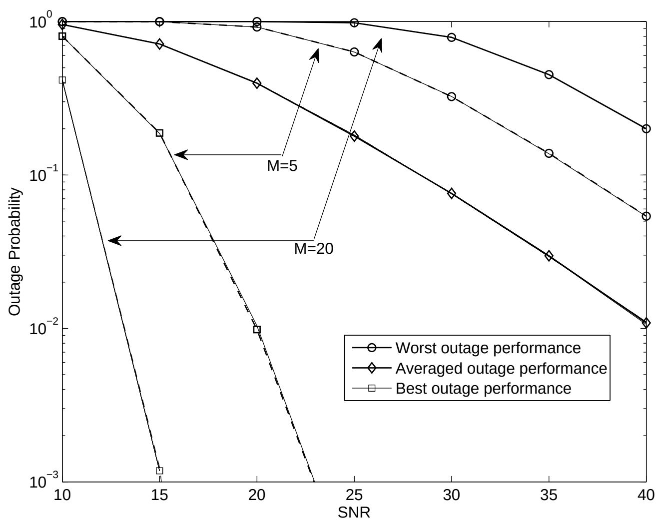
In this section, computer simulations will be carried out to evaluate the performance of those energy harvesting relaying protocols described in the previous sections.

We first study the accuracy of the developed analytical results. Specifically in Fig. 1, the outage performance achieved by the individual transmission scheme and the equal power allocation scheme is shown as a function of SNR. All the channel coefficients are assumed to be complex Gaussian with zero means and unit variances. The targeted data rate is $R=2$ bits per channel use (BPCU), and the energy harvesting efficiency is set as $\eta=1$. As can be seen from the figures the developed analytical results exactly match the simulation results, which demonstrates the accuracy of the developed analytical results.

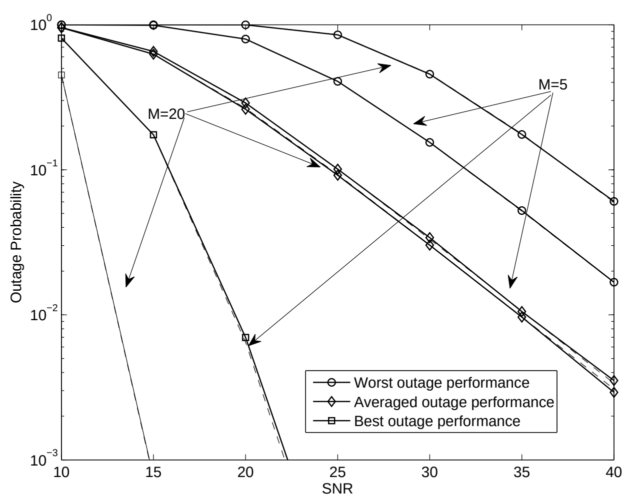
Comparing the two cases in Fig. 1, we find that the use of the equal power allocation strategy improves the outage performance. Consider the average outage performance as an example. When the SNR is 40 dB, the use of the individual transmission scheme realizes outage probability of 1×10^{-2}, whereas the equal power allocation scheme can reduce the outage probability to 3×10^{-3}. Such a phenomenon confirms the asymptotic results shown in Section IV.A. Specifically the outage probability achieved by the individual transmission scheme decays with the SNR at a rate $\frac{1}{SNR}(1+2\ln SNR)$, but the equal power allocation scheme can achieve a faster rate of decay, $\frac{1}{SNR}$.

When more source-destination pairs join in the transmission, it is more likely to have some nodes with extreme channel conditions, which is the reason to observe the phenomenon in Fig. 1 that with a larger number of user pairs, the best outage performance improves but the worst outage performance degrades. The impact of the number of user pairs on the average outage performance can also be observed in the figure. For the individual transmission scheme, there is no cooperation among users, so the number of user pairs has no impact on the average outage performance. On the other hand, it is surprising to find that an increase in the number of users yields only a slight improvement in the performance of the equal power allocation scheme, which might be due to the fact that power allocation can improve the transmission from the relay to the destinations, but not the source-relay transmissions.

In Fig. 2, the performance of the water filling scheme is studied. The same simulation setup as in the previous figures is used. Firstly the upper and lower bounds developed in Propositions 4 and 5 are compared to the simulation results in

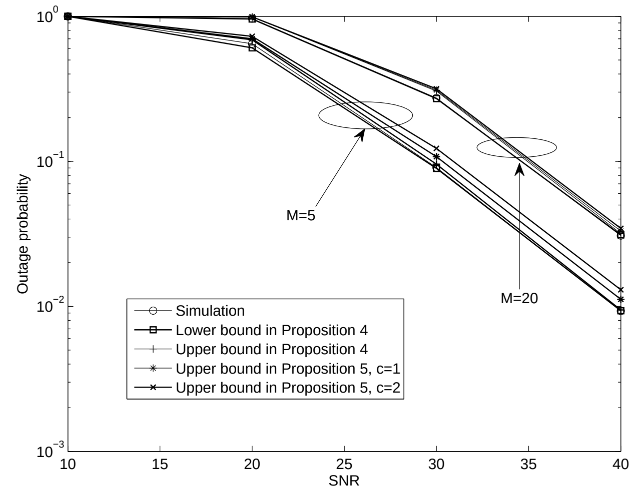


(a) Individual Transmission

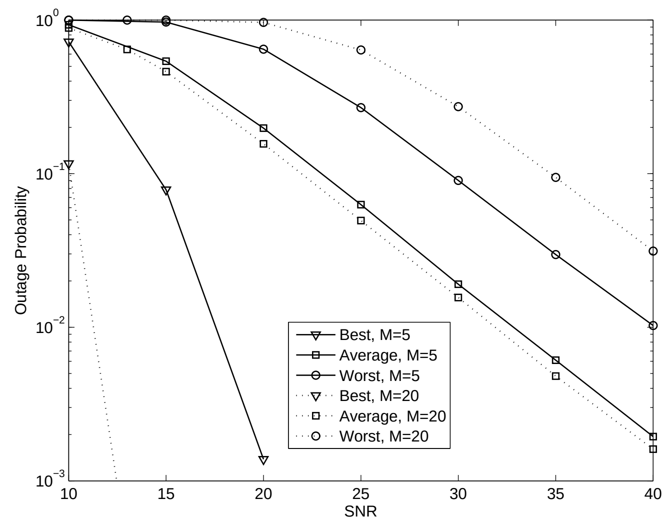


(b) Equal Power Allocation

Fig. 1. Outage probabilities achieved by the individual transmission scheme and the equal power allocation scheme. $R = 2$ BPCU. The solid curves are for the simulation results and the dashed ones are for the analytical results.



(a) Bounds for worst outage performance



(b) Outage performance

Fig. 2. Outage probability achieved by the water filling transmission scheme. $R = 2$ BPCU.

Fig. 2.a. As can be seen from the figure, the lower bound developed in (18) and the upper bound in (17) are very tight. Recall that the reason for the bounds in (17) and (18) are tight is because the dominant factor in the summation $\sum_{i=1}^{M} z_i$ is $z_{(M)}$. As shown at the end of Section III, $z_{(M)}$ is an unbounded variable, whereas the variance of the other variables, i.e. $z_{(m)}$, $m < M$, are always bounded. In Fig. 2.b, the outage performance based on different criteria is shown for the water filling scheme. Comparing Figures 1 and 2, we can see that the use of the water filling scheme yields the best performance. When there is more than one relay, relay selection can be carried out to further improve the performance of the three schemes, as can be observed in Fig. 3. Specifically a straightforward criterion for relay selection can be described in the following. Each relay reports the worst-user outage probability it realizes and the relay that minimizes the worst-user outage performance will be selected. The performance of the cooperative network might be further improved by designing more sophisticated criteria of relay selection.

In Figures 4 and 5, we focus on the comparison among the different power allocation strategies described in this paper. The targeted data rate is $\frac{1}{2}$ BPCU, and there are 20 user pairs, i.e., $M = 20$. Different choices of the energy harvesting efficiency coefficient η are used. Channels are assumed to be Rayleigh fading with path loss attenuation. Particularly the path loss factor is 2 and it is assumed that the distance from the sources to the relay is 2m, the same as the distance from the relay to the destinations. In Fig. 4, we study the outage performance for the user with the worst channel conditions. The water filling scheme can outperform the individual transmission and equal power strategies, consistent with the observations from the previous figures. The auction based strategy can achieve performance close to that of the water filling scheme for the case of $\eta = 1$. As indicated in Lemma 1, the water filling scheme is optimal in terms of the outage performance for the user with the worst channel conditions, which implies that the auction based scheme can achieve per-

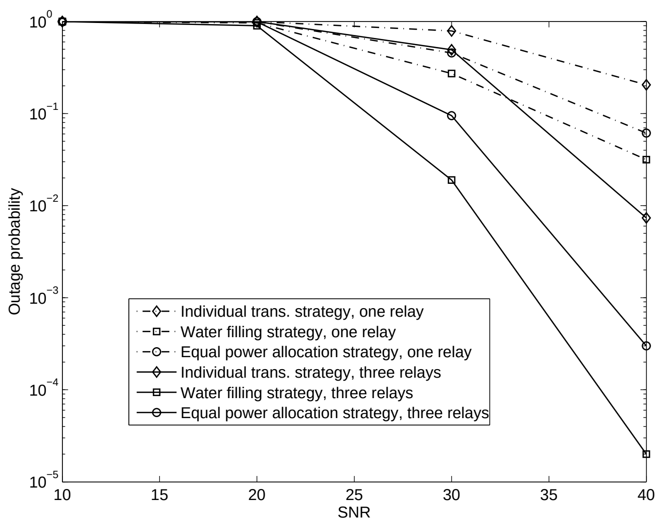


Fig. 3. The impact of relay selection on the outage probability for the user with the worst channel conditions. $R = 2$ BPCU. $M = 20$.

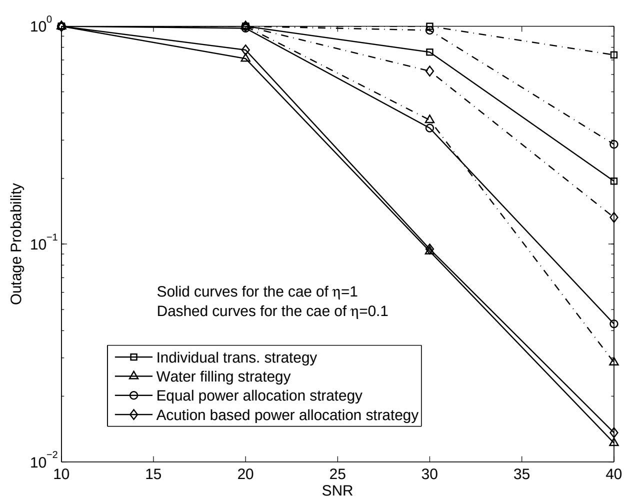


Fig. 4. Outage probability for the user with the worst channel conditions. $R = \frac{1}{2}$ BPCU and $\eta = 1$.

formance close to the optimal for the case of $\eta = 1$. When the energy harvesting efficiency is reduced, the performance of all power allocation schemes is degraded. However, the individual non-cooperative transmission scheme is particularly sensitive to the choice of η, whereas the sequential water filling strategy can still provide a reasonable outage performance even if the energy harvesting efficiency is very low. Furthermore, the auction based power allocation scheme can always outperform the equal power allocation scheme, irrespective of the choice of η.

In Fig. 5 the average outage performance achieved by the addressed transmission schemes is shown. Similar to Fig. 4, the water filling scheme can achieve the best performance, and the auction based scheme has performance close to the water filling scheme. The reason for the superior performance of the water filling scheme is due to the fact that this strategy can achieve the optimal performance for the destination with the best channel condition and also take fairness into consideration, as shown in Lemma 1. However, it is worth recalling

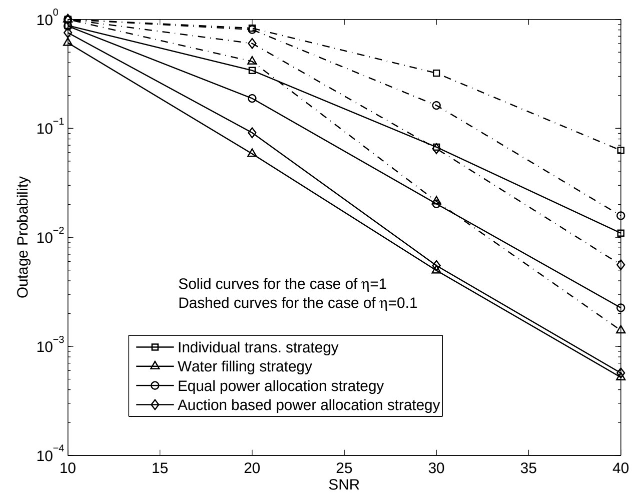


Fig. 5. Average outage performance achieved by the studied transmission protocols. $R = \frac{1}{2}$ BPCU and $\eta = 1$.

that the water filling scheme requires global CSI at the relay, whereas the other schemes, such as the auction based and equal power strategies, can be realized in a distributed way. Finally in Fig. 6, the convergence of the auction based strategy is studied by focusing on the average outage performance. As can be observed from the figure, with one iteration, the auction based scheme can outperform the individual transmission scheme, and with 5 iterations the auction strategy can achieve better performance than the equal power allocation scheme. After the number of iterations is larger than 10, performing more iterations does not offer much performance gain. In practice, the number of iterations can be dynamically changed to achieve a balanced tradeoff between the quality of service and system complexity.

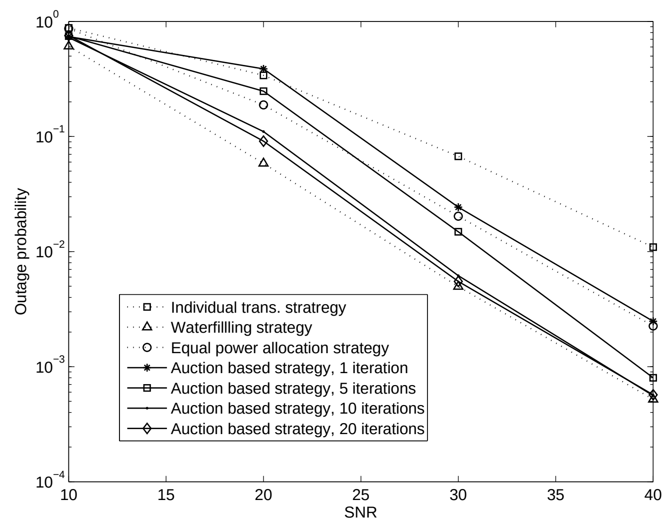


Fig. 6. Convergence of the auction based scheme. $R = \frac{1}{2}$ BPCU and $\eta = 1$.

VII. Conclusion

In this paper, we have considered several power allocation strategies for a cooperative network in which multiple source-destination pairs communicate with each other via an energy harvesting relay. The non-cooperative individual transmission scheme results in a outage performance decaying as $\frac{\log SNR}{SNR}$, the centralized power allocation strategies ensure that the outage probability decays at a faster rate $\frac{1}{SNR}$, and the water filling scheme can achieve optimal performance in terms of a few criteria. An auction based power allocation scheme has also been proposed to achieve a better tradeoff between the system performance and complexity. Individual source power constraints have been used in this paper, and carrying out power allocation among the sources may be helpful to further improve the energy efficiency since intuitively more power should be sent by a source with a better source-relay channel condition, which could yield an increase in the energy harvested at the relay. However, it is worth pointing out that such source power allocation could lead to a situation that some sources need to use very large transmission power, and hence the source batteries drain faster, an important factor to consider in the context of energy constrained communications. Another promising approach to further improve the system performance is that the relay could use all the signals from one source for energy harvesting, if the channel from this source to the relay is weak. However, such an approach cannot be applied to non-coherent detection receivers and may also cause some unfairness among the users. In addition, a failure of decoding is due to the poor source-relay channel condition, which means the energy harvested from such a channel could also be limited. The study of such different energy harvesting approaches is a promising future direction for further performance improvement.

VIII. Acknowledgments

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Appendix

Proof of Theorem 1: According to the instantaneous realization of the channels, we can group destinations into two sets, denoted by $\mathcal{S}_1$ and $\mathcal{S}_2$. $\mathcal{S}_1$ includes the destinations whose corresponding sources cannot deliver their information reliably to the relay, and $\mathcal{S}_2$ includes the remaining destinations; thus the size of $\mathcal{S}_2$ is N, i.e. $|\mathcal{S}_2| = N$. Therefore the outage probability for the i-th destination is

$$\mathcal{P}_{i,II} = \sum_{n=1}^{M} \Pr\left(\frac{1}{2}\log\left(1+\frac{P_r}{n}|g_i|^2\right) < R, N=n, i\in\mathcal{S}_2\right) + \Pr(i\in\mathcal{S}_1).$$

The second probability on the righthand side of the above equation can be calculated as $(1-e^{-\epsilon})$ by analyzing the error event $|h_i|^2 < \epsilon$. The probability of the event $i\in\mathcal{S}_2$ is $\frac{n}{M}$, conditioned on the size of the subset N, so the first factor in the above equation can be rewritten as

$$\mathcal{P}_{i,II} = \sum_{n=1}^{M} \frac{n}{M}\Pr\left(\frac{1}{2}\log\left(1+\frac{P_r}{N}|g_i|^2\right) < R \middle| N=n\right) \times \Pr(N=n). \tag{40}$$

The total available energy given N, the size of $\mathcal{S}_2$, is

$$P_r = \sum_{i=1}^{n} \eta P_s |h_i|^2\theta_i = \sum_{i=1}^{n} \eta\left(P_s|h_i|^2 - 2^{2R}+1\right).$$

Define $Q_1 \triangleq \Pr\left(\frac{1}{2}\log\left(1+\frac{P_r}{N}|g_i|^2\right) < R \middle| N=n\right)$. Using the independence among the channels, Q_1 can be evaluated as

$$Q_1 = \Pr\left(\left(P_s\sum_{i=1}^{n}|h_{n_i}|^2 - na\right)|g_i|^2 < \frac{na}{\eta} \middle| |h_{n_i}|^2 > \epsilon, \; 1\leq i\leq n\right), \tag{41}$$

where $\{n_1,\cdots,n_M\}$ is a perturbation of $\{1,\cdots,M\}$. Define $Y = \sum_{i=1}^{n}|h_{n_i}|^2$. To evaluate the above probability, it is important to find the density function of the sum of n exponentially distributed variables, Y, with the condition that each variable is larger than ϵ.

Conditioned on $|h_{n_i}|^2 > \epsilon$, we can find the Laplace transform of the density function of $|h_{n_i}|^2$ as

$$\begin{aligned}\mathcal{L}_{|h_{n_i}|^2>\epsilon}(s) &= \frac{1}{1-F_{|h_i|^2}(\epsilon)}\int_{\epsilon}^{\infty} e^{-xs} f_{|h_i|^2}(x)dx \\ &= e^{\epsilon}\int_{\epsilon}^{\infty} e^{-xs}e^{-x}dx = \frac{1}{1+s}e^{-s\epsilon}.\end{aligned}$$

Given the independence among the channels, conditioned on $|h_{n_i}|^2 > \epsilon, 1\leq i\leq n$, the density function of the sum of these channel coefficients has the following Laplace transform:

$$\mathcal{L}_{\sum_{i=1}^{N}|h_{n_i}|^2}(s) = \left(\mathcal{L}_{|h_{n_i}|^2>\epsilon}(s)\right)^n = \frac{e^{-n\epsilon s}}{(1+s)^n}. \tag{42}$$

By inverting the Laplace transform, the pdf of the sum, conditioned on $|h_{n_i}|^2 > \epsilon, 1\leq i\leq n$, is obtained as

$$f_{\sum_{i=1}^{N}|h_{n_i}|^2}(y) = \frac{(y-n\epsilon)^{n-1}}{(n-1)!}e^{-(y-n\epsilon)}u(y-n\epsilon). \tag{43}$$

A special case is when $\epsilon = 0$, in which case the above expression reduces to the classical chi-square distribution. Now the addressed probability can be calculated as

$$\begin{aligned}Q_1 &= \int_{n\epsilon}^{\infty}\left(1-e^{-\frac{b_n}{Py-na}}\right)\frac{1}{(n-1)!}(y-n\epsilon)^{n-1}e^{-(y-\epsilon)}dy \\ &= \frac{1}{(n-1)!}\left((n-1)! - 2\left(\frac{b_n}{P}\right)^{\frac{n}{2}}\mathbf{K}_n\left(2\sqrt{\frac{b_n}{P}}\right)\right).\end{aligned}$$

So the overall outage probability can be obtained after some algebraic manipulations by using the following result:

$$\Pr(N=n) = \frac{M!}{n!(M-n)!}e^{-n\epsilon}\left(1-e^{-\epsilon}\right)^{M-n}. \tag{44}$$

And the theorem is proved. ■

Proof of Lemma 1: The lemma can be proved by first developing a power allocation strategy optimal to the worst user outage performance and then showing that such a scheme achieves the same worst user outage probability as the water filling strategy.

Suppose that there are n sources that can deliver their signals successfully to the relay. The power allocation problem, which is to optimize the worst user outage performance, can be formulated as follows:

$$\begin{aligned}\max_{P_{ri}} \quad & \min\{R_{d,1},\cdots,R_{d,n}\} \\ s.t. \quad & \textstyle\sum_{i=1}^{n} P_{ri} = P_r.\end{aligned} \tag{45}$$

In order to find a closed-form expression for its solution, this optimization problem can be converted into the following equivalent form by introducing an auxiliary parameter:

$$\begin{aligned} \max_{P_{ri}} \quad & t \\ s.t. \quad & \tfrac{1}{2}\log(1+P_{ri}|g_i|^2) > t \\ & \textstyle\sum_{i=1}^{n} P_{ri} = P_r. \end{aligned} \tag{46}$$

By applying the Karush-Kuhn-Tucker conditions [19], a closed form expression for the optimal solution can be obtained as

$$P_{ri} = \frac{2^{2t}-1}{|g_i|^2}. \tag{47}$$

And the parameter t can be found by solving the following equation based on the total power constraint:

$$\sum_{i=1}^{n} \frac{2^{2t}-1}{|g_i|^2} = \sum_{i=1}^{n} \eta P_s |h_i|^2 \theta_i, \tag{48}$$

which yields

$$\begin{aligned} t &= \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{n}\eta P_s|h_i|^2\theta_i}{\sum_{i=1}^{n}\frac{1}{|g_i|^2}}\right) \\ &= \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{n}\eta(P_s|h_i|^2-2^{2R}+1)}{\sum_{i=1}^{n}\frac{1}{|g_i|^2}}\right). \end{aligned} \tag{49}$$

By using this closed form solution, the worst user outage probability can be written as in (50). On the other hand, for the addressed water filling strategy, the outage event for the user with the worst performance rises either because at least one of the source messages cannot be detected at the relay, $N < M$, or there is not enough power for all users, which means that the outage probability will be

$$\begin{aligned} \mathcal{P}_{worst,III} = \Pr\left(\sum_{i=1}^{M}\eta\left(P_s|h_i|^2-a\right) < \sum_{i=1}^{M}\frac{2^{2R}-1}{|g_i|^2}\right., \\ \left. N=M\right) + \sum_{n=0}^{M-1}\Pr(N=n). \end{aligned} \tag{51}$$

Comparing (50) and (51), we find that the two strategies achieve the same worst outage performance, and the lemma is proved. ■

Proof of Proposition 4 : The expression for the outage probability of the user with the worst channel conditions achieved by the water filling strategy is given in (51). The first factor in the expression, denoted by Q_4, can be expressed as

$$\begin{aligned} Q_4 = \Pr\left(\sum_{i=1}^{M} z_i > \frac{1}{a}\sum_{i=1}^{M}\eta\left(P_s|h_i|^2-a\right)\middle| N=M\right) \\ \times\Pr(N=M). \end{aligned} \tag{52}$$

To obtain some insightful understandings for the water filling scheme, we consider the following bounds:

$$\begin{aligned} \mathcal{P}_w\left(z_{(M)} > w\right) < \mathcal{P}_w\left(\sum_{i=1}^{M} z_i > w\right) \le \mathcal{P}_w\left(z_{(M)}\right. \\ \left. +(M-1)z_{(M-1)} > w\right), \end{aligned} \tag{53}$$

where $w = \frac{1}{a}\sum_{i=1}^{M}\eta\left(P_s|h_i|^2-a\right)$, $\mathcal{P}_w(\cdot)$ denotes the probability conditioned on a fixed w, and the condition $N = M$ has been omitted to simplify notation. The upper bound can be written as

$$\begin{aligned} &\mathcal{P}_w\left(z_{(M)} + (M-1)z_{(M)} > w\right) = \mathcal{P}_w\left(z_{(M)} > w\right) \\ &+\mathcal{P}_w\left(z_{(M)} + (M-1)z_{(M-1)} > w, \frac{w}{M} < z_{(M)} < w\right), \end{aligned}$$

where the condition $z_{(M)} > \frac{w}{M}$ is due to the fact that $z_{(M)}$ is the largest among the M ordered variables. Denote the second probability on the righthand side of the above equation conditioned on a fixed w by Q_3. Recall that the joint pdf of two order statistics $z_{(i)}$ and $z_{(j)}$, $1 \le i < j \le M$, can be written as [20]

$$\begin{aligned} f_{z_{(i)},z_{(j)}}(u,v) = \frac{M!}{(i-1)!(j-1-i)!(M-j)!} f(u)f(v) \\ \times(F(u))^{i-1}(F(v)-F(u))^{j-1-i}(1-F(v))^{M-j}, \end{aligned} \tag{54}$$

where the pdf and cumulative distribution function (CDF) are defined in (16) and the subscript z_i has been omitted for simplicity. Based on such a pdf, the probability Q_3 can be written as

$$\begin{aligned} Q_3 &= M(M-1)\int_{\frac{w}{M}}^{w}\int_{\frac{w-v}{M-1}}^{v} f(u)F(u)^{M-2}du f(v)dv \\ &= M\int_{\frac{w}{M}}^{w}\left(F(v)^{M-1} - F\left(\frac{w-v}{M-1}\right)^{M-1}\right) f(v)dv. \end{aligned}$$

Substituting the density function of z_i, we obtain

$$Q_3 = \left(e^{-\frac{M}{w}} - e^{-\frac{M^2}{w}}\right) - M\int_{\frac{w}{M}}^{w} e^{-\frac{(M-1)^2}{w-v}}\frac{1}{v^2}e^{-\frac{1}{v}}dv. \tag{55}$$

The probability $\mathcal{P}_w\left(z_{(M)} > w\right)$ can be obtained by applying the pdf of the largest order statistics as $\mathcal{P}_w\left(z_{(M)} > w\right) = 1 - e^{-\frac{M}{w}}$. So conditioned on a fixed w, the upper bound can be expressed as

$$\mathcal{P}_w\left(z_{(M)} + (M-1)z_{(M)} > w\right) \le 1 - e^{-\frac{M}{w}} + Q_3.$$

On the other hand, conditioned on M source messages successfully decoded at the relay, the density function of $w = (\eta\frac{1}{\epsilon}\sum_{i=1}^{M}|h_i|^2 - M\eta)$ can be obtained from (43) as $f_w(w) = \frac{1}{(M-1)!}\left(\frac{\epsilon}{\eta}\right)^M w^{M-1}e^{-\frac{\epsilon}{\eta}w}$. So the upper bound can be expressed as

$$\begin{aligned} \int_0^{\infty}\left(1 - e^{-\frac{M}{w}} + Q_3\right) f_w(w)dw \cdot \Pr(N=M) \\ + \sum_{n=0}^{M-1}\Pr(N=n), \end{aligned}$$

and the first part of the proposition is proved. The lower bound can be proved by using the steps similar to those used in the proof of Proposition 5, and will be omitted here. ■

$$\begin{aligned}\mathcal{P}_{out} &= \Pr\left(\min\left\{\frac{1}{2}\log(1+P_s|h_i|^2), i\in\{1,\cdots,M\}\right\} > R, N=M\right) + \sum_{n=0}^{M-1}\Pr(N=n) \quad (50)\\ &= \Pr\left(\frac{\sum_{i=1}^{M}\eta(P_s|h_i|^2-2^{2R}+1)}{\sum_{i=1}^{M}\frac{1}{|g_i|^2}} < 2^{2R}-1, N=M\right) + \sum_{n=0}^{M-1}\Pr(N=n).\end{aligned}$$

Proof of Proposition 5 : Recall that the upper bound for the water filling scheme is

$$\mathcal{P}_{worst,III} < e^{-M\epsilon}\int_0^\infty \left(1-e^{-\frac{M^2}{w}} - M\underbrace{\int_{\frac{w}{M}}^{w}\frac{e^{-\frac{(M-1)^2}{w-v}-\frac{1}{v}}}{v^2}dv}_{Q_5}\right) f_w(w)dw + 1 - e^{-M\epsilon}. \quad (56)$$

To obtain a more explicit expression for this upper bound, the factor Q_5 can be rewritten as

$$\begin{aligned}Q_5 &= \frac{1}{w}\int_0^{M-1} e^{-\frac{(y+1)(M-1)^2}{wy}-\frac{y+1}{w}}dy\\ &= \frac{1}{w}\int_0^{M-1} e^{-\frac{a(y)}{w}}dy, \quad (57)\end{aligned}$$

where $a(y)=(y+1)\left(\frac{(M-1)^2+1}{y}\right)$. An important observation from (57) is that $a(y)$ is not a function of w. Furthermore, the integration range in (57) is also not a function of w. As a result, we can first calculate the integral for w by treating y as a constant. First substituting (57) into the probability expression to obtain the following:

$$\begin{aligned}\mathcal{P}_{worst,III} &< 1-e^{-M\epsilon}+e^{-M\epsilon}\int_0^\infty\left(1-e^{-\frac{M^2}{w}}\right. \quad (58)\\ &\left.-\frac{M}{w}\int_0^{M-1}e^{-\frac{a(y)}{w}}dy\right) f_w(w)dw. \quad (59)\end{aligned}$$

We focus on the integral of the third factor in the bracket, denoted by Q_6, which is

$$\begin{aligned}Q_6 &\triangleq e^{-M\epsilon}\int_0^\infty\frac{M}{w}\int_0^{M-1}e^{-\frac{a(y)}{w}}dy f_w(w)dw\\ &= Me^{-M\epsilon}\int_0^{M-1}\int_0^\infty\frac{1}{w}e^{-\frac{a(y)}{w}}\frac{1}{(M-1)!}\left(\frac{\epsilon}{\eta}\right)^M\\ &\quad\times w^{M-1}e^{-\frac{\epsilon}{\eta}w}dwdwdy\\ &= \frac{M}{(M-1)!}e^{-M\epsilon}\left(\frac{\epsilon}{\eta}\right)^M\int_0^{M-1}\int_0^\infty w^{M-2}\\ &\quad\times e^{-\frac{a(y)}{w}-\frac{\epsilon}{\eta}w}dwdwdy\\ &= \frac{M}{(M-1)!}e^{-M\epsilon}\left(\frac{\epsilon}{\eta}\right)\int_0^{M-1}2\left(\frac{a(y)\epsilon}{\eta}\right)^{\frac{M-1}{2}}\\ &\quad\times\mathbf{K}_{M-1}\left(2\sqrt{\frac{a(y)\epsilon}{\eta}}\right)dy.\end{aligned}$$

Similarly the integrals of other components in (58) can be evaluated, the upper bound on the worst outage probability is obtained, and the proposition is proved. ■

Proof for Proposition 7 : The proposition can be proved by noting the first derivative of the payoff function is

$$\frac{\partial U_i(b_i;\mathbf{b}_{-i},\pi)}{\partial b_i} = \left(\frac{|g_i|^2}{2\ln 2(1+P_{ri}|g_i|^2)}-\pi_i\right)\frac{\partial P_{ri}}{\partial b_i}, \quad (60)$$

where $\frac{\partial P_{ri}}{\partial b_i} = \frac{\sum_{j=1,j\neq i}^{M} b_j+\xi}{(\sum_{j=1}^{M} b_j+\xi)^2}$. The first factor in the brackets is a strictly decreasing function of b_i, and $\frac{\partial P_{ri}}{\partial b_i}$ is always positive, so the payoff function is a strictly quasi-concave function of b_i, which indicates that there exists at least one NE. The unique best response for each player can be obtained by setting $\frac{\partial U_i(b_i;\mathbf{b}_{-i},\pi)}{\partial b_i}=0$, and a desirable outcome for the power allocation game is

$$P_{ri} = \left[\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}\right]^+, \quad (61)$$

where $(x)^+$ denotes $\max\{x,0\}$. By using the fact that the power that each user can get is bounded, i.e. $0\leq P_{ri}\leq P_r$, the first part of the proposition can be proved.

The uniqueness of the NE can be proved by studying the contraction mapping of the best response functions. Consider $\pi<\min\{\pi_{u,i}, i\in\mathcal{N}\}$, and define $\varrho_i(\pi)=\frac{\left(\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}\right)}{P_r-\frac{1}{2\ln 2\pi}-\frac{1}{|g_i|^2}}$. Therefore it is necessary to prove that there exists $\nu\in(0,1)$ such that for any $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^+$, $||BR(\mathbf{x})-BR(\mathbf{y})||_2\leq\nu||x-y||_2$, where $BR(\mathbf{b})=BR_1(\mathbf{b}_{-1})\times\cdots\times BR_N(\mathbf{b}_{-N})$, the Cartesian product of the best response function of each user and $||x||_2$ denotes the norm operation. Consider two distinct possible action sets, $\mathbf{x}$ and $\mathbf{y}$. From (37), $||BR(\mathbf{x})-BR(\mathbf{y})||_2$ can be expressed as

$$\begin{aligned}&\left(\sum_{i=1}^{N}\left(BR_i(\mathbf{x}_{-i})-BR_i(\mathbf{y}_{-i})\right)^2\right)^{\frac{1}{2}} \quad (62)\\ &= \left(\sum_{i=1}^{N}\varrho_i^2(\pi)\left(\sum_{j\neq i}^{N}(x_j-y_j)\right)^2\right)^{\frac{1}{2}}\\ &= \left(\sum_{i=1}^{N}\varrho_i^2(\pi)\left(\zeta-(x_i-y_i)\right)^2\right)^{\frac{1}{2}},\end{aligned}$$

where $\zeta=\sum_{j=1}^{N}(x_j-y_j)$. The above expression can be bounded as

$$\begin{aligned}
& \left(\sum_{i=1}^{N} \varrho_i^2(\pi)\left(\zeta-(x_i-y_i)\right)^2\right)^{\frac{1}{2}} \qquad (63)\\
\leq \; & \left(\sum_{i=1}^{N} \varrho_i^2(\pi)\left(|\zeta|+|x_i-y_i|\right)^2\right)^{\frac{1}{2}} \\
\underset{(a)}{\leq} \; & \left(\sum_{i=1}^{N} \varrho_i^2(\pi)\right)^{\frac{1}{2}} \left|\sum_{j=1}^{N}(x_j-y_j)\right| \\
& + \left(\sum_{i=1}^{N} \varrho_i^2(\pi)\left(x_i-y_i\right)^2\right)^{\frac{1}{2}} \\
\underset{(b)}{\leq} \; & N^{\frac{1}{2}} \left(\sum_{i=1}^{N} \varrho_i^2(\pi)\right)^{\frac{1}{2}} \left(\sum_{j=1}^{N}(x_j-y_j)^2\right)^{\frac{1}{2}} \\
& + \left(\sum_{i=1}^{N} \varrho_i^2(\pi)\left(x_i-y_i\right)^2\right)^{\frac{1}{2}} \\
\leq \; & \mu \left(\sum_{j=1}^{N}(x_j-y_j)^2\right)^{\frac{1}{2}},
\end{aligned}$$

where $|x|$ denotes the absolute value of x, $\varrho_{max}(\pi) = \max\{\varrho_1(\pi), \cdots, \varrho_N(\pi)\}$, $\mu = \left(N^{\frac{1}{2}}\left(\sum_{i=1}^{N} \varrho_i^2(\pi)\right)^{\frac{1}{2}} + \varrho_{max}(\pi)\right)$, the step (a) follows from the Minkowskis inequality and the step (b) follows from the Cauchy inequality. Since $\varrho_i(\pi)$ is a decreasing function of π, there exists a threshold such that when π is larger than this threshold, $\mu < 1$ and

$$\left(\sum_{i=1}^{N}\left(BR_i(\mathbf{x}_{-i})-BR_i(\mathbf{y}_{-i})\right)^2\right)^{\frac{1}{2}} < \left(\sum_{j=1}^{N}(x_j-y_j)^2\right)^{\frac{1}{2}}, \qquad (64)$$

which means that the best response function is a contraction mapping, and therefore there exists a unique NE [21]. Thus the proposition is proved. ■

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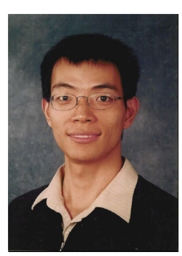

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