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# COUPLED HR NEURONS : ON THE INFLUENCE OF THE COUPLING STRENGTH ON THE NEURON DYNAMICS

Stefan Balev and Nathalie Corson <sup>\*†</sup>

**Keywords.** Hindmarsh-Rose Neurons, Bifurcation Diagram, Asymptotic Dynamics, Coupling Strength

## 1 Introduction

Studies concerning coupled neurons have already led to very interesting results on emergent global behaviour such as synchronization phenomena (see for example [6, 7]). While working on this kind of problems, it appears that if coupling neurons could make some collective behaviour appear, it could also drastically change each neuron dynamics. This paper deals with the influence of the coupling strength on one neurone behaviour through bifurcation diagrams and numerical analysis of the asymptotic dynamics of a neuron connected to another.

## 2 Coupling HR Neurons

The HR model (see [2, 3]) reads as follows,

$$(HR) \begin{cases} \dot{x} &= y + ax^2 - x^3 - z + I \\ \dot{y} &= 1 - dx^2 - y \\ \dot{z} &= \epsilon(b(x - c_x) - z) \end{cases} \quad (1)$$

Parameters  $a$ ,  $b$  and  $d$  are experimentally determined,  $c_x$  is the equilibrium  $x$ -coordinate of the two-dimensional system given by the first two equations of (1) when  $I = 0$  and  $z = 0$  and parameter  $I$  corresponds to the applied current. Finally, parameter  $\epsilon$  represents the ratio of time scales between fast and slow fluxes across the membrane of a neuron. This HR neuron model can exhibit most of biological neuron behaviour, such as *spiking* or *bursting*. With appropriate parameter settings, the HR model exhibits periodic behavior characterized by fast periods of spiking called bursts, followed by slow quiescent inter-burst periods, as shown in Fig. 1.

Hereafter, for all numerical experiments, we use HR system with the following coordinate changes, see [4],  $y =$

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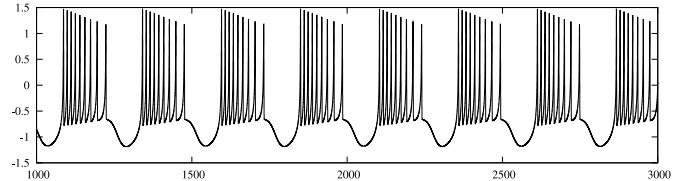


Figure 1: Time series  $(t, x)$  of (2) when parameters are fixed as in (5). For this set of parameters values, a HR system exhibit a periodic bursting behaviour.

$1 - y$ ,  $z = 1 + I + z$ ,  $d = a + \alpha$ ,  $c = -1 - I - bx_c$ . Applying this transformation, we obtain,

$$\begin{cases} \dot{x} &= ax^2 - x^3 - y - z \\ \dot{y} &= (a + \alpha)x^2 - y \\ \dot{z} &= \epsilon(bx + c - z) \end{cases} \quad (2)$$

Let us consider a network composed by  $n$  HR neurons. These neurons are coupled by their first variable  $x_i$ . A model of this network is given by

$$\begin{cases} \dot{x}_i &= ax_i^2 - x_i^3 + y_i - z_i - \sum_{j=1}^n c_{ij}h(x_i, x_j) \\ \dot{y}_i &= (a + \alpha)x_i^2 - y_i \\ \dot{z}_i &= \epsilon(bx_i + c - z_i) \end{cases} \quad (3)$$

for  $i = 1, \dots, n$ , where  $h$  is the coupling function and  $\{c_{ij}\}$  is the network adjacency matrix. When the neurons are chemically coupled, the coupling function  $h$  is given by [4] and reads as

$$h(x_i, x_j) = g_{\text{syn}} \frac{(x_i - V)}{1 + \exp(-\lambda(x_j - \Theta))} \quad (4)$$

where  $g_{\text{syn}}$  is the coupling strength,  $\Theta$  is the threshold reached by every action potential for a neuron. Parameter  $V$  is the reversal potential and must be larger than  $x_i(t)$  for all  $i$  and all  $t$  since synapses are supposed excitatory. The parameters are fixed as follows throughout this paper,

$$a = 2.8, \alpha = 1.6, c = 5, b = 9, \epsilon = 0.001 \quad (5)$$

$$V = 2, \lambda = 10, \Theta = -0.25 \quad (6)$$

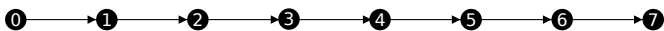


Figure 2: A chain of eight coupled HR neurons

It appears that when neurons are non linearly coupled, complete synchronization can be exhibited only if all the neurons of a network receive the same number of inputs. Moreover, if it is not the case, and under some conditions, a burst synchronization phenomenon can arise. In some previous papers (see [6, 7]), we presented a numerical method for the detection of this burst synchronization phenomenon and some results concerning the value of the coupling strength required to obtain burst synchronization.

### 3 Main Results

Consider a chain of  $n$  non-linearly coupled HR neurons as shown in Fig. 2. In the simplest case  $n = 2$ , we have two neurons, the first sending signal to the second. The signal received from the second neuron drastically changes its behavior. This change depends on the coupling strength  $g_{\text{syn}}$  as shows Fig. 3. Note that when the coupling strength is 0 (or equivalently, the neurons are not coupled) all of them have periodic behavior as shows the phase portrait on the left of Fig. 3. Therefore the first interesting result is that a chaotic behavior arises from the coupling for small values of the coupling strength. When we keep increasing this parameter, we observe periodic behavior interval around 1.5, followed by a new chaotic interval. Finally, for large parameter values, the periodic behavior is restored.

The bifurcation diagrams on the bottom of Fig. 3 show similar transition from chaotic to periodic behavior for neurons at higher level of the chain, except that there is no intermediate periodic interval.

Since in [7] we studied the coupling strength needed to make  $n$  neurons of a chain synchronize their bursts, we wanted to see if a correlation exists between periodic behavior and burst synchronization. Fig. 3 shows that these two phenomena seem not to be correlated. Surprisingly, in shorter chains neurons synchronize even before they become periodic, but this is not the case in longer chains.

In this extended abstract, we only present results for one kind of network: a chain of non-linearly coupled neurons. Among others, arise the questions of the influence of the network topology and the type of the coupling function on this kind of results.

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## Coupling strength as a bifurcation parameter

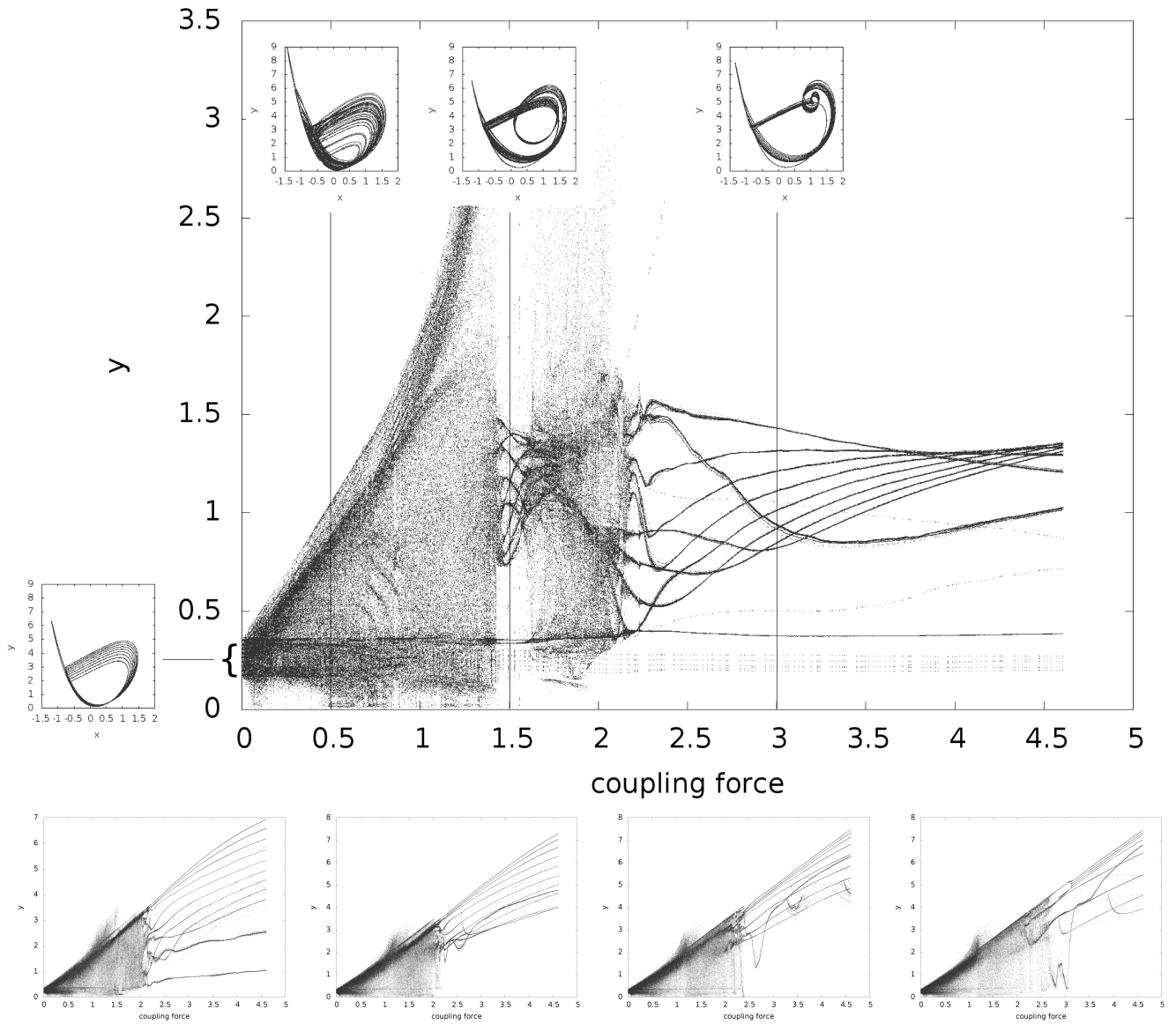


Figure 3: Top: Bifurcation diagram of the second neuron in a chain as a function of the coupling strength parameter. Phase portraits are given for selected coupling strength values: 0, 0.5, 1.5 and 3.0. Bottom: Bifurcation diagrams (from left to right) of the 4th, 8th, 15th and 32nd neuron in a chain.

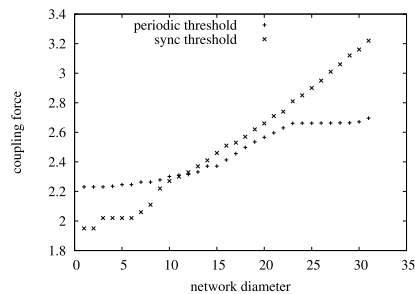


Figure 4: Behavior of a chain of  $n$  neurons. For each  $n$  we give the minimum coupling strength needed to pass from chaotic to periodic behavior (periodic threshold), as well as the minimum coupling strength needed to observe burst synchronization in the chain (sync threshold).