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# Study choices in an evolutionary game

Cyrille Piatecki

<sup>a</sup>LEO, University of Orléans and ALPTIS

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## Abstract

Since Becker's (1964) contribution, the accumulation of human capital has been developed only within the framework of individual choices based upon wage expectations. For over forty years, this approach showed itself fruitful. However, with the rarefaction of jobs during periods of crisis, it seems light to explain most of the decisions regarding human capital accumulation. Indeed, because nobody can be sure to obtain an employment which will yield a positive return to education, the decision to acquire further education can only be apprehended as a strategic decision taken in a situation of imperfect information. This individual decision will depend on the answer to the following simple question: *With the level of studies which I intend to acquire, will I have bigger chances to obtain a job than my classmates who stopped their studies at a lower level?*

*Key words:* Labor Economics, Evolutionary Game Theory, Strategic Interactions

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## 1 Introduction

In economics, many topics should be considered from the complexity point of view.

First of all, while it took approximately four hundred years to establish a utility theory from the early works by Edme Mariotte<sup>2</sup> — see Mariotte (1717) —, thirty years were sufficient to make the building vacillate on its basis.

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<sup>1</sup> I would like to thank A. Clark, J.-B. Desquilbet, C. Nieradzik and C. Garrouste for their helpful comments. All remaining errors are the sole responsibility of the author.

<sup>2</sup> Mariotte is the second name to appear in the appendix of Jevons (1871) dedicated to mathematico-economic books, memoir, and other published writings.

Among other sciences, Behavioral Psychology and Neurosciences question the very existence of a utility function derived from preferences and used to construct practical decisions. The assumption that human behavior can generate rational decisions<sup>3</sup> has been denied by new experimental evidences from the recent access to the brain through magnetic resonance imaging (MRI) and molecular biology. Thanks to the MRI, it is now proven that decisions can not be taken without reference to emotions and that we are sensitive to chemical substances which alter profoundly our behavior and decisions — see Damásio (1994), Lehrer (2009).

At the same time, with the emergence of a new type of society where information is huge and plethoric, we have begun to understand that the brain is, in some aspect, a wired electrical network with limited transmission speed and limited capacity to treat information. Furthermore, since Poincaré, but more effectively since Edward Lorenz, we know that linearity, through which one believes to understand the most essential structure of our universe, is misleading and that a very small departure from it can produce some processes that are truly indistinguishable from randomness — Poincaré (1890), Lorenz (1964) and also Devaney (2003).

At last, one begins to understand that strategic interactions, which are mainly described in the setting of standard game theory may not be understood from rationality itself.

Nevertheless, despite all these new evidences, many analysts are globally satisfied with their inherited instrument and are not yet ready to change paradigm. For instance, the core of the modelling of economic markets has not been fundamentally changed in thirty years.

For instance, let us have a look at labor economics. Since the development of the *Human Capital Theory* — Schultz (1961), Becker (1964) —, economists have concentrated their attention essentially on the main advantage that the education confers to individuals: an increase in their income.

The mechanics highlighted under this original human capital theory allowed, in a standard neo-classic framework, to propose an alternative explanation to the theory of *compensatory differences* that was first advanced by Adam Smith and in which individuals were considered as indistinguishable.

Two research areas have caught the attention and the energy of economists. On the one hand, following Mincer's works — see Mincer (1974) —, an

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<sup>3</sup> This assumption originated from the great greek philosophers (essentially Aristotle), followed by Daniel Bernoulli, Emmanuel Kant (for the moral point of view), Jeremy Bentham, Stanley Jevons and Von Neumann, among many others

effort has been made to estimate empirically education returns through some earning functions. That approach enables to estimate a return curve for each educational level and to make international comparisons. On the other hand, more recently, human capital has been introduced in models of endogenous growth.

Following Lucas (1988), macroeconomists have favored the stock of human capital over technology to explain growth, and the differences of rates in its accumulation to explain the persistence of international differences in growth rates. In particular, because of the externalities generated by its human capital, for a given level of qualification, a worker is more productive in a country already strongly endowed with human capital. Therefore, he is better paid. This mechanism could explain the strong migratory pressures of the South towards the North.

Without questioning the legitimacy of the Beckerian individualistic approach of human capital accumulation, it seems, nevertheless, important to reinterpret the contemporary logics of its accumulation in the context of jobs competition. In the Beckerian logic, the rise of the level of education appears as a real solution against unemployment, from the individual point of view, because the level of education is positively correlated with the speed of transition towards employment. This transition advantage is also assumed by the filter theory developed by Arrow (1973) and the signal theory developed by Spence (1974).

Other things being equal, considering a homogeneous population, this transition advantage given by education is expected to motivate all individuals to apply for further education and training. Hence, the ultimate consequence of this dynamic process is the deletion of the number of non-graduates.

In practice, we observe a conflicting logic which substitutes itself to the Beckerian individualistic approach as soon as such a situation is reached or, at least, as soon as a critical threshold of graduates is exceeded. Indeed, the diploma competitive advantage tends to disappear with an increase in the number of its graduates. Outside a conflicting approach of the human capital accumulation, we can only predict a desertion of the educational sphere: education does not generate any more a return on investment which can be evaluated in comparison to an investment in stocks.

However, from the empirical point of view, it is rare to observe parents preferring to donate the cost of their education to their children in the form of a financial endowment because it seems to them higher and less risky than that of the studies that their children undertake. Indeed, in spite of its depreciation, human capital can become the entry key to active life.

So, on the contrary to the predictions of the Beckerian model, the model of the conflicting logic seems to predict a wild race towards the accumulation, the non-accumulation appearing as an almost insuperable handicap.

However, if a model which rests on a conflicting logic ends in an extreme distribution — either every player graduates or nobody graduates —, it must be rejected as empirically not relevant because, as far as one can observe, in populations where the individual characteristics are distinguished with difficulty, the coexistence of the two logics of accumulation of human capital — *i.e.*: the maximalist logic of the acquisition of the highest diploma and the minimalist logic of the acquisition of the lowest diploma.

This paper proposes to examine the evolution of the graduate population within the framework of the theory of evolutionary games. For every generation, for a given unemployment rate, the percentage of graduates is predetermined by their expected remuneration relative to the previous generation. The graduates, in a context where the diploma gives essentially a priority access to jobs, are all the stronger as there are few qualified people. Consequently and *a priori*, we should expect that the global educational level does not stop to grow excluding the possibility of a mixed balance, that is a balance of population in which coexists both categories of agents. However, when the trained population is numerous, the high training costs decrease the relative advantage conferred by the studies and can incite to disinvest in education.

The problem hereby underlined is a real problem that all developed countries seem to encounter. For instance, in France, for the year 2000, it has been revealed that one third of the youth undergo a downgrading. Even if the apex of the phenomena has been observed between 1986 and 1995 for the graduates at the Baccalauréat (High School exit examination) level, and between 2001 and 2004, at the BAC+2 level (two-year post-secondary or tertiary level degree), due to the overall decrease in employment, even the higher-level graduates got heavily impacted. The chances for a newly graduate to get a position as an executive fell from 85% to 70%. Only 26% of the youth have a perfectly matched job according to their formation, in level and in speciality. At the same time, we still observe that the higher the level of the diploma, the higher the smoother the transition towards the first job.

In the first section, we consider the static evolutionary game and its dynamics when the matching of the young job-seekers is realized pairwise. The second section considers explicitly the effect of the unemployment growth on the population dynamics. The third section studies the formation of the long-term balances when irrational mutations appear. The fourth and last section is devoted to the commandability of the education

system<sup>4</sup>.

## 2 Pair Matching dynamics

Here we study more particularly a game in which the players have two strategies:

- ① undertake short studies — strategy (C);
- ② undertake long studies — strategy (L);

At first, we are interested in two players that compete for a job, both of them assigning to it a common present value  $V$ . The agent having adopted the L strategy will be given priority access to the job compared to the agent having adopted the C strategy. This preference for the most qualified refers to the human capital theory — under its shape filter or under its shape signal. Because our focus is on the advantage conferred by education, we suppose that the level of the remuneration  $V$  is independent from the educational level.

Labor markets are segmented and it is notable that the multiplication of the graduates at every level of study entailed a toughening of the dualism — for a definition of the dualism, we can consult<sup>5</sup>, for example, Taubman & Wachter (1986). Young people that are excluded from markets, which access conditions correspond exactly to their diplomas, are thus brought to look for an outlet on a market for which they are overqualified. This downgrading search creates an additional bottleneck for their companions who had acquired the diploma required to enter on the secondary market.

We can unwind this reasoning up to the markets which require no specific training and which are, in turn, assailed by young graduates. This logic leads to a more or less important eviction of the non-graduates of the labor market<sup>6</sup>. Naturally, later in their active life, some individuals will

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<sup>4</sup> See the survey published by the CÉREQ at <http://www.cereq.fr/gsenew/concours2008/cereq/G98ind/premierepage.htm>.

<sup>5</sup> Nevertheless, one can follow Piore (1978). According this theory, the labor market is divided in two sector : a *primary* which give right to better paying, promotion, more stables opportunities and a *secondary* which contains the poor paying unsecure and otherwise unattractive jobs.

<sup>6</sup> It is not in our objective to explain why the non-qualified market can not be itself balanced, while we describe a balancing process by downgrading rejection of populations towards the least qualified jobs, and while the existence of an involuntary unemployment is a necessary condition for the justification of the model.

manage to return to their desired market, but they will certainly represent an exception to the rule of the strict dualism. So, on an isolated market, we can expect to observe applications emanating from young graduates possessing the minimum level of required diplomas — see the figure 1.

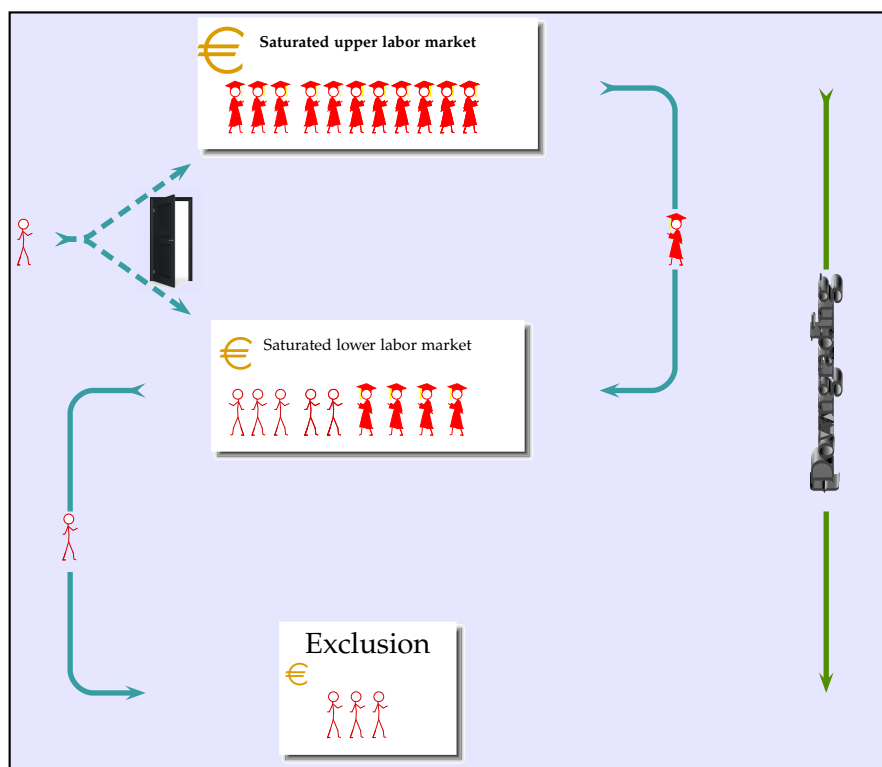


Fig. 1. THE GRADUATES' DOWNGRADING PROCESS

A rather reliable indicator of these observations would be established by the ratio between the level of education of the candidates at public service job competitions and the education level required by the examination. We observe that a large share of the candidates have a must higher level of education and qualification than required by the examination. As a consequence, these overqualified candidates systematically evict the least endowed without benefiting of a more generous salary scale. On the other hand, on the analytical point of view, we can imagine that the status of the labor market is such that companies are in a position to monopolize a rent or to make sure of the honesty of their employees. This point was developed in particular by Shapiro & Stiglitz (1984).

Returning to the model, let us assume that initially all players were used to short studies. As *ex ante* none of the players is able of determining the stemming from the overcoming conflict, by adopting Laplace's insufficient reason principle, it is possible to suppose that they anticipate to each have a  $1/2$  probability to monopolize the desired job. The couple of payoffs in a conflict between two individuals having adopted the short strategy is thus:  $(0.5V, 0.5V)$ .

Let us suppose that after a punctual mutation<sup>7</sup>, a fraction  $1 - \epsilon$  of the population decides to opt for a new strategy consisting of a cycle of longer studies. These players can be perceived as predators by all those players who have decided not to change strategy. Indeed, we suppose that it is of common knowledge that, if a long player is accidentally mated to a short player, the first one will obtain the employment and will be paid  $V$ , while the second will have to pursue its job search. However, we suppose that, compared to strategy **C** (short studies), the long studies yield a fixed additional cost  $C$ . The couple of payoffs bound to such a confrontation is then:  $(V - C, 0)$ .

Obviously, a mutant can be mated with an other mutant. As previously, in this case, they will obtain the couple of remuneration  $(0, 5(V - C); 0, 5(V - C))$ . Presented under normal form, the bi-matrix of the payoffs, associated with this game, can then be spell as in figure 2.





	<b>C</b> 	<b>L</b> 
<b>C</b> 	$(\frac{V}{2}, \frac{V}{2})$	$(0, V - C)$
<b>L</b> 	$(V - C, 0)$	$(\frac{V-C}{2}, \frac{V-C}{2})$

Fig. 2. PAYOFF MATRIX FOR THE GAME

In what follows we will postulate that  $0 \leq C \leq V \leq 2C$ . This hypothesis is as fundamental as natural. First of all, we imagine badly a negative cost. Then, because a cost lower than the present value of the income of the promised job is a necessary and sufficient condition to ensure that some individuals will wish to bear it. And finally, one has to impose that  $V \geq 2C$ . Indeed, let us suppose the opposite:  $V > 2C$ . In this case,  $(V - C)/C$  which is the *ex post* rate of return of longer studies is lower than the unit. Now, if the return was greater than 100%, there would be no student to not undertake longer studies.

<sup>7</sup> The term of mutation, borrowed from the biologist vocabulary, is here excessive. We can imagine that this phenomenon comes upstream from a human capital investment behavior or, more simply from the invasion of the labor market for the non-qualified by individuals stemming from a qualified labor market which has reached saturation, hypothesis which is defended in the body of the text.

## 2.1 The game as a perfect information game in normal form

In pure strategies, under the hypothesis  $V < 2C$ , there are two Nash-equilibria in this game :  $ENSP_1 = (C, C)$  and  $ENSP_2 = (L, L)$ . Both equilibria are symmetric — *i.e.* : both players must choose the same strategy —, which in itself is not so astonishing since the players are indistinguishable. To those pure strategy equilibria one must add a mixed strategy equilibria. One finally has :

$$ENSM_1 = (0,0), \quad ENSM_2 = \left( \frac{V-C}{C}, \frac{V-C}{C} \right) \quad \text{et} \quad ENSM_3 = (1,1)$$

The figure 3 give the three equilibria as intersections of the players' reaction correspondences.

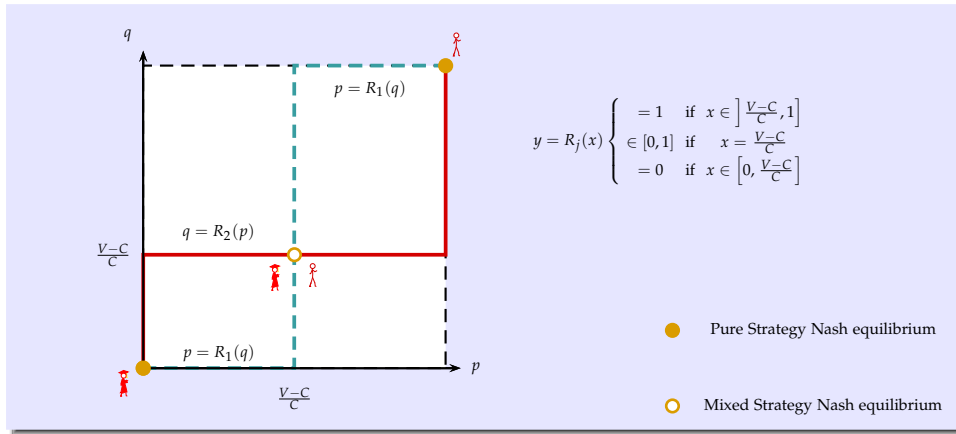


Fig. 3. THE SVASTIKA FOR NASH EQUILIBRIA

In front of the embarrassment induced by the multiplicity of the Nash equilibria, game theorists have developed a number of procedures aiming at extracting a single equilibrium, which will most probably be played by the player<sup>8</sup>. The first seducing approach consists in postulating that the players are going to notice that one of the two pure equilibria is socially more favorable than the other one, in the sense that the expected payoff is higher there — here, it is about the  $(C, C)$  which gives a payoff equal to  $V/2$  which is greater to the payoff of  $(V - C)/2$  associated to the second pure strategies equilibrium.  $(C, C)$  is called the Pareto-dominant equilibrium.

A standard approach of the selection of the multiple equilibria was proposed by Harsanyi & Selten (1992). This approach to equilibrium selec-

<sup>8</sup> In a finite game, every pure strategy Nash equilibrium is also a mixed strategy Nash equilibrium.

tion is known as the *risk dominance*. This approach of equilibrium selection presents the interest of a double interpretation. On the one hand, a player can realize an experiment in which he tries to guess the strategy adopted by the opponent with whom he will be mated — this is a game theory interpretation. On the other hand, he can, *ex ante*, try to conceive the characteristics of the distribution of the players between both strategies as members of the population to which they belong — this is a sociological interpretation.

Let us begin by replacing us in a frame where only pure strategies could be elected and let us endorse the clothes of one for the two matched players. If he thinks that his opponent will choose the **C** strategy with probability  $q$  and the **L** strategy with probability  $1 - q$ , his expected utility is :

$$\mathbb{E}U_1(s) = \begin{cases} (1 - q) \left(\frac{V}{2}\right) & \text{if } s = \mathbf{L} \\ q \left(\frac{V}{2}\right) + \left(\frac{V-C}{2}\right) & \text{if } s = \mathbf{C} \end{cases}$$

The player will prefer to adopt the **C** strategy — the **L** strategy — over the **L** strategy — the **C** strategy —, if :

$$\mathbb{E}U(\mathbf{C}) < (>) \mathbb{E}U(\mathbf{L})$$

Writing  $\bar{q}$ , the  $q$  value which makes the player indifferent between both strategies, we find that :

$$\bar{q} = \frac{C}{V + C}$$

Now, the player's choice depends of his mental scheme which drives him to postulate a  $q$  specific value. However it's clear that if  $1 - \bar{q} \geq q$  — or, more simply, if  $\bar{q} \geq 1/2$  —, which is the present case since  $C \leq V$ , the opponent will take a greater risk in postulating that the player will choose the **C** strategy over the **L** one. As this is a symmetric game what is valuable for one player is also valuable for the other. So one will say with Harsanyi and Selten that the (**L**, **L**) equilibrium *risk dominate* the (**C**, **C**) equilibrium if  $\bar{q} > 1 - \bar{q}$ , which means that there is a greater likelihood that both players coordinate on the (**L**, **L**) equilibrium than on the (**C**, **C**) one. In this game, according to the *risk dominance concept*, the players should be incited to adopt a coordination over long studies.

## 2.2 The static evolutionary game

The application of traditional game theory to the duration of education raises two problems:

- ① On the one hand, we cannot consider two isolated players, because the game is played by a very large number of players simultaneously.
- ② On the other hand, the players have to make decisions without knowing which type of player they will later be mated with — an **C** player or an **L** player. Now, the yield of a strategy *ex ante* depends on the percentage of players which endorse it in the population. In particular, even if there are **C** players in the population, a player can be interested in being an **L** player because the probability to be mated to an **C** player is low and because, by choosing to be an **L** player, he does not have to support the additional cost of the extra time of studies.

We begin by studying the pure strategies game. Let us note  $\epsilon$ , the percentage of players who use the **C** strategy. In such a way, the expected payoff of a **C** player in a mixed population is :

$$\mathbb{E}U(\mathbf{C}|\mathbf{L}) = \epsilon U(\mathbf{C}, \mathbf{C}) + (1 - \epsilon)U(\mathbf{C}, \mathbf{L})$$

where  $U(\mathbf{X}, \mathbf{Y})$  is the utility of a **X** player matched with a **L** player. In the same way, one can write :

$$\begin{aligned} \mathbb{E}U(\mathbf{L}|\mathbf{L}) &= \epsilon U(\mathbf{C}, \mathbf{C}) + (1 - \epsilon)U(\mathbf{C}, \mathbf{L}) \\ &= \epsilon(V - C) + (1 - \epsilon) \left( \frac{V - C}{2} \right) = \left( \frac{1 + \epsilon}{2} \right) (V - C) \end{aligned}$$

In the evolutionary game terminology, a strategy **X** is *evolutionary stable* if :

$$U(\mathbf{X}, \mathbf{Y}) > U(\mathbf{Y}, \mathbf{Y})$$

for every other **Y** strategy. This means that **X** offers a better fit<sup>9</sup> to the game than every other **Y**. Maynard-Smith (1974) has shown that **X** is evolutionary stable<sup>10</sup> if the two following conditions are satisfied :

- ① A Nash Equilibrium Condition :  $U(\mathbf{X}, \mathbf{X}) > U(\mathbf{X}, \mathbf{Y})$ .

<sup>9</sup> In the sense of greater expected utility.

<sup>10</sup> One speaks of an *Evolutionary Stable Strategy* — ESS.

② *A stability Condition* : if  $U(\mathbf{X}, \mathbf{X}) = U(\mathbf{X}, \mathbf{Y})$  then  $U(\mathbf{X}, \mathbf{Y}) > U(\mathbf{Y}, \mathbf{Y})$ .

It is obvious that the pure strategies Nash equilibria are *ESS*, which here was predictable as far as the game is symmetric. Consequently, by neglecting the fact that they are immersed in a larger population than that of both matched players, and as far as they manage to coordinate on an equilibrium — for example the risk dominating equilibrium —, the players protect themselves against the invasion of mutant players, who would have adopted the other strategy. Now, in the same way as in the traditional interpretation, what happens if one allows mixed strategies? If one redefines the utilities in terms of mixed strategies, that is:

$$\mathbb{E}U(\mathbf{p}|\mathbf{q}) = \epsilon \left( \mathbf{p}^\top \mathbf{U} \mathbf{p} \right) + (1 - \epsilon) \left( \mathbf{p}^\top \mathbf{U} \mathbf{q} \right)$$

where  $\mathbf{p} = [p, 1 - p]^\top$  is the mixed strategy of the first player,  $\mathbf{q} = [q, 1 - q]^\top$  the mixed strategy of the second one and  $\mathbf{U}$  is defined as earlier. Evaluating in the same way  $\mathbb{E}U(\mathbf{q}|\mathbf{p})$ , we can define an *Evolutionary Stable Mixed Strategy* — *ESMS* — in the same way as for the *ESS*. In that case the Maynard-Smith theorem gives that  $\mathbf{p}$  is and *ESMS* if two conditions are verified:

- ① *A Nash Equilibrium Condition* :  $\mathbf{p}^\top \mathbf{U} \mathbf{p} > \mathbf{p}^\top \mathbf{U} \mathbf{q}$ .
- ② *A stability Condition* : if  $\mathbf{p}^\top \mathbf{U} \mathbf{p} = \mathbf{p}^\top \mathbf{U} \mathbf{q}$  then  $\mathbf{p}^\top \mathbf{U} \mathbf{q} > \mathbf{q}^\top \mathbf{U} \mathbf{q}$ .

Now for exactly the same reason as in the pure strategy case, the three Nash equilibria are *SMES*.

### 2.3 Population dynamics

The direct *ESS*<sup>11</sup> or *SMES* approach are clearly insufficient. Indeed, unlike the classic theory of antagonistic games, evolutionary game theory does not postulate any form of rationality from the players. Generally, it considers that they inherit their behavior. On the labor market, and in an unstable environment, the only way of making a decision more or less justified, is, from one period to the next, when the information is available, to observe the relative fit of a strategy with regard to an other one. Therefore, on a macroscopic plan, one shall notice an increasing evolution on behalf of the population which adopts the strategy **C** if that strategy gives, on average, a satisfactory payoff in the matching with the upper strategy **L**.

<sup>11</sup> It is important to distinguish both because the first one is clearly less debated than the second. Numerous economists refuse to consider mixed strategies.

If students can observe that the **C** strategy can offer a greater utility than the average utility associated with a population divided out of  $\epsilon(t)$  percent of their own type and  $1 - \epsilon(t)$  percent of the other, then in all likelihood,  $\epsilon(t)$  is going to rise. This leads to a Malthusian population dynamics of the **C** players called *replication dynamics*. This dynamics is describe by the following differential equation <sup>12</sup> :

$$\begin{aligned}\frac{\dot{\epsilon}(t)}{\epsilon(t)} &= \mathbb{E}U(\mathbf{C}) - [\epsilon\mathbb{E}U(\mathbf{C}) + (1 - \epsilon(t))\mathbb{E}U(\mathbf{L})] \\ &= \frac{C}{2}(1 - \epsilon(t))(\epsilon(t) - \bar{\epsilon})\end{aligned}$$

with  $0 < \bar{\epsilon} = (V - C)/C < 1$ . This dynamics <sup>13</sup> to translate the idea according to which the **C** players' population growth rate is proportional to their relative payoff in a mixed population. Now, the average payoff in a mixed population is defined by the sum of the product between payoffs and probabilities of the realisation of a specific matching. This replication dynamics may be also written as :

$$\dot{\epsilon}(t) = \epsilon(t)[1 - \epsilon(t)][\mathbb{E}U(\mathbf{C}) - \mathbb{E}U(\mathbf{L})]$$

which gives also the following interpretation : the increase of the **C** players population is proportionate to the expected utility differential gap between both strategies, conditioned by the emergence of a mixed matching <sup>14</sup>.

Define the  $h(\epsilon(t))$  function as :

$$h(\epsilon(t)) = \frac{C}{2}\epsilon(t)(1 - \epsilon(t))(\epsilon(t) - \bar{\epsilon})$$

which leads to rewrite the replication dynamics as :

$$\dot{\epsilon}(t) = h(\epsilon(t))$$

<sup>12</sup> Only a small bunch of studies was interested in the justification of the recourse to the replication dynamics in the economic environment. Nevertheless, we can quote Weibull (1995), who shows how the dynamics of replication can appear as approximation for imitation behavior.

<sup>13</sup> To obtain  $\mathbb{E}U(\mathbf{C})$  and  $\mathbb{E}U(\mathbf{L})$  one needs only to substitute  $\epsilon(t)$  to  $q$  — *cf supra*.

<sup>14</sup> It would be possible to generalize the dynamics by the use of an increasing function of the right side of the replication dynamics, but this does not change the qualitative behavior of the dynamics.

The  $h(t)$  function possesses three fixed points:  $\epsilon_0 = 0$ , which is the case for the degenerated population composed exclusively of **L** players;  $\epsilon_1 = \bar{\epsilon}$ , which corresponds to a mixed population; and  $\epsilon_2 = 1$ , which is the case for the degenerated population composed exclusively of **C** players. These three fixed points are the three *SMES*. As the derivative of  $h(\epsilon)$  is  $h_\epsilon(\epsilon) = (C/2)[-3\epsilon^2 + 2(1 + \bar{\epsilon})\epsilon - \bar{\epsilon}]$ , it is very easy to show that  $\epsilon_0$  and  $\epsilon_1$  are locally stable equilibria<sup>15</sup> when  $\epsilon_2$  is locally unstable, the **C** strategy is completely evinced in favor of the **L** strategy. From a formal point of view, we can develop a symmetric argumentation<sup>16</sup>. So there are two attraction manifolds for the locally stable equilibria — see figure 4:

- ① a first attraction manifold  $\mathcal{B}_0 = [0, \bar{\epsilon}[$  for  $\epsilon_0$ .
- ② a second attraction manifold  $\mathcal{B}_1 = ]\bar{\epsilon}, 1]$  for  $\epsilon_1$ .

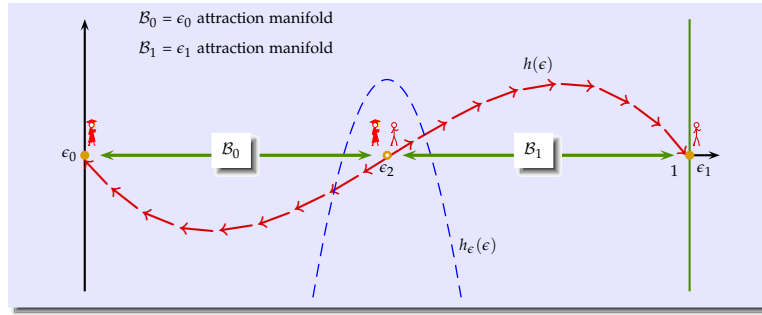


Fig. 4. THE REPLICATION DYNAMICS

Thus, according to its initial distribution —  $\epsilon(0) \in \mathcal{B}_0$ ,  $\epsilon(0) = \epsilon_0$  or  $\epsilon(0) = \epsilon_1$  —, the population converges towards the Pareto-dominant equilibrium, remains in the internal equilibrium — if by chance it was yet at this equilibrium —, or converges towards the risk dominant equilibrium.

We can propose a simple explanation for this result. For an initial population such as  $\epsilon(0) < \bar{\epsilon}$ , the players are matched with numerous players who, on average, have made long studies rather than short. The cost of longer studies is more than compensated by the low probability to be mated with an other long studies player. The expected relative advantage of a strategy over another determines its adoption.

Hence, at the final stage of the dynamics, the **C** strategy is completely evinced in favor of the **L** one. From a formal point of view, one can develop a symmetric argumentation<sup>17</sup> for the case where  $\epsilon(0) > \bar{\epsilon}$ .

However, this symmetry is only visible. Indeed, if we suppose that, initially, only the short studies existed and that a spontaneous mutation has

<sup>15</sup> If we note  $x_j(t) = \epsilon(t) - \epsilon_j$ , in the neighborhood of  $\epsilon_j$ , we get  $\dot{x}_j(t) = h_\epsilon(\epsilon_j)x_j$ .

<sup>16</sup> Since  $h(0) = 0$  and  $h_\epsilon(0) < 0$ , as  $\epsilon_2 \in ]0, 1[$ ,  $h_\epsilon(\epsilon_2)$  is, by necessity, positive.

<sup>17</sup> When  $\epsilon(0) = \bar{\epsilon}$ , the population repartition is temporally invariant.

moved a small percentage of players towards the **L** strategy, it is not very likely, that this percentage makes tip over the initial condition to the attraction manifold of the other *SMES*, i.e. to the long studies attraction manifold.

While the risk dominance argumentation is more convincing than the Pareto-dominance argumentation, in the final equilibrium choice decided by the population of players, it seems that the replication dynamics pleads clearly in favor of the eviction of long studies from the education system, unless, it is possible to produce an initial mutation which drives the initial population in the other attraction manifold — *i.e.* : which moves  $\epsilon(0)$  over  $\bar{\epsilon}$ . A classic argument is then to suggest the appearance of behavior determined by a rationality still weakened by the number of agents.

## 2.4 *Population dynamics with continuous mutations*

As pointed out in Orlean (1995), this dynamics leaves aside an important aspect of the evolutionary logic, namely the constant presence of mutational factors. Indeed, two elements can overlap to explain that the moving decision of a number of players is not directly driven by the relative advantage of a strategy. First of all, there is always a certain percentage of the population which has a very restricted access to information — in the replication dynamics, the access to information is supposed free and without cost for the players. Similarly, a certain percentage of the population is likely to decide to undertake or not to undertake long studies only based on its interest for the studied fields.

Then, the State, for reasons which run from the adequacy of the population to jobs which require an accumulation of human capital always more important for the prolonged, to the young people withdrawal of the labor market in period of under-employment, to make migrate sub-populations having adopted a particular strategy to an other one in a permanent way. So, a second dynamic system is build upon the first one, and this new system does not answer the arguments of the replication dynamics.

This dynamics translates into the fact that, over a lapse of time of length  $h$ , a percentage  $m$  of the players go from the **L** strategy to the **C** strategy and a percentage  $l$  goes from the **C** strategy to the **L** strategy. In  $t + h$ , the percentage of the population that has adopted the **C** strategy is then determined by the recursive equation:

$$\epsilon(t+h) = \underbrace{[1 - mh\epsilon(t)]}_{\text{probability not to migrate for a C player}} \epsilon(t) + \underbrace{[lh\epsilon(t)]}_{\text{probability to migrate for a L player}} (1 - \epsilon(t))$$

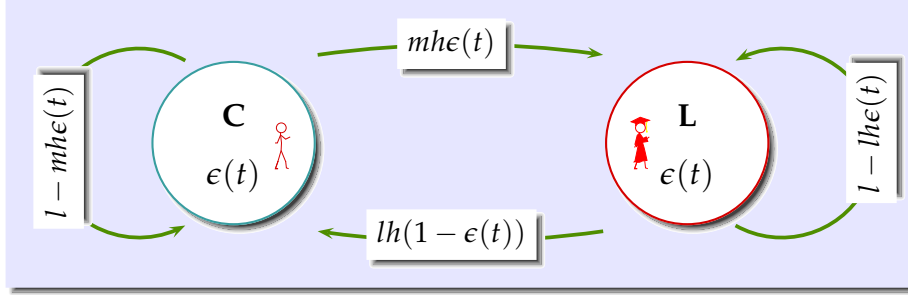


Fig. 5. THE CONTINUOUS MUTATIONS POPULATION DYNAMICS

By making  $h$  stretch to  $dt$ , one finally obtains:

$$\dot{\epsilon}(t) = [l - (m + l)\epsilon(t)]\epsilon(t) = k(\epsilon(t))$$

This dynamics has two fixed points  $\epsilon_0 = 0$  and  $\epsilon_4 = l/(m + l)$ . As  $k_\epsilon(\epsilon) = l - 2(m + l)\epsilon$ ,  $\epsilon_0$  is locally unstable, unless  $l \equiv 0$ , and  $\epsilon_4$  is locally stable.  $\epsilon_1 = 1$  is no more an equilibrium. It could be under the restriction  $m \equiv 0$ , in which case  $\epsilon_4 = \epsilon_1 = 1$ . This has a very simple explanation : if at each period of time some **C** players migrate to become **L** players, it is impossible that the final population-state be fixed in an all **C** players population. This dynamics, which could be called *a rational extreme dynamics*<sup>18</sup>, in opposition to the replication dynamics, insists on the central role of an equilibrium where **C** players and **L** players coexist — see the figure 6.

Let us consider the general population dynamics corresponding to the juxtaposition of the two types of behavior: *the strategic behavior* and the *pre-determined behavior*. It takes the form :

$$\begin{aligned} \dot{\epsilon}(t) &= h(\epsilon(t)) + k(\epsilon(t)) \\ &= \left[ \frac{C}{2} \epsilon(t)(1 - \epsilon(t))(\epsilon(t) - \bar{\epsilon}) \right] + ([l - (m + l)\epsilon(t)]\epsilon(t)) \\ &= \epsilon(t) \left( \frac{C}{2} [(1 + \bar{\epsilon})\epsilon(t) - \bar{\epsilon} + \epsilon^2(t)] + [l - (m + l)\epsilon(t)] \right) \\ &= H(\epsilon(t)|m, l) \end{aligned} \quad (\text{dm1})$$

<sup>18</sup> This type of dynamics is not derived from the rationality of learning as in the evolutionary game and its associated replication dynamics. Nothing forbids the agents to base this type of behavior on another logic, even a logic of maximization.

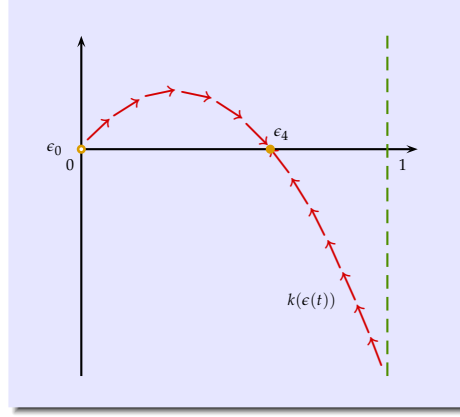


Fig. 6. THE EXTREME ARATIONAL DYNAMICS

Here the mutations remind a *trembling hand process* — see Orlan (1995). They constitute a permanent and autonomous source of variability inside the labor market system.

Then one can also envision a new population dynamics in which,  $\alpha$  percent of the population share a *quasi-rational* behavior, when the other  $1 - \alpha$  has adopted an *irrational* one. In this case, the dynamics becomes:

$$\begin{aligned}
 \dot{\epsilon}(t) &= \alpha h(\epsilon(t)) + (1 - \alpha)k(\epsilon(t)) \\
 &= \alpha \left[ \frac{C}{2} \epsilon(t)(1 - \epsilon(t))(\epsilon(t) - \bar{\epsilon}) \right] + (1 - \alpha) ([l - (m + l)\epsilon(t)]\epsilon(t)) \\
 &= K(\epsilon(t)|m, l) \tag{dm2}
 \end{aligned}$$

It is particularly noticeable that both dynamics are not reducible. Indeed, in the dynamics (drn1), the irrational dynamics appears as stacked over the replication dynamics, whereas in the dynamics (dm2) they work in parallel. In this case, for one unit value of  $\alpha$ , we get back to the replication dynamics and for  $\alpha = 0$ , we find the irrational dynamics. However, in spite of these differences, the two dynamics approach the same phenomenon in an increased complexity with regard to both elementary dynamics.

In what follows, we will adopt the subsequent strategy: we will pose  $\alpha = 1$ ,  $m = l = 0$  from the replicator dynamics, and study the deformations induced on it from small variations of the parameters.

#### 2.4.1 The (dm1) dynamics

Let us note that the (dm1) dynamics is identical to the replicator dynamics for  $m = l = 0$  because:  $H(\epsilon(t)|0, 0) = h(\epsilon(t))$ . As the presence of two new

parameters fairly complicates the problem, let us begin by assuming that  $l \equiv 0$ . In such a way, we start with three equilibria:  $\epsilon_0=0$ ,  $\epsilon_1$  and  $\epsilon_2$ .

$$\underline{l \equiv 0}$$

In this case, let us leave from the replication dynamics to study the way a tiny variation apply on it. An equilibrium of the (dm1) dynamics is characterized by:  $H(\epsilon(t)|0,0) = 0$ . Let us suppose that the value of  $m$  increases from  $m = 0$ .

The equilibria move in the neighborhood of  $\epsilon_i$  —  $i = 1, 2, 4$  — in agreement with the equation:

$$\begin{aligned} H_\epsilon(\epsilon(t)|0,0)d\epsilon + H_m(\epsilon(t)|0,0)dm &= 0 \\ \iff \left. \frac{d\epsilon(t)}{dm} \right|_{m=0} &= -\frac{H_m(\epsilon_i|0,0)}{H_\epsilon(\epsilon_i|0,0)} \end{aligned}$$

Now:

$$\begin{cases} H_m(\epsilon_i|0,0) = -\epsilon_i \\ H_\epsilon(\epsilon_i|0,0) = \left(\frac{C}{2}\right) [-3\epsilon_i^2 + 2(1-\bar{\epsilon})\epsilon_i - \bar{\epsilon}] \end{cases}$$

In consequence of what, a small variation of  $m$  induces — the results are shown in the (a)-quadrant of the figure 7 —:

- ① an invariance of  $\epsilon_0$  since in that case  $H_m(\epsilon_i|0,0) = 0$ .
- ② an increase of  $\epsilon_2 = \bar{\epsilon}$  since :

$$\left. \frac{d\epsilon}{dm} \right|_{m=0} = \frac{\bar{\epsilon}}{(C/2)\bar{\epsilon}(1-\bar{\epsilon})} > 0$$

- ③ a decrease of  $\epsilon_2 = 1$ , which is driven inside  $[0,1]$  since :

$$\left. \frac{d\epsilon}{dm} \right|_{m=0} = \frac{-\bar{\epsilon}}{(C/2)\bar{\epsilon}(1-\bar{\epsilon})} < 0$$

So, a small variation of  $m$  in the neighborhood of  $m = 0$  when  $l = 0$ , eliminates the all in C equilibrium, in which all the players become C players — *i.e.* : eliminates the Pareto-dominant equilibrium. In fact, the disappeared equilibrium swerves inward, which gives now two internal equilibrium, the new one being stable and at the heart of its attraction manifold.

As long as  $m$  remains smaller than  $\tilde{m} = (C/2)[(1 - \bar{\epsilon})/2]^2$ , nothing changes. Then, at  $m = \tilde{m}$ , there is a drastic qualitative change<sup>19</sup> because, even if  $\epsilon_0$  remains stable, both interior equilibria merge to become a saddle point equilibrium — noted  $\epsilon_5$  —, which will be reached from every starting point  $\epsilon(0) > \epsilon_5$  but not from a starting point  $\epsilon(0) < \epsilon_5$  — see the (b) quadrant in figure 6. Finally, if  $m > \tilde{m}$ ,  $\epsilon_0$  remains an equilibrium. The (d) quadrant in figure 6 — called the *equilibrium bifurcation diagram* — summarizes the qualitative properties of the equilibria.

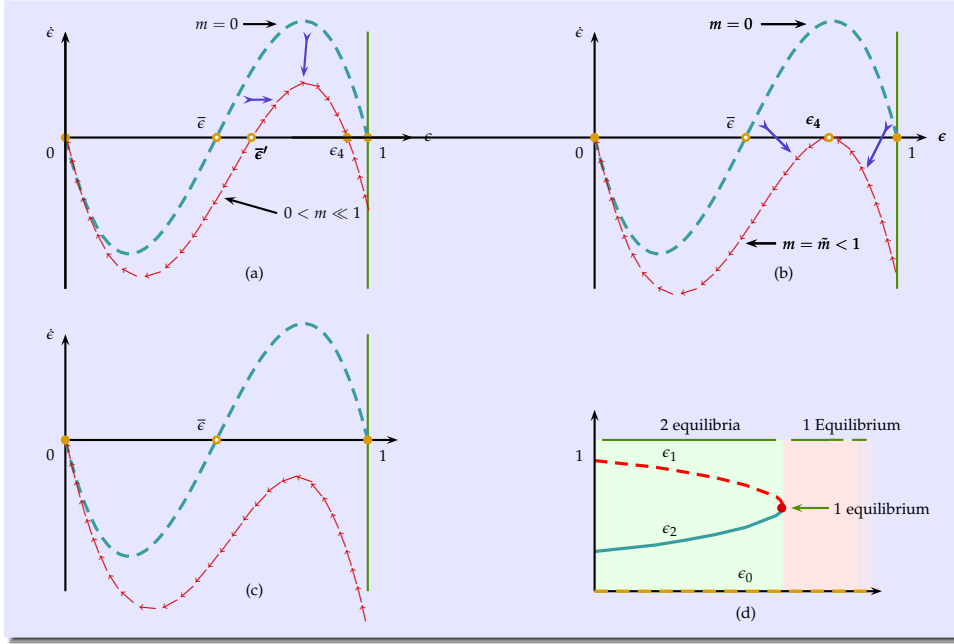


Fig. 7. THE MIXED DYNAMICS FOR  $l \equiv 0$

The main lesson induced by the perturbation of the replicator dynamics, with the exclusive irrational flight from the **C** players to the **L** players, is that, when  $m$  rises, the attraction manifold of the all **L** players equilibrium, is the emergence of a mixed stable equilibrium which may vanish with a further increase of  $m$  beyond  $\tilde{m}$ .

In this scenario, the risk dominant equilibrium tends to win over the Pareto equilibrium. One has :

- ① 3 equilibria for  $m \in [0, \tilde{m}[$ ,
- ② 2 equilibria for  $m = \tilde{m}$ ,
- ③ 1 equilibrium for  $m \in ]\tilde{m}, 1]$ .

Furthermore, we can observe that it is really the irrational dynamics —

<sup>19</sup>  $H(\epsilon|m, 0)$  has  $\epsilon_0 = 0$  and  $\epsilon_{\pm} = (2C)^{-1}[C(1 + \bar{\epsilon}) \pm \sqrt{\Delta}]$  with  $\Delta = C^2\bar{\epsilon}^2 - 2C^2\bar{\epsilon} + C^2 - 8Cm$ . One can see that, when  $m = \tilde{m}$ :  $\epsilon_+ = \epsilon_-$ .

a pure mutation dynamics —, which pollutes the replication dynamics because the subset of  $m$  value for which one has a stable mixed equilibrium is very small. This tends to confirm that the replication dynamics plays the central role in this scenario.

$$\underline{m \equiv 0}$$

Now that the scenarii analysis tools have been set up, we will proceed with a graphic analysis based on figure 7. This time, a small variation of  $l$  induces the immediate disappearance of the equilibrium  $\epsilon_0 = 0$ . In that case, it could not be any more the case of the convergence towards a state where nobody chose to become a **C** player.  $\epsilon_2$  remains a stable equilibrium in all the declinations of this situation. At first, two internal balances coexist, the equilibrium  $\epsilon_5$  arisen from the shift of the defunct  $\epsilon_0$  being stable, while the equilibrium  $\bar{\epsilon}'$ , born from the  $\bar{\epsilon}$  equilibrium remains unstable. But, when  $l$  reaches the limit value  $\tilde{l} = (C/2)(\bar{\epsilon}/2)^2$ , there is a fork which compacts both internal equilibria in a unique saddle point, which immediately disappears. Only one equilibrium remains. One has :

- ① 3 equilibria for  $l \in [0, \tilde{l}[$ ,
- ② 2 equilibria for  $l = \tilde{l}$ ,
- ③ 1 equilibrium for  $l \in ]\tilde{l}, 1]$ .

Here again, it is the irrational dynamics which pollutes the replication dynamics because, when  $m$  rises,  $\bar{\epsilon}$  and its siblings — interior equilibria generated by the  $m$  induced sliding of  $\bar{\epsilon}$ , change only at the margin in their qualitative behavior.

Naturally, the most realistic situation corresponds to  $m \neq 0$ ,  $l \neq 0$  and  $m \neq l$ . However, this situation is particularly complex and a more precise approach could be obtained in posing  $l = m$ . In fact, one can rely on intuition to assume that the values of  $m$  and  $l$  are not too much apart from each other. First of all, they are mutation rates and, by nature, they are restrained on  $[0, 1]$ . Then, if those mutations are irrational, i.e. induced by purely subjective elements without any reference to any strategy from mutants, in all likelihood one can suppose that, in a great population, we will observe very similar percentages of mutant from both populations.

$$\underline{m = l = k}$$

$$\dot{\epsilon}(t) = \frac{C}{2}\epsilon(t)(1 - \epsilon(t))(\epsilon(t) - \bar{\epsilon}) + k[1 - 2\epsilon(t)]$$

Once again, let us study the properties of this case from a purely graphical

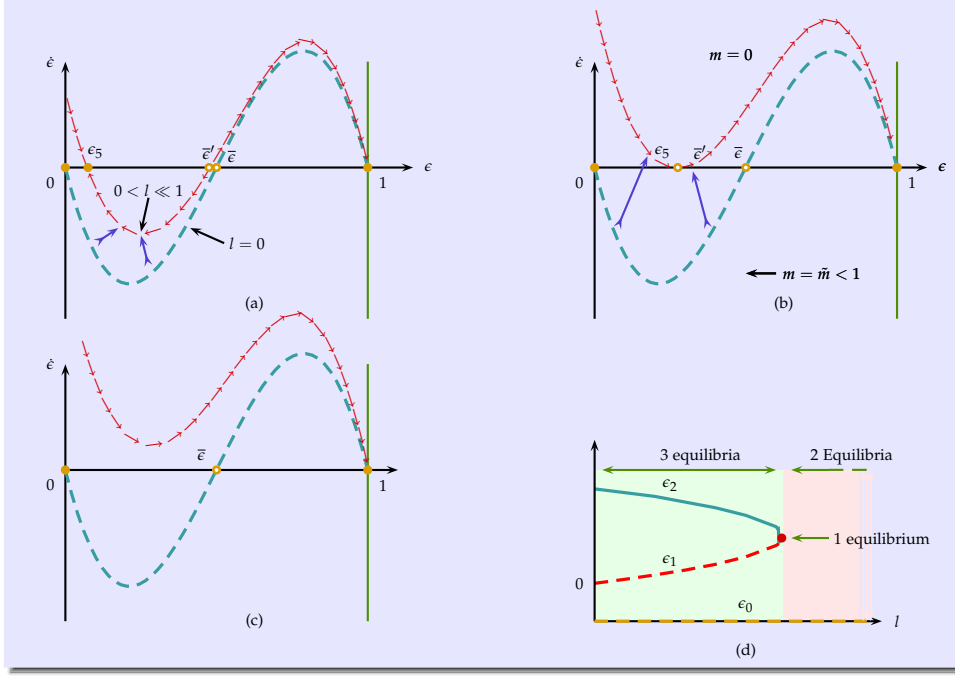


Fig. 8. THE MIXED DYNAMICS FOR  $m \equiv 0$

point of view — see figure 8. In contrast with the two polar cases studied above, the focus in this case is concentrated on the internal mixed equilibrium. Indeed, not only the increase in  $k$  drives both extreme equilibria to their disappearance but the internal equilibrium nature is modified by the variations of  $k$ . For  $k < \tilde{k} = (C/2)(\bar{\epsilon}^2 - \bar{\epsilon} + 1)$ , it is unstable, while stable for  $k > \tilde{k}$ . In fact, in  $\tilde{k}$  there is a bifurcation of the dynamics and the internal equilibrium is the only equilibrium to remain — see figure 10 for the separation hyperbola as a function of  $\bar{\epsilon}$  for  $C$  given — one must not forget that  $\bar{\epsilon}$  is a function of  $C$  and  $V$ .

Consequently, as soon as  $k$  exceeds  $\tilde{k}$ , the replication dynamics is absorbed by the irrational dynamics, which imposes its properties on the replication dynamics equilibrium. As far as a stable mixed equilibrium seems to better correspond to the observed situation of the labor market than any other equilibrium, the model preaches in favour of a significant pollution of the replication dynamics by the irrational dynamics. Contrary to what asserts Orlean (1995), with the increase of  $k$ , the attraction manifold of the risk dominant equilibrium — but also the attraction manifold of the Pareto-dominant equilibrium —, if it can rise for a moment, eventually disappears to make way for a unique stable equilibrium.

This conclusion applies to the general model, i.e. to the model for which  $m \neq l$ . Indeed, by the methodology of the tiny variations in the neighborhood of  $m = l = 0$ , if the measure or the variation of  $m$  is equal to that of  $l$ ,

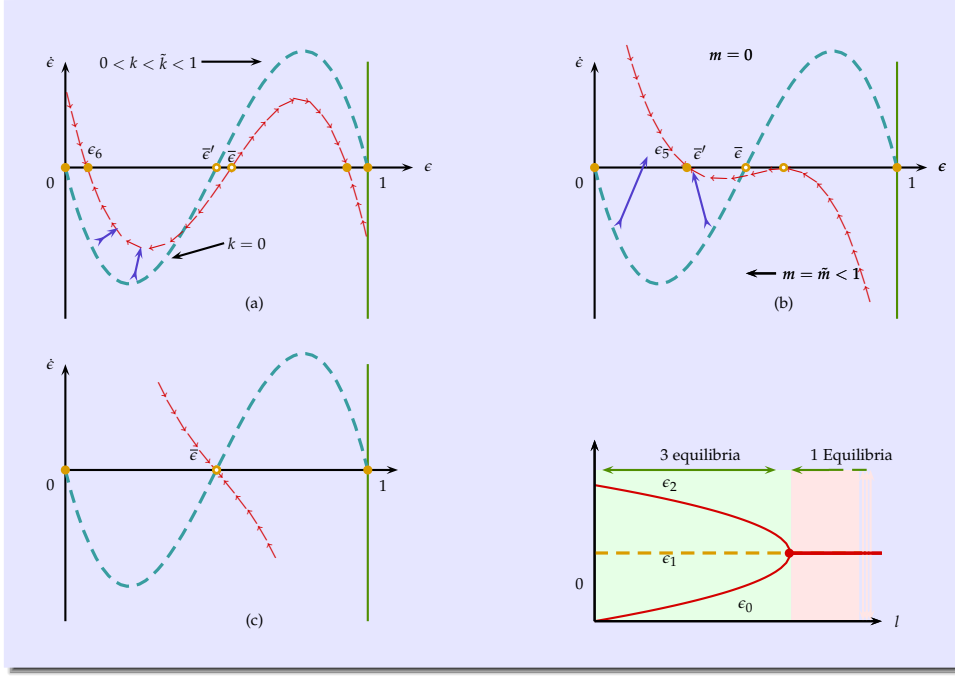


Fig. 9. THE MIXED DYNAMICS FOR  $m = l = k$

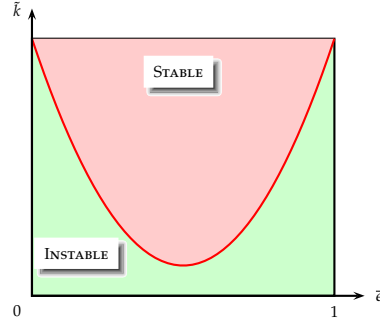


Fig. 10. THE BIFURCATION FRONTIER FOR THE MIXED EQUILIBRIUM WHEN  $l = m$

one finds the same result as the one from which we had just moved from. It would therefore be useful to explore in detail the case  $m \neq l$ .

#### 2.4.2 The $(dm2)$ dynamics

We can immediately observe that in the case of this representation of the pollution of the replication dynamics by the irrational dynamics, only one value of  $\alpha = 1$  allows to find the pareto-dominating balances and the dominant risk anew. The general case pleads for internal equilibria, even for a unique stable internal stable equilibrium when the irrational dynamics dominates the replication dynamics. Some simulations — which are

not presented here — prove that the irrational dynamics dominates very quickly the replication dynamics. Consequently, the (dm2) dynamics seems to confirm the results obtained during the study of the (dm1) dynamics — see figure 11.

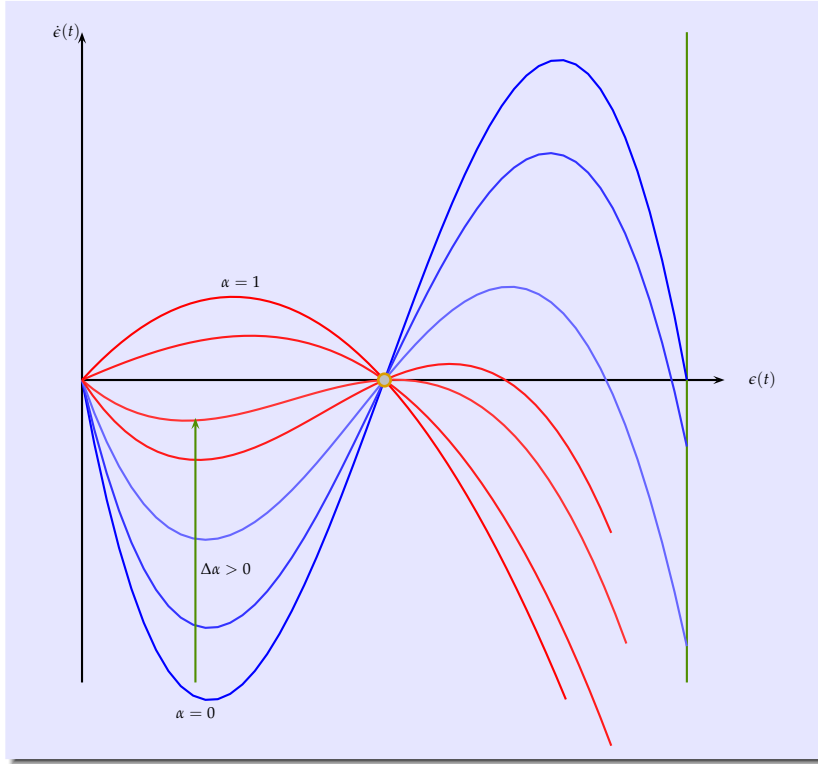


Fig. 11. THE (DM2) DYNAMICS

In fact, this description is only true on the surface, because the dynamics embed a *local emergence phenomenon*. While one should expect that with the decrease of  $\alpha$  the only observed behavior of the dynamics is the change of sign of its slope leading the internal equilibrium from instability to stability, we can observe for a very peculiar range of values of  $\alpha$  the emergence of an unexpected unstable equilibrium .

Unfortunately, we can not show this emergence other than by specifying the parameter values. We have chosen  $\bar{\epsilon} = .5$  and  $l = m = 0.1$ . For this specific range, a simple simulation shows that for the approximate value of their bounds, we have the emergence of an unstable internal equilibrium on  $]0, \bar{\epsilon}[$  when  $\alpha \in ]0.167667, 0.285667[$  — see figure 12 .

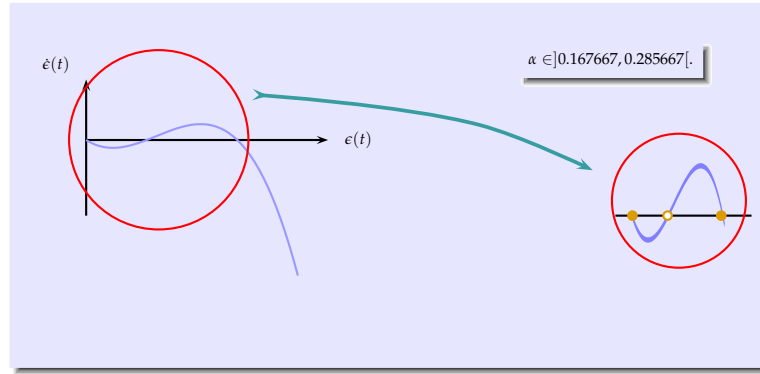


Fig. 12. LOCAL EMERGENCE OF AN INSTABLE EQUILIBRIUM

### 3 Conclusion

In this paper, one has tried to explain the persistence of a mixed equilibria composed of **C** players and **L** players. It is no way, at least in the evolutionary game formalism, a trivial affair since one is obliged to pollute the standard evolutionary game by arational behavior to generate such an equilibrium. One even shows that there is an embedded emergence phenomenon inside the mixed model.

Many extensions to this approach are under study: First of all, a differential wage distinguishing job introduce according to their place in the dual hierarchy of the labor market can easily be introduced. Then one can also improve the description of the matching by introducing two distinct effects: agents can run several things at once, in that they can not receive a simple but several jobs and / or not be matched in pairs but by N-tuple. One can also introduce rigid agents who never change their mind according to the length of the study. Finally, the information available on the labor market is necessarily dated and it would be interesting to see what is the impact on the evolutionary dynamics of a delayed reaction

In all the case, this approach reveal the complexity of the specific decision wether to undertake studies even in a stationary world.

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