Delay-Adaptive Full-State Predictor Feedback for Systems With Unknown Long Actuator Delay

MIROSLAV KRSTIC AND DELPHINE BRESCH-PIETRI
Department of Mechanical and Aerospace Engineering,
University of California, San Diego, La Jolla, CA 92093-0411, USA

***Abstract*— Stabilization of unstable systems with actuator delay of substantial length and of completely unknown value is an important problem that has never been attempted. We present a Lyapunov-based adaptive control design that achieves global stability, without a requirement that the delay estimate be near the true delay value. We solve the problem by employing a framework where the actuator delay is represented as a transport PDE, by estimating the delay value as the reciprocal of the convection speed in the transport PDE, and by using full state predictor-based feedback.**

I. INTRODUCTION

Adaptive control in the presence of actuator delays is a hard problem. To our knowledge, the only existing results are the 1988 result by Ortega and Lozano [26] and the 2003 results by Niculescu and Annaswamy [24] and Evesque et al [5]. These results deal with the problem where the plant has unknown parameters but the delay value is known.

The remaining theoretical frontier, and a problem of great practical relevance, is the case where the actuator delay value is unknown and highly uncertain. This problem is open in general even in the case where no parametric uncertainty exists in the ODE plant. The importance of problems with unknown delays was highlighted in [4], where a simple scheme for delay estimation and controller gain adjustment to preserve closed-loop stability was also presented. An attempt at adaptive design for unknown delay was also made in [14] by applying the Pade approximation, however, while the design was (predictably) successful for the approximate problem, it was not successful for a model with an actual delay of significant length.

In this paper we present the first systematic adaptive control design for a system with unknown actuator delay by focusing on the case

$$\dot{X}(t) = AX(t) + BU(t-D), \tag{1}$$

where the full state—both the ODE plant state $X \in \mathbb{R}^n$ and the infinite-dimensional actuator state $U(\eta), \eta \in [t-D,t]$—are available for measurement, and where the ODE plant parameters are known, but where the delay length D is unknown (though constant) and can have an arbitrarily large value. This problem can be formulated around an actuator delay model given by a *transport equation* (convective/first-order hyperbolic PDE), namely,

$$\dot{X}(t) = AX(t) + Bu(0,t) \tag{2}$$

$$Du_t(x,t) = u_x(x,t) \tag{3}$$

$$u(1,t) = U(t), \tag{4}$$

where $u(x,t)$ is the state of the actuator, the domain length is known (unity) but the propagation speed $1/D$ is unknown. The actuator state is related to the input through the following equation

$$u(x,t) = U(t + D(x-1)), \tag{5}$$

which, in particular, gives $u(1,t) = U(t)$ and $u(0,t) = U(t-D)$. The control law around which we build a delay-adaptation mechanism is a predictor-based feedback law,

$$U(t) = K\left[\mathrm{e}^{AD}X(t) + D\int_0^1 \mathrm{e}^{AD(1-y)}Bu(y,t)dy\right], \tag{6}$$

which achieves exponential stability at $u \equiv 0, X = 0$ by performing perfect compensation of the actuator delay, and which has been employed in many control design and analysis studies for systems with actuator delays over the last three decades [1], [5], [6], [7], [8], [9], [10], [11], [12], [13], [17], [18], [19], [20], [21], [22], [23], [24], [25], [27], [30], [31], [32], [33], [34], [35], [36].

Within this framework we obtain a global adaptive stabilization result, for an arbitrarily large and unknown actuator delay value (Sections III and IV).

Without a question, an even more relevant and challenging problem is the one where the full state is not available for measurement, more specifically, when the state of the transport PDE $u(x,t)$, i.e., the actuator state, is not measured. A yet more challenging problem is when, in addition, only an output of the ODE system

$$Y(t) = CX(t) \tag{7}$$

is measured, rather than the full state $X(t)$, and, finally, the most challenging in this string of problems is when the ODE plant has parametric uncertainty, i.e., $A(\theta), B(\theta), C(\theta)$, where θ is unknown. (For an exhaustive categorization of adaptive control problems with actuator delay, please see Section II). However, as restrictive as the requirement for measurement of $u(x,t)$ may seem, we do not believe that any delay-adaptive problem without the measurement of $u(x,t)$ is solvable globally because it cannot be formulated as linearly parametrized in the unknown delay D. As a consequence, when the controller uses an estimate of $u(x,t)$, not only do the initial values of the ODE state and the actuator state have to be small, but the initial value of the delay estimation error also has to be small (the delay value is allowed to be large but the initial value of its estimate has to be close to the true value of the delay). This local result is actually proven in our companion paper [2].

In our global full-state feedback design we require only one bit of a priori knowledge about the length of the delay:

Assumption 1: An upper bound $\bar{D}$ on the unknown $D>0$ is known.

This upper bound is used in two ways. An adaptation algorithm employing projection keeps the delay estimate below the a priori bound. In addition, based on the upper bound for the delay length, the adaptation gain is selected to be sufficiently small, and a normalization parameter is selected to be sufficiently large, to ensure that adaptation is sufficiently slow to guarantee closed-loop stability. The approach for update law design (Section III) and for the corresponding stability analysis (Section IV) is based on the ideas that we introduced in [16] for Lyapunov-based adaptive control of parabolic PDEs. The adaptation and normalization gain choices are conservative. The relevant part of the design is the structure of the adaptation law, not the exact gain values employed in the analysis.

In this paper the only parametric uncertainty considered is the unknown delay. This is done for clarity of presentation, as the presence of unknown parameters in the plant would obscure presentation of new tools for handling the unknown delay. In another companion paper [3] we present an extension with unknown plant parameters and where the control objective is not regulation to zero but trajectory tracking.

We start this paper with Section II in which we categorize all the combinations of delay-adaptive, ODE parameter-adaptive, full-state, and output-feedback problems arising in the area of adaptive control in the presence of delay.

II. Categorization of Adaptive Control Problems With Actuator Delay

A finite-dimensional system with actuator delay may come with

- unknown delay (D)
- unmeasured actuator state (u)
- unknown parameters in the finite-dimensional part of the plant (A)
- unmeasured state of the finite-dimensional part of the plant (X).

Each one of these situations introduces a design difficulty, which needs to be dealt with by using an estimator (a parameter estimator or a state estimator). We point out that a state estimator of the actuator state is trivial when the delay is known (one gets the full state by waiting one delay period), however this estimation problem is far from trivial when the delay is also unknown.

The symbols D,u,A,X will be helpful as we try to categorize all the problems in which one, two, three, or all four of these design difficulties may arise. For example, (D,u,X) denotes the case where only the ODE plant parameters are known, whereas the delay is unknown and the state of the actuator and the ODE are unmeasurable.

There are a total of fourteen combinations arising from the four basic problems, (D), (u), (A), and (X). We focus exclusively on problems where the delay is present and is of significant length to require the use of predictor feedback (rather than being treated as a small perturbation through some form of small gain argument). The following list categorizes the fourteen control problems and gives the status of each them:

1) (X), (u), (u,X)—non-adaptive problems solvable using observer-based predictor feedback [17];
2) (A,X), (A)—solved in [26], [24], [5] but with relative degree limitations;
3) (u,A), (u,A,X)—tractable using the techniques from [26], [24], [5];
4) (D)—the main result of the present paper (Sections III and IV);
5) (D,X)—tractable as in Point 4 (by adding a standard ODE observer) but not highly relevant;
6) (D,A)—the subject of our companion paper [3];
7) (D,A,X)—tractable using the techniques in Point 6 combined with adaptive backstepping and Kreisslemeier observers;
8) (D,u), (D,u,A), (D,u,A,X)—not tractable globally because of lack of linear parametrization in any situation involving (D) and (u) simultaneously; the case (D,u) is studied in our other companion paper [2].

If this combinatorial complexity hasn't already overwhelmed the reader, we should point out that in each of the cases involving unknown parameters, namely (D) and (A), multiple choices exist in terms of design methodology (Lyapunov-based, estimation/swapping-based, passivity/observer-based, direct, indirect, pole placement, etc.). In addition, in output-feedback adaptive problems, namely problems involving (A) and (X), the relative degree plays a major role in determining the difficulty of a problem. Finally, trajectory tracking requires additional tools, as compared to problems of regulation to zero.

So, the present paper addresses only a subset among important problems in adaptive control with actuator delay, but in our opinion the most relevant among the tractable problems.

III. Delay-Adaptive Predictor Feedback With Full-State Measurement

We consider the system (2)–(4) where the pair (A,B) is completely controllable. Before we proceed, for a reader familiar with our prior work we point out that the representation (3), (4) is different than the representation $\check{u}_t(\check{x},t)=\check{u}_x(\check{x},t), \check{u}(D,t)=U(t), \check{u}(0,t)=U(t-D), \check{u}(\check{x},t)=U(t+x-D)$, which we used in [17], [13], and which would be less convenient for adaptive control as it is not linearly parametrized in D.

When D is unknown, we replace (6) by the adaptive controller

$$U(t)=K\left[\mathrm{e}^{A\hat{D}(t)}X(t)+\hat{D}(t)\int_0^1 \mathrm{e}^{A\hat{D}(t)(1-y)}Bu(y,t)dy\right] \quad (8)$$

with an estimate $\hat{D}$ governed by the update law

$$\dot{\hat{D}}(t)=\gamma\mathrm{Proj}_{[0,\bar{D}]}\{\tau(t)\} \quad (9)$$

where

$$\tau(t)=-\frac{\int_0^1(1+x)w(x,t)K\mathrm{e}^{A\hat{D}(t)x}dx\,(AX(t)+Bu(0,t))}{1+X(t)^TPX(t)+b\int_0^1(1+x)w(x,t)^2dx}, \quad (10)$$

the standard projector operator is given by

$$\mathrm{Proj}_{[0,\bar{D}]}\{\tau\}=\tau\begin{cases}0, & \hat{D}=0 \text{ and } \tau<0\\ 0, & \hat{D}=\bar{D} \text{ and } \tau>0\\ 1, & \text{else}\end{cases} \quad (11)$$

the matrix P is the positive definite and symmetric solution of the Lyapunov equation

$$P(A+BK)+(A+BK)^TP=-Q \quad (12)$$

for any positive definite and symmetric matrix Q, the constant b is chosen to satisfy the inequality

$$b\geq\frac{4|PB|^2\bar{D}}{\lambda_{\min}(Q)}, \quad (13)$$

the transformed state of the actuator is given by

$$\begin{aligned}w(x,t) &= u(x,t)-\hat{D}(t)\int_0^x K\mathrm{e}^{A\hat{D}(t)(x-y)}Bu(y,t)dy\\ &\quad -K\mathrm{e}^{A\hat{D}(t)x}X(t), \end{aligned} \quad (14)$$

and the positive adaptation gain γ is chosen "sufficiently large."

For this adaptive controller, the following result holds.

Theorem 1: Consider the closed-loop system consisting of the plant (2)–(4), the control law (8), and the parameter update law defined through (9)–(14). Let Assumption 1 hold. There exists $\gamma^*>0$ such that for any $\gamma\in(0,\gamma^*)$, the zero solution of the system $(X,u,\hat{D}-D)$ is stable in the sense that there exist positive constants R and ρ (independent of the initial conditions) such that for all initial conditions satisfying $(X_0,u_0,\hat{D}_0)\in\mathbb{R}^n\times L_2(0,1)\times[0,\bar{D}]$, the following holds:

$$\Upsilon(t)\leq R\left(\mathrm{e}^{\rho\Upsilon(0)}-1\right), \quad \forall t\geq 0, \quad (15)$$

where

$$\Upsilon(t)=|X(t)|^2+\int_0^1 u(x,t)^2dx+\tilde{D}(t)^2. \quad (16)$$

Furthermore,

$$\lim_{t\to\infty}X(t)=0, \quad \lim_{t\to\infty}U(t)=0. \quad (17)$$

IV. Proof of Stability for Full-State Feedback

In this section we prove Theorem 1. We start by considering the transformation (14), along with its inverse

$$\begin{aligned}u(x,t) &= w(x,t)+\hat{D}(t)\int_0^x K\mathrm{e}^{(A+BK)\hat{D}(t)(x-y)}Bw(y,t)dy\\ &\quad +K\mathrm{e}^{(A+BK)\hat{D}(t)x}X(t).\end{aligned} \quad (18)$$

After a careful calculation, the transformed system can be written as

$$\dot{X}(t)=(A+BK)X(t)+Bw(0,t) \quad (19)$$

$$Dw_t(x,t)=w_x(x,t)-\tilde{D}(t)p(x,t)-D\dot{\hat{D}}(t)q(x,t) \quad (20)$$

$$w(1,t)=0, \quad (21)$$

where $\tilde{D}(t)=D-\hat{D}(t)$ is the parameter estimation error, and

$$\begin{aligned}p(x,t) &= K\mathrm{e}^{A\hat{D}(t)x}(AX(t)+Bu(0,t))\\ &= K\mathrm{e}^{A\hat{D}(t)x}((A+BK)X(t)+Bw(0,t))\end{aligned} \quad (22)$$

$$\begin{aligned}q(x,t) &= \int_0^x K\left(I+A\hat{D}(t)(x-y)\right)\mathrm{e}^{A\hat{D}(t)(x-y)}Bu(y,t)dy\\ &\quad +KAx\mathrm{e}^{A\hat{D}(t)x}X(t)\\ &= \int_0^x w(y,t)\Big[K\left(I+A\hat{D}(t)(x-y)\right)\mathrm{e}^{A\hat{D}(t)(x-y)}B\\ &\quad +\hat{D}(t)\int_y^x K\left(I+A\hat{D}(t)(x-\xi)\right)\mathrm{e}^{A\hat{D}(t)(x-\xi)}BK\\ &\quad \times\mathrm{e}^{(A+BK)\hat{D}(t)(\xi-y)}Bd\xi\Big]dy\\ &\quad +\Big[KAx\mathrm{e}^{A\hat{D}(t)x}+\int_0^x K\left(I+A\hat{D}(t)(x-y)\right)\\ &\quad \times\mathrm{e}^{A\hat{D}(t)(x-y)}BK\mathrm{e}^{(A+BK)\hat{D}(t)y}dy\Big]X(t).\end{aligned} \quad (23)$$

Now we consider a Lyapunov-Krasovskii type (non-quadratic) functional

$$V(t)=D\log N(t)+\frac{b}{\gamma}\tilde{D}(t)^2, \quad (24)$$

where

$$N(t)=1+X(t)^TPX(t)+b\int_0^1(1+x)w(x,t)^2dx. \quad (25)$$

Taking a time derivative of $V(t)$, we obtain

$$\begin{aligned}\dot{V}(t)=&-\frac{2b}{\gamma}\tilde{D}(t)\left(\dot{\hat{D}}(t)-\gamma\tau(t)\right)\\ &+\frac{D}{N(t)}\Big(-X(t)^TQX(t)+2X(t)^TPBw(0,t)\\ &-\frac{b}{D}w(0,t)^2-\frac{b}{D}\|w(t)\|^2\\ &-2b\dot{\hat{D}}(t)\int_0^1(1+x)w(x,t)q(x,t)dx\Big),\end{aligned} \quad (26)$$

where we have used integration by parts and $\|w(t)\|^2$ denotes $\int_0^1 w(x,t)^2dx$. Using the assumption that $\hat{D}(0)\in[0,\bar{D}]$ and the update law (9)–(11) with the help of [15, Lemma

E.1] or [17, Lemma 3], we get

$$\dot{V}(t) \leq \frac{D}{N(t)}\left(-X(t)^T QX(t)+2X(t)^T PBw(0,t)\right.$$
$$-\frac{b}{D}w(0,t)^2-\frac{b}{D}\|w(t)\|^2$$
$$\left.-2b\dot{\hat{D}}(t)\int_0^1(1+x)w(x,t)q(x,t)dx\right), \tag{27}$$

as well as that $\hat{D}(t)\in[0,\bar{D}], \forall t\geq 0$, and $\dot{\hat{D}}^2 \leq \gamma^2\tau^2$. Then, applying Young's inequality and employing (13), we obtain

$$\dot{V}(t) \leq -\frac{D}{2N(t)}\left(\lambda_{\min}(Q)|X(t)|^2+\frac{b}{D}w(0,t)^2+2\frac{b}{D}\|w(t)\|^2\right.$$
$$\left.+4b\dot{\hat{D}}(t)\int_0^1(1+x)w(x,t)q(x,t)dx\right), \tag{28}$$

and, finally, substituting (9), we arrive at

$$\dot{V}(t) \leq -\frac{D}{2N(t)}\left(\lambda_{\min}(Q)|X(t)|^2+\frac{b}{D}w(0,t)^2+2\frac{b}{D}\|w(t)\|^2\right)$$
$$+2Db\gamma\frac{\int_0^1(1+x)|w(x,t)||p(x,t)|dx}{N(t)}$$
$$\times\frac{\int_0^1(1+x)|w(x,t)||q(x,t)|dx}{N(t)}. \tag{29}$$

Then, a lengthy but straightforward calculation, employing the Cauchy-Schwartz and Young inequalities, along with (22) and (23), yields

$$\int_0^1(1+x)|w(x,t)||p(x,t)|dx$$
$$\leq Me^{m\hat{D}(t)}\left(|X(t)|^2+\|w(t)\|^2+w(0,t)^2\right) \tag{30}$$

and

$$\int_0^1(1+x)|w(x,t)||q(x,t)|dx$$
$$\leq Me^{m\hat{D}(t)}\left(|X(t)|^2+\|w(t)\|^2\right), \tag{31}$$

where M,m are sufficiently large positive constants given by

$$\begin{aligned} M &= \max\{2|K|^2|A+BK|^2, 2|K|^2|B|^2,\\ &\quad 1+2|K|(1+|A|\bar{D})|B|(1+\bar{D}|BK|),\\ &\quad |K|^2(|A|+|(1+|A|\bar{D})|BK|)^2\} \end{aligned}\tag{32}$$
$$m = |A|+|A+BK|. \tag{33}$$

Introducing these two bounds into (29), we get

$$\dot{V}(t) \leq -\frac{D}{2N(t)}\left(\lambda_{\min}(Q)|X(t)^2+\frac{b}{D}w(0,t)^2+2\frac{b}{D}\|w(t)\|^2\right.$$
$$\left.-\gamma\frac{4bM^2e^{2m\bar{D}}}{\min\{\lambda_{\min}(P),b\}}\left(|X(t)|^2+\|w(t)\|^2+w(0,t)^2\right)\right), \tag{34}$$

and, finally,

$$\dot{V}(t) \leq -\frac{D}{2}\left(\min\left\{\lambda_{\min}(Q),\frac{b}{D}\right\}-\gamma\frac{4bM^2e^{2m\bar{D}}}{\min\{\lambda_{\min}(P),b\}}\right)$$
$$\times\frac{|X(t)|^2+\|w(t)\|^2+w(0,t)^2}{N(t)}. \tag{35}$$

By choosing

$$\gamma^* = \frac{\min\{\lambda_{\min}(Q),\frac{b}{D}\}\min\{\lambda_{\min}(P),b\}}{4bM^2e^{2m\bar{D}}} \tag{36}$$

and $\gamma\in(0,\gamma^*)$ we make $\dot{V}$ negative semidefinite, and hence

$$V(t)\leq V(0), \quad \forall t\geq 0. \tag{37}$$

From this result we now derive a stability estimate.

From (14) and (18) we show that

$$\|u(t)\|^2 \leq r_1\|w(t)\|^2+r_2|X(t)|^2 \tag{38}$$
$$\|w(t)\|^2 \leq s_1\|u(t)\|^2+s_2|X(t)|^2, \tag{39}$$

where r_1,r_2,s_1,s_2 are sufficiently large positive constants given by

$$r_1 = 3\left(1+\bar{D}^2|K|^2e^{2|A+BK|\bar{D}}|B|^2\right) \tag{40}$$
$$r_2 = 3|K|^2e^{2|A+BK|\bar{D}} \tag{41}$$
$$s_1 = 3\left(1+\bar{D}^2|K|^2e^{2|A|\bar{D}}|B|^2\right) \tag{42}$$
$$s_2 = 3|K|^2e^{2|A|\bar{D}}. \tag{43}$$

From (24), (25) the following two inequalities readily follow:

$$\tilde{D}^2 \leq \frac{\gamma}{b}V \tag{44}$$
$$|X|^2 \leq \frac{1}{\lambda_{\min}(P)}\left(e^{V/D}-1\right). \tag{45}$$

Furthermore, from (24), (25) and (38) it follows that

$$\|u\|^2 \leq \frac{r_1}{b}\left(e^{V/D}-1\right)+r_2|X|^2. \tag{46}$$

Combining (44)–(46) we get

$$\Upsilon(t) \leq \left(\frac{1+r_2}{\lambda_{\min}(P)}+\frac{r_1}{b}+\frac{\gamma}{Db}\right)\left(e^{V(t)/D}-1\right). \tag{47}$$

So, we have bounded $\Upsilon(t)$ in terms of $V(t)$, and thus, using (37), in terms of $V(0)$. Now we have to bound $V(0)$ in terms of $\Upsilon(0)$. First, from (24), (25) it follows that

$$V \leq D\left(\lambda_{\max}(P)|X|^2+2b\|w\|^2\right)+\frac{b}{\gamma}\tilde{D}^2. \tag{48}$$

Using (39) we get

$$V \leq \left(D\lambda_{\max}(P)+2bDs_2\right)|X|^2$$
$$+2bDs_1\|u\|^2+\frac{b}{\gamma}\tilde{D}^2. \tag{49}$$

and hence

$$V(0) \leq \left(D\lambda_{\max}(P)+2bDs_2+2bDs_1+\frac{b}{\gamma}\right)\Upsilon(0). \tag{50}$$

Denoting

$$R = \frac{1+r_2}{\lambda_{\min}(P)}+\frac{r_1}{b}+\frac{\gamma D}{b} \tag{51}$$
$$\rho = \lambda_{\max}(P)+2bs_2+2bs_1+\frac{b}{\gamma D}, \tag{52}$$

we complete the proof of the stability estimate (15).

Finally, to prove the regulation result we will use (35) and Barbalat's lemma. However, we first discuss the boundedness of the relevant signals. By integrating (37) from $t=0$ to $t=\infty$, and by noting that $N(t)$ is uniformly bounded, it follows that $X(t)$, $\|w(t)\|$, and $\hat{D}(t)$ are uniformly bounded in time. Using (38) we also get the uniform boundedness of $\|u(t)\|$ in time. With the Cauchy-Schwartz inequality, from (8) we get uniform boundedness of $U(t)$ for $t \geq 0$. From (5) we get the uniform boundedness of $u(0,t)$ for $t \geq D$. Using (2) we get uniform boundedness of $d|X(t)|^2/dt$ for $t \geq D$. From (35) it follows that $X(t)$ is square integrable in time. From this fact, along with the uniform boundedness of $d|X(t)|^2/dt$ for $t \geq D$, by Barbalat's lemma we get that $X(t) \to 0$ as $t \to \infty$.

What remains is to prove the regulation of $U(t)$. From (35) it follows that $\|w(t)\|$ is square integrable in time. Using (38) we get that $\|u(t)\|$ is also square integrable in time. With the Cauchy-Schwartz inequality, from (8) we get that $U(t)$ is also square integrable. To complete the proof of regulation of $U(t)$ by Barbalat's lemma, all that remains to show is that $dU(t)^2/dt$ is uniformly bounded. Towards this end, we calculate

$$\frac{d}{dt}U(t)^2 = 2U(t)K\left[e^{A\hat{D}(t)}\dot{X}(t) + \dot{\hat{D}}(t)G_1(t) + \frac{\hat{D}(t)}{D}G_2(t)\right], \tag{53}$$

where

$$\begin{aligned} G_1(t) =& A e^{A\hat{D}(t)} X(t) \\ &+ \int_0^1 \left(I + A\hat{D}(t)(1-y)\right) g(y,t)dy \end{aligned} \tag{54}$$

$$\begin{aligned} G_2(t) =& BU(t) - Be^{A\hat{D}(t)}u(0,t) \\ &+ \int_0^1 A\hat{D}(t) g(y,t)dy \end{aligned} \tag{55}$$

and

$$g(y,t) = e^{A\hat{D}(t)(1-y)}Bu(y,t)\,. \tag{56}$$

The signal $\dot{\hat{D}}(t)$ is uniformly bounded over $t \geq 0$ according to (9)–(11). By using also the uniform boundedness of $X(t), \dot{X}(t), \|u(t)\|, U(t)$ over $t \geq 0$, and of $u(0,t)$ over $t \geq D$, we get uniform boundedness of $dU(t)^2/dt$ over $t \geq D$. Then, by Barbalat's lemma, it follows that $U(t) \to 0$ as $t \to \infty$.

V. Simulations

We present the simulation results for the state-feedback scheme in Section III, namely, for the closed-loop system consisting of the plant (2)–(4), the control law (8), and the parameter update law defined through (9)–(14).

We focus on highlighting the most important aspect of our scheme—the ability to handle long delays, in the presence of a large uncertainty on the delay. For this reason we focus on the case of a scalar but unstable ODE (2), with $A = 0.75$ and $B = 1$. We take the delay as $D = 1$, which is larger than A. So, the system's transfer function is $X(s)/U(s) = e^{-s}/(s-0.75)$. We assume that the known upper bound on the delay is $\bar{D} = 2$. We take the nominal control gain as $K = -A - 1 = -1.75$ (which means that

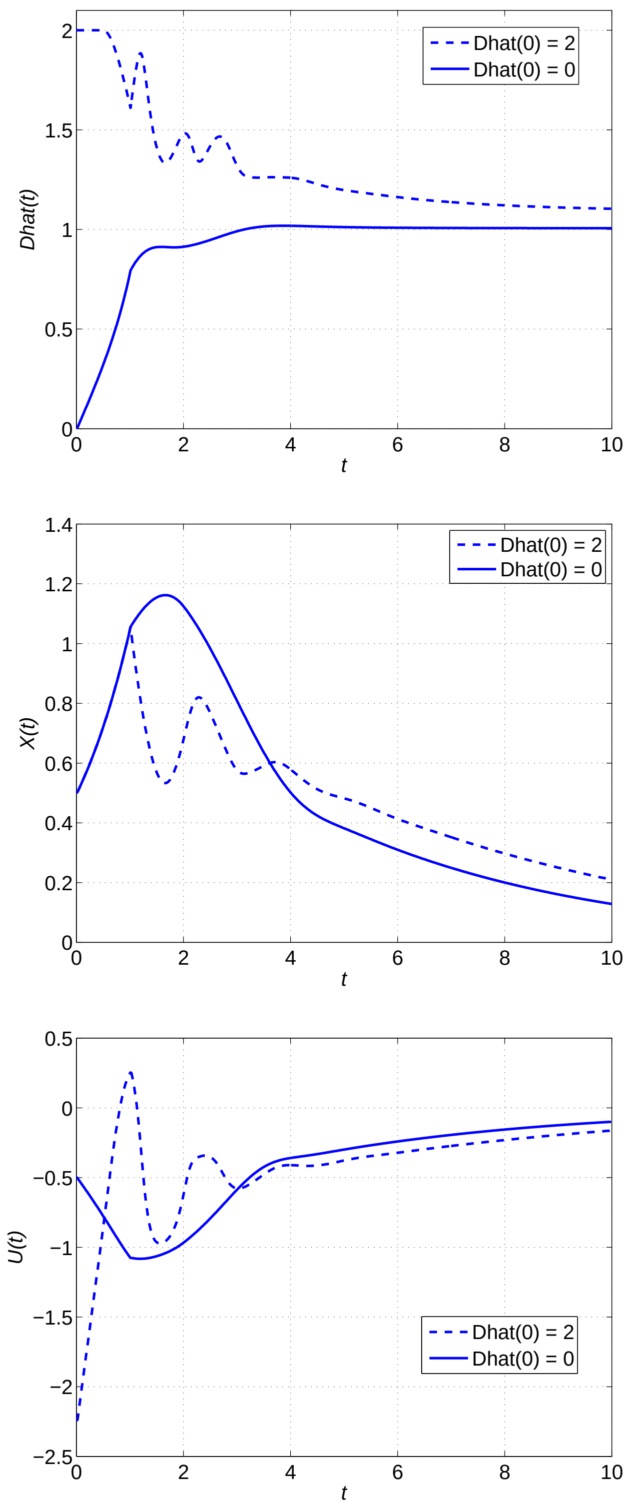

Fig. 1. The system response of they system (2)–(4), (8)–(14) for $D = 1$ and for two dramatically different values of initial estimate, $\hat{D}(0) = 0$ and $\hat{D}(0) = \bar{D} = 2D = 2$.

$P = 1, Q = 2$). We take the adaptation gain as $\gamma = 23$ and the normalization coefficient as $b = \frac{4|PB|^2\bar{D}}{\lambda_{\min}(Q)} = 2\bar{D} = 4$. We take the actuator initial condition as $u_0(x) \equiv 0$, i.e., as $U(\theta) \equiv 0, \forall\theta \in [-D, 0]$, and the plant initial condition as $X(0) = 0.5$.

Hence, the closed-loop system responds to $X(0)$ and to $\hat{D}(0)$. We perform our tests for two distinctly different values of $\hat{D}(0)$—at one extreme we take $\hat{D}(0) = 0$ and at the other extreme we take $\hat{D}(0) = \bar{D}$.

The responses are shown in Figure 1. First, they show that, for both initial estimates, the adaptive controller achieves regulation of the state and input to zero. Second, they show that in both cases the estimate $\hat{D}(t)$ converges towards the true D and settles in its vicinity. The perfect convergence is not achieved in either of the two cases, since the regulation problem does not provide persistency of excitation for parameter convergence. Third, the dashed plot for $\hat{D}(t)$ shows that the projection operator is active during the first 0.7 seconds. Fourth, we can observe that by about 3 sec, the evolution of the estimate $\hat{D}(t)$ has been completed.

Fifth, the plots for $X(t)$ and $U(t)$ are very informative in showing four distinct intervals of behavior of the controller and of the closed-loop system. During the first 1 sec, the delay precludes any influence of the control on the plant, so $X(t)$ shows an exponential open-loop growth. At 1 sec, the plant starts responding to the control and its evolution changes qualitatively, resulting also in a qualitative change of the control signal. When the estimation of $\hat{D}(t)$ ends at about 3 seconds, the controller structure becomes linear. However, due to the delay, the plant state $X(t)$ continues to evolve based on the inputs from 1 second earlier, so, a non-monotonic transient continues until about 4 seconds. From about 4 seconds onwards, the (X, U) system is linear and the delay is sufficiently well compensated, so the response of $X(t)$ and $U(t)$ shows a monotonically decaying exponential trend of a first order system.

We want to stress that the plots presented here do not show the best performance achievable with the scheme. Quite on the contrary, the plots have been selected to illustrate the less than perfect behaviors, with non-monotonic evolution of all the states in the closed-loop system, that one would obtain when γ and b are not highly tuned.

VI. CONCLUSIONS

As we have explained in Section II, the problem of full state stabilization with known ODE plant parameters but with unknown delay is the central problem in adaptive control of systems with actuator delays. The other problems in the lengthy catalog of problems are extensions of the this central problem. Some of them are solvable globally and some of them only locally.

We present a globally stabilizing adaptive controller which employs the measurement of actuator state and then prove that, when the actuator state is replaced by its adaptive estimate, local stability and regulation are achieved. A stepping stone towards the latter result is a nonadaptive linear robustness result with respect to the delay value employed in the predictor feedback. We have presented the linear result in more detail, to help the reader's intuition regarding the proof of the local adaptive result, which is considerably more complex and presented with a limited amount of detail due to space limitations.

The simulations show the effectiveness of the Lyapunov-based adaptive controller. Whether the initial estimate of the delay is zero or 100% above the true value, the estimator drives the estimate towards the true value, which in turn results in the stabilization of the closed-loop system by the predictor-based adaptive controller.

When the actuator state is not measured but it is estimated, local stability is achieved, as proved in our companion paper [2]. The extension to the problem with unknown ODE plant parameters and to trajectory tracking is presented in our other companion paper [3].

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