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Reconstructing Faces from their Signatures using RBF Regression.

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Abstract

The increasing popularity of social networks raises critical issues regarding privacy and protection of personal information. One typical example – which is the motivation of the paper – is face images and face recognition, face recognition being ubiquitous on the web now. Assuming someone with bad intentions steals a database containing face signatures, would it be possible to reconstruct the corresponding face images, revealing who was in the database? Would these reconstructed images be good enough to allow them to gain access via a face recognition system? This paper brings a contribution to this topic by proposing a face reconstruction algorithm, based on RBF-regression in eigenspace, which is able to reconstruct face images from their signatures (*i.e.* neither knowing the identity of the persons nor their true facial appearance). We show that in addition to being visually realistic, the images generated by the proposed method can fool a state-of-the-art face recognition algorithm.

1 Introduction

Faces are now omnipresent on the Internet (Facebook profiles, photo sharing sites, etc.), and so are face recognition technologies (Google, Facebook, etc. now use face recognition to automatically tag people). Even if the identity/name of the persons appearing in most of the photographs are not explicitly given, face matching technologies have become powerful enough to discover the links between people through faces and, thus, are able to aggregate the information scattered at different places. Such databases use and store face signatures (or *face templates* as they are called in the biometrics literature) as keys for organizing the data.

One important question related to privacy is to know if the information encoded by such templates might be reverse engineered, revealing the identities of the persons in case the database is stolen or made publicly available by mistake. In other words, do face signatures (or face templates, we interchangeably use the two expressions) contain personal information?

This is also a topic of interest for biometric applications where the way templates are encoded is critical [28]. Indeed, stolen templates should not allow reconstruction of images that can be used to break in a system. People have believed for a while that reconstructing images from their signatures was impossible, but several researchers have shown recently it is actually feasible (*e.g.* [2]). As face representation and face verification technologies

have progressed a lot during the last five years, we believe it is important to know, if and to what extent, people can be recognized from their face templates once their face has been reconstructed.

In this context, the objective of this paper is twofold: (i) to propose a simple but efficient method for reconstructing face images from face templates with unknown identities and (ii) to show that these reconstructed images allow the recognition of people and can fool modern face recognition algorithms. We believe that the proposed approach could be used, each time a new method for representing faces is proposed, to evaluate how anonymity is preserved by the encoding.

In practice, we chose to use the I-LPQ approach of [33] – which was, at the moment we wrote the paper, the state-of-the-art unsupervised approach on the LFW [15] and FERET [25] datasets – as a baseline approach for computing face templates. Another reason for using this approach is that it is a good representative of modern approaches for face recognition, most of them being based on histograms of LBP/SIFT/Gabor-like features computed on a regular grid [6, 23, 37].

The proposed reconstruction method is based on radial basis function regression in the face eigenspace. During a training phase, face/signature pairs are used to learn a non linear mapping between images and their signatures. This mapping minimizes the reconstruction error of the eigenface representations of the training images. The mapping can then be used directly to reconstruct faces from their signatures.

In addition of showing reconstructed images and comparing them with the original ones, the proposed approach is experimentally validated on the two aforementioned datasets, *i.e.* LFW and FERET. For these two datasets, we have reproduced the standards protocols of evaluation defined by the datasets, with the difference that instead of using original images we used reconstructed images. We show that the drop in performance is rather low, allowing us to conclude that the signatures do contain enough information to reconstruct face images that can fool – with a good chance – an automatic face recognition system.

2 Related Work

Face templates are compact digital representations of the essential features of facial images. Early works in face representations were mainly relying on global image transformations, projecting faces into meaningful subspaces by the means of PCA or LDA [4, 32]. These representations, so-called *e.g.* Eigen and Fisher faces, were encoding the coordinates of faces with respect to these subspaces. Most of the face reconstruction methods have been experimented in this context (*e.g.* [1, 2, 21]), explaining why they observed that simple gradient based optimization allows to reconstruct faces easily in their cases.

However, the majority of best performing modern methods use distributions of local features for characterizing the facial traits, such as [6], which vector quantizes local pixels and aggregates local histograms, or [26, 27] which use spatially localized Gabor filters in multi-layer frameworks. Histograms of LBPs, LTPs or LQPs are also commonly used in recent approaches, either extracting them from raw pixel intensities [3, 31, 36] or from images of gradient orientations [22, 33, 35, 39]. These features (LBP, SIFT, *etc.*) can be also combined, at the decision level (*e.g.* using Multiple Kernel Learning) to improve the performance of the final classifier [6, 18, 26, 31, 36, 38]. To our knowledge, no face reconstruction approaches have considered such modern representations using histograms of local features.

Regarding the question of reconstructing face images from face templates, it has been claimed in the past that biometrics data such as fingerprints or face templates can be consid-

ered as non-identifiable data, allowing to allay concerns about privacy. However, since the beginning of the 2000's, several authors have proposed reconstruction approaches, following the *hill-climbing* approach of [1, 2]. Starting from a generic initial image, these approaches update iteratively the initial image in such a way that the match score is increasingly better (*i.e.* in such a way that the similarity between the target template and the template of the reconstructed image is higher). Different variants of this approach have been applied to different biometric tasks [9, 10, 19, 34].

However, as pointed out by [21], this hill-climbing optimization is rather inefficient as it involves the optimization of a non-convex criterion and requires a large number of attempts before succeeding. Consequently, [21] proposed to use a different paradigm. They first model the face recognition algorithm by an affine transformation approximating the distances between pairs of training faces by applying multi-dimensional scaling (MDS). The training templates embedded in this affine space are then linearly fitted to the face images represented in their eigenspace. The distance between the templates to be reconstructed and templates coming from the set of training faces are then computed, allowing to embed targeted templates in an affine space. The image of the targeted template is then reconstructed using a linear transformation, obtaining better results than hill-climbing, and at a lower cost.

Building on the pioneering work of [21], our reconstruction method gets inspiration from the literature on *radial basis function (RBF) least squares*. The different RBF methods differ from each other mainly by (*i*) the presence or absence of regularization or bias terms, (*ii*) the nature of the basis functions and (*iii*) how the centroids are chosen.

One of the first RBF least squares method [12], known as the multiquadric interpolation, has been introduced to model the topography of fields. Its name comes from the use of the so-called multiquadric function. It has been used in various applications [13], from surface interpolation [7] to numerical solutions of differential equations [16, 17]. In addition, Micchelli showed that the problem was solvable for any conditionally positive (or negative) definite kernels among which is the multiquadric function [20]. Some studies have shown that multiquadric interpolations tend to be more robust than interpolations based on others RBF kernels, especially than the widely used Gaussian kernel [8]. More generally the advantage of using RBF kernels is that they allow to approximate any functions or boundaries [24].

When applied to noisy data, RBF interpolation might suffer from over-fitting. Broomhead *et al.* [5] tried to overcome this issue by reducing the number of centroids with respect to the number of data points. They also give an interpretation of the approach as a neural network and called their model *RBF networks*.

Other methods use regularization to handle over-fitting. This is the case, for instance, of the *least squares support vector regression (LS-SVR)* method proposed by Suykens *et al.* [30] and the *kernel ridge regression (KRR)* of Saunders *et al.* [29]. KRR differs from LS-SVR in the absence of the bias term.

Contrasting with the MDS-based method of [21], our approach (*i*) allows to learn efficiently non-linear relationships between faces and templates and (*ii*) does not need to build an intermediate Euclidean space since it (*iii*) models directly the transformation from templates to images in the eigenspace, leading to a much more efficient reconstruction framework.

3 Reconstructing faces from templates

Formally, the problem we address can be stated as follows. We assume having a face encoder working as a black box (*i.e.* the way it encodes faces is not known). This encoder performs a mapping between the image and template space, denoted as the function $f : \mathcal{I}_w \times \mathcal{h} \rightarrow \mathbb{R}^D$.

This function takes an image \mathbf{y} of size $w \times h$ and computes its corresponding D -dimensional template $\mathbf{x} = f(\mathbf{y})$. In the following (and to simplify the notations) images are considered as P -dimensional vectors of raw pixel intensities, with $P = w \times h$, obtained by concatenating image rows. If images are in color, they are converted to grayscale

We also assume that the similarity between two faces is measured as the Euclidean distance between their templates, which is the case for recent face recognition approaches such as [6] or [33]. In addition and in any case, this is not an unrealistic assumption since for positive definite metrics face templates can always be embedded into an Euclidean space, *e.g.* by using kernel PCA. Furthermore, when the similarity measure does not correspond to a valid metric, approaches such as the ones proposed in [21] can be used to enforce the desirable properties.

Our approach relies on the use of *training faces*, which can be any set of face images (obviously, it is even better if the training faces have same illumination, range of pose or scales as the images to be reconstructed). Since the encoder f is provided, pairs of image/templates denoted $(\mathbf{y}_i, \mathbf{x}_i)$ can be computed from the training images. These pairs will allow us to learn the decoding map $\varphi(\mathbf{x}) \simeq f^{-1}(\mathbf{x})$, which will approximate the inverse of the mapping f .

RBF least squares reconstruction. Given a set of N samples $\mathbf{x}_i \in \mathcal{X} = \mathbb{R}^D$ with associated target values $\mathbf{y}_i \in \mathcal{Y} = \mathbb{R}^P$, RBF least squares models the mapping $\varphi : \mathcal{X} \rightarrow \mathcal{Y}$ as a set of P independent functions φ_k (one different function per pixel):

$$\varphi_k(\mathbf{x}) = w_{0k} + \sum_{j=1}^M \phi_j(\|\mathbf{x} - \mathbf{c}_j\|) w_{jk} \quad (1)$$

where the scalars w_{jk} ($k \in [0 \dots N]$) are the parameters of the model, the vectors \mathbf{c}_j are a set of centroids and ϕ_j are functions associated to each centroid.

By using the following notations,

$$\begin{aligned} Y &= [\mathbf{y}_1 \dots \mathbf{y}_N]^T \\ \Phi &= \begin{pmatrix} \phi_1(\|\mathbf{x}_1 - \mathbf{c}_1\|) & \dots & \phi_M(\|\mathbf{x}_1 - \mathbf{c}_M\|) \\ \vdots & \ddots & \vdots \\ \phi_1(\|\mathbf{x}_N - \mathbf{c}_1\|) & \dots & \phi_M(\|\mathbf{x}_N - \mathbf{c}_M\|) \end{pmatrix} \\ W &= \begin{pmatrix} w_{11} & \dots & w_{1P} \\ \vdots & \ddots & \vdots \\ w_{M1} & \dots & w_{MP} \end{pmatrix} \\ \mathbf{w}_0 &= [w_{01} \dots w_{0P}]^T \end{aligned}$$

W and \mathbf{w}_0 can be estimated by solving the following optimization problem:

$$\min_{W, \mathbf{w}_0} \|\Phi W + \mathbf{1}_M \mathbf{w}_0^T - Y\|_F^2 + \mathcal{R}(W) \quad (2)$$

where $\|\cdot\|_F$ is the Frobenius norm, $\mathcal{R}(W)$ is a regularization term and $\mathbf{1}_M$ is the M -dimensional vector with all elements equal to 1.

As stated in the previous section, this model generalizes several least-square based methods. For instance, multiquadric interpolation [12] is a special case of equation (2), with no regularization term and centroids \mathbf{c}_j chosen as the training samples \mathbf{x}_i . The functions ϕ_j

correspond in this case to the multiquadric function $\phi_j(x) = \sqrt{s_j^2 + x^2}$ where s_j is a scale parameter chosen to reflect the distribution in the local neighborhood of each centroid.

On the other hand, generalizing LS-SVR to deal with vector valued targets would lead to the following optimization problem:

$$\min_{W, \mathbf{w}_0} \|\Phi W + \mathbf{1}_N \mathbf{w}_0^T - Y\|_F^2 + \lambda \text{Tr}[W^T \Phi W] \quad (3)$$

where the scalar $\lambda > 0$ controls the amount of regularization. The centroids are chosen the same way as for multiquadric interpolation but, in this case, the functions ϕ_j are all the same, a common choice being the gaussian kernel.

However, in practice, the reconstructions obtained this way suffer from the fact that pixels are reconstructed independently from each other, which motivates what follows.

Face reconstruction in the eigenspace. As explained before, we believe that reconstructing pixels independently is not optimal, and suggest instead to reconstruct faces as linear combination of eigenfaces. The reconstruction is then the set eigenface weights. In the eigenspace, most of the energy lies in the first few eigenfaces, which roughly correspond to the macro details of the faces. However, recent face recognition algorithms are usually based on tiny local features (*e.g.* LBPs computed on 3×3 neighborhoods). Consequently, and to encode more accurately those local features, we instead work in the *whitened eigenspace*. We remind that the whitening transformation transforms random variables (of known covariance matrix) into random variables whose covariance is the identity matrix.

Let us start by centering Y and Φ , which can be done by computing $\bar{Y} = J_N Y$ and $\bar{\Phi} = J_N \Phi J_N$, with $J_N = I_N - \frac{1}{N} \mathbf{1}_N \mathbf{1}_N^T$ being a centering matrix with I_N the identity matrix of size $N \times N$.

If the singular value decomposition of \bar{Y} is denoted as $\bar{Y}^T = U S v^T$, our reconstruction is done by solving the following problem:

$$\min_W \|\bar{\Phi} W - \bar{Y} U S^{-1}\|_F^2 = \min_W \|\bar{\Phi} W - v\|_F^2 \quad (4)$$

If $\Phi_{ij} = \sqrt{s^2 + \|\mathbf{x}_i - \mathbf{x}_j\|^2}$ (multiquadric kernel), the literature on radial basis function (RBF) least squares has demonstrated that matrix Φ is invertible [20]. Because of the centering, $\bar{\Phi}$ is of rank $N - 1$ and W is given by (in the least squares sense):

$$W = \bar{\Phi}^+ v \quad (5)$$

where $\bar{\Phi}^+$ denotes the pseudo-inverse of $\bar{\Phi}$.

Once W is computed, the reconstruction of new faces from their templates is done as follows. Let \mathbf{x}'_i be the set of M templates for which we want to synthesize images, Φ' is computed as $\Phi'_{ij} = \sqrt{s^2 + \|\mathbf{x}'_i - \mathbf{x}_j\|^2}$. Centering Φ' with respect to Φ gives $\bar{\Phi}' = (\Phi' - \frac{1}{N} \mathbf{1}_M \mathbf{1}_N^T \Phi) J_N$. The corresponding images Y' are then given by $Y' = \bar{\Phi}' W v^T Y + \frac{1}{N} \mathbf{1}_M \mathbf{1}_N^T Y$. Hence:

$$Y' = \left(\bar{\Phi}' (\bar{\Phi}^+ v v^T) + \frac{1}{N} \mathbf{1}_M \mathbf{1}_N^T \right) Y \quad (6)$$

The choice of the multiquadric kernel was guided by Franke's observations [7], which led to the conclusions that the multiquadric kernel is more robust and less sensitive to the choice of the scale parameter than other kernels such as the Gaussian Kernels. In the following experiments, s has been fixed to 1.

4 Experiments

The first motivation of the paper is to investigate whether people can be recognized from their template, once an image of their face has been synthesized by the proposed approach. We therefore present some qualitative results comparing original images with their reconstructions.

In addition, and in order to have an objective measure of the quality of the reconstruction, we also performed face recognition experiments in which the reconstructed faces are used instead of the real images, and evaluated how this replacement affects the recognition rates. Good performances would mean that synthesized images are good enough to break in a system which uses this face recognition technology to control access.

As stated in the introduction, we have used the face encoder of [33] for these experiments. The face template is thus a vector containing histograms of Local Quantized Patterns (LQP) computed over a grid centered on the face, resulting in vectors of 1100 and 1400 dimensions respectively for FERET and LFW. We have used our own implementation of the encoder based on the details given in [33]. This particular encoder was chosen because it is unsupervised (*i.e.* the representation is independent of the task) and because it gives state-of-the-art results (unsupervised settings) on the LFW dataset.

Datasets and protocols

The experimental validation is done on two datasets: (*i*) FERET which is a good representative of biometrics data and (*ii*) LFW which contains the same type of real images that can be found on the Internet. We start by presenting these datasets and give some implementation details (settings, *etc.*) specific for each dataset.

FERET. The FERET database [25] was released as a benchmark for face recognition algorithms. FERET is made of different views of 1195 persons, split into different non overlapping subsets. One of these subsets is used as the gallery (*i.e.* the set representative of the enrolled faces in an actual biometric scenario) and the others are used as probes (*i.e.* the collection of images that need to be recognized or identified by matching). In practice, we used the `fb` set as the probe, reproducing exactly the experimental protocol of [33]. The encoder [33] is trained on the gallery set and then used to encode the whole set of faces. Matching probe faces with gallery faces, with our implementation, gives a recognition accuracy of 99.2%. This is consistent with the results of [33] and thus validates our implementation.

To learn the face decoder, we have used images from an external database. This ensures that our face decoder is trained with completely different people from FERET (otherwise the reconstruction would be made artificially too easy). The external database is MultiPIE [11] from which we have selected the image folds acquired under similar conditions as those from FERET (frontal poses with same lighting conditions). Face templates are extracted with the same encoder as the one used for the FERET images, giving the image/template pairs needed for learning our decoder. We have trained the face decoder with different amount of training data in order to evaluate the amount of training images required to obtain accurate reconstructions. In practice, we have used 300, 400, 600 and 921 (*i.e.* all MultiPIE images) images.

Labeled Faces in the Wild. Labeled Faces in the Wild [14] is a widely used database to benchmark face recognition algorithms. While in FERET faces are acquired with normalized poses and lighting conditions, LFW faces are captured *in the wild*, exhibiting large variations



Figure 1: Reconstructing FERET images. Top rows are real faces while bottom ones are faces reconstructed with a decoder trained with 921 training images.

in pose, expressions and lighting conditions. This makes LFW much more challenging than FERET.

The recognition task on LFW consists of predicting if pairs of face images represent the same person or not (so called face verification). The performance is reported as the average accuracy of pairs classification.

Here again, we follow the experimental protocol and settings given in [33]. The `view2` of LFW is divided in 10 folds. People present in one fold are never occurring in any of the other 9 folds. Each fold contains 300 positive pairs (same person) and 300 negative pairs (different persons). Performances are evaluated as a 10 fold cross validation process. For each round, a different fold is used for testing while the 9 remaining ones are used for training.

Our decoder is trained with images coming from the folds used for training, and is used to reconstruct images of the remaining fold, giving the guarantee that people whose faces are used to train the decoder are not in the set of images to be reconstructed. At test time, the first element of each pair of the test set is replaced by the image reconstructed from the corresponding descriptor.

As with FERET, we have reconstructed face images using different numbers of training images. We have used more images than with FERET, due to the greater variability in face appearances, *i.e.* 500, 1000, 2000 and 3000 images.

Visual comparison of original images and their reconstructions

Figure 1 presents pairs of original/reconstructed images. The reconstructions are computed from the 1100-dimensional templates using the proposed method.

One can notice strong artifacts, which make the reconstructed faces difficult to confuse with real images, at least for a human being. The most obvious artifact seems to be due to the presence of a lot of people wearing different kind of glasses in the MultiPIE database (we remind that MultiPIE is used to learn the face decoder). As a consequence, some shadowed glasses are visible on each synthesized image, even when no glasses are present on the

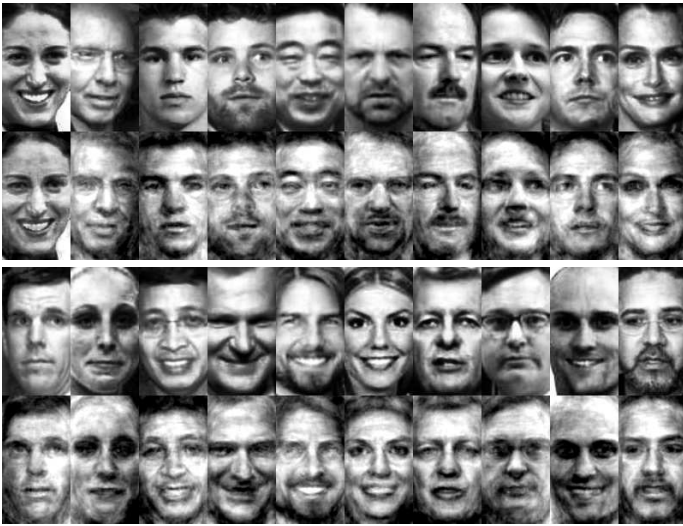


Figure 2: Reconstructing LFW images. Top rows are real faces while bottom ones are faces reconstructed with a decoder trained with 3000 training images.



Figure 3: Reconstructing LFW images. For each face and from left to right: reconstructions with 300, 500, 1000, 2000, 3000 training images, and, finally, the real image.

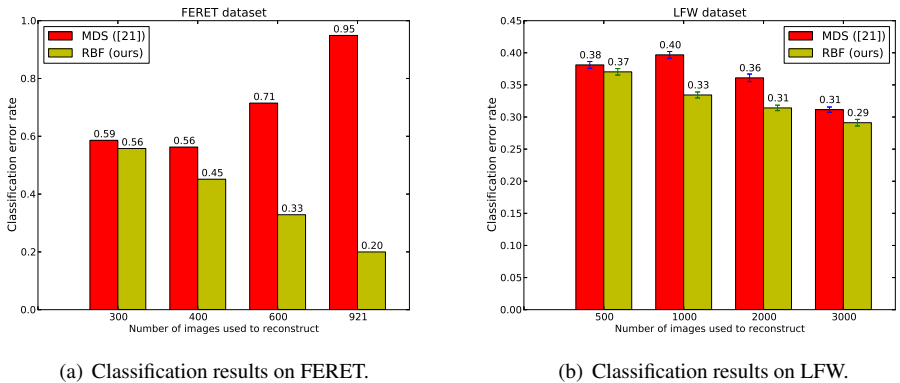
original image. However, some specific traits of the face such as the shape of the nose or the line of the eyebrows are rather well reconstructed.

One would expect that using training images with a broader variability, more realistic results would be obtained.

While MultiPIE and FERET are rather homogeneous with respect to the acquisition conditions, LFW exhibits a large range of pose and lighting conditions. Figure 2, shows some sample images from the LFW database with their reconstructions. While, as for FERET, the reconstructions can not be confused with real photographs, the final result is much more realistic and, in most of the cases, the persons can be easily identified from the reconstructed images. As expected, the quality of the reconstruction mainly depends on the amount of training images available. This is illustrated by Figure 3 which shows some reconstructions given by a decoder trained with different number of images; reconstructed faces become visually very close to original images when a sufficiently large number of training images is used.

Face recognition using reconstructed images

One interesting question is to know whether a set of face templates can be used to break in a face recognition system, by using the images synthesized from the templates. For doing these experiments we substitute the original probe images (with FERET) or one of the two



(a) Classification results on FERET.

(b) Classification results on LFW.

Figure 4: Classification results on FERET and LFW using the images synthesized from the templates.

test images (with LFW) with images synthesized using their corresponding template, and see how the performance of the face recognition is affected. We also provide comparison with the reconstructions given by the approach of [21], which is the state-of-the-art approach at the moment. In the following, we denote [21] as MDS, as the approach is based on multi-dimensional scaling. Our approach is denoted as RBF.

Figure 4(a) gives the classification error rate obtained using our reconstructed faces as probes on FERET, and compares it with [21]. One can see that the performance of MDS dramatically drops when the number of training images raises, while the error rate of our method regularly decreases, as expected. The bad behavior of MDS seems to be due to numerical instabilities since the range of gray levels in the output images diverges from the acceptable values (typically 0–255) when the number of training images increases. The error rate of 0.20, obtained by our method (with a face decoder trained with 921 images from MultiPIE), is to be compared with the error rate of 0.08, obtained with the original face images.

Figure 4(b) presents the classification error rates obtained on LFW. As in the experiments on FERET, the performance of our method (RBF) improves when the number of training images decreases and remains significantly better than the one obtained with MDS [21]. Unlike what was observed on FERET, the performance of MDS does not drop when the number of training images increases. This may be due to the regularization effect induced by the variability of the LFW database. Finally, when training our decoder with 3000 training images, the classification error rate is of 0.29, which is to be compared with 0.18, obtained using the original images.

In conclusion, for both datasets, even if reconstructed images can not be recognized as well as the original ones, the error rate is low enough to prove that the chance to break in a system using synthesized images is significant.

5 Conclusions

At a time when more and more face images are available on-line on the Internet and when face technologies are spreading quickly, it is important to understand to what extent the face signatures manipulated and stored by face recognition applications respect user’s privacy. In other words, we wanted to know if the information encoded in face signatures was sufficient

to allow the reconstruction of face images and to allow the recognition of the persons they represent. For this purpose, we have proposed a simple and original reconstruction method based on RBF regression in the face eigenspace. The method has been experimentally validated on two datasets, on which we have shown that the reconstructed images (i) can be surprisingly visually similar to original ones and (ii) can fool a modern face recognition algorithm.

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