A decision support tool for procurement planning process under uncertainty

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Abstract: This communication presents a method to support the customer in the choice of a procurement plan when the gross requirements are ill-known, in a context of collaboration with the supplier. A general model of imperfect parameter representation is suggested, imperfection gathering uncertainty (through various scenarios) and imprecision (through quantities and dates expressed by possibility distribution). A method to compute the possible quantities required to satisfy the gross requirements under the supplier delivering constraints is proposed. From this value, a set of possible supplied quantities is computed to support the decision making of the customer. The decision maker then evaluates the procurement plan with the possible evolution of the inventory.

Keywords: supply chain, uncertainty, collaboration, theory of possibility.

1. INTRODUCTION

Enterprises are nowadays more and more tightly integrated in supply chains whereas the globalization of the market and the reduction of the product life cycles increases the uncertainty on the customer's demand. Thus, taking into account uncertainty in supply chains is becoming a major issue.

If no historical data allowing to build a stochastic model is available, uncertainty is often taken into account in the literature by possibility theory and fuzzy set theory (Dubois and Prade, 1988) (see for instance (Peidro *et al.,* 2009), (Lan *et al.,* 2009), (Aliev *et al.,* 2007)). These methods have for instance been used to model the imprecision on the coefficients of a cost function (Demirli and Yimer, 2006) or the weights of constraints (Mula, 2007).

Moreover, in an uncertain environment, a collaborative process between actors of the supply chain (customer, supplier) is a powerful tool to reduce the risk of the bullwhip effect (Galasso *et al.,* 2006). One of the references of collaborative processes in the industry is CPFR (Collaborative Planning forecasting replenishment (Ireland and Crum, 2005)). Within this framework, data such as inventory levels, forecast, etc. are exchanged between the supplier and the customer.

In order to take into account more explicitly the uncertainty related to the demand, a method is suggested in this communication aiming at supporting the decision maker (the customer) in building a procurement plan for the supplier. This decision is made under uncertainty on the gross requirements, in collaborative context (the supplier sends to the customer its delivering capacity constraints). To support the decision maker in finding a feasible procurement plan, we build the set of all the possible procurement plans. This method is composed of three steps:

◊ 1) the computation of the required inventory for each period,

◊ 2) the computation of the possible procured quantity under the delivering constraints.

◊ 3) the evaluation of the proposed procurement plan according to the possible evaluations of the inventory.

This paper is organized as follows: the first section presents a model of imperfect gross requirement. In the second one, a method to compute the required inventory is suggested while in the third section, a method to compute the possible procured quantity for a period is described, together with the framework of a decision support process aiming at building a procurement plan. In the last section, this decision support process is illustrated on an example.

2. IMPRECISE AND UNCERTAIN GROSS REQUIREMENTS

Our main hypothesis is that the customer has knowledge on gross requirements including imperfections (uncertainty and imprecision), often empirically modelled through expertise. So, the gross requirements can be given by the customer as a set of scenarios, with an imprecision on the value (e.g. "Not more than twenty parts, but certainly between 15 and 20"). Possibility theory is then used to model the uncertainty and imprecision on the demand in a more analytical way. Without loss of generality, uncertainty is represented as often by a possibility level (definition 1) and the imprecision by trapezoidal possibility distributions (definition 2 and figure 1). All the scenarios are synthesized in a graph (definition 3, figure 2) where a path stands for a possible scenario. The method to compute this graph of gross requirement is developed in (Guillaume *et al.*, 2010).

Definition 1: A possibility level denotes to what extent the occurrence of event $e \in E$ if possible. The possibility level t is an upper bound of the probability that the event *e* will appear $\Pr(e) \le \Pi(e)$ and $\exists e \middle| \Pi(e) = 1$.

Definition 2: A possibility distribution π_v of *v* quantifies the plausibility of the information *v*. π_v is a function of *S* in *L* such as $\forall s \in S, \pi_v(s) \in L$ and $\exists s \middle| \pi_v(s) = 1$ with *v* denoting an ill-known value in *S*, and *L* the scale of plausibility ([0,1] for the theory of possibility) (Dubois and Prade, 2006).

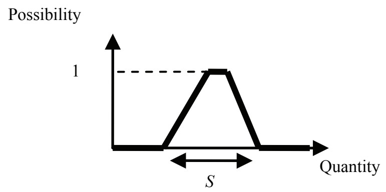

Fig. 1. Trapezoidal distribution of possibility

Definition 3: The graph of the gross requirements is a directed arc weighted layer graph $G = < V, E >$ with quantities on node defined as:

Node set $V = V_1 \cup ... \cup V_T$ with T the number of layers such as $V_i \cap V_j = \phi$ where $V = \{\widetilde{v}_1; \widetilde{v}_2; ..; \widetilde{v}_n\}$ with $\widetilde{v}_i$ trapezoidal possibility distribution.

Arc set $E = (v, w)$ such as $v \in V_i$ & $w \in V_{i+1}$ and the weights of arc are defined by a matrix $\mathbf{M} = (\Pi_{v,w})_{(v,w) \in E}$ with $\max(\Pi_{v,w}, v \in V_i \,\&\, w \in V_{i+1}) = 1$ (consequence of possibility theory).

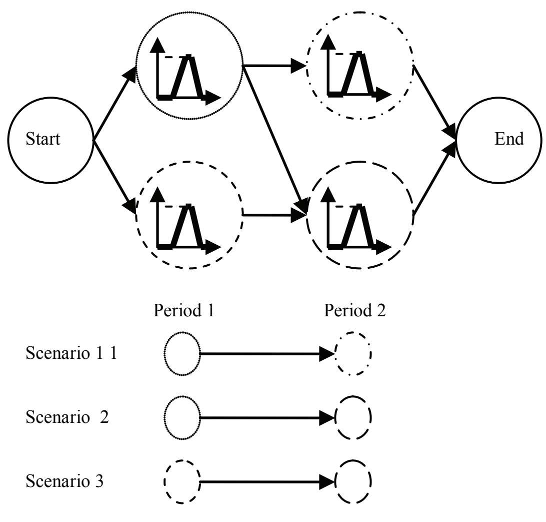

Fig. 2. Example of graph for a horizon of 2 period and 3 scenarios

To represent a fuzzy number, an interval is used where the extreme values are gradual numbers. This representation enables to apply the interval analysis without using the α-cuts (Fortin *et al.*, 2008). We also use a set of two linear gradual numbers to represent the trapezoidal possibility distribution. Figure 2 illustrates the model for a fuzzy number $\widetilde{A}$. The gradual number is represented by two values $\widetilde{A}^- = [\widetilde{A}^-(0); \widetilde{A}^-(1)]$, so a fuzzy number is represented by 4 values.

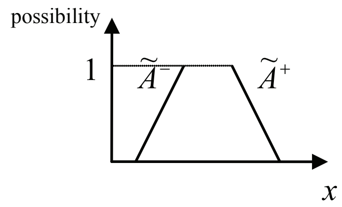

Fig. 3. Fuzzy number $\widetilde{A} = [\widetilde{A}^-; \widetilde{A}^+]$

The data are denoted as follows:

t: period with $t \in [1,T]$

c_t: index of fuzzy gross required quantity on period *t* with $c_t \in [1, C_t]$

$\widetilde{Gr}_{t,c_t} = [\widetilde{Gr}^-_{t,c_t}; \widetilde{Gr}^+_{t,c_t}]$: fuzzy gross required quantity on node (t, c_t)

$w_{c_t, c_{t+1}, t}$: possibility level of the arc linking the node (t, c_t) with the node $(t+1, c_{t+1})$

$L_t = [l_t^-; l_t^+]$: set of possible quantity of deliveries l_t on period *t*

3. COMPUTATION OF THE REQUIRED INVENTORY

The first step, which consists in computing the required inventory (*definition 4*), is presented in this section.

Definition 4: The required inventory is the quantity required in the inventory to fulfil the gross quantities required under supplier delivering constraints.

This method uses an iterative algorithm, from period *T* to 1.

The dependant variables are denoted:

$\widetilde{Ir}_t^* = [\widetilde{Ir}_t^{*-}; \widetilde{Ir}_t^{*+}]$: fuzzy inventory required for period *t* which satisfies all the possible gross requirements,

$\widetilde{Ir}_{*t} = [\widetilde{Ir}_{*t}^-; \widetilde{Ir}_{*t}^+]$: fuzzy inventory required for period *t* which satisfies one of the possible gross requirement,

$\widetilde{Ir}_{t,c_t}{}^{*} = [\widetilde{Ir}_{t,c_t}{}^{*-}; \widetilde{Ir}_{t,c_t}{}^{*+}]$: fuzzy inventory required for node (t,c_t) which satisfies all the possible gross requirements,

$\widetilde{Ir}_{*t,c_t} = [\widetilde{Ir}_{*t,c_t}{}^{-}; \widetilde{Ir}_{*t,c_t}{}^{+}]$: fuzzy inventory required for node (t,c_t) which guarantee the respect of one or more possible gross requirements.

When the gross requirements are imperfect (uncertain and imprecise), the required inventory is imperfect too. To deal with imprecision, each required quantity becomes a possibility distribution. To deal with uncertainty, only the two extremes cases are computed: the required inventory which satisfies all the possible gross requirements and the required inventory which satisfies one of the possible gross requirements.

The operator used to back-propagate the required inventory depends on the type of required inventory:

- minimum: used for the inventory which satisfies one of the possible gross requirements,
- maximum: used for the inventory which satisfies all the possible gross requirements

It is considered that the required inventory for period $T+1$ is equal to 0. To compute a required inventory (for example: which satisfy all possible gross requirements $\widetilde{Ir}_{t,c_t}{}^{*}$), the optimization problem (equations 1 to 3) is solved for each node (t,c_t) (from (T, C_T) to $(1,1)$) and for the four values of the required inventory (for example: $\widetilde{Ir}_{t,c_t}{}^{*-}(0)$; $\widetilde{Ir}_{t,c_t}{}^{*-}(1)$; $\widetilde{Ir}_{t,c_t}{}^{*+}(1)$; $\widetilde{Ir}_{t,c_t}{}^{*+}(0)$).

$$\text{minimize}(\widetilde{Ir}^{*}{}_{t,c_t}) \quad (1)$$

s.t.

$$\widetilde{Ir}^{*}{}_{t,c_t} - \widetilde{Gr}_{t,c_t} + l_t^{+} \geq \widetilde{Ir}^{*}{}_{t+1} \quad (2)$$

$$\widetilde{Ir}^{*}{}_{t,c_t} \geq 0 \quad (3)$$

The same method is applied for the inventory which satisfies one of the possible gross requirements (with I^* becoming I_*).

The extreme required inventory (optimist $= \widetilde{Ir}^{*-}{}_{t,c_t}$; pessimist $= \widetilde{Ir}^{*+}{}_{t,c_t}$) corresponds to the extreme bound of the set of the gross required quantity for the next period ($\widetilde{Ir}^{*}{}_{t+1}$) and the extreme bound of set the gross requirement ($\widetilde{Gr}_{t,c_t}$), so we have $\widetilde{Ir}^{*-}{}_{t,c_t}$ for $\widetilde{Gr}^{-}{}_{t,c_t}$ and $\widetilde{Ir}^{*-}{}_{t+1}$; and $\widetilde{Ir}^{*+}{}_{t,c_t}$ for $\widetilde{Gr}^{+}{}_{t,c_t}$ and $\widetilde{Ir}^{*+}{}_{t+1}$.

The required inventory is the minimum inventory needed to satisfy a gross requirement, so the minimum required inventory respecting the delivery constrains (whatever the quantity delivered by the supplier is) corresponds to the maximum possible quantity procured: l_t^{+}.

To back-propagate the maximum (minimum) required inventory level and keep the representation of the possibility distribution by 4 values (figure 4), an approximate method is used. This method maximises (minimises) two gradual numbers by a linear function which guarantees that all the values of the exact solution are lower (upper) than the approximate solution (for the minimum: equations 4 to 6 and figure 5). In fact, the maximum (minimum) between two values has in the worst case one intersection point (figure 4).

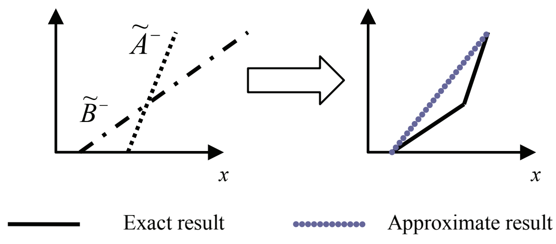

Fig. 4. Exact/Approximation of minimum

$$\widetilde{Ir}_t{}^{*+}(0) = \max_{w_{c_t,c_{t+1},t} \neq 0} \left(\widetilde{Ir}_{t,c_t}{}^{*+}(0)\right) (4)$$

$$\widetilde{Ir}_{t,c_t}{}^{*+}(1) \Leftarrow \widetilde{Ir}_{t,c_t}{}^{*+}(1) + \frac{1 - w_{c_t,c_{t+1},t}}{w_{c_t,c_{t+1},t}} \times \left(\widetilde{Ir}_{t,c_t}{}^{*+}(1) - \widetilde{Ir}_t{}^{*+}(0)\right) (5)$$

$$\forall w_{c_t,c_{t+1},t} \,\middle|\, 0 < w_{c_t,c_{t+1},t} < 1$$

$$\widetilde{Ir}_t{}^{*+}(1) = \max_{w_{c_t,c_{t+1},t} \neq 0} \left(\widetilde{Ir}^{*}_{t,c_t}{}^{+}(1)\right) (6)$$

Figure 5 illustrates the equations defining the minimum between to gradual numbers $\widetilde{A}^{-}$ and $\widetilde{B}^{-}$.

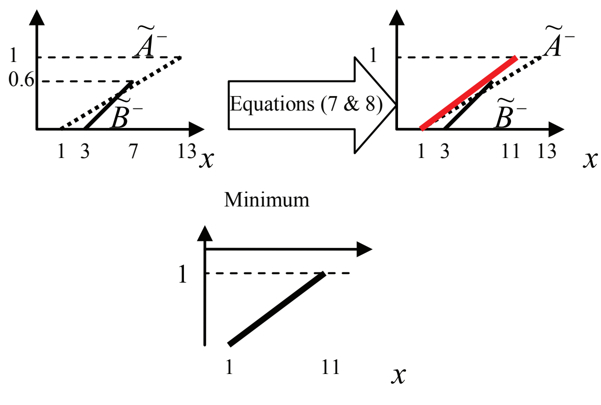

Fig. 5. Illustration of the approximation of the minimum

4. CHOOSE A PROCUREMENT PLAN

The second and the third steps of the method, respectively the computation of the possible procured quantity and the evaluation of the possible inventory level for a given procured quantity, are described in this section. These two steps are performed consecutively from the period 1 to T. The dependant variables are denoted:

$\widetilde{P}_{t,c_t}{}^{*} = [\widetilde{P}_{t,c_t}{}^{*-}; \widetilde{P}_{t,c_t}{}^{*+}]$: fuzzy procurement quantity for node (t,c_t) which satisfies all the possible gross requirements,

$\widetilde{P}_{*t,c_t} = [\widetilde{P}_{*t,c_t}{}^{-}; \widetilde{P}_{*t,c_t}{}^{+}]$: fuzzy procurement quantity for node (t,c_t) which satisfies one of the possible gross requirements,

$\widetilde{P}_t^{*} = [\widetilde{P}_t^{*-}; \widetilde{P}_t^{*+}]$: fuzzy procurement quantity for period t which satisfies all the possible gross requirements,

$\widetilde{P}_{*t} = [\widetilde{P}_{*t}{}^{-}; \widetilde{P}_{*t}{}^{+}]$: fuzzy procurement quantity for period t which satisfies one of the possible gross requirement,

$\widetilde{I}_t^{*}$: fuzzy possible inventory at the end of period t in the case satisfy all the possible gross requirements are satisfied,

$\widetilde{I}_{*t}$: fuzzy possible inventory at the end of period t in the case when one of the possible gross requirements is satisfied,

x_t : value proposed by the decision maker for the period t

δ: difference between the required inventory of period t and the possible inventory for the period t.

4.1 Computation of fuzzy possible procured quantities

This method is applied for the two extreme cases concerning all the gross requirements or one of the possible gross requirements. The decision maker receives so a fuzzy set for each case. In order to compute the two fuzzy procured quantities, the following optimization model is solved (equations 7 to 9) for the four values (from $\widetilde{P}_{t,c_t}{}^{*-}(0)$ to $\widetilde{P}_{t,c_t}{}^{*+}(0)$).

$$\text{minimize}(\delta)\ (7)$$

s.t.

$$\delta = \left|\widetilde{P}_{t,c_t}{}^{*-}(0) + \widetilde{I}_{t-1}^{*-}(0) - \widetilde{G}r_{t,c_t}^{-}(0) - \widetilde{I}r_t^{-}(0)\right|\ (8)$$

$$l_t^{-} \le \widetilde{P}_{t,c_t}{}^{*-}(0) \le l_t^{+}\ (9)$$

The required inventory for period 0 is different from the real inventory at the beginning of period 0, which is known. So we are looking for the procured quantity which satisfies the required inventory for the next period (period 1). We associate a possibility level to the possible delivering quantity, corresponding to the possibility level that the quantity will be required.

We compute then the fuzzy quantity delivered for the period. In the case of the procurement plan which satisfies all the possible gross requirements, we merge the nodes with the max operator, in the other case with the min operator (equation 10: procurement plan which satisfies all the possible gross requirements)

$$\widetilde{P}_t^{*} = \max_{c_t}(\widetilde{P}_{t,c_t}^{*})\ (10)$$

4.2 Computation of fuzzy possible procured quantities

From the possible procured quantities, a crisp value is proposed according to a risk level in terms of inventory. For example, the lower bound of the most possible value for the case which satisfies all gross requirements can be proposed as a first suggestion since it is a prudent solution.

Then, two resulting inventories are computed (one aiming at satisfying all the possible gross requirements; one aiming at satisfying one of the possible gross requirements), using the minimum operator for the first one, and the maximum operator for second one. The minimum and the maximum are computed like the required inventory (equations 4, 5 and 6). Equations 11 and 12 compute the inventory levels which satisfy all the possible gross requirements (with x_t standing for the proposed value by the decision maker).

$$x_t + \widetilde{I}_{t-1}^{*} - \widetilde{G}r_{t,c_t} = \widetilde{I}_{t,c_t} \quad \forall c_t\ (11)$$

$$\widetilde{I}_t^{*} = \min_{c_t}(\widetilde{I}_{t,c_t}^{*})\ (12)$$

Then, the fuzzy possible procured quantities are computed for the next period.

From the first proposition of procurement plan (x_1 to x_T), the decision maker analyses the evolution of the inventory and modifies the procurement plan until it gives him satisfaction.

5. EXAMPLE

In this section, we illustrate on an example the method of computation of the procurement plan under uncertainties.

5.1 Data

The considered planning horizon is 2 periods and the inventory at period 0 is equal to 0. The imperfect gross requirements are represented in Figure 6 and Table 1, and the delivering constraints in Table 2.

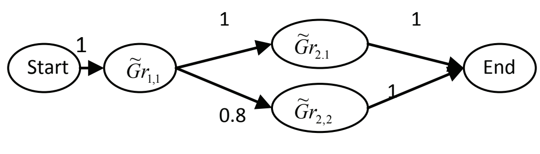

Fig. 6. Imperfect gross requirement

Table 1. Gross requirement

t, c_t	$\widetilde{G}r_{t,c_t}^{-}(0)$	$\widetilde{G}r_{t,c_t}^{-}(1)$	$\widetilde{G}r_{t,c_t}^{+}(1)$	$\widetilde{G}r_{t,c_t}^{+}(0)$
2,1	12	17	17	25
2,2	15	16	20	22
1,1	6	7	7	8

Table 2. Delivery constraints

	Period 1	Period 2
Delivery quantity	$\left[l_1^1; l_1^+\right] = [5;17]$	$\left[l_2^1; l_2^+\right] = [5;15]$

From these data, the required inventory is computed.

5.2 Computation of the required inventory

It is considered that the required inventory of period 3 is (0;0;0;0). Then, the required inventory for the node 1 of period 2 is computed as follows.

For the last period, the required inventory which satisfies all the gross requirements is equal to the required inventory which satisfies one of the possible gross requirements ($\tilde{I}r^{*}{}_{2,1} = \tilde{I}r_{*2,1}$ & $\tilde{I}r^{*}{}_{2,2} = \tilde{I}r_{*2,2}$).

The lower bound is then computed:

$$\text{minimize}(\tilde{I}r^{*-}{}_{2,1})$$

s.t.

$$\tilde{I}r^{*-}{}_{2,1} - \tilde{G}r^{-}{}_{2,1} + l_2^{+} \geq \tilde{I}r^{*-}{}_{3}$$

$$\tilde{I}r^{*-}{}_{2,1} \geq 0$$

In this case, the equation (5) becomes:

$$\tilde{I}r^{*-}{}_{2,1}(0) - 12 + 15 \geq 0 \Rightarrow \tilde{I}r^{*-}{}_{2,1}(0) = 0$$

$$\tilde{I}r^{*-}{}_{2,1}(1) - 17 + 15 \geq 0 \Rightarrow \tilde{I}r^{*-}{}_{2,1}(1) = 2$$

The upper bound is:

$$\tilde{I}r^{*+}{}_{2,1}(0) - 25 + 15 \geq 0 \Rightarrow \tilde{I}r^{*+}{}_{2,1}(0) = 10$$

$$\tilde{I}r^{*+}{}_{2,1}(1) - 19 + 15 \geq 0 \Rightarrow \tilde{I}r^{*+}{}_{2,1}(1) = 2$$

So, $\tilde{I}r^{*}{}_{2,1} = \tilde{I}r_{*2,1} = (0;2;2;10)$ and in the same way, the node (2,2) is computed : $\tilde{I}r^{*}{}_{2,2} = \tilde{I}r_{*2,2} = (0;1;5;7)$.

From these results, the required inventory of the node 1 of the period 1 is computed. The back-propagation of the maximum required by the inventory of the period 2 is used for the case which satisfy all the gross requirements (otherwise, the minimum). We apply the equation (4), then the equation (5) and the equation (6). Figure 7 illustrates the calculation.

$$\begin{aligned}\tilde{I}r_2^{*+}(0) &= \max\left(\tilde{I}r_{2,1}^{*+}(0); \tilde{I}r_{2,1}^{*+}(0)\right)\\ &= \max(10;7) = 10\end{aligned}$$

$$\tilde{I}r_{2,2}^{*+}(1) = 5 + \frac{1-0.8}{0.8} \times (5-10) = 3.75$$

$$\begin{aligned}\tilde{I}r_2^{*+}(1) &= \max\left(\tilde{I}r_{2,1}^{*+}(1); \tilde{I}r_{2,1}^{*+}(1)\right)\\ &= \max(2;3.75) = 3.75\end{aligned}$$

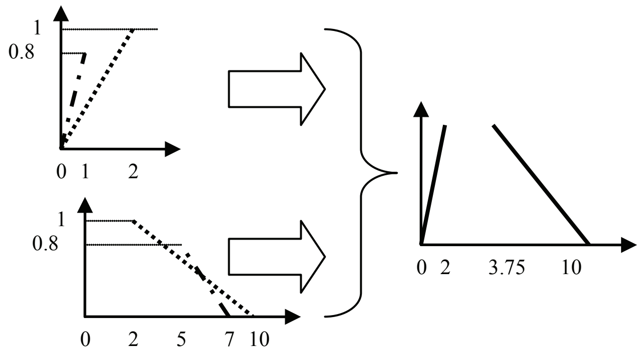

Fig. 7. Maximum Required inventory of period 1

In the same way, but using the min operator, we compute the required inventory which satisfies one of the possible gross requirements (Figure 8). The result of the computation of the required inventory is illustrated in Figure 9.

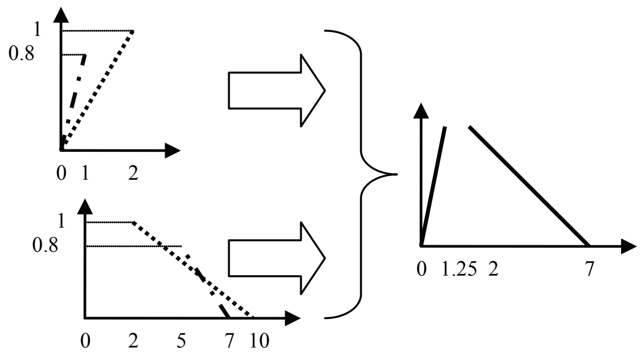

Fig. 8. Minimum Required inventory of period 1

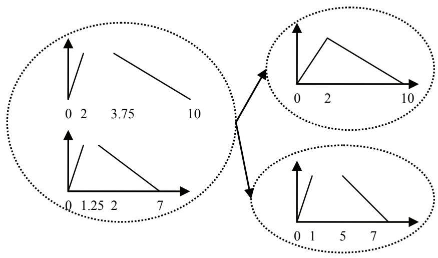

Fig. 9. Required inventory

5.3 Computation of fuzzy possible procured quantities

First, we compute the fuzzy procurement plan which satisfies all the gross requirements. We apply the equations 11, 12 and 13 at the period 1:

$$\delta = \left|\tilde{P}_{1,1}^{*-}(0) + 0 - 6 - 0\right|$$

$$5 \leq \tilde{P}_{1,1}^{*-}(0) \leq 17$$

so $\tilde{P}_{1,1}^{*-}(0) = \tilde{P}_{1}^{*-}(0) = 6$

In the same way, we have: $\widetilde{P}_1^{*-}(1) = 9$ $\widetilde{P}_1^{*+}(1) = 10.75$ $\widetilde{P}_1^{*+}(0) = 17$.

5.4 Computation of fuzzy possible procured quantities

On the left of FA prudent solution is proposed: the lower bound of the most possible value for the case, which satisfies all gross requirements (value=9). On the left of Figure 10 a set of fuzzy procured quantity are proposed. On the right, the resulting inventory level for the proposition is represented with the required inventory level for the next period.

Then, the decision maker can modify the first crisp value and the right side to evaluate the new proposition. In the example, he proposes the quantity 11 which leads to a less risked solution .

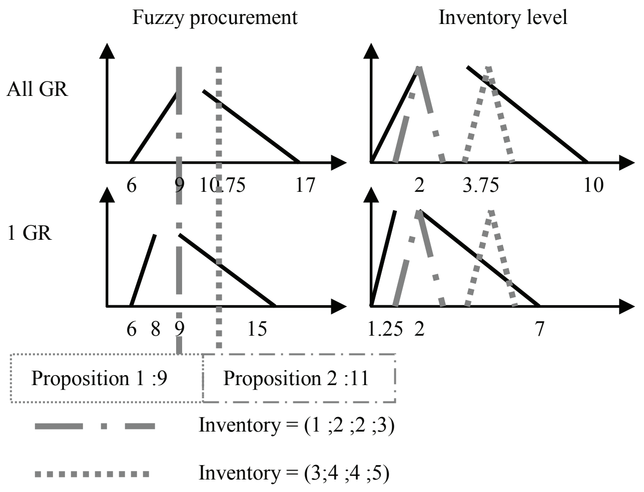

Fig. 10. Dashboard of the decision maker for period 1

6. CONCLUSIONS

In this paper, we propose a decision support tool for helping a human decision maker in building a procurement plan under uncertainty. The decision maker knows a set of possible quantities evaluated by possibility levels and can see the consequences of his choices through the inventory evolution. This decision support tool allows the decision maker to take decision in knowledge of the facts and also to evaluate the risk of the chosen procurement plan.

As a perspective, we want to solve this problem as an optimization problem using robust optimization (as minmax or minmax*Regret* criteria). Those criteria minimize the maximum possible cost over all possible scenarios of the gross requirement. Thus, this method enables to minimize the risk in term of backordering/obsolete inventory in fuzzy uncertain context.

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