Modelling of ill-known requirements and integration in production planning

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Abstract – Making decisions on the base of uncertain forecasts is one of the key challenges for efficient Supply Chain Management. This article suggests the use of the theory of possibility for building a procurement plan on the base of ill-known requirements. These requirements, expressed in quantities by date, may come from various sources: forecasts or orders for instance. The possible types of imperfection pervading requirement are analysed and a unified representation model is suggested. A method is then described for calculating a plausible demand by period without loss of information; it is illustrated on an example in the last section.

Keywords – requirement, demand, imprecision, uncertainty, theory of possibility.

1 INTRODUCTION

Today's industrial context is characterised by an increased uncertainty on the final customer's demand. In many supply chains, and especially in the aeronautical industry, the supply times are much longer than the manufacturing times. In this context, an important challenge is to manage the procurement process in spite of the uncertainty on the demand.

Collaborative processes aim at reducing this supply risk. Vendor Management Inventory (VMI®) (Marques *et al.,* 2008) and Collaborative Planning, Forecasting and Replenishment (CPFR®) (Ireland and Crum*,* 2005) are for instance widely used in an industrial context. These techniques aim at creating short and reactive decision loops between customers and suppliers in order to cope with the growing uncertainty on demand forecasting, due to the shortening of the product life cycle and to customers' versatility. Nevertheless, they can only deal with quite standard requirements, which is for instance the case in the automotive industry. Taking into account the uncertainty of the demand in the planning processes can be an alternative solution for more specific requirements, e.g. in the spatial and aeronautical industry for which the problem is still critical.

In this study is considered a dyadic elementary link of a supply chain, involving two actors (customer/supplier). Within these two partners, it will be shown that an uncertain demand can be considered in order to build a "fuzzy" procurement plan, firstly internally processed then transmitted to the upstream suppliers. Like a classical one, this procurement plan is expressed in quantities per periods, but does not only describe one (most plausible) situation, but a set of situations, with a quantification of their possibility of occurrence. It should then allow to make more informed decisions concerning the risk taken by ordering a given quantity.

The article is organized as follows: section 2 presents a state of the art on the application of possibility theory to production planning problems. The basis of possibility theory and the problem of representation of imperfect information are introduced in the following section, whereas a way to represent "imperfection" (uncertainty, imprecision…) on the requirements is proposed in section 3. The integration in the Material Requirement Planning of imperfect requirements, using the suggested method, is described in section 4. The approach is illustrated on an example in section 5.

2 LITERATURE REVIEW

In a manufacturing context, production planning aims at optimizing the use of the resources to fulfil the customers' demand. Usually, forecasts are estimated on the base of historical data, and allow to build a Sales and Operation Plan, implicitly considered as certain but periodically revised (SOP step of the MRP (Manufacturing Resource Planning) method). A Master Production Schedule then details the SOP, listing quantities of required products per periods of time. Using the bill of materials, these "independent" requirements are then used to generate requirements on components, also by periods. In an uncertain context, the problem is to build a feasible requirement plan on the base of an ill-known demand.

In that purpose, a large part of the literature focuses on rolling horizon planning methods: an initial planning is periodically updated in order to take into account the most recent knowledge on the demand. Strategies like adjusting the re-planning horizon size (Galasso *et al.,* 2009), or freezing a part of the plan (Zhao and Xie*,* 1998) may be used in that purpose. On the other hand, production planning under uncertainty aims at modelling this uncertainty in order to provide the decision maker with a richer view on the various possible situations which may arise. Within this strategy, stochastic and possibilistic models have been mainly used to represent imperfect data. Stochastic models need a huge

amount of historical data, and require that the context in which the data have been collected remains still valid in the future. Since this is more and more rare, we focus here on the case when it is difficult to use such methods. In that case, uncertainty may often be modelled on the base of the human expertise. Fuzzy logic and theory of possibility have shown their interest for representing subjective information and some works have already been performed on the representation of a subjectively assessed uncertainty on the demand.

Within these works, it is important to distinguish production planning under flexible constraints from production planning with ill-known parameters (Dubois et al., 2003). In the literature on production planning under flexible constraints (Mula *et al.,* 2007, Jamalnia and Soukhakian*,* 2009), fuzziness models a level of satisfaction of constraints expressed by imprecise terms (often in natural language), or a degree of tolerance (Bellman and Zadeh, 1970). Several surveys (Kacprzyk and Orlovski*,* 1987, Delgado *et al.,* 1994, Baykasoglu and Göçken*,* 2008) show how fuzziness can in that case be accommodated with linear programming.

In the literature on production planning under uncertainty, fuzziness is used for modelling the level of possibility that an event may occur. Both approaches refer to the possibility theory (Zadeh, 1978, Dubois and Prade, 1988) and to decision under fuzzy uncertainty (Dubois *et al.,* 2001).

Production planning under uncertainty based on possibility theory may allow to compute a supply plan considering an uncertain demand, modelled by possibility distributions. Some authors suggest to make decisions on the base of a fuzzy plan, while others use defuzzification techniques for obtaining a crisp plan. In the first category can be found (Fargier and Thierry*,* 2000), in which the MPS (Master Production Schedule) considers an uncertain demand. A method is proposed to support subcontracting decisions or negotiation with the customer. In the same way, in (Grabot *et al*., 2005), the imperfection on the customer's demand is taken into account in the Manufacturing Resource Planning through a method called F-MRP (Fuzzy-MRP), resulting in a "fuzzy plan".

Another solution is to model first the uncertain plan, then to defuzzify in order to obtain a crisp equivalent demand (demand after defuzzification) (Niu and Dartnall, 2008, Liang, 2008, Peidro *et al.,* 2009).

An alternative is to build a plan which takes into account a level of possibility, necessity or credibility in order to guarantee or optimise a "level of chance" that the plan is feasible. In (Lan *et al.,* 2009), the chance that the inventories will be superior to zero is guaranteed by the constraints while in (Sun and

Liu, 2008), the objective to minimize is the chance than the cost function is inferior to a given threshold. In (Aliev *et al.,* 2007), the objective function to minimize is the chance that the demand will not be satisfied, combined with the profit.

(Liu, 2009) presents a set of other models for optimisation under uncertainty with fuzzy parameters, which could be used in supply chain planning.

When dates related to precise orders and durations of tasks are considered, scheduling under uncertainty may result in a fuzzy schedule (Chen and Huang, 2006, Balasubramanian and Grossmann, 2003). These references are summarised in Table 1.

Table 1. Classification of previous works

References	Use of fuzziness	type of model
(Mula *et al.,* 2007) (Jamalnia and Soukhakian, 2009)	Flexible constraints	planning
(Fargier and Thierry, 2000) (Grabot *et al*., 2005)	Integration of uncertainty	planning
(Niu and Dartnall, 2008) (Liang, 2008) (Peidro *et al.,* 2009)	Defuzzification then optimisation	planning
(Lan *et al.,* 2009) (Sun and Liu, 2008) (Aliev *et al.,* 2007)	Optimisation under uncertainty	planning
(Chen and Huang, 2006) (Balasubramanian and Grossmann, 2003)	Optimisation under uncertainty	scheduling

Even if possibility theory has been intensively used for modelling uncertainty in planning processes, it is interesting to notice that none of these methods addresses the basic industrial problem in its more generic form: if the imperfection of the demand concerns both the quantities (i.e. the level of a requirement may change) and the dates (the requirement may be advanced or postponed), finding how many components may be required in a given period in order to fulfil this demand becomes a complex task. In order to address this problem, this article first suggests a method aiming at characterising and representing "uncertainty" (imperfection, uncertainty and imprecision) on the independent requirements, then shows how to turn these "imperfectly known" requirements into dependent requirements (a supply plan composed of required quantities of components by periods), in a way which stays as close as possible to industrial practices. This supply plan will be considered for deciding the quantities ordered to the upstream partners of the supply chain.

3 BASIC FRAMEWORK

The objective of this section is to position the various concepts covering the vague notion of "imperfect information", and to present some models allowing to represent them. Besides are introduced the main results of the theory of possibility which will be used afterwards.

3.1 Definition of imperfection

Even if the word "uncertainty" is commonly used, a distinction has to be made between several concepts: uncertainty, imprecision and incompleteness. The term "imperfection" will be used to gather these three concepts, as suggested in (Bouchon-Meunier, 1995).

- ***Uncertainty*** occurs when it is not possible to state whether a particular statement is definitely true or false.

- ***Imprecision*** denotes the impossibility to enounce a piece of knowledge including crisp values in a sure manner.

- ***Incompleteness*** means in most cases that an information source does not contain full information about the values of the attributes of the real-world it describes (some properties may be missing from the description).

As stated above, two types of mathematic models can represent imperfect data: stochastic and possibilistic models (Dubois and Prade, 2006). Because of the highly dynamic context of today's manufacturing, we consider in this article the case when no previous observation is available for building a stochastic model. In that case, possibility theory has shown its interest for modelling uncertainty on the base of expert knowledge.

3.2 Representation of imperfection

3.2.1Representation of uncertainty

For each event A from a subset S, the uncertainty may be evaluated through two levels: the possibility $\Pi(A)$, denoting to what extent the occurrence of event A if possible, and the certainty $\mathrm{N}(A)$, denoting to what extend the occurrence of event A is certain, with:

$$\forall A \subseteq S, \mathrm{N}(A) \leq \prod(A) \qquad (1)$$

and

$$\forall A \subseteq S, \mathrm{N}(A) = 1 - \Pi(\overline{A}) \qquad (2)$$

If the occurrence of A is certain $\Pi(A)=1$ and $N(A)=1$; if the occurrence of A is impossible: $\Pi(A)=0$ and $N(A)=0$. If there is no knowledge available: $\Pi(A)=1$ and $N(A)=0$ (the event is fully possible but not necessary at all).

3.2.2 Representation of imprecision

An imprecise information may be defined as $v \in A$ where A is a subset of S which contains more than one element. The imprecision may be expressed by a disjunction of values (Dubois and Prade, 2006) defined by a possibility distribution on S. $v \in A$ means that all values from v outside A are supposed to be impossible.

A possibility distribution π_v of v quantifies the plausibility of the information v. π_v is a function of S in L such as $\forall s \in S, \pi_v(s) \in L, \textit{and } \exists s, \pi_v(s) = 1$ with v denoting an ill-known value in S, and L the scale of plausibility ([0,1] for the theory of possibility).

3.2.3 Representation of uncertainty and imprecision

Information can be pervaded with both uncertainty and imprecision. Uncertainty *and* imprecision is denoted by $v \in AUB$ where A and B are two disjoints subset of S, one expressing the possibility that the event does not occur, the other defining the (possibly imprecise) content of the event if it occurs. The possibility distribution associated to the information $v \in AUB$ is noted $\pi(v)$ with:

$$\begin{aligned} \Pi(A) &= \max(\pi(v) | v \in A) \\ \Pi(B) &= \max(\pi(v) | v \in B) \end{aligned} \quad (3)$$

These two events being mutually exclusive, they are characterised by relation (4):

$$\begin{aligned} &\max(\Pi(A), \Pi(B)) = \max(\Pi(A), \Pi(\overline{A})) \\ &= \max(\Pi(A), 1 - N(A)) = 1 \end{aligned} \quad (4)$$

3.3 Basic Operations

In order to describe the events by possibility distributions, trapezoidal distributions (cf. Figure 1), denoted (*a, b, c, d, h*), can be used without an important loss of generality. Indeed, these sets intend to model an expertise which suggests a global shape rather than a precise function.

For performing a Material Requirement Planning, we need first to define an operator of addition, which is made according to (Zadeh, 1978) and (Dubois and Prade, 1988).

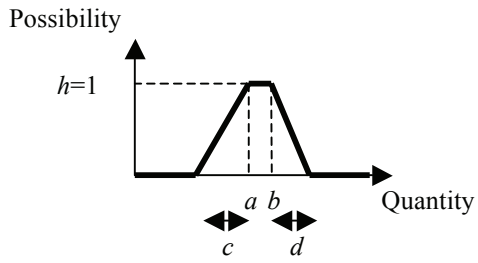


Figure 1. Trapezoidal distribution of possibility

A possibility distribution π_x is defined as the membership function μ_A of A, A being defined as a fuzzy set, i.e. a set of elements x which belong to A with a degree $\mu_A(x)$.

The sum of two trapezoidal distributions A_i and A_j defined by the quintuplets A_i (a_i, b_i, c_i, d_i, h_i) and A_j (a_j, b_j, c_j, d_j, h_j) is defined by (Dubois and Prade, 1988):

$$A_i \oplus A_j = (a, b, c, d, h) \quad (5)$$

with:

$$h = \min(h_i, h_j)$$
$$c = h \cdot \left(\frac{c_i}{h_i} + \frac{c_j}{h_j} \right)$$
$$d = h \cdot \left(\frac{d_i}{h_i} + \frac{d_j}{h_j} \right)$$
$$a = a_i + a_j - c_i - c_j + c$$
$$b = b_i + b_j + d_i + d_j - d$$

If $A = A_1 \cup A_2 \ldots \cup A_n$ and $B = B_1 \cup B_2 \ldots \cup B_p$ with A_i and B_j defined by the quintuplets (a_i, b_i, c_i, d_i, h_i) and (a_j, b_j, c_j, d_j, h_j)

$$A \oplus B = \bigcup_{i=1..n, j=1..p} A_i \oplus B_j \quad (6)$$

Calculating gross requirements consists in allocating quantities of components to periods. If the quantity is imprecise or uncertain, we need to compute the possibility and the necessity that a set belongs to an interval.

The possibility of the event "$x \in F$", denoted $\Pi(x \in F)$, is the degree of intersection between A and F, defined by equation (7). The dual measure of necessity of "$x \in F$", denoted $N(x \in F)$, evaluates to what extent "$x \in F$" is certainly true. It is defined by equation (8).

$$\Pi(x \in F) = \sup_u (\min(\mu_A(u), \mu_F(u))) \quad (7)$$

$$N(x \in F) = \inf_u (\max(1 - \mu_A(u), \mu_F(u)) = 1 - \Pi(x \in F^c)) \quad (8)$$

where F^c denotes the fuzzy complement of F ($\mu_F = 1 - \mu_{F^C}$)

If x is a real variable and A a fuzzy interval, *tmin* and *tmax* represent respectively the minimum and the maximum limit of the interval (see Figure 2). The possibility that x is between the limits of the interval is given by equation (9):

$$\Pi(t\min \le x \le t\max) = \Pi(x \in t) = \sup_{t\min \le u \le t\max} (\mu_A(u)) \qquad (9)$$

On the opposite, the possibility that x does not belong to the interval is given by equation (10):

$$\Pi(x \notin t) = \max(\Pi(x \in (-\infty, t\min]), \Pi(x \in [t\max, +\infty)) = \max(\sup_{u \le t\min} (\mu_A(u)), \sup_{t\max \le u} (\mu(A))) \qquad (10)$$

The necessity that x belongs to the interval is given by (11):

$$\mathrm{N}(t\min \le x \le t\max) = \mathrm{N}(x \in t) = 1 - \Pi(x \notin t) \quad (11)$$

The results of equations (9) to (11) are illustrated in Figure 2.

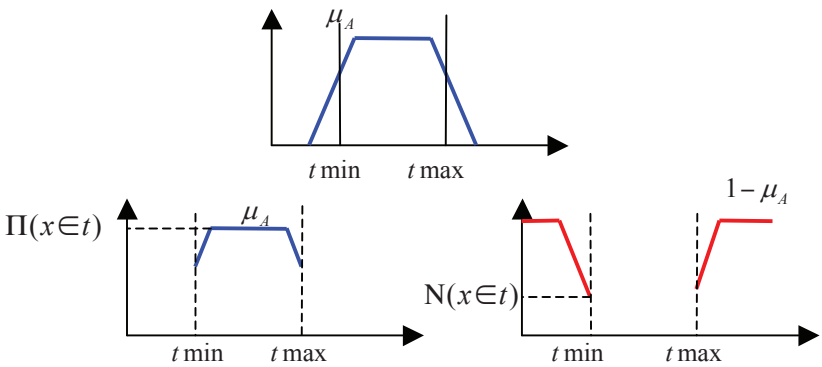


Figure 2. Computation of possibility and necessity levels that an element belongs to an interval

It will be shown in next section how these basic operations allow to build a possible gross requirement plan.

4 CALCULATION OF THE GROSS REQUIREMENTS

Figure 3 illustrates the process allowing to compute the gross requirements. Ill-known requirements (coming from forecasts or orders) are firstly considered as input of the process ((1) in Figure 3). These requirements are modelled through possibility distributions (step (2)). The result of this step is that each requirement is now modelled by a fuzzy quantity on a fuzzy date (3). The possibility and necessity that each quantity belongs to a period of the plan is then computed (4). Since a quantity may have non null possibilities to belong to several periods, it is then possible to define various scenarios for each requirement (depending on the period on which its occurrence is considered) (5). The decision maker will then suggest criteria for extracting one or several scenarios (most possible, optimistic, pessimistic, etc.) (6). These criteria are used to filter the possible scenarios (7). The decision maker can then make a decision on the base of the provided information.

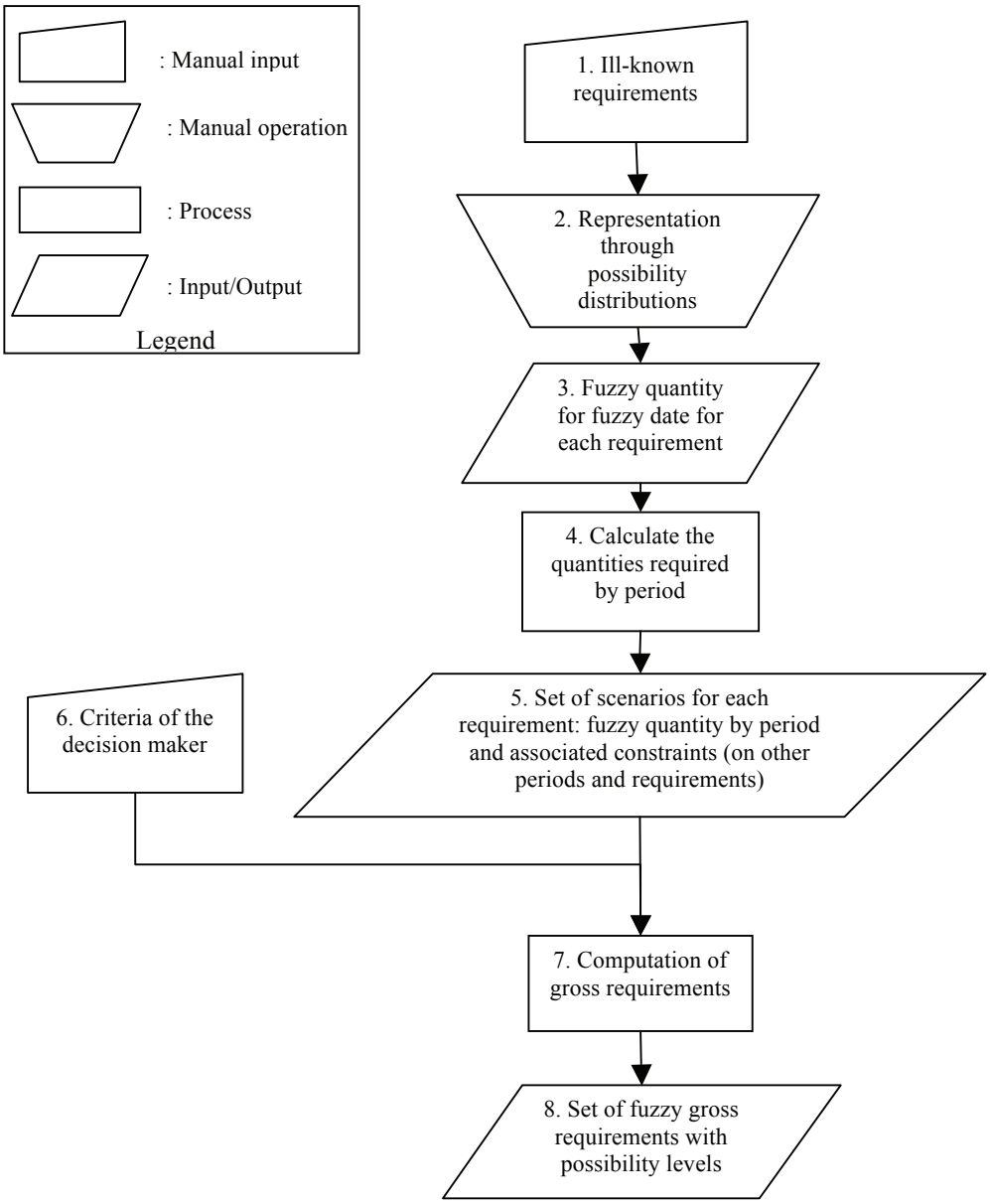


Figure 3. Flowchart of the computation of gross requirements

These steps will now be explained with more details.

4.1 Representation of ill-known requirements

A requirement o is characterised by a quantity Q_o and a date (due date) τ_o.

4.1.1 *Types of imperfections*

The imperfection on the requirements may be on the requirement itself, i.e. the requirement can be uncertain. For example, if the requirement comes from forecasts, it can be confirmed or cancelled through time. The imperfection may also affect the attributes of the requirement: the quantity or the

date. For each attribute, the two different types of imperfections defined above (imprecision and uncertainty) can be considered.

4.1.2 *General model of imprecision on the requirements*

It is possible to define the imprecision of a quantity by two intervals: one defining the most possible values, and the second all the possible values (Grabot *et al.,* 2005) or to infer a possibility distribution from very few measurements (Mauris, 2008), then to compute the trapezoidal fuzzy number which approximates this possibility distribution (Grzegorzewski, 2008). In order to be consistent with the expert knowledge available in the companies, we suggest to represent the imperfection (uncertainty and imprecision) on the requirements using two fuzzy sets:

- the first fuzzy set represents the imperfection (uncertainty and imprecision) on the required quantity, and the uncertainty of the occurrence of the requirement itself (possibility that the required quantity is 0),
- the second fuzzy set represents the imperfection on the due date.

The considered fuzzy sets are so defined as unions of trapezoidal fuzzy sets, which preserves the generality of the representation. In the following figures are introduced the various types of elementary sets used to represent the requirement pervaded by different types of imperfection.

Figure 4 represents a precise value (a), of possibility *h*. Figure 5 shows an imprecise quantity with a maximum possibility equal to *h*. Figure 6 represents the uncertainty on the occurrence of a requirement; the value zero (which means that the requirement is cancelled) has a possibility level equal to *h'*.

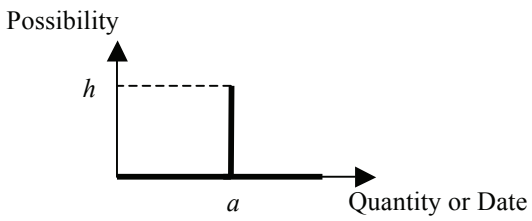


Figure 4. Representation of (*a; a; 0; 0; h*)

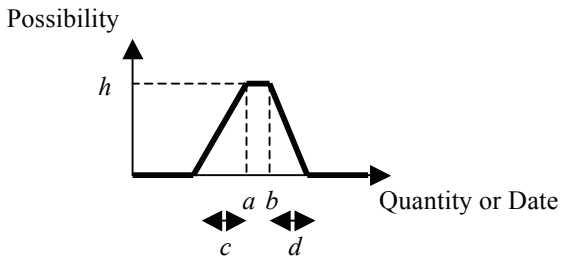


Figure 5. Representation of (*a; b; c; d; h*)

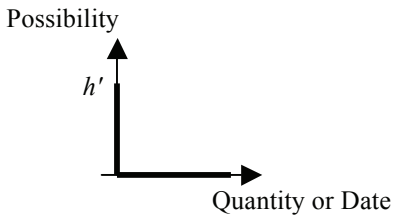


Figure 6. Representation of (*0; 0; 0; 0; h'*)

These figures do not yet define consistent requirements, since the definition of a possibility distribution sets the constraint that max(π(*A*))=1 (at least one event has to be completely possible), which is not the case for these sets if *h* and *h'* ≠1. In order to define an imprecise requirement which cancellation is more plausible than its confirmation, the union of the fuzzy sets of Figure 5 and 6 has to be defined, with (for instance) *h*=0.8 and *h'*=1.

Table 2. Representation a of quantity

	Certain quantity		Uncertain Quantity	
	Precise quantity	Imprecise quantity	Precise quantity	Imprecise quantity
Certain requirement	*(qa; qa; 0; 0; 1)*	*(qa; qb; qc; qd; 1)*	$\bigcup_{i=1}^{n}(qa_i; qa_i; 0; 0; qh_i)$ $\max_{j=1}^{n}(h_j)=1$	$\bigcup_{i=1}^{n}(qa_i; qb_i; qc_i; qd_i; qh_i)$ $\max_{j=1}^{n}(h_j)=1$
Uncertain requirement	*(qa; qa; 0; 0; qh)U (0; 0; 0; 0; h') max(h',qh)=1*	*(qa; qb; qc; qd; qh) U (0; 0; 0; 0; h') max(h',qh)=1*	$\bigcup_{i=1}^{n}(qa_i; qa_i; 0; 0; qh_i)$ $\cup(0; 0; 0; 0; h')$ $\max_{j=1}^{n}(h_j, h')=1$	$\bigcup_{i=1}^{n}(qa_i; qb_i; qc_i; qd_i; qh_i)$ $\cup(0;0;0;0;h')$ $\max_{j=1}^{n}(h_j, h')=1$

Table 2 summarises the way the different models can be combined in order to define the imperfections on the quantities, *qa*, *qb*... being the values allowing to define the fuzzy number which represents the quantities.

Table 3 summarises the combinations of models which may represent the various imperfections on the dates. The values allowing to represent the fuzzy numbers modelling a date are denoted *τa, τb*...

Table 3. Representation of the requirement date

	Certain requirement date		Uncertain requirement date	
	Precise date	Imprecise date	Precise date	Imprecise date
Certain requirement	*(τa; τa; 0; 0; 1)*	*(τa; τb; τc; τd; 1)*	$\bigcup_{i=1}^{n}(\tau a_i;\tau a_i;0;0;\tau h_i)$ $\max_{j=1}^{n}(\tau h_j)=1$	$\bigcup_{i=1}^{n}(\tau a_i;\tau b_i;\tau c_i;\tau d_i;\tau h_i)$ $\max_{j=1}^{n}(\tau h_j)=1$
Uncertain requirement	*(τa; τa; 0; 0; τh)* *max(h';τh)=1*	*(τa; τb; τc; τd; τh)* *max(h'; τh)=1*	$\bigcup_{i=1}^{n}(\tau a_i;\tau a_i;0;0;\tau h_i)$ $\max_{j=1}^{n}(\tau h_j)=1$	$\bigcup_{i=1}^{n}(\tau a_i;\tau b_i;\tau c_i;\tau d_i;\tau h_i)$ $\max_{j=1}^{n}(\tau h_j)=1$

4.2 Material requirement planning

Models of requirements based on unions of two sets have been introduced. One of the sets represents the imperfection on the quantity and the uncertainty on the requirement, whereas the other represents the imperfection on the date. The problem is now to compute a gross requirement, which is a set of quantities expressed for each period of the horizon.

Different scenarios can be deduced from the elementary requirements expressed as in Tables 2 and 3, since a requirement may belong to different periods (according to the imprecision on its date) and may be composed of different quantities (according to its uncertainty and imprecision).

To compute the gross requirements, let us define the following notations:

Index

o: requirement with $o \in [1,O]$

t: period with $t \in [1,T]$

j_o: index of fuzzy set of requirement o with $j_o \in [1,J_o]$

u_o: period in which the requirement o could exist with $u_o \in [U_{ao}, U_{zo}]$

c: number of combination with $c \in [1,C]$

Variables

Q_o: possible required quantity of requirement o,

τ_o: possible date of requirement o,

$E_{o,j_o,u_o,t}$: quantity for period t of the fuzzy set j_o of requirement o which belongs to period u_o

S_{o,j_o,u_o} : scenario in which the fuzzy set indexed by j_o of the requirement o belongs to period u_o

$D_{c,t}$: gross requirement quantity for combination c for period t.

4.2.1 *Design of requirement scenarios*

For each requirement, the number of elementary scenarios is equal to the number of periods to which the requirement may belong. As a first step, the possibility for each requirement *o* to belong to each period *t* is calculated (equation 12). This means that the requirement exists in a given period *t* with a given level of possibility:

$$\Pi\ (\tau_o \in t) = \max(\pi(\tau_o) | \tau_o \in t) \qquad (12)$$

As a second step, for each requirement *o*, the set $\mathcal{T}_o$ of the periods on which the requirement may occur is selected (equation 13):

$$\mathcal{T}_o / u_o \in \mathcal{T}_o \ if\ \Pi(\tau_o \in u_o) \neq 0 \qquad \text{with } u_o \in [\,U_{ao}\,, U_{zo}] \ (13)$$

Example: let us consider a requirement *A* on $\tau_A = (6;\ 6;1;2;1)$ and period 1= [0;7[, period 2= [7;14[and period 3= [14;21[.

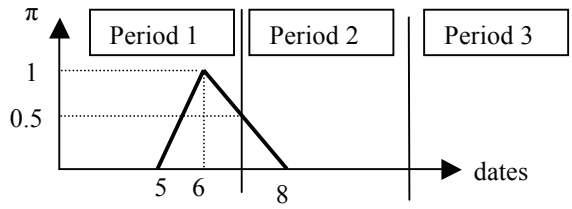


Figure 7. Representation of τ_A

$$\Pi(\tau_A \in 1) = \max\left(\pi(\tau_A) | \tau_A \in [0;7[\right) = 1$$

$$\Pi(\tau_A \in 2) = \max\left(\pi(\tau_A) | \tau_A \in [7;14[\right) = 0.5$$

$$\Pi(\tau_A \in 3) = \max\left(\pi(\tau_A) | \tau_A \in [14;21[\right) = 0$$

So $\mathcal{T}_A = \{1;2\}$

The requirement belongs to period 1 with a degree of possibility 1, and to period 2 with a degree of possibility 0.5 (see Figure 7).

In the third step, the required quantity of the different scenarios is calculated. It can be noticed that the possibility degree that a requirement *o* belongs to period *t* ($\Pi(\tau_o \in u_o)$) expresses than the requirement can exist in this period, but also that it cannot exist in other periods of $\mathcal{T}_o$.

Let us denote $Q_o = \bigcup_{j_o=1}^{J_o} A_{o,j_0}$ the quantity of the requirement, with j_o the index of the fuzzy set (imprecise value); $j_o \in [1,J_o]$. The different scenarios w_o of requirement o can be defined by $w_o = (j_o; u_o)$.

Table 4 represents the values of the required quantities for each period, by taking into account that the requirement can exist in one and only one period of the horizon.

U_o being the number of periods of $\mathcal{T}_o$, we have $S_o = U_o \times J_o$ scenarios for each requirements o.

Let us denote $S_{o,j_o,u_o} = \bigcup_{t \in T_o} E_{o,j_0,u_o,t}$

Table 4. Representation of the possible quantities

Quantity of requirement in other periods than u_o		Quantity of requirement in period u_o	
U_{ao}	$o \subseteq U_{ao}$ with $\Pi(\tau_o \in U_{ao})$ $\Downarrow$ $\overline{B}_{o,v,u_{ao}} = (0;0;0;0;\Pi(\tau_o \in U_{ao}))$ $\forall v \in \mathcal{T}_o - \{U_{ao}\}$	$j_o = 1$	$B_{o,1,U_{ao}} = (a_{A_{o,1}}; b_{A_{o,1}}; c_{A_{o,1}}; d_{A_{o,1}}; h_{A_{o,1}} \times \Pi(\tau_o \in u_{ao}))$
		....	
		$j_o = J_o$	$B_{o,J_o,U_{ao}} = (a_{A_{o,J_o}}; b_{A_{o,J_o}}; c_{A_{o,J_o}}; d_{A_{o,J_o}}; h_{A_{o,J_o}} \times \Pi(\tau_o \in u_{ao}))$
	...		
U_{zo}	$o \subseteq U_{zo}$ with $\Pi(\tau_o \in U_{zo})$ $\Downarrow$ $\overline{B}_{o,v,U_{zo}} = ..$	$j_o = 1$	$B_{o,1,U_{ao}} = (a_{A_{o,1}}; b_{A_{o,1}}; c_{A_{o,1}}; d_{A_{o,1}}; h_{A_{o,1}} \times \Pi(\tau_o \in u_{ao}))$
		....	
		$j_o = J_o$	$B_{o,J_o,U_{ao}} = (a_{A_{o,J_o}}; b_{A_{o,J_o}}; c_{A_{o,J_o}}; d_{A_{o,J_o}}; h_{A_{o,J_o}} \times \Pi(\tau_o \in u_{ao}))$

Example: let us consider the same requirement A

$Q_A = (10;12;2;2;1) \cup (20;24;4;4;0.8)$ We have:

$A_{A,1} = (10;12;2;2;1)$ and $A_{A,2} = (20;24;4;4;0.8)$ $\mathcal{T}_A = \{1;2\}$

The set of scenarios is $\{w_A\} = \{(1;1);(1;2);(2;1);(2;2)\}$

We compute for each scenario w_A the quantity $B_{A,j,u}$ which belongs to the period u and the quantity $\overline{B}_{A,v,u}$ which belongs to period v knowing that A belongs to u. The resulting scenarios are summarized in Table 5.

Table 5. Results for the requirement A

$r_A \subseteq 1$ $Q_A \in \mathcal{T}_A \neq 2$ $\overline{B}_{A,2,1} = (0;0;0;0;1)$	$B_{A,1,1} = (10;12;2;2;1)$ $B_{A,2,1} = (20;24;4;4;0.8)$
$r_A \subseteq 2$ $Q_A \in \mathcal{T}_A \neq 1$ $\overline{B}_{A,1,2} = (0;0;0;0;0.5)$	$B_{A,1,2} = (10;12;2;2;0.5)$ $B_{A,2,2} = (20;24;4;4;0.4)$

4.2.2 *Computation of the gross requirements*

From the various scenarios, a set of gross requirements, which is a set of required quantities for each period on a planning horizon, can be computed.

From Table 4, we generate all the combinations of the requirement scenarios. We have $C = \prod_{o=1}^{O} S_{ox}$ combinations. Let $\mathcal{C}$ be the set of these C combinations.

For a combination c of the requirement scenarios, the total required quantity $D_{c,t}$ for a period t is the sum of the required quantities $E_{o,j_o,u_o,t}$ (equation 14) concerning all the scenarios of requirements w_o of this combination belonging to this period t (equation 16).

Thus, for each scenario w_o and each period t in $\mathcal{T}_o$ we first compute $E_{o,j_o,u_o,t}$. If the requirement o belongs to the considered period t, the quantity is equal to B_{o,j_o,u_o}. Else, the quantity is equal to $\overline{B}_{o,t,u_o}$. B_{o,j_o,u_o} is the quantity indexed by j_o of requirement o weighted by the possibility to exist in the period u_o, and $\overline{B}_{o,t,u_o}$ denotes that requirement o belongs to another period than t (quantity=0, possibility denoting that the requirement belong to another period u_o).

$$
\begin{aligned}
&\forall w_o, \forall t \in \mathcal{T}_o \\
&E_{o,j_o,u_o,t} = B_{o,j_o,u_o} \forall u_o = t \\
&E_{o,j_o,u_o,t} = \overline{B}_{o,t,u_o} \forall\, u_o \neq t \\
&\textit{with} \\
&\quad \overline{B}_{o,v,u_o} = (0;0;0;0;\Pi(\tau_o \in u_o)) \;\; \forall v \in \mathcal{T}_o - \{u_o\} \\
&\quad \forall j_o \in [1, J_o], \quad B_{o,j_o,u_o} = (a_{A_{o,j_o}}; b_{A_{o,j_o}}; c_{A_{o,j_o}}; d_{A_{o,j_o}}; h_{A_{o,j_o}} \times \Pi(\tau_o \in u_o))
\end{aligned}
\tag{14}
$$

In order to manage the combinatory explosion of elementary scenarios, the decision maker may select a plausible set of combinations and thus may only consider the combinations for which the possibility is higher than a given value *p*.

The set of combination is then reduced to $\mathcal{C}_p$ (see equation (15)).

$$\begin{aligned} &C_p = \{A_c\} = \prod_{o=1}^{O} \{w_o\} \forall j_o \in J_o \, and \, \forall u_o \in \mathcal{U}_o \\ &w_{o,c} = (j_{o,c}; u_{o,c}) \\ &A_c = \{w_{o,c}\} \; \forall o \in [1, O] \end{aligned} \tag{15}$$

Each combination is characterised by a level of possibility (equation 17). The combinations computed are thus classified according to their degree of possibility Pos_comb_c.

We compute the gross requirement of the combination *c* and the total required quantities for the period t of this combination:

$$D_{c,t} = \bigoplus_{o / w_{o,c} = (j_{o,c}; u_{o,c}) \in A_c}^{O} (E_{o, j_{o,c}, u_{o,c}, t}) \; \forall t, \forall c \tag{16}$$

$$Pos\_comb_c = \min_t (\Pi(D_{c,t})) \tag{17}$$

The combinatory is directly linked to the level of imperfection of the requirements, and to the threshold introduced by the decision maker.

Example: let us consider the same requirement *A*

The set of scenarios is $\{w_A\} = \{(1;1);(1;2);(2;1);(2;2)\}$

We compute the quantity for period {1; 2} of the scenario quantity w_A (in this case $E_{A,j,u,1}$ and $E_{A,j,u,2}$)

$$E_{A,1,1,1} = B_{A,1,1}; \; E_{A,2,1,1} = B_{A,2,1}; \; E_{A,1,2,2} = B_{A,1,2;} \; E_{A,2,2,2} = B_{A,2,2}$$

$$E_{A,1,1,2} = \overline{B}_{A,2,1}; \; E_{A,2,1,2} = \overline{B}_{A,2,1}; \; E_{A,1,2,1} = \overline{B}_{A,1,2;} \; E_{A,2,2,1} = \overline{B}_{A,1,2}$$

Then, we compute the gross requirements:

$$D_{1,1} = E_{A,1,1,1} = (10;12;2;2;1); \quad D_{1,2} = E_{A,1,1,2} = (0;0;0;0;1)$$

$$D_{2,1} = E_{A,1,2,1} = (0;0;0;0;0.5); \quad D_{2,2} = E_{A,1,2,2} = (10;12;2;2;0.5)$$

$$D_{3,1} = E_{A,2,1,1} = (20;24;4;4;0.8); \quad D_{3,2} = E_{A,2,1,2} = (0;0;0;0;0.8)$$

$$D_{4,1} = E_{A,2,2,1} = (0;0;0;0;0.4); \quad D_{4,2} = E_{A,2,2,2} = (20;24;4;4;0.4)$$

---

5 ILLUSTRATIVE EXAMPLE

In this example, we consider four requirements concerning material C. Each requirement has its origin in four different products P1, P2, P3 and P4, C being one of their components. These requirements are imperfectly known. The time horizon is 28 days and starts at day 0. Each period is composed of 7 days. The gross requirements of material C has to be computed.

The following qualitative information on each requirement is available (depending for instance on the schedule of assembly):

Requirement 1:

- Two quantities are possible 300 (very possible) and 600 (could be possible). These quantities are given at ± 20 units
- The requirement date is 20. This date is given at ± 2 days

Requirement 2:

- The most possible quantity is between 400 and 450, but the requirement is not totally sure. This quantity is given at ± 10 units
- The requirement date is between 14 and 16 and this date is given at ±2 days

Requirement 3:

- The most possible quantity required is between 350 and 360. This quantity is given at ± 10 units
- Two dates are possible:
    - The most possible: around 5 (given at ±1 days)
    - The less possible: around 15 (given at ±3 days)

Requirement 4:

- The most possible requirement quantity is between 600 and 620. This quantity is given at ± 20 units
- The most possible date is between 25 and 26; it is possible to need this quantity up to two days before, but not later than the most possible date.

These pieces of information are represented in Figures 8 and 9.

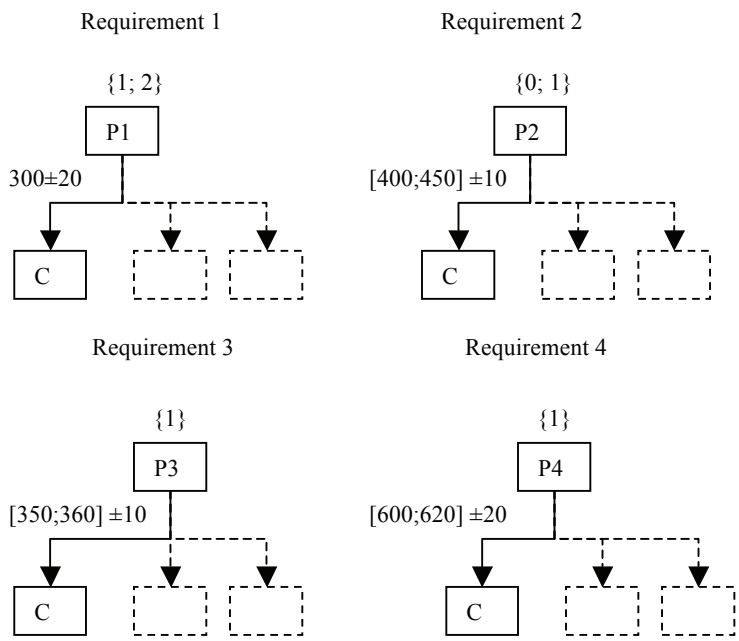


Figure 8. Information on quantities from the expert

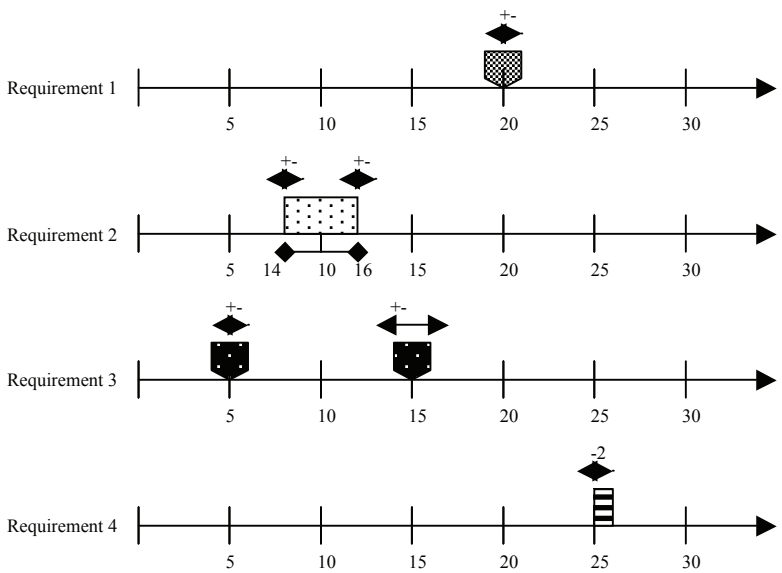


Figure 9. Information on dates from the expert

5.1 Possibilistic model of the ill-known requirements

The first step of the method is to build the possibilistic models describing each requirement, on the base of their "expert" description. The result, obtained according to the principles described in section 4, are given for Requirements 1 and 2 in Table 6 and 7 (requirements 3 and 4 are similar).

Table 6. Model for Requirement 1

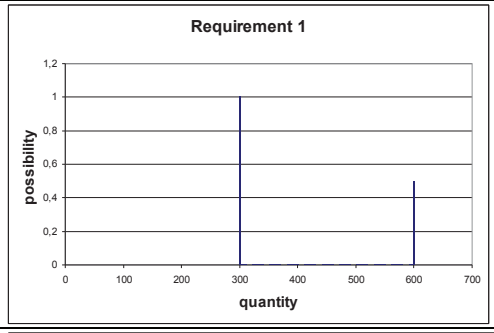
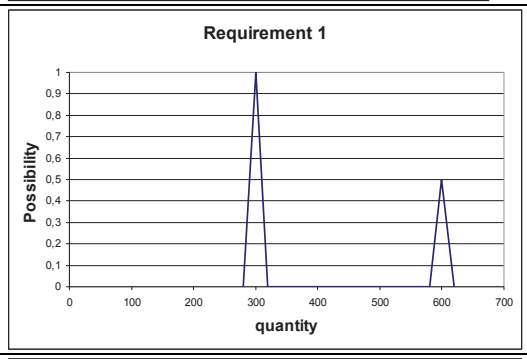
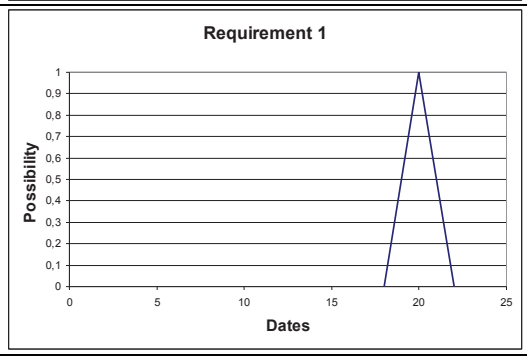
Expression	Model	Representation
Two values are possible for the required quantities.	Possibility to need 300 = 1 Possibility to need 600 = 0.5.	
The required quantities are given with a tolerance	Triangular fuzzy numbers are used.	
The date is a crisp value given with a tolerance	Triangular fuzzy number is used.	

Table 7. Model for Requirement 2

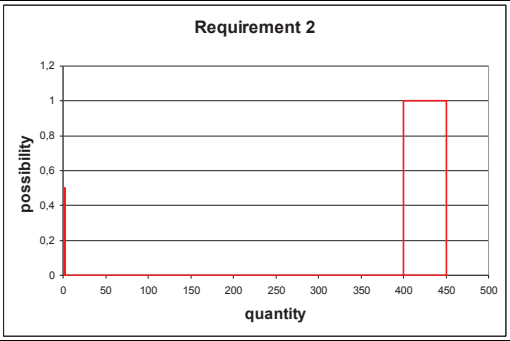
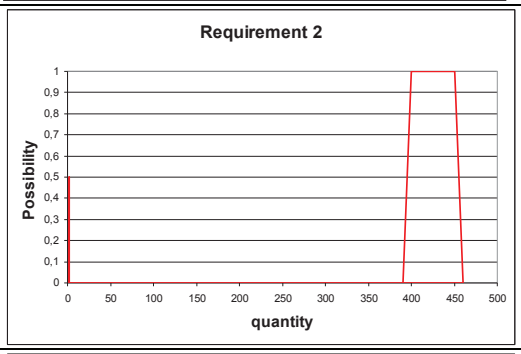
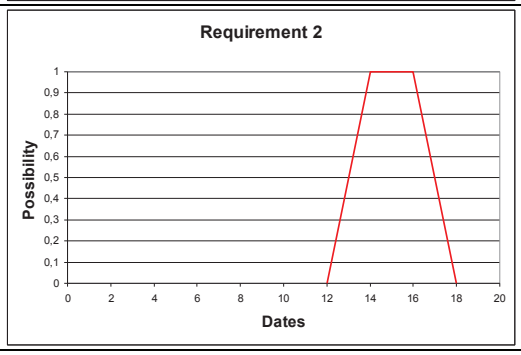
Expression	Model	Representation
Requirement can be cancelled. The quantity belongs to an interval	Possibility to be cancelled = 0.5 Possibility to need 400=1; Possibility to need 450=1	
The quantities are given with a tolerance	Trapezoidal fuzzy number is used.	
The date is belong a interval and is given with a tolerance	Trapezoidal fuzzy number is used.	

Table 8 summarizes the model whereas Figures 10 and 11 represent graphically the models of the required quantities. Figure 12 summarizes the models of the requirements dates (black lines represent the periods).

Table 8. Requirements

	Quantity	Requirement date
r_1	(300; 300; 20; 20; 1)U (600; 600; 20; 20; 0.5)	(20; 20; 2; 2; 1)
r_2	(400; 450; 10; 10; 1)U (0; 0; 0; 0; 0.5)	(14; 16; 2; 2; 1)
r_3	(350; 360; 10; 10;1)	(5; 5; 1; 1; 1)U (15; 15; 3; 3; 0.5)
r_4	(600; 620; 20; 20; 1)	(25; 26; 2; 0; 1)

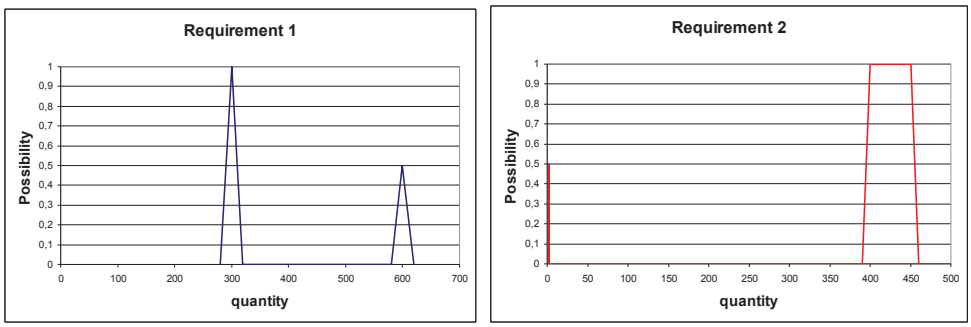


Figure 10. Representation of the quantities for requirements 1 and 2

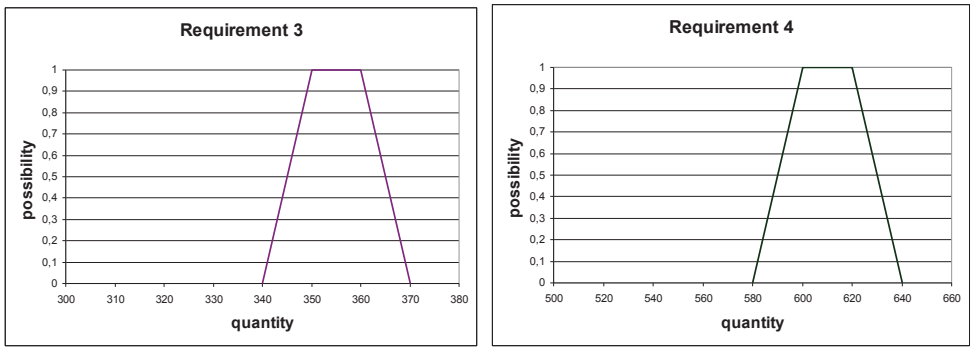


Figure 11. Representation of the quantities for requirements 3 and 4

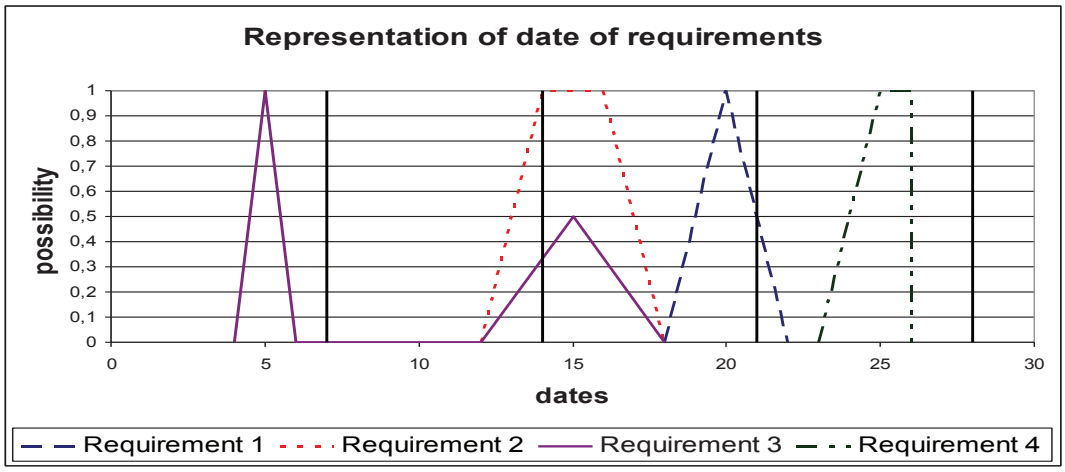


Figure 12. Representation of the dates of requirement

5.2 Material Requirement Planning

5.2.1 *Design of requirement scenarios*

The next step consists in building the scenarios according to the requirements belonging to the different time periods of the gross requirement horizon. $\mathcal{T}_o$ being the set of the possible periods for requirement *o*, the level of possibility that each requirement belongs to the different periods has to be computed (Table 9).

Table 9. Possibility than the requirements belong to the different periods

	Requirement 1	Requirement 2	Requirement 3	Requirement 4
Period 1	$\Pi(\tau_1 \in 1) = 0$	$\Pi(\tau_2 \in 1) = 0$	1	0
Period 2	$\Pi(\tau_1 \in 2) = 0$	1	0.33	0
Period 3	1	1	0.5	0
Period 4	0.5	0	0	1

From Table 9, we determine the set of possible periods for each requirement $\mathcal{T}_1 = \{3;4\}$, $\mathcal{T}_2 = \{2;3\}$, $\mathcal{T}_3 = \{1;2;3\}$ and $\mathcal{T}_4 = \{4\}$.

Then, we can build the different scenarios for each requirement (Tables 10 to 13):

Table 10. Results for the requirement 1

$r_1 \subseteq 3$ $Q_1 \in \mathcal{T}_1 \neq 3$ $\overline{B}_{1,3,4} = (0;0;0;0;1)$	$B_{1,1,3} = (300;300;20;20;1)$ $B_{1,2,3} = (600;600;20;20;0.5)$
$r_1 \subseteq 4$ $Q_1 \in \mathcal{T}_1 \neq 4$ $\overline{B}_{1,4,3} = (0;0;0;0;0.5)$	$B_{1,1,4} = (300;300;20;20;0.5)$ $B_{1,2,4} = (400;400;20;20;0.25)$

Table 11. Results for the requirement 2

$r_2 \subseteq 2$ $Q_2 \in \mathcal{T}_2 \neq 2$ $\overline{B}_{2,2,3} = (0;0;0;0;1)$	$B_{2,1,2} = (400;450;10;10;1)$ $B_{2,2,2} = (0;0;0;0;0.5)$
$r_2 \subseteq 3$ $Q_2 \in \mathcal{T}_2 \neq 3$ $\overline{B}_{2,3,2} = (0;0;0;0;1)$	$B_{2,1,3} = (400;450;10;10;1)$ $B_{2,2,3} = (0;0;0;0;0.5)$

Table 12. Results for the requirement 3

$r_3 \subseteq 1$ $Q_3 \in \mathcal{T}_3 \neq 1$ $\overline{B}_{3,1,2} = \overline{B}_{3,1,3} = (0;0;0;0;1)$	$B_{3,1,1} = (350;360;10;10;1)$
$r_3 \subseteq 2$ $Q_3 \in \mathcal{T}_3 \neq 2$ $\overline{B}_{3,2,1} = \overline{B}_{3,2,3} = (0;0;0;0;0.33)$	$B_{3,1,2} = (350;360;10;10;0.33)$
$r_3 \subseteq 3$ $Q_3 \in \mathcal{T}_3 \neq 3$ $\overline{B}_{3,3,1} = \overline{B}_{3,3,2} = (0;0;0;0;0.5)$	$B_{3,1,2} = (350;360;10;10;0.5)$

Table 13. Results for the requirement 4

$r_4 \subseteq 4$ $Q_4 \in \mathcal{T}_4 \neq 4$	$B_{4,1,4} = (600;620;20;20;1)$

5.2.2 *Computation of the gross requirements*

From Tables 10 to 13, the different gross requirements are obtained for some combinations of the requirement scenarios (Figure 13). There are 48 possible gross requirements, but only the gross requirements with level of possibility 1 are computed, so that the less possible gross requirement which could happen in the worst case: possibility = 0.25 (Table 14 and Table 15).

Table 14. Combinations

	Requirement 1	Requirement 2	Requirement 3	Requirement 4
Gross requirements combination 1	$r_1 \subseteq 3 \& A_1$	$r_2 \subseteq 2 \& A_1$	$r_3 \subseteq 1 \& A_1$	$r_4 \subseteq 4 \& A_1$
	(300;300;20;20;1)	(400;450;10;10;1)	(350;360;10;10;1)	(600;620;20;20;1)
Gross requirements combination 2	$r_1 \subseteq 3 \& A_1$	$r_2 \subseteq 3 \& A_1$	$r_3 \subseteq 1 \& A_1$	$r_4 \subseteq 4 \& A_1$
	(300;300;20;20;1)	(400;450;10;10;1)	(350;360;10;10;1)	(600;620;20;20;1)
Gross requirements combination 3	$r_1 \subseteq 4 \& A_2$	$r_2 \subseteq 2 \& A_2$	$r_3 \subseteq 2 \& A_1$	$r_4 \subseteq 4 \& A_1$
	(400;400;20;20;0.25)	(0;0;0;0;0.5)	(350;360;10;10;0.33)	(600;620;20;20;1)

Table 15. Gross requirement for a combination

	Period 1	Period 2	Period 3	Period 4
$Pos\_comb_1 = 1$	(350;360;10;10;1)	(400;450;10;10;1)	(300;300;20;20;1)	(600;620;20;20;1)
$Pos\_comb_2 = 1$	(350;360;10;10;1)	(0;0;0;0;1)	(700;750;30;30;1)	(600;620;20;20;1)
$Pos\_comb_3 = 0.25$	(0;0;0;0;0.33)	(350;360;10;10;0.33)	(0;0;0;0;0.33)	(985;1035;25;25;0.25)

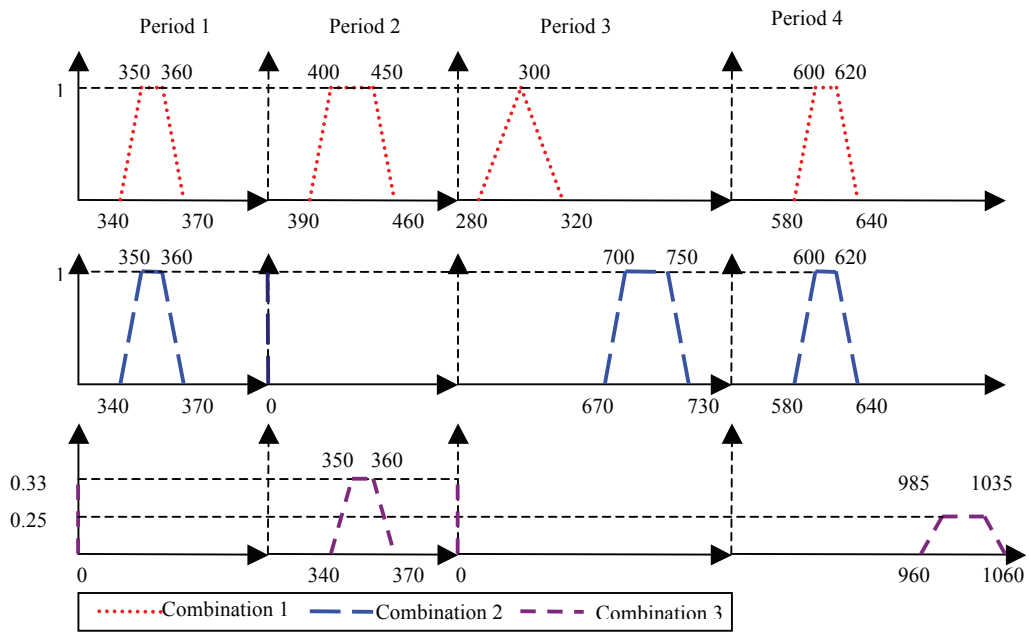


Figure 13. Fuzzy gross requirements

Now, let us consider that the four requirements have been considered as crisp for making possible a "classical" gross requirement calculation. In that case, the four requirements will be certainly considered as follows (see summary in Table 16):

Requirement 1: quantity 300, date 20; Requirement 2: quantity 425, date 15; Requirement 3: 355, date 5; Requirement 4: 610, date 25. Thus the crisp gross requirement is given in Table 16.

Table 16. "Crisp" gross requirement

period	1	2	3	4
quantity	355	0	725	610

It can be seen that the "crisp" scenario of Table 16, obtained by neglecting any knowledge on the imprecision of the input information, is close to combination 2. Therefore, a first result is that our method is consistent with a "good sense" defuzzification obtained by considering only the most plausible cases. What is more important is that another scenario, obtained with combination 1, has the same possibility (=1): as a matter of fact, there is a possibility equal to one that some requirements are also present in period 2, and as a consequence that the gross requirements are relatively balanced on the four considered periods. In addition, it can be seen that there are slight possibilities (0.33 and 0.25) that the gross requirements are completely unbalanced, with a quantity around 350 during period 2 and a huge quantity, around 1000, during period 4.

It is so clear that the method may allow to obtain a much more informative set of possible situations, together with an estimation of their possibility of occurrence. On that base, the decision maker is able to make a really "informed" decision, considering not only the most plausible case but also an estimation of the risk taken regarding to the possibility of occurrence of other situations.

6 CONCLUSION

Through a literature review, we have shown that the idea to model an imprecise demand on final products using possibility theory has sometimes been considered, but that the practical way for dealing with such a demand in the first steps of the MRP method (Material Requirement Planning) has not yet been fully explored. However, this step is the base for generating a supply plan for the upstream partners of the supply chain, which should take benefit from more precise information.

In this paper, uncertainty and imprecision on the requirements have been analysed and a model allowing to take them into account has been proposed. Based on this model, a method has been

suggested for modelling uncertain gross requirements (describing plausible quantities by period). The result stresses that the resulting uncertain quantity which is required is not a simple fuzzy set, but describes a set of situations, which is much more informative than the result obtained by considering only the most plausible requirements. Indeed, this model does not lose any information on the relation between period and quantity.

As a following step, two different options are possible: the decision maker may choose a supply plan after assessing its feasibility, and may compute the risk taken (shortage or excess of inventories) if other situations occur. In order to do this, he can defuzzify the gross requirement before sending it to his supplier. The second solution is to transmit the set of the most plausible supply plans to the supplier as an element for negotiation. Indeed, the described situations are a good starting point for assessing how the risk taken by preparing a situation more than another can be shared between the two partners.

In this article, the incompleteness of the requirements has not been considered. Considering this other type of imperfection is an interesting perspective for the study, since the knowledge on the requirements may be incomplete for far periods. In that case, any evaluation of the required quantity linked to forecasts could be false. An incompleteness factor could be added to the requirements already known through long term contracts in order to address this problem.

Finally, fuzzy demands can be considered in other cases, such as economic production quantity models (Lee and Yao*,* 1997), inventory control (Petrovic *et al.,* 2008) or optimal inventory (Chiang and Wen-kai*,* 2000), also requiring a demand defined by quantities by period. As a perspective, the integration of our model in those methods could be another interesting issue.

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