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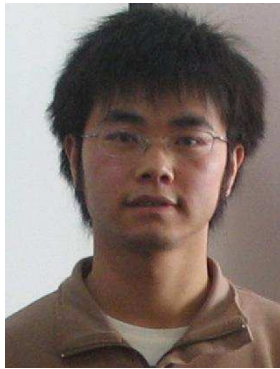
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## ASSESSING CONFIDENCE INTERVALS OF EXTREME TRAFFIC LOADS



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### Abstract

For short- to medium-span bridges, truck gross vehicle weights (GVW) and axle loads are of great importance to assess extreme load effects, as extreme load events are obtained by combining the extremes of GVWs or axle loads. In this paper, three known prediction methods are applied, a normal distribution fitting to the sample distribution upper tail, the block maxima method and the peaks-over-threshold method, to extrapolate extreme traffic loads. Moreover, two methods, the delta method and the profile likelihood method, are introduced to assess the confidence in these extrapolations. Results show that generalized Pareto distribution based peaks-over-threshold method is the best among these three methods to assess the extreme traffic loads.

**Keywords:** WIM data, gross vehicle weight, return period, return level, confidence interval, delta method, profile likelihood method, generalized extreme value distribution, generalized Pareto distribution.

### Résumé

Pour les ponts de courte ou moyenne portée, le poids total et le poids par essieu des camions sont cruciaux pour déterminer les sollicitations extrêmes induites dans la structure. En effet, les cas de charges extrêmes sont obtenus en combinant les maxima de ces deux types de charges. Trois méthodes classiques, un ajustement de gaussienne sur la queue de la distribution d'échantillon, la méthode des maxima de blocs et celles des pics au-dessus d'un seuil, sont appliquées pour extrapoler les charges extrêmes. En outre deux méthodes, la méthode Delta et la méthode de vraisemblance de profil, sont introduites pour déterminer le niveau de confiance de l'extrapolation. Les résultats montrent que la distribution de Pareto généralisée issue de la méthode des pics au-dessus d'un seuil est la mieux adaptée pour extrapoler les charges du trafic.

**Mots-clés:** Données de pesage en marche, poids total, période de retour, intervalle de confiance, méthode Delta, méthode de vraisemblance de profil, la distribution des valeurs extrêmes généralisées, distribution de Pareto généralisée.

## 1. Introduction

Bridges need to provide safe crossing for all vehicles, with respect to ultimate and serviceability limit states. Thus accurately assessing traffic loads on existing bridges during their lifetime is important. However, using the load models of the current design standards, like Eurocode 1991-2 (CEN, 2003) in Europe, which are made for designing new structures, may often lead to the conclusion that the bridge will fail. Indeed, these load models include a safety margin, which has been designed to take into account any possible future modification in the traffic (weights and dimensions of the trucks for example). Therefore, site-specific traffic loads and corresponding load effects are required to assess an existing bridge (O'Connor et al., 2002). There is considerable potential for reducing the assessed traffic actions by considering actual traffic loads which can be obtained by weigh-in-motion (WIM) or bridge weigh-in-motion (B-WIM) systems.

The yearly maximum or lifetime maximum distribution function is a major component to build the limit state function in reliability assessment, which is provided by extreme value theory (EVT). However, it is usually more convenient to interpret extreme value models in terms of quantiles or return levels relative to individual parameter values. The standard statistical method adopted to assess traffic load or traffic load effect return levels assumes that the distribution of yearly maximum traffic load effects can be approximated by a Gumbel distribution (O'Connor et al. 2002; Caprani, 2005), a Weibull distribution (Bailey, 1996) or a generalized extreme value distribution (GEVD) (Caprani, 2005). The R-year return level estimation is then a certain quantile of the underlying distribution corresponding to this R-year return period, commonly referred as 1000-year return level in Eurocode 1991-2 (CEN, 2003), which is the quantile corresponding to a 10% probability of exceedance during 100 years.

Since the observations are drawn from random variables, repetitions of the measurements would generate different observations, e.g. different traffic load effects, and hence different estimates of the R-year return level. Thus, the sampling process induces randomness in the estimator. Quantifying estimate accuracy can usually be made more explicit by calculating a confidence interval (CI) in statistics.

This paper focuses on applying fitting distribution method to upper tail, block maxima, and peaks-over-threshold (POT) approach to obtain maximum-likelihood (ML) estimates for GVW return levels. Their variability is investigated by delta method and profile likelihood method to build confidence intervals.

Section 2 introduces the two sets of WIM data used in the analysis. Section 3 describes the theory of the three extrapolation methods and of the two methods used in the part about the methods of building confidence intervals, giving their advantages and drawbacks. Section 4 provides comparison and comments based on the 1000-year return level of GVW for two sets of WIM data.

## 2. WIM Data

Traffic load data were collected from January to May 2010 by a piezo-ceramic WIM system on the A9 motorway at Saint-Jean-de-Védas, near Montpellier in South-East of France (0). The motorway has 4 traffic lanes (2 in each direction) but only the north bound traffic lanes were recorded. 846,019 trucks (GVW>3.5 t) were recorded, which gives an average daily

truck traffic (ADTT) of 6,130 trucks. Some suspicious data (outliers) were eliminated according to accepted criteria (Sivakumar, 2010) such as axle spacing greater than 20 m. Finally 835,468 trucks were kept for the analysis. Another set of WIM data was recorded from April to May 2010 on the A31 motorway near Loisy, in the East of France, for comparison. This sample contains 374,119 trucks after filtering the outliers.

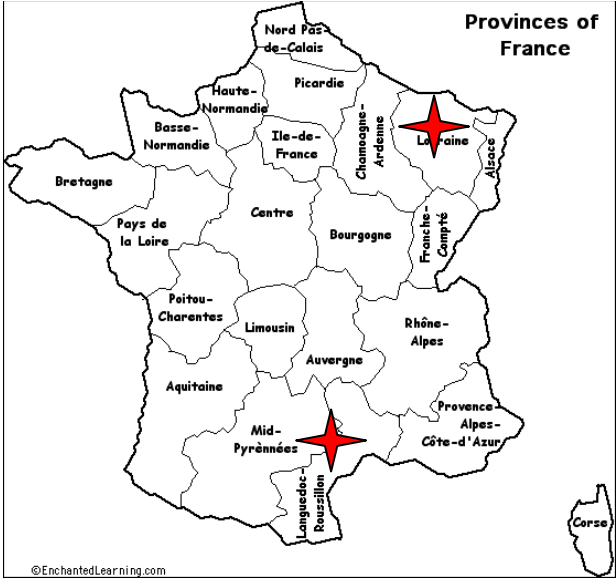


Figure 1 - The measurement locations

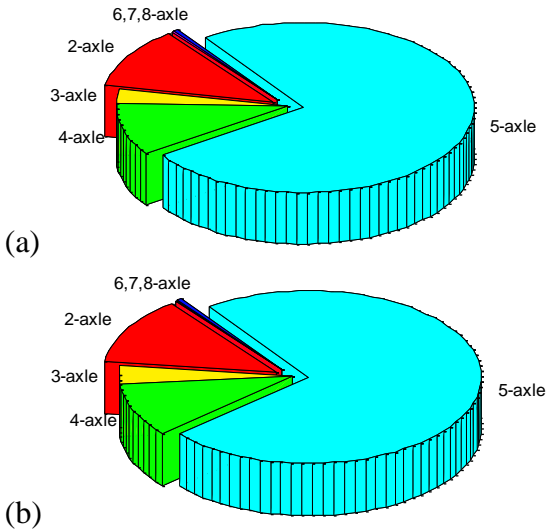
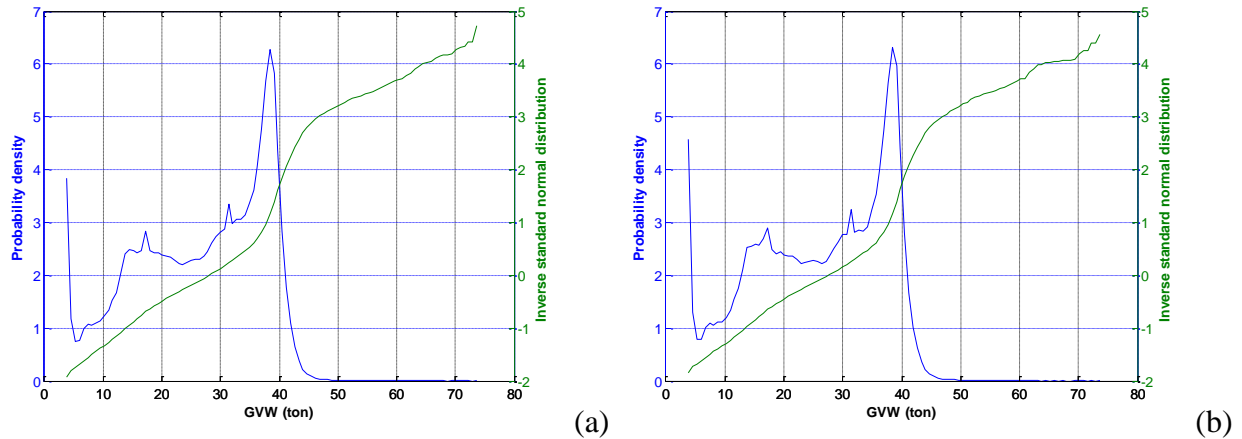


Figure 2 - Proportion of vehicle types (a) Saint-Jean-de-Vedas, (b)Loisy

Table 1 - Gross vehicle weight statistics

Statistics	Location of WIM station			
	Saint-Jean-de-Vedas		Loisy	
	All	Weekdays	All	Weekdays
Number of surveyed days	138	98	61	43
Total number of trucks	835,468	656,974	374,119	323,470
Maximum of GVWs (tons)	74	74	74	74
Mean of daily maxima (tons)	57.5	58.5	57.5	60.9
COV of daily maxima (%)	12.7	12.2	13.4	10.5
Mean of top 1% trucks (tons)	44.9	44.8	44.9	45.0
COV of top 1% trucks (%)	7.8	8.0	7.7	8.1
Mean of top 0.5% trucks (tons)	46.7	46.7	46.7	47.0
COV of top 0.5% trucks (%)	9.0	9.2	8.8	9.2

Nearly 75% of the trucks are 5-axle articulated trucks in both sites (0). Both samples have similar statistics on GVW: maximum, mean value, and coefficient of variation (COV) (0 and 0), although the sample sizes are quite different. The statistics of the whole samples or of the week day only are different, above all for small sample size.



**Figure 3 - Single vehicle GVW relative frequencies (a) St. Jean de Vedas, (b) Loisy**

### 3. Theory of Extrapolation

#### 3.1 Methods for extrapolating R-year Level

In bridge engineering, the GVW of heavy trucks is particularly important to assess load effects on structures, as single-heavy-truck combined with common trucks governs the traffic loading scenarios for short- to medium-span bridges. The R-year return period level of GVW based upon a set of samples can be extrapolated with the maximum distribution of the sample. Let  $X$  be a random variable and  $F$  its cumulated distribution function. Let's denote  $x_1, \dots, x_n$  an identically and independent distributed sample of  $F$ . The maximum value over the "n-observation" period is  $M_n = \max x_1, \dots, x_n$ , and the distribution of  $M_n$  is  $\Pr M_n = F z^n$ .

The distribution of  $M_n$  can be exactly derived according to the parent distribution function  $F$  of the sample (Coles, 2001). However, only the upper tail of parent distribution function really contributes to the distribution function of the maximum, obtained by rising  $F$  to the power  $n$ . Thus, (Nowak, 1994) fits Normal distribution to the upper tail of the ratio of load effect to HS20 load effect to extrapolate average 75-year maximum load effect. (Jacob and Maillard, 1991) adopts half-Normal distribution and Gumbel distribution, which were used in the background study of the Eurocode 1991-2 (Flint and Jacob, 1996).

It is noticed that very small discrepancies in the estimate of  $F$  can lead to substantial discrepancies for  $F^n$ . Statisticians have found that  $F^n$  asymptotically approximates to the three extreme value distributions (EVD): Gumbel, Frechet and Weibull, which makes it possible to avoid rising power of parent distribution function. But a decision needs to be made on selecting the distribution type of EVD. A unification of the three families of EVD into a single family known as GEVD, was widely used in recent years to avoid choosing which of the three families is the most appropriate for the data (Coles, 2001). Periodic maxima consisting of the largest values drawn from blocks which are defined by measurement periods, day or month, are recorded, and this block maximum method is very practical to fit GEVD, if the block length is sufficient, and if enough blocks can be obtained.

In recent years, especially in hydrology, finance and wind engineering, it was agreed that the discard of some of the largest observation in the block, given that only the block maximum is considered, represents a loss of information if the maximum values of other blocks are lower

than these rejected values. Peaks-over-threshold method is an approach that avoids having to decide the distribution type and efficiently using upper tail data. Indeed the excess over the threshold conforms to generalized Pareto distribution (GPD). (Crespo-Minguillon and Casas, 1997) uses POT approach to study weekly maximal traffic load effects.

Therefore these three methods of fitting distribution to upper tail data, i.e. block maxima method for daily maxima, peaks-over-threshold method for excesses over high threshold, and extrapolating extreme values have been applied on the WIM data observed to calculate the extreme traffic loads.

### 3.2 Confidence Intervals for Return Period Levels

Practically, a finite number of samples are used to estimate the distribution parameters. The question is to know how close the estimates are to the real-values. Quantifying the accuracy of an estimator can usually be made by calculating a confidence interval. For theory purpose (Shao 2003) wrote:

$$P \theta \in C X = [\underline{\theta} X, \bar{\theta} X] = 1 - \alpha \quad (1)$$

where  $[\underline{\theta} X, \bar{\theta} X]$  is the confidence interval, for the  $1 - \alpha$  confidence level. The lower and upper bounds of this confidence interval must be determined. All parameters of the above considered prediction methods are estimated by maximum likelihood estimation (MLE) methods, while delta method and profile likelihood function based method are commonly used for building confidence intervals of ML estimates (Coles, 2001).

The delta method is a common technique to determine confidence intervals for functions of ML estimates, when direct evaluation of that function variance is not feasible. Generally, the ML estimate, denoted by  $\hat{\theta}$ , follows an asymptotic Normal distribution:

The principle of the profile likelihood method is to invert a likelihood-ratio test to obtain a CI for the considered parameter. Let a statistical model with parameter  $\theta$  that can be partitioned into two component,  $\theta^{(1)}, \theta^{(2)}$ , of which  $\theta^{(1)}$  is the k-dimensional vector of interest and  $\theta^{(2)}$  is the remaining parameters in the model. Maximizing the log-likelihood function  $l(\theta^{(1)}, \theta^{(2)})$  with respect to  $\theta^{(2)}$  for fixed  $\theta^{(1)}$  leads to the profile log-likelihood  $l_p(\theta^{(1)}) = \max_{\theta^{(2)}} l(\theta^{(1)}, \theta^{(2)})$ , and  $D_p(\theta^{(1)}) = 2(l(\hat{\theta}_0) - l_p(\theta^{(1)}))$ , where  $l(\hat{\theta}_0)$  denotes the maximized log-likelihood, conforms to Chi-square distribution with k-degree of freedom according to the statistical theory (Coles 2001). For the concerned variable,  $z = G(\theta) = G(\theta^{(1)}, \theta^{(2)})$ , a function of  $\theta$ , the re-parameterization of the parameters of interest  $\theta^{(1)} = G^{-1}(z, \theta^{(2)})$  is obtained. The profile likelihood,  $l_p(z)$ , for  $z$  is then obtained by maximizing the  $l(G^{-1}(z, \theta^{(2)}), \theta^{(2)})$  with respect to the remaining  $\theta^{(2)}$ , and the CI for  $z$  is constructed with the asymptotic  $\chi_1^2$  distribution.

#### 4. Comments and Comparison

In this section, we present the results of our calculations: in 0, you may find the 1000-year return period level of the GVW of the WIM data of the two locations we presented earlier. For each location, the three methods are applied and in the last column you may find the results of the return period level. In table 3, you will find the corresponding confidence intervals for this 1000-year return period level, assessed for each location and with the two methods explained in the previous section. Indeed,

General comments can be made:

- The results for the two WIM sites show that larger sample size ensures more reliable estimates (narrower CI); moreover, using only the weekday data leads to narrower CI.
- A simple fitting of a normal distribution to the data upper tail provides much higher estimates than the other two extreme value theory based methods. This is surely due to the selected normal distribution: the shape parameters ( $\xi$ ) of the two EVT based methods are negative, which means that the underlying parent distribution has a bounded tail, while the normal shape has an exponential infinite bound.
- The slightly larger mean and COV causes larger estimates. Using data excluding weekend days induce smaller estimates of return levels ( $\theta$  and  $\theta$ ). The same phenomenon occurs with the confidence interval length.
- Among the three prediction methods, the block maxima method (GPD) mostly has much larger length of confidence intervals than the other methods. It must be caused by the larger COV of the daily maxima than of initial data, and less data are involved in estimation.
- The estimated confidence interval from delta method is greater than the one obtained by the profile likelihood method; this demonstrates the statement by Coles (2001) that profile likelihood method usually gives more accurate estimates than delta method.

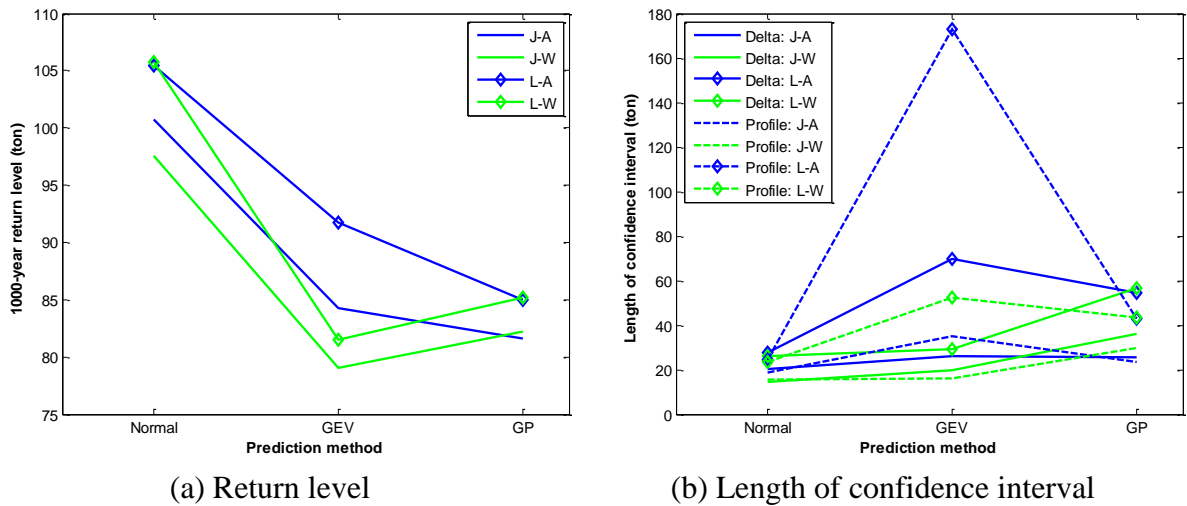
**Table 2 - 1000-year return period level of GVW (in tons)**

Site	Method	Distribution parameter			Return level
		Shape, $\xi$	Scale, $\sigma$	Location, $\mu$	
Saint-Jean-de-Vedas	Normal		8.72 (7.56)	47.29 (51.61)	100.70 (97.60)
	GEVD	-0.22 (-0.27)	6.90 (6.96)	54.76 (55.97)	84.24 (79.09)
	GPD	-0.20 (-0.19)	5.57 (5.33)		81.63 (82.25)
Loisy	Normal		10.53 (10.88)	40.99 (39.41)	105.49 (105.79)
	GEVD	-0.16 (-0.26)	6.91 (6.15)	54.40 (58.64)	91.70 (81.50)
	GPD	-0.17 (-0.17)	6.37 (6.39)		84.99 (85.16)

Note: numbers in brackets denote the result based on weekdays' data, as in the table below.

**Table 3 - 95% confidence interval for 1000-year return level of GVW (in tons)**

Site	CI	Normal		GEVD		GPD	
		Delta	Profile	Delta	Profile	Delta	Profile
Saint-Jean-de-Vedas	Lower	90.62 (90.28)	92.69 (91.53)	71.27 (71.25)	76.51 (75.06)	68.93 (64.18)	76.08 (76.05)
	Length	20.15 (14.65)	18.75 (15.36)	25.94 (19.92)	35.22 (15.92)	25.40 (36.14)	23.37 (29.76)
Loisy	Lower	91.79 (92.73)	94.89 (95.48)	56.79 (66.84)	75.73 (74.40)	57.70 (56.83)	76.38 (76.45)
	Length	27.39 (26.12)	24.64 (23.52)	69.83 (29.31)	172.83 (52.30)	54.58 (56.66)	42.80 (43.26)



**Figure 4 - Comparison (J-A: Saint-Jean-de-Vedas all days, J-W: Saint-Jean-de-Vedas weekdays; L-A: Loisy all days, L-W: Loisy weekdays)**

## 5. Conclusion

Various prediction methods are applied to extrapolate GVW 1000-year return levels, for two sets of WIM data measured on two different motorway sites, and different period lengths. Comparison of these methods requires a careful choice of the distribution type to extrapolate the remote future value, and the two EVT based methods should be the better approach to extrapolation as they give quite similar findings. However, the difference between the two EVT based methods has been revealed through building confidence intervals. Small discrepancy between the block maxima, excluding weekend, generates noticeable changes, while POT method adopting a more realistic technique to draw data from the sample leads to more stable results. Thus, the threshold model is the better extrapolation approach among the studied methods, and the profile likelihood approach provides narrower confidence interval length than the delta method.

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