Boundary Element Method for transient viscoelastic flow: the MPTT model

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This paper deals with the modelling of viscoelastic flows in transient regimes using the Modified Phan Tien and Tanner (MPTT) model. This model is the most recent one which can predict stress overshoot in unsteady shear flow and is based on the description of the liquid microstructure rather than empirical and mathematical developments. Here, the analysis of the squeeze film between parallel plates is presented. It is an alternative in quantifying and understanding the two main factors which may enhance the carrying capacity (inertia effect and viscoelastic behavior) situation which can occur, for instance, in the case of overloading in journal bearings. A boundary element method is used here to model such complex flow. The main factors have been extracted and the restrictions due to a rough estimate of the lubricant's flow are shown.

KEY WORDS

Hydrodynamic Lubrication, Squeeze-Film Lubrication, Non-Newtonian Lubricants, Inertia Effects, Viscoelastic Lubricants

INTRODUCTION

Modern lubricants generally incorporate a range of additives including soluble long-chain polymers. These are called viscosity index (VI) improvers and were originally introduced to limit the drop in viscosity of the base oils with temperature. However, due to these polymers, the oil behavior becomes non-Newtonian. The most widely recognized influence is the shear-thinning effect which results in a decrease of viscosity at high shear rate 10^5–$10^8\ s^{-1}$ (*1*). This effect on its own should reduce the magnitude of the drag force and, in addition, the load. However, several experiments in dynamically and steadily loaded bearings (*2*)–(*4*) have shown that the use of additive oils not only reduces the friction coefficient but also increases the carrying capacity compared to Newtonian oils.

Recent correlations between the rheological parameters of the lubricant and the minimum film thickness, in an operating bearing, have further supported the argument of the significance of viscoelastic effects in improving characteristics (*3*), (*5*). Several studies (*3*), (*6*)–(*8*) address the effect of the extensional stresses due to severe leakage in the case of a short bearing. Such stresses would give additional normal stresses on the shaft. However, Lodge (*9*) notes that the measurement of the first normal stress difference is not high enough to predict an additional viscoelastic contribution comparable to the viscous contribution. Other arguments have also been put forward, such as those of Bouldin and Berker (*10*), who suspect that the cavitation phenomena may be influential. There is no overall consensus to explain the enhancement of the carrying capacity and the decrease in friction coefficient due to a lack of clear theoretical justification. However, the theoretical basis reveals several major difficulties in defining the response of the fluid:

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NOMENCLATURE

- n = parameter of the MPTT model
- L = length of the contact
- L/h_0 = ratio of length of the contact to the minimum film thickness
- t = time
- $tr(S)$ = trace of matrix S
- U_i = velocity component
- V_p = squeeze velocity of the upper surface
- W_i = Weissemberg number
- x_i = coordinate in the i direction
- ∇ = gradient operator
- ε = parameter of the MPTT model
- Γ = parameter of the MPTT model
- λ = relaxation time
- δ_{ij} = Kronecker symbol
- ξ = parameter of the MPTT model

1. The choice of a correct constitutive law.
2. The evaluation of the inertia terms.
3. The search for a proper boundary condition which will take into account cavitation with a viscoelastic fluid.
4. The analysis of the effect of temperature and pressure on the viscosity.
5. The resolution of a time-dependent problem by considering these previous points.

Furthermore, because the stresses are a priori unknown, it becomes difficult to simplify the equations in the same way as in the lubrication theory (*11*) and to solve the flow in a complex journal bearing directly. In order to simplify this analysis and to keep a realistic transient viscoelastic problem, consideration of the squeeze film between parallel plates presents one alternative in quantifying and in understanding the two main factors which may enhance the load: the inertia effect and the viscoelastic behavior. This flow may be regarded as the main component in lubrication mechanisms where the load, or the velocities, suddenly change as a function of time. Such conditions are present in crankshaft journal bearings or occur when the fracture of a component generates a sudden overload, e.g., the loss of a blade from a turbine shaft.

From a numerical point of view, the research of a global method in order to solve problems involving viscoelastic flows leads to the elimination of the finite difference because it is specific for only one single rheological law and only one single domain of resolution. Nevertheless, an alternative is proposed with the integral methods such as the finite element and boundary element methods which give more possibilities to solve these kinds of problems. During the past 10 years, a lot of problems related to viscoelastic flows have been solved using the finite element method as it offers specific attractive advantages from the scientific and industrial points of view. Nevertheless, this method presents some deficiencies when strong nonlinearities appear from the rheological laws. A good compromise between the complexity of the resolution and the general aspect of the problem can be given by the boundary element method formulated by Bush et al. (*12*). The non-linearities here are averaged on the entire domain of resolution and are treated as pseudo-forces. Furthermore the numerous works in Refs. (*12*)–(*19*) have shown the strength of the numerical scheme which comes essentially from the continuum equation, which is implicitly taken into account, and the uncoupling of the equilibrium equations from the constitutive equation.

INTEGRAL FORMULATION

Basic Equations

The authors consider here the case of an isothermal laminar flow of a viscoelastic incompressible fluid in a domain Ω of boundary Φ. In a Galilean system of axis, using Cartesian coordinates, the equilibrium equations and the equation of continuity read:

$$\frac{\partial T_{ij}}{\partial x_j} + \rho(f_i - a_i) = 0 \qquad [1]$$

$$\frac{\partial U_i}{\partial x_i} = 0 \qquad [2]$$

where f_i are the components of the volumic forces, a_i are the convective terms $U_k \frac{\partial U_i}{\partial x_k}$, T_{ij} are the components of the total stress tensor, and ρ is the specific mass.

The fluid is a viscoelastic liquid. Only one mode of relaxation is considered in this numerical approach. The rheological law is of the Maxwell type. It can be described with the following general differential equation:

$$\begin{cases} T_{ij} p' \delta_{ij} + 2\eta_s D_{ij} + S_{ij} \\ \lambda \frac{\mathcal{D} S_{ij}}{\mathcal{D} t} + R_{ij} = 0 \end{cases} \qquad [3]$$

where S_{ij} are the components of the non-linear stress tensor, R_{ij} are the components of the tensor which characterizes the behavior law, D_{ij} are the components of the rate of deformation tensor $D = 0.5(L + L')$ and $L = \nabla U^T$ the transposed velocity vector gradient, p' is an arbitrary pressure, η_s is the viscosity of the solvent, and λ is the relaxation time of the polymer.

The differential term $\frac{\mathcal{D} S_{ij}}{\mathcal{D} t}$ corresponds to an objective derivative of the extra-stress tensor to the time.

Bush et al. have shown (*12*) that it is possible and preferable to decompose the total stress tensor T_{ij} in a linear part and in a second non-linear part E_{ij} during the numerical treatment. Crochet et al. also postulate this decomposition because of the addition of a viscous term in the equilibrium equations which reduces the numerical instabilities (*20*).

Therefore, $E_{ij} = S_{ij} - 2\eta_p D_{ij}$, where $\eta_n = \eta_s + \eta_p$ is the total viscosity and η_p the viscosity at shear rate equals to zero for the viscoelastic model. Then, the ratio of viscosities β can be introduced:

$$\beta = \frac{\eta_p}{\eta_p + \upsilon_s}$$

Modified Phan-Tien and Tanner Model (MPTT)

Reviewing numerous publications (*21*) shows that both shear-thinning and other viscoelastic effects where stress overshoot appears must be taken into account in transient lubrication problems. One of the most recent models which can predict stress overshoot in unsteady shear flow in contrast with many other works is the MPTT model. As rheological studies have progressed over the years, they have become increasingly based on the description of the liquid microstructure rather than the empirical and mathematical developments. Several interpretations have been given to explain the stress overshoot phenomena (*22*):

- An important resistance to the macromolecule's uncurling which leads to a stretching movement rather than a global rotation in the lubricant.
- A perturbation of the flow of the lubricant induced by the macromolecules's movement.

The MPTT model is based on the idea of the effective velocity gradient. It comes from the polymer's network theory. Nevertheless, this model presents some restrictions due to a rough estimate of the lubricant's flow (*22*).

In the case of an MPTT model the differential term $\frac{\mathcal{D}S}{\mathcal{D}t}$ can be written as follows:

$$\frac{\mathcal{D}S}{\mathcal{D}t} = \frac{\partial S}{\partial t} + u\nabla.S - \mathcal{L}.S - S\mathcal{L}^t \qquad [4]$$

where $\mathcal{L} = \mathbf{L} - \xi\mathbf{D}$. The R_{ij} tensor from Eq. [3] can be expressed as:

$$R_{ij} = h(tr(S))S_{ij} - 2\eta(\mathring{\gamma})D_{ij} \qquad [5]$$

with:

$$h(tr(S)) = \exp\left(\varepsilon\frac{\lambda}{\eta_p}tr(S)\right) \qquad [6]$$

Also,

$$\eta(\mathring{\gamma}) = \eta_p\frac{1 + \xi(2 - \xi)\lambda^2\mathring{\gamma}^2}{(1 + \Gamma^2\mathring{\gamma}^2)^{(1-n)/2}} \text{ with } \mathring{\gamma}^2 = 2\ tr(D^2) \qquad [7]$$

where ξ, ε, n and Γ are the parameters of the MPTT model.

Boundary Conditions

The external boundary of the domain of the resolution can be divided into two parts, as Fig. 1 shows.

On $\varnothing$ the boundary conditions can be written in terms of imposed velocities $\hat{U}_i$:

$$-U_i = \hat{U}_i \text{ on } \varnothing_1$$

On the second part $\varnothing_2$, the authors applied a strain vector $\hat{t}_i$:

$$-t_i = T_{ij}n_j = \hat{t}_i \text{ on } \varnothing_2$$

where n is the normal outward to the surface.

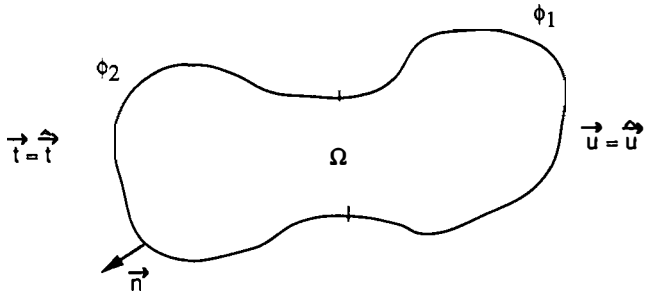

Fig. 1—Boundary conditions.

Formulation of the Problem—The Weighted Residual Method

The problem represented by Eqs. [1]–[3] can be substituted for by an equivalent integral formulation which can be obtained from the weighted residual method. This method consists of the evaluation of the U, T and p functions which verify the necessary conditions of derivation and integration on the domain of resolution Ω and its boundary Φ so that the integral equation defined on Φ and Ω can be determined:

$$\int_\Omega\left(\frac{\partial T_{ij}}{\partial x_j} + \rho(f_i - a_i)\right)U_i^*d\Omega + \int_\Omega\frac{\partial U_i}{\partial x_i}p^*d\Omega = \int_{\varnothing_1}(\hat{U}_i - U_i)t_i^*d\varnothing + \int_{\varnothing_2}(\hat{t}_i - t_i)U_i^*d\varnothing \qquad [8]$$

where U^* and t^* are the vectorial weighted functions, and p^* is the scalar weighted function defined on the domain Ω and its boundary Φ.

Equation [8] presents three types of integrals:

- The first one on the left-hand side represents the weighted residual of the equilibrium equations on the domain Ω with the weighted vector function U^*.
- The second integral corresponds to the weighted residual of the equation of continuity on the domain Ω with the weighted scalar function p^*.
- The right-hand side of Eq. [8] weighs on the boundary Φ the errors of approximation between the approximate solutions and the boundary conditions with the weighted vectorial functions U^* and t^*.

The following relations result:

$t_i^* = T_{ij}^*n_j$ the strain vector

$T_{ij}^* = -p^*\delta_{ij} + 2\eta_n D_{ij}^*$ the stress tensor

$D_{ij}^* = \frac{1}{2}\left(\frac{\partial U_i^*}{\partial x_j} + \frac{\partial U_j^*}{\partial x_i}\right)$ the rate of deformation tensor

Integrating by parts leads to the following equation:

$$-\int_\Omega T_{ij}\frac{\partial U_i^*}{\partial x_j}d\Omega - \int_\Omega\frac{\partial U_i}{\partial x_i}p^*d\Omega + \int_\Omega\rho(f_i - a_i)U_i^*d\Omega = \int_{\varnothing_1}(\hat{U}_i - U_i)t_i^*d\varnothing - \int_{\varnothing_1}t_iU_i^*d\varnothing - \int_{\varnothing_2}\hat{t}_iU_i^*d\varnothing \qquad [9]$$

After some manipulations on the two first integrals on the right-hand side, one obtains the following reciprocity principle:

$$\int_\Omega T_{ij}\frac{\partial U_i^*}{\partial x_j}d\Omega - \int_\Omega\frac{\partial U_i}{\partial x_i}p^*d\Omega = \int_\Omega T_{ij}^*\frac{\partial U_i}{\partial x_j}d\Omega - \int_\Omega\frac{\partial U_i^*}{\partial x_i}pd\Omega + \int_\Omega E_{ij}\frac{\partial U_i^*}{\partial x_j}d\Omega \qquad [10]$$

Integrating by parts again and introducing the result in Eq. [8], one finally obtains the following equation:

$$-\int_{\Omega}\frac{\partial T_{ij}^{*}}{\partial x_j}U_i d\Omega - \int_{\Omega}\frac{\partial U_i^{*}}{\partial x_i}p d\Omega - \int_{\Omega}\rho(f_i - a_i)U_i^{*}d\Omega$$

$$+\int_{\Omega}E_{ij}\frac{\partial U_i^{*}}{\partial x_j}d\Omega = -\int_{\varnothing_1}U_i t_i^{*}d\varnothing \qquad [11]$$

$$-\int_{\varnothing_2}\hat{U}_i t_i^{*}d\varnothing + \int_{\varnothing_1}t_i U_i^{*}d\varnothing + \int_{\varnothing_2}\hat{t}_i U_i^{*}d\varnothing$$

A particular solution U^* to supress the first term of the left-hand side of Eq. [11] must now be found. Then, the unknowns U and t only appear in the boundary integrals. This can be done by finding the fundamental solution. Thus U^*, t^* and p^* have to satisfy the following equations:

$$\frac{\partial T_{ij}^{*}(P,Q)}{\partial x_j} = -\Delta(P,Q)\delta_{ik} \qquad [12]$$

$$\frac{\partial U_i^{*}(p,Q)}{\partial x_i} = 0 \qquad [13]$$

The index k indicates the direction of application of the concentrated force. This problem involves:

- P: source point, point of the domain where the concentrated force is applied.
- Q: description point, point of the domain where the velocities and the stresses induced by the concentrated force are evaluated.
- Δ: the delta dirac function:
  $\Delta(P,Q) = 0$ if $P \neq Q$
  $\Delta(P,Q) = \infty$ if $P = Q$

The analytical solution of this particular flow is well defined U^* is usually known as Stokeslet. When the flow is bidimensional and if r is the distance between the source point P and the description point Q, the velocity Cartesian components of the Stokeslet are given by:

$$U_{kj}^{*} = \frac{1}{4\pi\eta_n}\left(-\ln(r)\delta_{kj} + \frac{\partial r}{\partial x_k}\frac{\partial r}{\partial x_j}\right) \qquad [14]$$

In the same way, when n is the outward normal to the surface located at the point Q and $(\vec{Q},\vec{n},\vec{s})$ a direct orthonormal referential, as shown in Fig. 2, the Cartesian components of the stress vector read:

$$t_{kj}^{*} = -\frac{1}{\pi r}\left(\frac{\partial r}{\partial n}\frac{\partial r}{\partial x_k}\frac{\partial r}{\partial x_j}\right) \qquad [15]$$

In this expression, the index j refers to the Cartesian component of the velocity vector or the stress vector and the index k to the direction of the application of the force.

Boundary Integral Equation

Using the equilibrium equations of the Stokeslet [12] and substituting in Eq. [11] the following is obtained:

$$\delta_{ki}U_i(P) = \int_{\Omega}\rho(f_i(Q) - a_i(Q))U_{ki}^{*}(P,Q)d\Omega$$

$$-\int_{\Omega}E_{ij}(Q)\frac{\partial U_{ki}^{*}(P,Q)}{\partial x_j}d\Omega \qquad [16]$$

$$-\int_{\varnothing}t_{ki}^{*}(P,Q)U_i(Q)d\varnothing + \int_{\varnothing}t_i(Q)U_{ki}^{*}(P,Q)d\varnothing$$

where $k = 1,2$.

When the source point P and the description point Q are the same, the integrals on Ω and Φ require specific treatment (*22*).

Numerical Procedure

Equation [16] gives a relation between the velocity vector of an arbitrary point of the domain, namely the source point P, and the velocity and stress vectors at the boundary. A numerical approximation using a discretization for the domain Ω as well as the boundary Φ is needed (*22*). The domain Ω is divided in cells and a linear approximation for the boundary Φ is used here as shown in Fig. 3. The nonlinear terms are treated as pseudo-forces and the discretization gives the following nonlinear system:

$$[A]\{X\} = [B]\{Y\} + \{b\} \qquad [17]$$

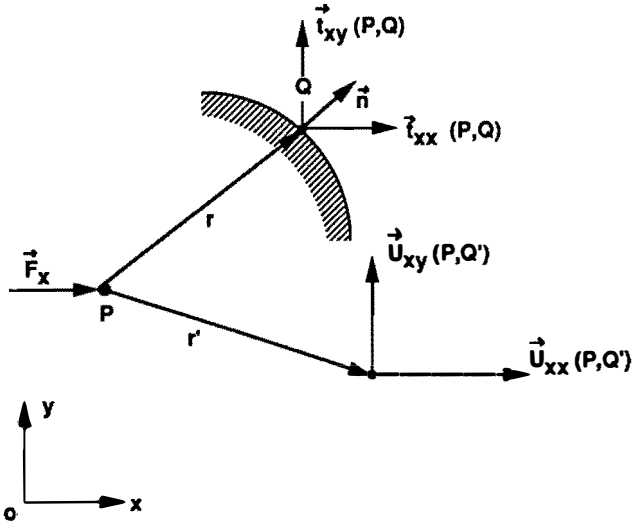

Fig. 2—Fundamental solution.

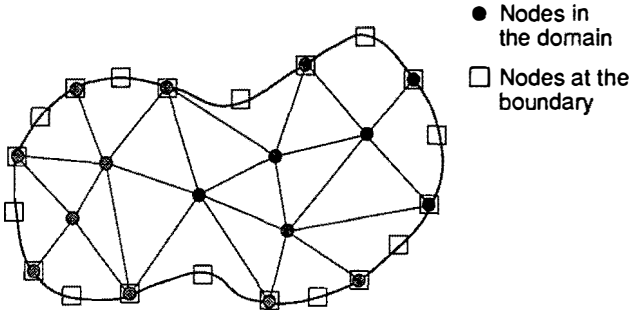

Fig. 3—Discretization.

where $\{X\}$ represents the vector of unknowns, $\{Y\}$ represents the known values of the velocity and stress vectors on the boundary, and $\{b\}$ is a vector including the contribution of the nonlinear terms. An iterative scheme is needed to obtain the final solution.

RESULTS

MPTT Model—The Squeeze Film Problem

The behavior of a viscoelastic fluid differs strongly from a Newtonian one (anisotropic normal stresses, stress overshoot, relaxation, etc.). It is described by fluid reactions, e.g., in the journal bearing, which can not be only resolved in terms of shear stresses and hydrostatic pressure generation. Then, if one considers, for example, overloading the effect of normal stresses, the consideration of four significant regions; shown in Fig. 4, is required:

1. The shearing zone which is located at the film formation where the Couette flow is predominant. Anisotropic normal stresses are generated there.
2. The stretching region which is located in the convergent zone near the minimum film thickness area. The fluid particle trajectories are complex and can be noted for the high eccentricities by a recirculated flow phenomenon. The flow is essentially of the Poiseuille type. The effect of stretching resistance is dominant.
3. The squeeze effect which is located at the minimum film thickness region. Due to the sudden movement of the shaft to the housing, the fluid is suddenly squeezed between two surfaces. The inertia effects as well as the relaxation time and the stress overshoot are strongly present.
4. The cavitation region which takes place in the divergent zone where the pressure is less than the atmospheric one which leads to the rupture of the lubricant flow (*23*).

The modelling of these problems is very complex. Nevertheless one can concentrate their attention by looking for the reaction of the fluid to a sudden overload to the squeeze film problem thus to extract the main factors. In this area the viscoelastic effects as well as the inertia ones are dominant.

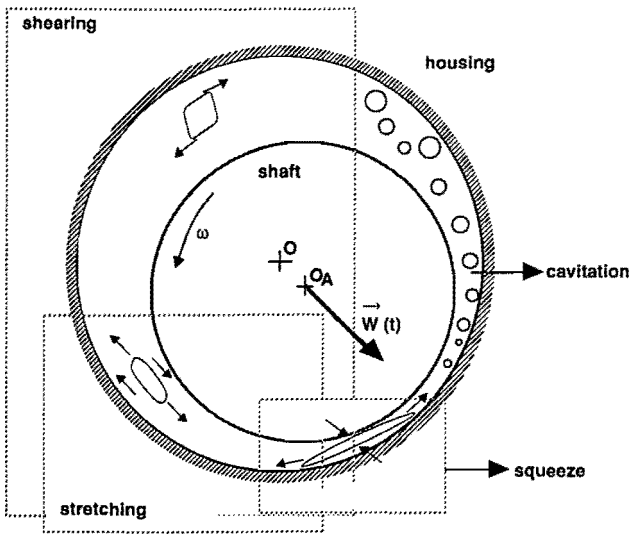

Fig. 4—Solutions in a journal bearing in the case of overloading.

Squeeze Film at Constant Speed

The aim of this approach is to point out the influence of the different parameters of the MPTT model and then to link the microstructural aspects to more global quantities represented mainly by the fluid reaction.

The fluid parameters are the following: $W_i = 0.4$, $\varepsilon = 0$, $\beta = 0.5$, $L/h_0 = 5$.

Figure 5 presents the load evolution for five values of ξ: $\xi = 0$ (fluid of Oldroyd-B type), $\xi = 0.1$, $\xi = 0.2$, $\xi = 0.3$, and $\xi = 0.4$. Except for $\xi = 0$ the evolution of the load against ξ presents an evolution greater than that given by a Newtonian fluid. This can be explained by the specific reaction of the MPTT fluid which gives an overshoot reaction with an amplitude increasing with the increase of the parameter ξ (*22*).

Weissemberg Number Influence (Figure 6)

The following parameters are used: $\xi = 0.4$, $\varepsilon = 0$, $\beta = 0.5$, $L/h_0 = 5$. The Weissemberg number takes the three following values: $W_i = 0.04$, $W_i = 0.2$, $W_i = 0.4$. These values correspond respectively to the following relaxation times for an upper plate speed equal to $V_p = 0.2\ \text{ms}^{-1}$ and a film thickness of $h_0 = 10^{-6}$ m: $\lambda = 10^{-5}$ s, $\lambda = 5.10^{-5}$ s, $\lambda = 10^{-4}$ s.

Contrary to an Oldryod-B fluid ($\xi = 0$), the magnitude of the load increases with the Weissemberg number. At low W_i

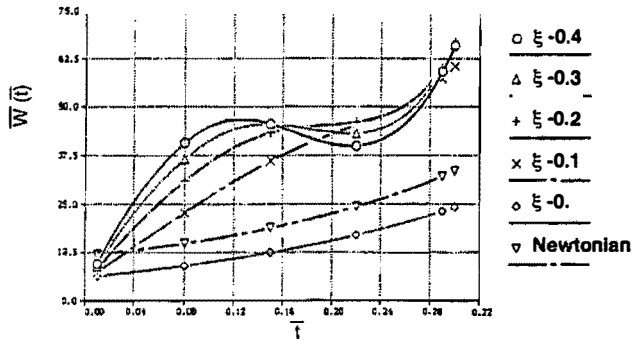

Fig. 5—Influence of ξ parameter. MPTT fluid: $W_i = 0.4$, $\beta = 0.5$, $\varepsilon = 0$, $\xi = 0$, $\xi = 0.1$, $\xi = 0.2$, $\xi = 0.3$, $\xi = 0.4$.

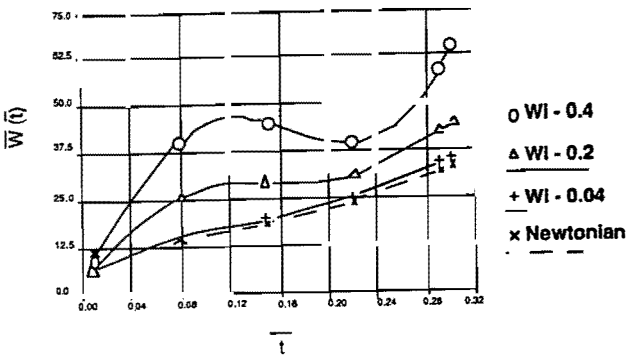

Fig. 6—Influence of Weissemberg number. MPTT Fluid: $\xi = 0.4$, $\beta = 0.5$, $\varepsilon = 0$, $n = 1$, $W_i = 0.04$, $W_i = 0.2$, $W_i = 0.4$.

values (W_i = 0.4) the fluid reaction leads to a Newtonian reaction. For higher Weissemberg numbers (W_i = 0.4), the load evolution shows an important overshoot. The maximum is reached when t = 0.12 for this case. This phenomenon is less pronounced for W_i = 0.2.

Influence of β Parameter (Figure 7)

The parameter β defines the ratio of the polymer viscosity and the total viscosity (polymer and solvent). When β approaches 1 the fluid contains only polymer. On the contrary, when β tends to be zero, the fluid becomes Newtonian. The following parameters have been used: W_i = 0.4, ξ = 0.4, ε = 0, L/h_0 = 5. β takes the following values: 0, 0.1, 0.3, 0.5, 0.7.

The results show that the difference of the load obtained with the Newtonian theory increases when the viscoelastic proportion in the fluid increases.

Influence of the L/h_0 Ratio (Figure 8)

Contrary to a Newtonian or an Oldroyd-B fluid, the dimensionless reaction of the MPTT fluid $W(t)$ depends on the ratio L/h_0. The comparison between the results obtained for W_i = 0.4, ξ = 0.4, n = 1, ε = 0, β = 0.5 for two values of L/h_0 (five and 10) shows that the load overshoot occurs rapidly when L/h_0 increases and that the magnitude of the overshoot becomes more important when L/h_0 increases. These results can be explained by the fact that when the ratio L/h_0 increases the rates of deformation become higher. Then, the viscoelastic effects are strengthened.

Squeeze Film at Constant Load

The influence of the different parameters for the MPTT fluid has been shown in the case of a squeeze film problem at constant speed. Nevertheless it is the study of the squeeze film at constant load which is closest to the case of a shaft submitted to a sudden load. The authors have to add to the system of Eqs. [1]–[3] a new equation which describes the upper plate movement. It takes the following form:

$$m\frac{d^2h(t)}{dt^2} = W(t)_{\text{fluid}} + W_{\text{imposed}} \qquad [18]$$

where m is the mass of the upper plate, $h(t)$ represents film thickness, $W(t)_{\text{fluid}}$ is the fluid reaction, and W_{imposed} represents imposed load constant in time.

The initial conditions are

- At time 0^- the fluid is in the rest, the stress state is nil.
- At time 0^+ the upper surface moves at V_0 velocity.

The dimensionless numbers used are the following:

1. The fluid characteristics: ξ, n, ε and β.
2. The Weissemberg number defined at the initial time: $W_i = \lambda\frac{V_0}{h_0}$.
3. The ratio L/h_0.
4. The dimensionless mass: $\overline{m} = \frac{m}{W_{\text{Newtonian}}}\frac{V_0^2}{h_0}$.
5. The dimensionless imposed load: $\overline{W}_{\text{imposed}} = \frac{W_{\text{imposed}}}{W_{\text{Newtonian}}}$.

$W_{\text{Newtonian}}$ is taken as a reference. It represents the reaction of a Newtonian fluid of $\eta_n = \eta_s + \eta_p$ viscosity to a squeeze flow at the initial time:

$$W_{\text{Newtonian}} = \frac{8\eta_n V_0 L^3}{h_0^3}B \qquad [19]$$

A numerical scheme of explicit Euler type is used to obtain the position, the velocity and the acceleration of the upper surface at each time step.

Two cases have been considered for MPTT fluid:

1. The imposed load corresponds to the weight of the upper plate. The acceleration at t = 0 is equal to 0, m = 81.5, W_{imposed} = 1.
2. The imposed load corresponds to the weight of the upper solid. The reaction of the fluid is taken equal to zero at t = 0 and then the acceleration is different from zero, m = 163, W_{imposed} = 1.

The parameters for the MPTT model are taken as: β = 0.5, ε = 0, L/h_0 = 5.

Case A (Figures 9 and 10)

Two values of W_i have been considered: W_i = 0.04 and W_i = 0.4. The authors take 0.4 and 0 for ξ values (Oldroyd-B fluid).

The instantaneous reaction of the viscoelastic fluid is only coming from the solvent. Then at $t = 0^+$ contrary to a Newtonian fluid of the same viscosity, the fluid reaction cannot balance the imposed load: $W(t = 0)_{\text{fluid}} = 0.5\ W_{\text{imposed}}$. This leads to an acceleration of the upper plate at $t = 0^+$ induced by the relaxation time of the viscoelastic fluid.

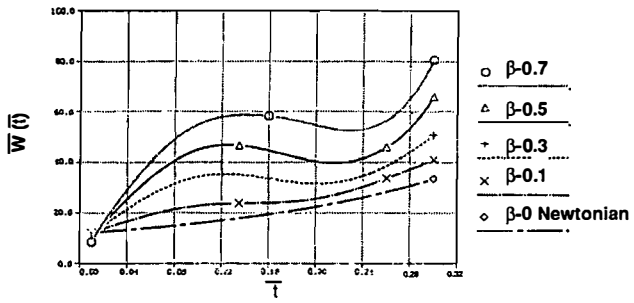

Fig. 7—Influence of β parameter. MPTT Fluid: W_i = 0.4, ξ = 0.4, n = 1, ε = 0, β = 0, β = 0.3, β = 0.5, β = 0.7.

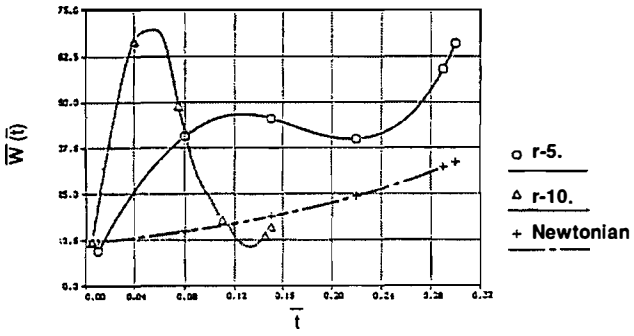

Fig. 8—Influence of L/h_0 parameter. MPTT Fluid: W_i = 0.4, ξ = 0.4, β = 0.5, n = 1, ε = 0, L/h_0 = 5, L/h_0 = 10.

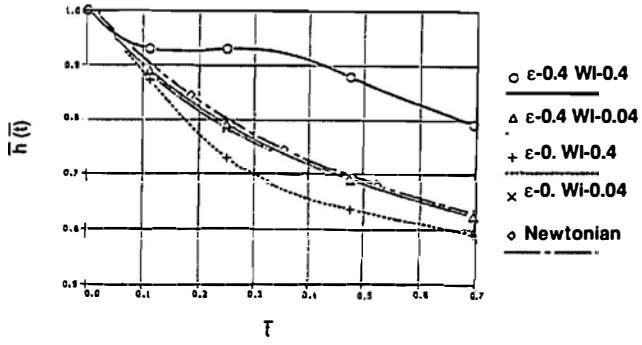

Fig. 9—$h(\bar{t})$, $\overline{m}$, = 81.5, $\overline{W}_{imposed}$ = 1; MPTT Fluid.

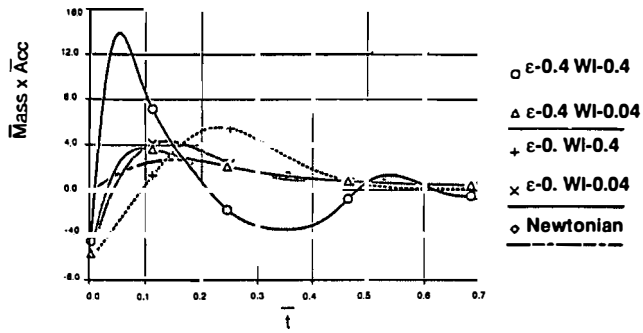

Fig. 10—Mass × Acceleration = $\overline{m}$ = 81.5, $\overline{W}_{imposed}$ = 1; MPTT Fluid.

When W_i = 0.04 the plate acceleration is rapidly counterbalanced by the fluid reaction which is slightly greater than the Newtonian reaction. The authors did not note a strong difference between the cases ξ = 0 and ξ = 0.4 compared to the Newtonian one. The relaxation time is sufficient to affect the evolution of the load which stays similar to the one obtained with a Newtonian fluid. Distinct differences are obtained with W_i = 0.4. However, this value is 10 times greater than the usual relaxation times of lubricants with additives.

Case B (Figures 11 and 12)

Only the cases for ξ = 0.4 have been treated. For W_i = 0.04 one finds the Newtonian aspect. The case W_i = 0.4 still shows a decline of the plate movement.

Squeeze Film at Constant Load Superimposed on a Couette Flow (Figures 13–15)

The lower plate moves at a translation speed U_p. This results in the drag effect present in the bearing problem. The upper plate has a squeeze velocity due to a vertical load constant in time. One must consider for this study a new dimensionless number U_p/V_0 and any symmetry cannot be assumed. The domain extends to $2L$, the length of the plate. The authors consider here L/h_0 = 5. The boundary conditions are then modified.

Case A has been considered. This is the case where the differences between the viscoelastic behavior and the Newtonian one are the most prominent. The following parameters for the MPTT fluid have been taken: W_i = 0.4, β = 0.5 and ε = 0.

When ξ = 0 (Oldroyd-B fluid), the cases U_p/V_0 = 0 and U_p/V_0 = 10 give the same results. The effect of the normal

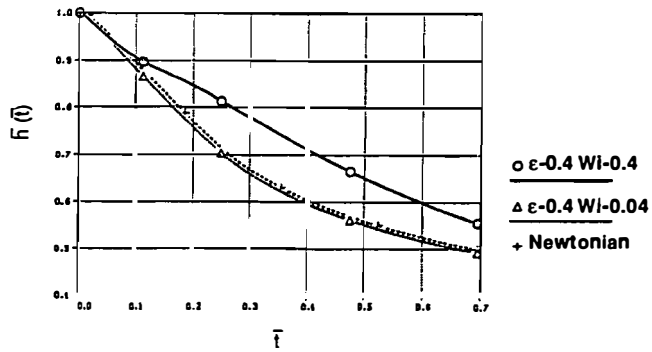

Fig. 11—$h(\bar{t})$, $\overline{m}$, = 163, $\overline{W}_{imposed}$ = 2; MPTT Fluid.

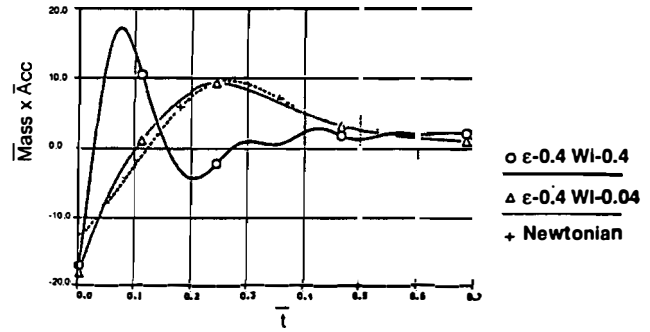

Fig. 12—Mass × Acceleration = $\overline{m}$ = 163, $\overline{W}_{imposed}$ = 2; MPTT Fluid.

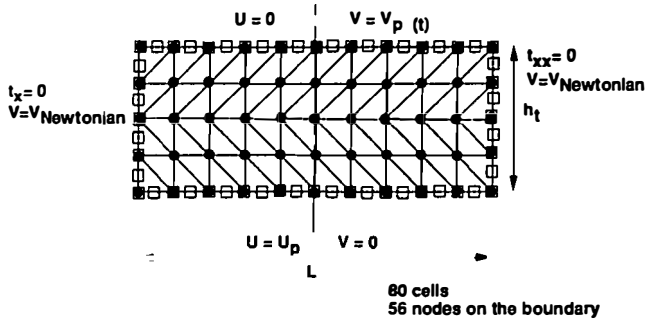

Fig. 13—Domain of study and boundary conditions.

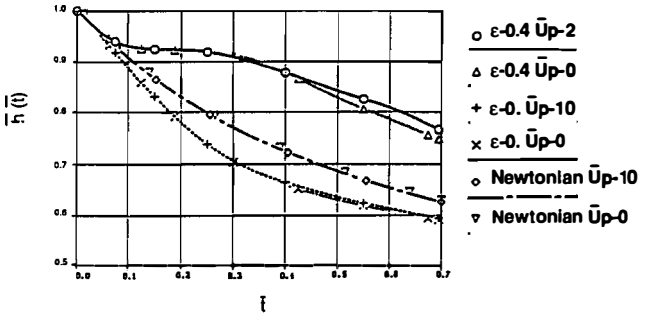

Fig. 14—$h(\bar{t})$, $\overline{m}$, = 81.5, $\overline{W}_{imposed}$ = 1; Newtonian fluid, MPTT fluid W_i = 0.4, β = 0.5, n = 1, ε = 0, ξ = 0 and ξ = 0.4.

stresses generated by the Couette flow is not significant.

For ε = 0.4 the additional shearing coming from the lower plate sliding movement increases the magnitude of the nor-

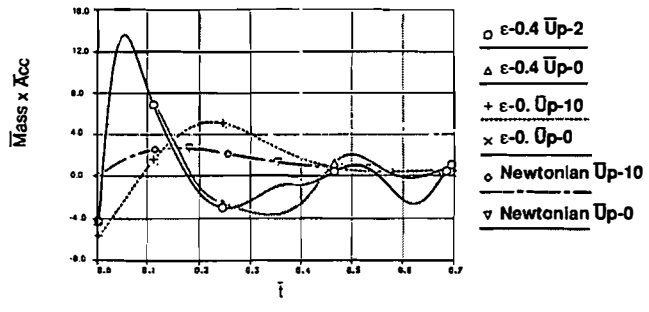

Fig. 15—Mass × Acceleration, $\bar{m}$ = 81.5, $\bar{W}_{imposed}$ = 1; Newtonian fluid, MPTT fluid W_i = 0.4, β = 0.5, n = 1, ξ = 0, ξ = 0 and ξ = 0.4.

mal stresses. This leads to an increase of the upper plate decline.

CONCLUSION

A boundary element method has been developed to study the transient viscoelastic flows. The viscoelastic terms have been considered as pseudo-forces. Taking into account these terms requires a numerical approach. The resolution procedure is based on a perturbation method of a Newtonian flow without inertia terms. This method does not allow the study of flows with strong nonlinearities (high Weissemberg number or Reynolds number). Nevertheless, with this method, the authors confirm the importance of the stress overshoot in the squeeze film problems:

- At constant speed a larger value of the carrying capacity is obtained compared to the Newtonian case.
- At constant load an important decline of the upper plate is obtained due to the viscoelastic effects.

These results have been obtained with a relaxation time 10 times greater than usual and with a ξ parameter not equal to zero due to the existence a second difference of the normal stresses. The hypothesis of such a fluid gives a solution to the problem of overloading on a shaft in its housing. Then, it can decline the shaft movement and avoid the generation of important pressure build up in the film.

The authors note that the MPTT model cannot be used successfully for high shear stresses in squeeze film problems. A microstructural analysis shows that the simplicity of the MPTT model is obtained using restrictive hypotheses concerning the modification of the polymer networks under quick solicitations.

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