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Multi-product valid inequalities for the DLSP with sequence-dependent changeover costs

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Abstract

We consider the multi-product discrete lot-sizing and scheduling problem with sequence-dependent changeover costs. We propose a new family of multi-product valid inequalities which enables us to better take into account in the MILP formulation the conflicts between different products simultaneously requiring production on the resource. We then present both an exact and a heuristic separation algorithm for these valid inequalities. We finally discuss some preliminary computational results which confirm the practical usefulness of the proposed valid inequalities at strengthening the MILP formulation.

1 Introduction

In the present paper, we focus on a variant of lot-sizing problems known as the multi-product single-resource discrete lot-sizing and scheduling problem or DLSP. The DLSP relies on a discrete production policy: it thus assumes that at most one product can be produced per period and that the resource processes either one product at full capacity or is completely idle. The costs to be minimized are the inventory holding costs and the changeover costs. We consider here the case where the changeover costs are sequence-dependent.

We aim at developing an exact solution approach based on a standard Branch & Bound procedure for this combinatorial optimization problem. However the computational efficiency of such a procedure strongly depends on the quality of the lower bounds used to evaluate the nodes of the research tree. Within the last thirty years, much research has been devoted to the polyhedral study of lot-sizing problems in order to obtain tight linear relaxations and improve the corresponding lower bounds:

see e.g. [3]. However, these procedures mainly focus on the underlying single-product subproblems and thus fail at capturing the conflicts between multiple products sharing the same resource capacity. This leads in some cases to significant integrality gaps for multi-product instances of the DLSPSD.

In what follows, we propose a new family of multi-product valid inequalities which enables us to partially remedy this difficulty and discuss both an exact and a heuristic algorithm to solve the corresponding separation problem. To the best of our knowledge, this is one of the first attempts focusing on improving the polyhedral description of the multi-product DLSP.

2 Multi-product valid inequalities

We first present the proposed new family of multi-product valid inequalities for the DLSPSD. The reader is referred to e.g. [1] for a description of the MILP formulation of the DLSPSD and its strengthening by single-product valid inequalities.

We denote $p = 1 \dots P$ the set of products to be processed on the resource over a planning horizon involving $t = 1 \dots T$ periods. Product $p = 0$ represents the idle state of the machine. d_{pt} is the demand for product p in period t and $D_{p,1t}$ is the cumulated demand for product p over time interval $[1, t]$. We introduce two types of binary decision variables to formulate the problem: y_{pt} where $y_{pt} = 1$ if product p is assigned to period t and 0 otherwise and w_{pqt} where $w_{pqt} = 1$ if there is a changeover from product p to product q at the beginning of t and 0 otherwise.

Proposition 1

Let $SP \subset \{0 \dots P\}$ and $SD \subset \{0 \dots P\}$ be two disjoint subsets of products.

Let $[1, \theta] \subset [1, T]$ be a time interval and $t \in [2, \theta - 1]$ be a period within this time interval. The following inequalities are valid for the multi-product DLSPSD.

$$\left[\sum_{q \in SD} D_{q,1,\theta} \right] \left[\sum_{p \in SP} y_{pt} \right] \leq \sum_{\substack{\tau=1 \dots t-2 \\ t+2 \dots \theta}} \left[\min \left(\sum_{q \in SD} y_{q,\tau}, \sum_{p \in SP} y_{p\tau} \right) \right] + \sum_{\substack{p \in SP \\ q \in SD}} (w_{q,p,t-1} + w_{p,q,t+1})$$

We briefly explain the underlying idea. Let consider a subset SP of products. If none of these products is assigned for production in period t (i.e. $\sum_{p \in SP} y_{pt} = 0$), all corresponding valid inequalities are trivially respected. But if one of these products is assigned for production in period t (i.e. $\sum_{p \in SP} y_{pt} = 1$), then we have to make sure that we are able to satisfy the total cumulated demand over the interval $[1, \theta]$ for the products in subset SD (i.e. to satisfy $\sum_{q \in SD} D_{q,1,\theta}$) on the remaining periods $1 \dots t - 1, t + 1 \dots \theta$. In this case, the right hand side of the inequalities computes an upper bound of the remaining production capacity available for the products in SD .

3 Solving the separation problem

The number of the proposed multi-product valid inequalities grows very fast with the problem size. Thus, for an instance involving e.g. $P = 10$ products and $T = 25$ periods, we have more than 27 millions of valid inequalities. It is therefore not possible to include them *a priori* in the MILP formulation of the problem. This is why we use a cutting-plane generation strategy to add to the MILP formulation only the most violated valid inequalities of the family.

This requires solving the corresponding separation problem which, given a fractional solution (\bar{y}, \bar{w}) of the DLSPSD, will either identify a violated valid inequality if there is one or prove that no such inequality exists. In our case, this amounts to looking, for each couple of periods (t, θ) , for the partition of $\{0 \dots N\}$ into 3 subsets SP , SD and $\{0 \dots P\} \setminus \{SP \cup SD\}$ which provides the largest difference between the left and the right hand side of the valid inequalities.

We investigate two solution approaches for this optimization problem:

- We first formulate it as a quadratic binary program and solve it to optimality by using a mixed-integer quadratic programming solver such as the one embedded in CPLEX 12.5.
- We then develop a heuristic solution approach based on a Kernighan-Lin type algorithm (see [2]). Our separation problem namely shares some common features with a graph partitioning problem known as the maximum cut problem and the Kernighan-Lin heuristic is known to be rather efficient at solving it.

4 Preliminary computational results

We now discuss the results of some preliminary computational experiments we carried out to evaluate the effectiveness of the proposed multi-product valid inequalities at strengthening the formulation of the problem and to assess their overall impact on the total computation time. We randomly generated instances using a procedure similar to the one described in [4] for the DLSP with sequence-dependent change-over costs. We considered 4 problem dimensions and used a set of 20 instances for each of them. Instances of sets A1-A4 have a general structure for the changeover cost matrix whereas instances of sets B1-B4 correspond to the frequently encountered case where products can be grouped into product families.

All tests were run on an Intel Core i5 (2.7 GHz) with 4 Go of RAM, running under Windows 7. We used a standard MILP software (CPLEX 12.5) with the solver default settings to solve the problems with one of the following formulations: DLSPSD1, the MILP formulation as discussed e.g. in [1], DLSPSD2e and DLSPSD2h, the formulation DLSPSD1 strengthened by multi-product valid inequalities, using either the exact or the heuristic separation algorithm. Table 1 displays the computational re-

Table 1: Preliminary computational results

	P	T	DLSPSD1		DLSPSD2e			DLSPSD2h		
			Gap_{LP1}	T_{IP1}	MPe	Gap_{LP2e}	T_{IP2e}	MPh	Gap_{LP2h}	T_{IP2h}
A1	4	15	2.6%	0.3s	9	0.0%	11.6s	9	0.0%	0.1s
A2	6	15	0.9%	0.3s	4	0.2%	13.5s	3	0.2%	0.2s
A3	4	20	2.6%	0.4s	15	0.2%	445.4s	15	0.2%	0.3s
A4	6	20	2.3%	0.5s	17	0.3%	534.6s	13	0.3%	0.4s
B1	4	15	11.5%	0.3s	16	0.1%	19.5s	16	0.1%	0.1s
B2	6	15	5.3%	0.3s	12	2.1%	60.1s	10	2.1%	0.3s
B3	4	20	8.3%	0.5s	24	0.4%	698.7s	23	0.4%	0.4s
B4	6	20	8.7%	0.7s	24	0.7%	609.5s	23	0.7%	0.4s

sults. We provide, for each set of 10 instances, information on the problem size (P , T), on the number of multi-product violated valid inequalities added in the formulation by the exact and heuristic separation algorithm (MPe, MPh), on the average percentage gap between the linear relaxation and the value of optimal integer solution for each formulation (Gap_{LP1} , Gap_{LP2e} , Gap_{LP2h}) and on the total computation time (T_{IP1} , T_{IP2e} , T_{IP2h}).

Results from Table 1 first show that the proposed valid inequalities are efficient at strengthening formulation DLSPSD1. Namely, the integrality gap is reduced from an average of 5.3% with formulation DLSPSD1 (see Gap_{LP1}) to an average of 0.5% with formulation DLSPSD2e (see Gap_{LP2e}). Moreover, it can also be noted that the heuristic separation algorithm is efficient at finding the violated valid inequalities as we have $Gap_{LP2h} = Gap_{LP2e}$ for all instance sets.

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