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# An orbital determination of Triton with the use of a revised pole model

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## ABSTRACT

In this paper, we used the 3108 Earth-based astrometric observations from the Natural Satellite Data Center over more than 30 yr time span from 1975 to 2006 for determining the epoch state vectors of the Neptunian largest satellite Triton. These observations almost contain all modern photo and CCD observations available. In integrating perturbation equation, the barycentric frame of Neptune–Triton system is adopted, and in considering the oblateness perturbation due to Neptune, a revised pole model describing the precession of the Neptune’s pole is used in our calculation.

**Key words:** astrometry – planets and satellites: general.

## 1 INTRODUCTION

Among all the moons of the Solar system, Triton is a large and remarkable planetary satellite due to its retrograde orbit about the primary, which not only can be used to explore the dynamical evolution of Neptunian system (Matija, Brett & Gladman 2005) but also that of the Solar system itself. The Neptunian tidal friction can affect the motion of Triton by transferring angular momentum between the orbiting Triton and the spinning Neptune.

Earlier orbits for Triton were given by Eichelberger & Newton (1926) and by Harris (1984). In those models, an inclined orbit precessing at a constant rate was adopted to represent Triton’s motion. To date, the best available orbit of Triton was completed by Jacobson (1990a, 2009) and by Jacobson, Reidel & Taylor (1991) in employing a precessing pole model of Neptune (Jacobson 1990b). These previous works used the observations over a century, containing the Earth-based visual, photographic and CCD observations with also some spatial observations from radio tracking of the Voyager spacecraft. It is reported that the orbit has been improved upon to present an accuracy better than 100 km for the satellite.

In the Neptunian system, the oblateness force depends upon the orientation of the pole of Neptune. The polar motion is driven primarily by the torque due to the gravitational attraction of the Triton on the planet’s equatorial bulge, which causes the orbit to actually precess at constant inclination to a plane about the Neptune pole. The significant feature of the perturbational model of Triton arises more complicated calculations than for other satellites. In the calculations of the force due to Neptune’s oblateness made by

Jacobson (1990a) and by Jacobson et al. (1991), the pole is modelled directly as a vector  $\mathbf{r}$  precessing about an axis, direction vector of the system angular momentum. However, in our reduction, we used a revised Neptune pole model presented here, in which the formulae used are different from Jacobson (1990b).

Also, in this paper we used the data base of astrometric observations from the Natural Satellite Data Center (NSDC) of the IMCCE (Arlot & Emel’yanov 2009), including the two largest sets of recent observations of Triton as the 1192 CCD observations taken by Veiga & Vieira Martins (1996, 1998) from 1989 to 1997, and the 943 modern CCD observations made in 1996–2006 by Qiao et al. (2007). The used observations almost cover the whole period since the observations used by Jacobson et al. (1991).

## 2 DYNAMICAL MODEL

### 2.1 Orbital theory and perturbation model

The current best theory of Triton’s motion is provided by Jacobson (1990a, 2009) and by Jacobson et al. (1991), in which almost all relevant dynamical parameters were solved. The satellite orbit is a numerical integration of the equation of motion proposed by Peters (1981).

In the force model, we have included the following forces: the central force of the primary; the perturbing force due to the Sun, Saturn, Jupiter and Uranus; the perturbation due to the Neptunian oblateness which is related to the orientation of the pole of Neptune that precesses and rotates at constant rate about angular momentum vector of the Neptunian system.

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**Table 1.** Neptunian system dynamical constants from Jacobson (private communication).

Constant	Value	Units
Neptune system GM	6836 524.433 737 406	(km <sup>3</sup> s <sup>-2</sup> )
Neptune GM	6835 096.902 831 996	(km <sup>3</sup> s <sup>-2</sup> )
Triton GM	1427.530 905 409 709	(km <sup>3</sup> s <sup>-2</sup> )
Jupiter GM	126 712 764.485 82	(km <sup>3</sup> s <sup>-2</sup> )
Saturn GM	3.794 058 500 00	(km <sup>3</sup> s <sup>-2</sup> )
Uranus GM	5.794 548 600 00	(km <sup>3</sup> s <sup>-2</sup> )
Sun GM	132 713 233 266.4355	(km <sup>3</sup> s <sup>-2</sup> )
Neptune $J_2$	3406.368 915 7168 $\times 10^{-6}$	
Neptune radius	25 225	(km)
Neptune pole right ascension	299.440 222 90	(deg)
Neptune pole declination	43.400 209 139	(deg)
Precession axis right ascension <sup>a</sup>	299.364	(deg)
Precession axis declination <sup>a</sup>	43.448	(deg)

<sup>a</sup>The values were taken from Jacobson (1990b).

In our previous calculations of perturbing forces that we made for Phoebe (Shen et al. 2005), the oblateness perturbations were not significant, but for Triton, they become here very important. This is mainly due to the important Triton's mass (see Table 1) and to the proximity to Neptune. Tides are raised by the gravitational attraction of Triton on Neptune's equatorial bulge, resulting in a transfer of angular momentum between the two bodies. This can affect the orientation of the pole of Neptune in inertial space, which precesses and nutates very quickly about the angular momentum vector of the Neptunian satellite system.

The details for calculating the perturbations on the satellite due to the oblateness of the primary are given in the description of the oblateness force by Jacobson (1990b). In Triton's equations of motion, the potential function  $\phi_{\text{obl}}$  for the  $n$ th zonal harmonics of the gravity field of the planet upon the satellite is assumed to be of the following form (Sinclair & Taylor 1985):

$$\nabla\phi_{\text{obl}} = A\mathbf{r} + B\hat{\mathbf{k}}. \quad (1)$$

Here,  $\nabla\phi_{\text{obl}}$  is the oblateness acceleration,

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k},$$

$\hat{\mathbf{k}}$  is a unit vector in the  $z$ -direction,

$$A = \frac{\mu_0 + \mu}{r^3} \left[ -\frac{3}{2} J_2 \frac{a_0^2}{r^2} (5\zeta - 1) \right]$$

$$B = \frac{\mu_0 + \mu}{r^3} \left[ -\frac{3}{2} J_2 \frac{a_0^2}{r^2} (5\zeta - 2) \right]$$

$$\zeta = \frac{z}{r},$$

where  $r^2 = x^2 + y^2 + z^2$ , subscript 0 corresponds to the equatorial radius of oblate primary planet.  $J_2$  is associated with the second harmonics of oblateness functions. Here, fourth harmonics are not included as they are negligible.

When specifying the orientation of the pole, the representation of the pole right ascension and declination angles may be any function of time. Jacobson (1990a) affirms that the representation of the pole right ascension and declination by linear function of time given by Peters (1981) is not valid over the time span of the observational data. So, an alternative pole orientation model was adopted here.

Our model defines the pole as a unit vector precessing at constant rate about the angular momentum vector and the precession rate. By the rotation matrix, the vector  $\mathbf{r}_i - \mathbf{r}_0$  is rotated into the Earth equatorial frame,

$$\mathbf{r} = \mathbf{C}^T(\mathbf{r}_i - \mathbf{r}_0). \quad (2)$$

Rotation matrix  $\mathbf{C}$  is given by

$$\mathbf{C} = \begin{pmatrix} -\sin\alpha & \cos\delta & 0 \\ -\cos\alpha \sin\delta & -\sin\alpha \sin\delta & \sin\delta \\ -\cos\alpha \cos\delta & \sin\alpha \cos\delta & \sin\delta \end{pmatrix}.$$

According to the description of Jacobson (1990b), the right ascension  $\alpha$  and declination  $\delta$  of Neptune pole with respect to International Celestial Reference Frame (ICRF) satisfy the following formulae:

$$\alpha = \alpha_p + 0:696 \sin\psi - 0:003 \sin 2\psi$$

$$\delta = \alpha_p + 0:506 \cos\psi + 0:001 \cos 2\psi,$$

where

(i)  $\psi = 352:099 + 52:318T$ ;

(ii)  $\alpha$  = the right ascension of the epoch pole of Neptune;

(iii)  $\delta$  = the declination of the epoch pole of Neptune;

(iv)  $\alpha_p = 298:953$  – right ascension of the pole of the invariable plane;

(v)  $\delta_p = 43:312$  – declination of the pole of the invariable plane;

(vi)  $T$  is measured in Julian centuries from JED244 7763.5.

The more detailed model was described in the appendix of Jacobson (1990a).

The partial derivatives required for orbit fitting were also generated by numerical integration. A variational equation is integrated for each parameter to be differentially corrected. These parameters are the initial state vectors for the satellite. Each equation is of the general form (Peters 1981):

$$\frac{\partial \dot{r}_i}{\partial P_k} = \sum_{j=1}^n \frac{\partial \dot{r}_i}{\partial r_j} \frac{\partial r_j}{\partial P_k} + \left( \frac{\partial \dot{r}_i}{\partial P_k} \right)_{\text{explicit}}, \quad i = 1, \dots, n \quad (3)$$

for  $P_k$  = initial positions for the integrated satellite. The starting values of the partial derivatives at the initial epoch are

$$\left( \frac{\partial \dot{r}_i}{\partial r_j} \right)_{t=0} = \delta_{ij} \mathbf{I}, \quad (4)$$

where  $\mathbf{I}$  is  $3 \times 3$  identity matrix and the Kroneker delta is defined by

$$\delta_{ij} = \begin{cases} \mathbf{I} & \text{for } i = j \\ 0 & \text{for } i \neq j. \end{cases} \quad (5)$$

The initial positions for  $\partial r_i / \partial P_j$  are zero for all other parameters.

## 2.2 Formulae used in our alternative pole model

In this work, we use a revised pole model presented here for a better representation of the pole direction with time than previous Peters (1981) representation which could not be valid over the whole time span of the observational data (Jacobson 1990a). Moreover, we checked such a better validity of the revised pole adopted here as we obtained a better convergence in Triton's orbit than in using Peters (1981) formulae. Here, we derive the accelerations and partial

derivatives of the acceleration upon a satellite due to the oblateness of Neptune in an arbitrary planetocentric reference.

We suppose that the direction cosines of the pole vector of the planet are defined as  $(\gamma_1, \gamma_2, \gamma_3)$ , and that the planetocentric coordinates of the satellite are  $x_1, x_2, x_3$ ,

$$r^2 = x_1^2 + x_2^2 + x_3^2$$

$$\zeta = \frac{\gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3}{r}. \tag{6}$$

The potential function for the effect of the  $n$ th zonal harmonic of the gravity field the planet upon the satellite is

$$\phi = -\frac{k(\mu_0 + \mu)}{r} \sum_{n=2}^4 J_n \left(\frac{a_0}{r}\right)^n P_n(\zeta) = K \frac{1}{r^{n+1}} \sum_{n=2}^4 J_n P_n(\zeta), \tag{7}$$

where  $K = -k(\mu_0 + \mu)J_n a_0^n$ ,  $\mu_0$  is the mass of the planet,  $\mu$  is the mass of satellite,  $a_0$  is the equatorial radius of the planet and  $J_n$  is the coefficient of the  $n$ th zonal harmonic.

The acceleration component of coordinate  $x_i$  is

$$F_i = \frac{\partial \phi}{\partial x} = \frac{K}{r^{n+2}} \left(-\frac{x_i}{r} P'_{n+1}(\zeta) + \gamma_i P'_n(\zeta)\right), \tag{8}$$

where we have used the identity

$$(n + 1)P_n(x) + x P'_n(x) \equiv P'_{n+1}(x). \tag{9}$$

The partial derivative of  $F_i$  with respect to  $x_j$  is obtained after algebraic process and the use of another identity between Legendre polynomial. It is found to be

$$\frac{\partial F_i}{\partial x_j} = \frac{K}{r^{n+3}} \left( (-\delta_{i,j} + (n + 3)\xi_i \xi_j) P'_{n+1}(\zeta) + \gamma_i \gamma_j P''_n(\zeta) + (\xi_i \xi_j \zeta - \xi_i \gamma_j - \xi_j \gamma_i) P''_{n+1}(\zeta) \right), \tag{10}$$

where  $\xi_k = x_k/r$ .

### 3 OBSERVATIONS AND REFERENCE FRAME

#### 3.1 Used observations

The observations applied in our calculation are taken from NSDC, as shown in Table 2, in which columns listed contain: period, observer, type of observation and number of observation ( $N_u$ ). In the last two columns also are listed the rms of the post-fit observation residuals and their means. Relative right ascension and declination are denoted by  $\Delta\alpha$  and  $\Delta\delta$  or  $X$  and  $Y$ , absolute right ascension and declination by  $\alpha$  and  $\delta$ . In the NSDC website (<http://www.imcce.fr/sat>),  $\Delta\alpha$  and  $X$  similarly represent the relative right ascension of Triton, but respectively expressed in seconds of time and in arcseconds. These observations include all of the available observations after 1970 modern observing development, in which a large amount of observation is astrometric accurate CCD observations.

**Table 2.** List of all the series of observations used here to fit our new orbit of Triton.  $N_u$  is the number of observations for each series. We also give the mean residuals  $\mu$ (arcsec), standard deviations  $\sigma$ (arcsec) of the O–C residuals about the mean. The O–C(S) residuals are computed from our new orbit and the O–C(J) residuals are from the ephemeris of Jacobson (2009).

Obs. code	Period of observation	Observer	Type of data	$N_u$	O–C(S)		O–C(J)	
					$\sigma$ (arcsec)	$\mu$ (arcsec)	$\sigma$ (arcsec)	$\mu$ (arcsec)
689	1975–1977	U.S.N.O	Phot., $X$	28	0.067	0.025	0.476	–0.096
			Phot., $Y$	28	0.087	0.206	0.379	–0.167
689	1979–1983	U.S.N.O	CCD, $X$	114	0.047	0.001	0.097	0.131
			CCD, $Y$	114	0.073	0.219	0.095	–0.041
689	1984–1986	Flagstaff	Phot., $X$	56	0.029	0.012	0.085	0.163
			Phot., $Y$	56	0.030	0.176	0.122	0.103
119	1986–1993	Abastumani	Phot., $\alpha$	54	0.418	–0.068	0.416	–0.061
			Phot., $\delta$	54	0.409	–0.182	0.416	–0.334
874	1989–1994	Veiga	CCD, $X$	433	0.128	0.073	0.126	0.129
			CCD, $Y$	433	0.139	0.014	0.198	0.050
188	1990	Majdanak	Phot., $\alpha$	5	0.067	0.081	0.115	0.183
			Phot., $\delta$	5	0.059	0.289	0.040	0.047
874	1995–1997	Veiga	CCD, $X$	759	0.152	–0.058	0.162	0.030
			CCD, $Y$	759	0.199	–0.109	0.183	–0.052
337	1996–2006	Qiao	CCD, $\alpha$	943	0.062	–0.072	0.092	0.042
			CCD, $\delta$	943	0.043	–0.073	0.044	–0.143
689	1998–2000	Flagstaff	CCD, $\alpha$	188	0.140	–0.324	0.161	–0.038
			CCD, $\delta$	188	0.114	–0.186	0.220	–0.152
673	1999, 2001	Table Moun.	CCD, $\Delta\alpha$	6	0.111	0.033	0.247	–0.134
			CCD, $\Delta\delta$	6	0.033	0.162	0.092	–0.322
673	1999, 2001	Table Moun.	CCD, $\alpha$	6	0.370	–0.145	0.092	0.207
			CCD, $\delta$	6	0.103	–0.209	0.030	0.033
874	2000–2002	Itajuba	CCD, $\alpha$	66	0.298	–0.183	0.207	–0.067
			CCD, $\delta$	66	0.241	0.077	0.243	0.040
689	2001–2005	Flagstaff	CCD, $\alpha$	306	0.184	–0.269	0.116	0.021
			CCD, $\delta$	306	0.194	–0.137	0.052	–0.241
689	2005–2006	Flagstaff	CCD, $\alpha$	144	0.117	–0.202	0.133	–0.008
			CCD, $\delta$	144	0.124	–0.033	0.147	–0.029
All	1975–2006		$\alpha$	3108	0.167	–0.040	0.135	0.045
All	1975–2006		$\delta$	3108	0.185	–0.051	0.238	–0.134

**3.2 Barycentric reference frame**

The initial state vectors of the satellite are referred to the Earth equator and equinox of J2000.0 and centred at the Neptunian system barycentre in the ICRF which is used as the integration reference frame.

It is said from Taylor et al. (1990) that the range of the motion of Neptune relative to Neptune–Triton barycentre is of the order 0.04 arcsec in right ascension and 0.03 arcsec in declination. In our calculation, we are considering such a difference. So, we need to translate the planetocentric coordinates of Neptune into barycentre coordinates as defined:

$$r_B = \frac{1}{M_n} \sum_{i=0}^n m_i r_i. \tag{11}$$

The coordinates and velocities of the Sun and other planets are taken from the JPL DE421 ephemeris (Folkner, William & Boggs 2009).

**4 REDUCTION**

**4.1 Initial values and dynamical constants**

We started our integration at 1982 August 19 0h TDT (JD244 5200.5). The initial state values of the satellite and its relevant system dynamical constants required in the integration are provided by Jacobson et al. (1991) and by Jacobson (private communication). They are appearing in Tables 1 and 3, respectively.

In connection with the short-period coverage of the observations that we used, the dynamical constants cannot be expected to acquire very high improvements so that they are fixed in the integrating process.

**4.2 Calculating process**

We carried out a numerical integration still using the same formulae to the 12th-order Runge–Kutta–Nyström as used in our previous work (Shen et al. 2005). Its advantages are very significant over general Runge–Kutta methods in most of astrodynamical cases in which the second-order differential equations  $\ddot{r} = a(t, r)$  are allowed for the direct integration due to independence on the velocity of the satellite. The method can be especially designed for easy stepsize control and be well adapted for high-accuracy requirements (Liu 1998; Montenbruck & Gill 2000). The positions of Neptune, Sun and planets are issued from the Ephemeris DE421 given in the ICRF reference frame. Our numerical integration is referred to the barycentre of the Neptune–Triton system, Sun, Saturn and Uranus. Consequently, the new orbit is also referred to the ICRF.

The theoretical positions of Triton at Julian ephemeris date were acquired from the numerical integration. As usual way, a least-squares program has been used in order to improve the orbit.

**Table 3.** Starting state vectors of Triton at Julian ephemeris date 244 5200.5 referred to the barycentre of the Neptunian system and to the Earth mean equator and equinox J2000.0, which were taken from Jacobson et al. (1991).

Position (au)	Velocity (au d <sup>-1</sup> )
0.201 965 291 6166D–02	–0.885 239 590 1161D–03
0.447 266 749 6641D–03	–0.124 423 124 2257D–02
–0.115 875 594 8953D–02	–0.202 316 195 8544D–02

**Table 4.** Resulting state vectors of Triton at Julian ephemeris date 244 5200.5 referred to the Earth mean equator and equinox J2000.0.

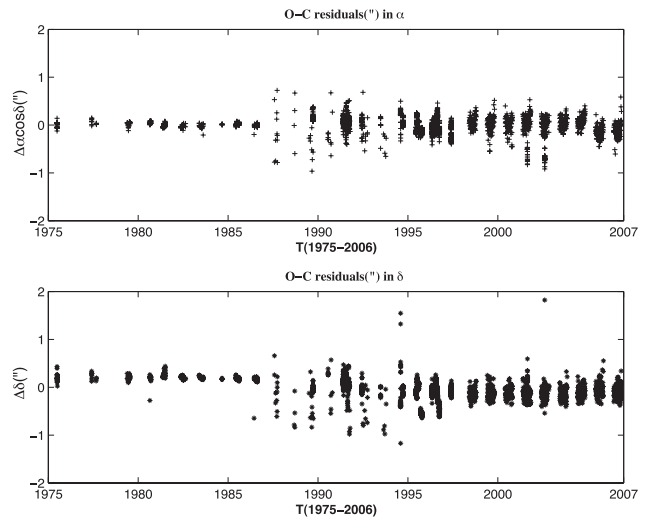
Position (au)	Velocity (au d <sup>-1</sup> )
0.201 966 383 9902D–02	–0.885 232 682 2536D–03
0.447 274 426 9529D–03	–0.124 422 037 1714D–02
–0.115 873 948 8470D–02	–0.202 316 799 7257D–02

Through repeated iterations we computed the values of the satellite epoch state vectors, which are listed in Table 4.

**4.3 Processing results**

Table 2 contains the residuals of all the sets of observations. The last two columns, respectively, give the standard deviations to the mean  $\sigma$  and the mean residuals  $\mu$ . Also, Fig. 1 shows the O–C residuals in right ascension and declination for all CCD and photographic used observations over the period 1975–2006. In the improving process, we chose a rejection level of 2 arcsec with which no observation was rejected. In Fig. 1, we can see that the old observations present slightly positive residuals in declination (up to + 0.1 arcsec) and oppositely, the new observations present slightly negative residuals (up to about –0.1 arcsec). One explanation could be that the old observed positions, until about 1990, are relative to the planet, and the new observed positions, made after 1990, are absolute, as it is mentioned in the NSDB of the IMCCE. This could be consequent to a drift in declination (up to –0.2 arcsec) of the DE421 ephemeris of Neptune that we have used to compare the absolute observed positions of Triton. Such a drift of the ephemeris of Neptune had previously been pointed out by Arlot, Dourneau & Le Campion (2008) concerning the DE405 ephemeris. Our result shows that the new DE421 could not have been completely corrected the drift of the previous DE405.

In Table 2, we can see that the set of 943 CCD observations made over the period 1996–2006 by Qiao et al. (2007) at the She-shan Station is the most accurate among all these observations sets with Flagstaff’s set made in the period 1984–1986 but which only contains 56 observations. So, this observation set by Qiao et al.



**Figure 1.** Plots of residuals O–C in right ascension (above) and declination (below) from integration fitting all CCD and photo observations over 1975–2006.

(2007) appears quite important to be used in this work for obtaining a precise determination of the orbit presented here.

The new orbit will be available for the scientific community on the Saimirror MULTI-SAT server of the IMCCE (Emel' Yanov & Arlot 2008) before the end of the year 2013 at the following address: [www.imcce.fr/hosted\\_sites/saimirror/nssephf.php](http://www.imcce.fr/hosted_sites/saimirror/nssephf.php). In the meantime, we propose to the readers to send us a mail at the address given in the text of the paper with the dates at which they want to obtain the positions of Triton computed by our new orbit. Then we will answer them as soon as possible.

### 5 COMPARISON WITH JACOBSON'S THEORY

For Triton, the previous investigations were primarily completed by Jacobson et al. (1991) and by Jacobson (2009). In order to check our ephemeris, we have used the ephemeris of Jacobson (2009) covering 1975–2006 period. For a better comparison of our new orbit to Jacobson (2009), we have calculated the O–C residuals from both of these ephemerides. The O–C residuals from Jacobson (2009) were obtained from the IMCCE website. The so-derived residuals are presented in Table 2. The results show a rather good agreement between the two ephemerides, within less than 0.1 arcsec, which is lower than the observational errors visualized in Fig. 1 and which can reach 0.2 arcsec or more. Table 2 emphasizes the high quality of our new orbit as its mean residuals computed for all the observations appear to be slightly lower in right ascension (−0.040 arcsec) and in declination (−0.051 arcsec) than those derived from the Jacobson (2009) ephemeris (0.045 and −0.134 arcsec, respectively). Also, we have used the Jacobson (2009) ephemeris as observing points covering the 1982–2006 period. Fig. 2 gives the differences between the two ephemerides, above in right ascension and below in declination. Fig. 2 shows a rather good agreement between the two orbits, as differences remain within about 0.1 arcsec and are lower than the observational accuracy. In Fig. 2, we can see some periodic differences, mainly in right ascension. With the help of Fourier analysis, we have obtained their following main periodic term:

$$\Delta\alpha(t)(\text{arcsec}) = 0.01175 \text{ arcsec} + 0.1385 \text{ arcsec} \cos(6.245t)$$

which may be attributed to the pole model used as it is different from Jacobson's.

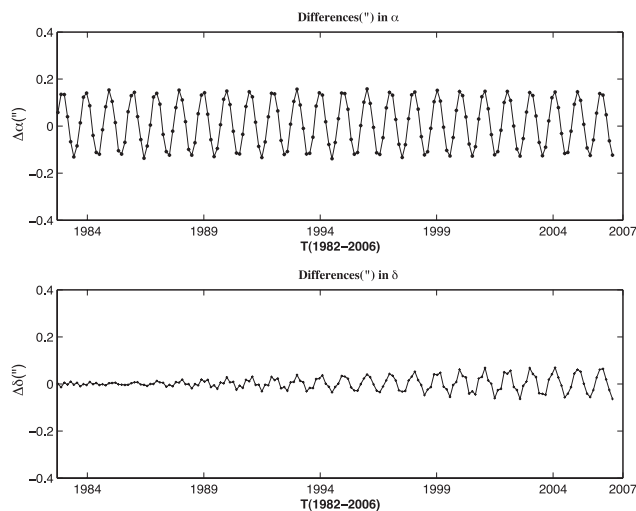


Figure 2. Plots of the differences in right ascension (above) and declination (below) from integration fitting Jacobson's ephemeris over 1984–2006.

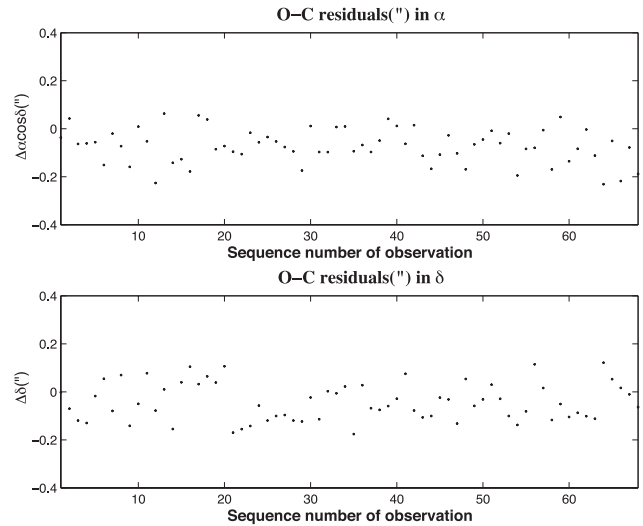


Figure 3. Plots of the residuals in right ascension (above) and declination (below) from fitting our integration to the 68 brand new observations that we have taken in 2012.9.13–19.

In order to estimate the accuracy of our new ephemeris in the recent period, Fig. 3 gives the plots of the residuals acquired from fitting our integration to 68 brand new observations that we made in 2012 September at Lintong station of NTSC by making use of a 36 cm catadioptric telescope. These data have not yet been published, so they cannot have been and used in any previous orbit improvement. In this specific fit, we have shown that the rms residuals are 0.098 arcsec in right ascension and 0.101 arcsec in declination. This result shows that our new orbit appears to be very accurate, within less than 0.1 arcsec for the recent period. This emphasizes that the new orbit has better outside agreement.

### 6 CONCLUSION

In this paper, we have presented a new determination of the orbit of Triton. We have fitted it, via numerical integration, to the new CCD and astrophotographical astrometric observations, including the largest sets of recent CCD observations of Triton as the data obtained in our recent observing campaign (Qiao et al. 2007) and the data collected by Veiga & Vieira Martins (1996, 1998). On one hand, we have translated the centre of coordinate system to the barycentric coordinate of Neptune–Triton system. On the other hand, we have considered the effect due to the precession of pole with a revised Neptune's pole model presented here. For both of those reasons, our orbit represents an improvement compared to the earlier reported orbits. First, this has been checked with a comparison of our new orbit to Jacobson (2009). We have shown that our new orbit presents slightly lower mean residuals than Jacobson's for all the observations spreading over the period 1975–2006. Also, we have emphasized the high accuracy of our new orbit for the recent period, within less than 0.1 arcsec, with another comparison to a brand new set of unpublished observations that we made in 2012 September. Moreover, we have pointed out a possible negative drift in declination, up to −0.2 arcsec, of the DE421 ephemeris of Neptune, similar to the drift already pointed out by Arlot et al. (2008) for the previous Neptune DE405 ephemeris. Obviously, it can be expected that the ephemerides based on the new orbit will support future scientific investigation of the Neptunian system, especially as

the new orbit will be soon available on the Saimirror MULTI-SAT server of the IMCCE.

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