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Dynamic Behavior of Cyclic Symmetrical N DOF System in the Presence of Dry Friction

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Abstract: In the frequency domain, the multi harmonic balance method is used to study the dynamic behavior of bladed assemblies with nonlinearities caused by dry friction dampers. Associated to a Macroslip contact model, this method is generalized to be adapted to a multi degrees of freedom mass-spring system characterized by the cyclic symmetrical property. The influence of contact parameters is investigated and their capacities in minimizing the magnitude of nonlinear responses of the system are studied. The effects of mistuning caused by contact parameters are discussed and its influence on the dynamic behavior of the structure is emphasized.

Keywords: Peak Flattening / Dry friction /Stick-Slip / Cyclic symmetry /Mistuned System Turbomachinery

INTRODUCTION

In turbomachines, devices involving dry friction (Popp et al, 2003, Csaba, 1998, Griffin, 1990) are often used to decrease maximal amplitudes of structures during resonance. Each device is characterised by a different behaviour depending on the geometry, as well as on the position and the dimensions of surfaces in contact. A common characteristic between all friction devices relies on the induced flattening of Frequency Response Function (FRF) peaks obtained when specific normal loads are applied to the contact.

Different numerical techniques can be used to solve such non linear problems. Because of the complexity of solutions adapted to the dynamics of systems integrating dry friction devices, these results were limited to extremely simplified models. On the other hand, it is also recognized that the approach based on direct time integration methods leads to computation costs that are too high when dealing with realistic models. Several authors have highlighted the significant difficulties involved when implementing the criteria of transition between different states of contacts (stick, slip or separation), usually leading to an expensive time step abnormally small to detect these phase changes accurately. As exact solutions are available only for very simplified models, and as the techniques of direct time integration lead to prohibitive costs, approximated methods have been continuously implemented during the last 20 years. Among the latter, the Harmonic Balance Method (HBM), developed by Nayfeh and Mook (1979) has been successfully applied to various structures damped by friction. This method is based on the assumption that the forced response of the system is harmonic. Using this approximation, the dynamic behavior of clamped free beams has been studied by various researchers, as for examples (Dowell, 1983, D'Ambrosio et al. (2004), Menq and Griffin, 1985). Sanliturk et al. (1997) applied the method of harmonic balance to the study of the forced response of blades in jet engines. Wang and Chen (1993), Girini and Zucca (1994) applied the MHBM to a discrete system with one degree of freedom, considering 3 harmonics.

Systems with contact and dry friction effects give the most practical and effective way to reduce high vibrational amplitudes. However, these sliding contacts are also a source of mistuning, due to the loss of the property of cyclic symmetry they induce, that may lead to dangerous localization of the vibration. The cyclic symmetrical property is altered by slight variations of the parameters associated to different blades or to interconnection devices. This alteration usually leads to peaks splitting and/or to the concentration of vibration on certain blades of the assembly, resulting in responses with high amplitudes (Ewins and Han, 1984). For mistuned bladed disc assemblies, the vibrational energy, instead of being distributed uniformly, just concentrate on a few blades generating amplitudes much larger than those predicted on the same tuned structure (thereby shortening their life duration). This phenomenon can be explained by considering the vibrational energy of the system as a wave of displacement. In a tuned system, the wave propagates freely across each identical sector leading to uniform amplitudes which differ only by their relative phase. In the mistuned case, the presence of "defaults" causes the reflection of waves on each sector. This can lead to confinement of vibrational energy over a small area and therefore, in the presence of forced vibration, certain blades may experience stress and amplitudes that exceed those provided by an analysis of the tuned structure.

Sextro et al. (2001) developed an approximate method to calculate the envelopes of the FRF of a bladed disk with cyclic symmetry including under platform devices when blades frequencies are statistically variable. This method is able to identify areas where localization can occur with a high probability. Castanier et al. (1997), Wei and Pierre (1989)

analysed the possibility of apparition of highly localized vibrations in mistuned cyclic systems under engine order excitations. They also discussed the effect of viscous damping and damping brought by dry friction and on localization. They showed that the degree of localization decreases as damping increases and that dry friction damped systems are more sensitive to localized vibrations. The mistuning of the blades is inherently uncertain (Bladh et al. (2002), Kenyon et al. (2003)) and is then difficult to locate and quantify with precision. In many papers on the subject, mistuning is assumed to be controlled and imposed intentionally by varying system characteristics such as mass, stiffness or frequency. The study of the dynamical behavior of bladed assemblies, considering the combined effects of mistuning and nonlinearities associated to dry friction is still relatively underdeveloped and improvements in the understanding of the phenomenon are always needed.

In this paper, the Method of Harmonic Balance (HBM) associated to a “Masing” Macroslip model are briefly presented and then adapted to a single dof system incorporating a dry friction device. The method is then applied to a system of 13 dof characterised by the property of cyclic symmetry. Parametric studies are then conducted in order to determine the influence of variation of contact parameters (such as friction coefficient, tangential stiffness and normal force) on the response. Finally, the behaviour of the mistuned system is analysed in the case of engine order excitation.

DEVELOPMENTS AND CONTACT MODEL

The prediction of responses of systems involving dry friction is complex due to their highly nonlinear behaviour. Different techniques are available to solve such non linear problems. Time domain solutions, based on numerical integrations, give precise results but are often costly and may induce large discrepancies if time steps are not properly chosen. Frequency based methods generally reduce drastically computational efforts by considering that the solution is periodic. Different approaches may be used, depending on the complexity of the structure and the level of non linearity. The classical Harmonic Balance Method is used in many applications dealing with the dynamics of periodically excited structures with contact and dry friction. Steady state response approximations of motion are obtained in frequency domain by decomposing the periodical response of the non linear system into a Fourier series, leading to a set of algebraic equations. The accuracy of this method to predict responses and friction forces is highly dependent upon the number of harmonics retained. But keeping a too large number of harmonics reduces the interest of the method.

Multi Harmonic Balance Method

Using the MHBM, differential equations of motion are transformed into a set of algebraic equations for which the Fourier coefficients are the unknowns of the problem. A single dof system (Fig. 1) associated with a Masing Macroslip contact model (Fig. 2) (Wang and Chen, (1993)) has been chosen in order to simplify the numerical procedures and to be able to conduct parametric studies. This system is supposed to model a blade to ground behaviour with Macroslip frictional contacts where parts in contact are either slipping or sticking. Contact separation and partial slips are not allowed here. A periodic excitation force $F_{ex}(t)=f_0\cos\omega t$ is applied to the system, and a normal load F_n is applied to the frictional damper to load the contact.

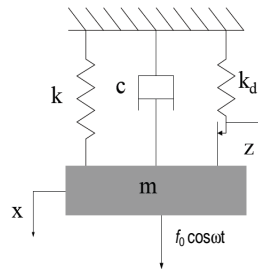


Figure 1: 1 DOF system

For such systems, the motion equation is:

$$m \ddot{\mathbf{x}}(t) + c \dot{\mathbf{x}}(t) + k \mathbf{x}(t) = F_{ex} - \mathbf{f}_{nl} \quad (1)$$

Structural parameters are: mass m , stiffness k and viscous damping c . The frictional device is represented by its stiffness k_d , the coefficient of friction μ at the contact interface and the normal load F_n which is supposed to be constant. $x(t)$ represents the time dependent displacement of the entire system while $z(t)$ is the displacement of the contact point with respect to ground. $y(t)$ is the relative displacement between the mass and the contact point $y(t)=x(t)-z(t)$. According to the Macroslip approach (Wang and Chen, (1993)), the friction force f_{nl} can be expressed as:

$$f_{nl}(t) = \begin{cases} +k_d(x(t) - z(t)) & \text{when } k_d|x - z| \leq \mu F_n \\ +\mu F_n \text{sign}(\dot{z}) & \text{when } k_d|x - z| \geq \mu F_n \end{cases} \quad (2)$$

The steady state associated to Eq. (1) may be obtained using the MHBM, which considers the total displacement as a superposition of harmonic components with frequency ω :

$$x(t) = \sum_{n=1}^N a_n \cos(n\theta) + b_n \sin(n\theta) \quad (3)$$

where $\theta = \omega t$, a_n and b_n are harmonic (cosine and sine) components and N is the number of harmonic retained. Wang and Chen (1993) showed that the friction force, when associated with the Macroslip model of Fig. 2, can be approximated as follows:

$$f_{nl}(t) = \begin{cases} \mu F_n + k_d(x(t) - A_{\max}) & \text{for stick contact state} \\ \mu F_n \text{sign}(\dot{x}) & \text{for slip contact state} \end{cases} \quad (4)$$

where A_{\max} is the highest amplitude of the total displacement and $x(t)$ is given as a function of θ_0 :

$$A_{\max} = x(\theta_0) = \sum_{n=1}^N a_n \cos(n\theta_0) + b_n \sin(n\theta_0) \quad (5)$$

The two parameters θ^* and θ_0 are introduced to determine whether the slider is within stick or slip states. Angle θ_0 is associated to point A of the hysteresis cycle where slip state stops and stick state begins. Angle θ^* is associated to point B where slip state begins and stick state stops. Taking into account these two parameters, defining contact states, the nonlinear force can be developed into two different expressions, one for the stuck state where $\theta_0 < \theta < \theta^*$ and another for the slip state where $\theta^* < \theta < \theta_0 + \pi$.

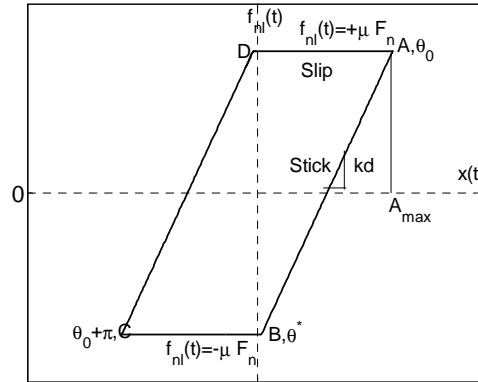


Figure 2: Hysteresis loop (Masing Model): Multi terms approximation. Parameters θ^* , θ_0

As observed in Fig. 2 and 3, stuck state is characterized by line (AB) and slipping state by line (BC). Changes in contact states from slip to stick are identified by points A and C, at $\theta = \theta_0$ and $\theta_0 = \pi + \theta_0$ respectively. Change from stick to slip occurs at point B where $\theta = \theta^*$. The non linear force due to dry friction is also approximated using a truncated Fourier series as:

$$f_{nl}(t) = \sum_{n=1}^N f_{c_n}(\theta) \cos(n\theta) + f_{s_n}(\theta) \sin(n\theta) \quad (6)$$

$f_{c_n}(\theta)$ and $f_{s_n}(\theta)$ are a function of the contact model chosen and, as the non linear force is antisymmetric, may be obtained from the following Fourier integrals:

$$f_{c_n}(\theta) = \frac{2}{\pi} \int_{\theta_0}^{\theta_0 + \pi} f_{nl}(t) \cos(n\theta) d\theta \quad (7)$$

$$f_{s_n}(\theta) = \frac{2}{\pi} \int_{\theta_0}^{\theta_0 + \pi} f_{nl}(t) \sin(n\theta) d\theta \quad (8)$$

Both expressions of the nonlinear force (Eq.2) in stuck and slip states, written as a function of the state parameters θ_0 and θ^* , are considered when integrating over the proper intervals. By introducing (Eq.3) and its derivatives and (Eq.4) into the equation of motion (Eq.1) and identifying the cosine and sine components, a system of N algebraic equations with $2N+2$ unknowns $a_1, b_1, \dots, a_n, b_n, \theta_0, \theta^*$ is obtained.

$$\begin{aligned}
& -m\omega^2 \left[\sum_{n=1}^N n^2 (a_n \cos(n\theta) + b_n \sin(n\theta)) \right] - c \left[\sum_{n=1}^N n (a_n \sin(n\theta) - b_n \cos(n\theta)) \right] \\
& + k \left[\sum_{n=1}^N a_n \cos(n\theta) + b_n \sin(n\theta) \right] + \sum_{n=1}^N f_{c_n} \cos(n\theta) + f_{s_n} \sin(n\theta) \\
& = f_0 \cos(\theta)
\end{aligned} \tag{9}$$

Two additional equations are required. The first one states that at $\theta = \theta_0$, displacement $x(t)$ reaches a maximal value A_{\max} and so, velocity is null. Null velocity is written as:

$$\left. \frac{\partial x}{\partial t} \right|_{\theta=\theta_0} = \omega \sum_{n=1}^N n (b_n \cos(n\theta_0) - a_n \sin(n\theta_0)) = 0 \tag{10}$$

At point B (Fig. 2), the two expressions of the nonlinear force given for stick and slip states are equal. Consequently, the second additional equation will be:

$$\begin{aligned}
& \mu F_n + k_d (x(t) - A_m) = -\mu F_n \quad \text{then} \\
& \mu F_n + k_d \sum_{n=1}^N (a_n (\cos(n\theta^*) - \cos(n\theta_0)) + b_n (\sin(n\theta^*) - \sin(n\theta_0))) \\
& = -\mu F_n
\end{aligned} \tag{11}$$

For each pulsation ω initial values are needed for θ^* and θ_0 . The unknown coefficients a_n, b_n, θ^* and θ_0 are determined by solving equations Eq. (9), Eq. (10) and Eq. (11) using a Newton-Raphson iterative method.

Validation with a 1 dof system

Validation is illustrated by considering the dynamical response of a single dof system. Fig. 3 gives the maximum amplitude of the response of the system excited by a sinusoidal force, as a function of the excitation frequency for different values of the normal force F_n . This system models a blade clamped on a disk and vibrating according its first bending mode. The simulation parameters are as follows: $f_0=1\text{N}$, $m=1\text{ Kg}$, $k=1e4\text{ N/m}$, $c=0.5\text{ N/m/s}$, $\mu=0.5$ and $F_n=100\text{ N}$, $k_d=1e4\text{ N/m}$. For $F_n = 0.01\text{ N}$, contact is slipping and the system has a quasi-linear behavior. The equivalent stiffness for the system is the stiffness k of the structure. For $F_n > 100\text{ N}$, contact is completely stuck (linear case). The equivalent stiffness is here $(k + k_d)$. Contact changes from completely slipping to stick /slip when the normal load increases from 0.01 to 3 N. Peaks flatten and the amplitudes become much smaller than those observed for the linear case.

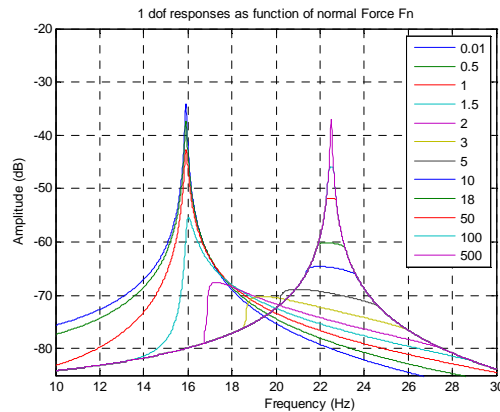


Figure 3: Maximal Amplitude responses as function as normal forces F_n .

As proved by Girini and Zucca (1994), comparisons of responses calculated with various harmonics (here noted MHBM), one harmonic (noted 1HBM) and using an exact time solution (Fig. 4) prove that, in this case, only one harmonic of the Fourier series is sufficient to obtain good results in terms of amplitude (specific phenomenon of peak flattening very well reproduced). In spite of the greater accuracy of the multi-HBM, the FRF obtained is very similar to

the one coming from the HBM. This means the one term-approximation is good enough to predict the dynamic behavior of the system studied.

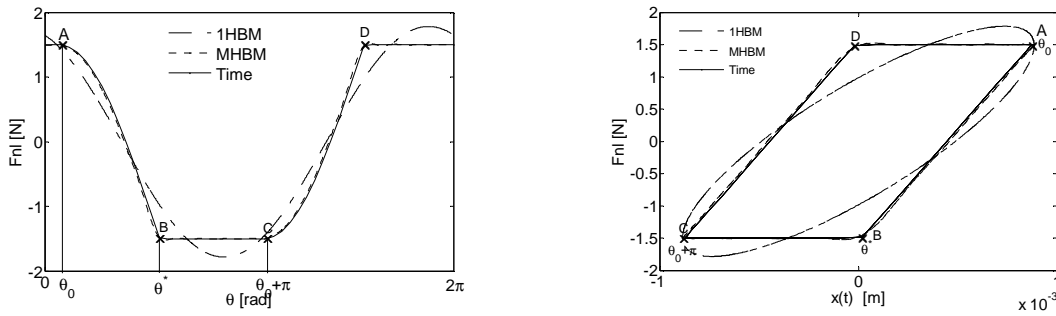


Figure 4: Nonlinear force versus time in 1HBM, MHBM and Time solution for $F_n=3N$, frequency=20Hz.

In spite of the greater accuracy of the multi-HBM, the FRF obtained is very similar to the one coming from the HBM. This means the one term-approximation is good enough to predict the dynamic behavior of the system studied. Consequently, due to the cost of calculations, only one term will be considered in what follows.

CYCLIC SYMMETRICAL SYSTEM WITH N DOF

The literature allows identifying the most important effects linked to mistuning. However, despite the current level of knowledge and understanding phenomena related to friction and mistuning, the overall behavior of mistuned bladed discs assemblies in the presence of inter-blade contacts remains quite poorly controlled. Figure 5 shows a 13 DOF mass-spring system with friction devices, arranged according to the cyclic symmetrical property.

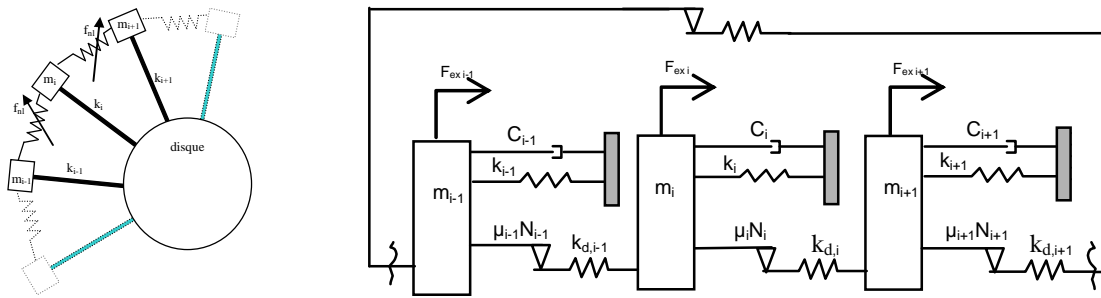


Figure 5: Cyclic symmetrical Masses-springs system linked by friction devices

Excitation type

Mono Point excitation

The simulation parameters are as follows: $f_0 = 1N$, $m_i = 1 \text{ kg}$, $k = 1e4 \text{ N / m}$, $c_i = 0.5 \text{ N / m / s}$, $\mu_i = 0.5$ and $F_n = 100 \text{ N}$, $k_{d,i} = 1e6 \text{ N / m}$. The periodically 13 blades system may be excited by a harmonic mono point force of constant amplitude f_0 applied on a single dof. Figure 5 shows the linear response of the system under such excitation. Each color on the graph is associated to a specific dof. Seven modes of resonance may be identified as nodal diameter modes of the cyclic symmetrical assembly. As well known, if NDDL represents the number of blades, the number of possible diametral modes is then equal to $(NDDL + 1) / 2 = (13 + 1) / 2 = 7$ here. This result confirms that mono point excitation is able to excite all the modes of the structure.

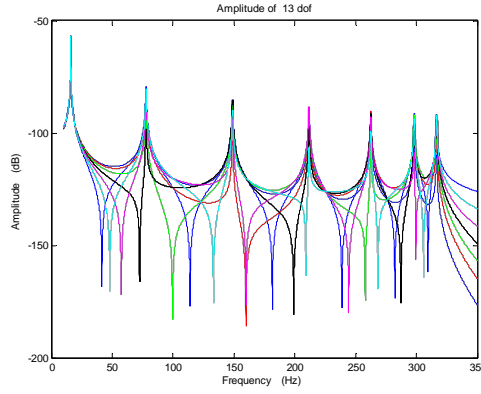


Figure 6: Responses of Cyclic symmetrical Mass-spring system for harmonic mono point force

Engine Order Excitation

In this case, an engine order excitation is applied to every dof of the system, as defined by the following relation where i represent the number of dof, n the total number of dof and EO the Engine Order.

$$F_{exc}(i) = f_0 \cdot \cos\left(EO \cdot \frac{2\pi}{n} \cdot i\right) + f_0 \cdot \sin\left(EO \cdot \frac{2\pi}{n} \cdot i\right) \quad (12)$$

Excitation with an engine order $EO = 0$ corresponds to a force of equal value and same sign on each dof. The corresponding response in the tuned case implies that all the DOF will vibrate together in the same direction (Fig. 7a).

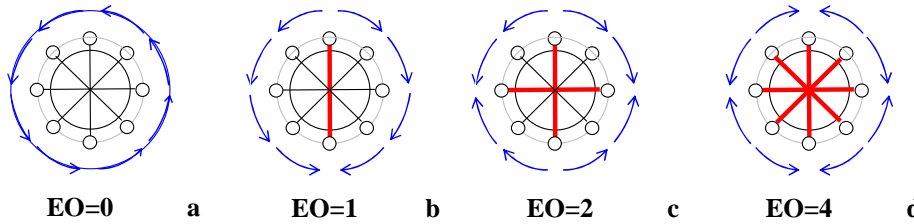


Figure 7: Nodal diameters for different engine order.

Engine order excitation $EO = 1$ implies a sinusoidal excitation of first order and then half of the DOF vibrating together in one direction and half in the opposite direction. This vibration pattern is characterized by a nodal diameter (symbolized by the red line) which divides the system into two parts (Fig. 7b). Higher values of EO (Fig. 7c-7d), induce vibrations according to EO diameters, until $EO = (NDDL + 1) / 2$ is reached (up to 7 in our case with 13 DOF).

TUNED CYCLIC SYMMETRICAL N DOF SYSTEM

Dynamic behaviour of the tuned system

Figures 8a to 8c show the frequency responses of the system for different values of Engine Order excitation EO (1, 3 and 6). The simulation parameters are: $f_0 = 1N$, $m_i = 1 \text{ kg}$, $k_i = 1e4 \text{ N / m}$, $c_i = 0.5 \text{ N / m / s}$, $\mu_i = 0.5$ and $F_n = 100 \text{ N}$, $K_{di} = 1e6 \text{ N/m}$. These graphs show the evolution of the absolute amplitudes of each blade (DOF) versus the excitation frequency. It appears that responses of the different dof are overlapped when the system is perfectly tuned.

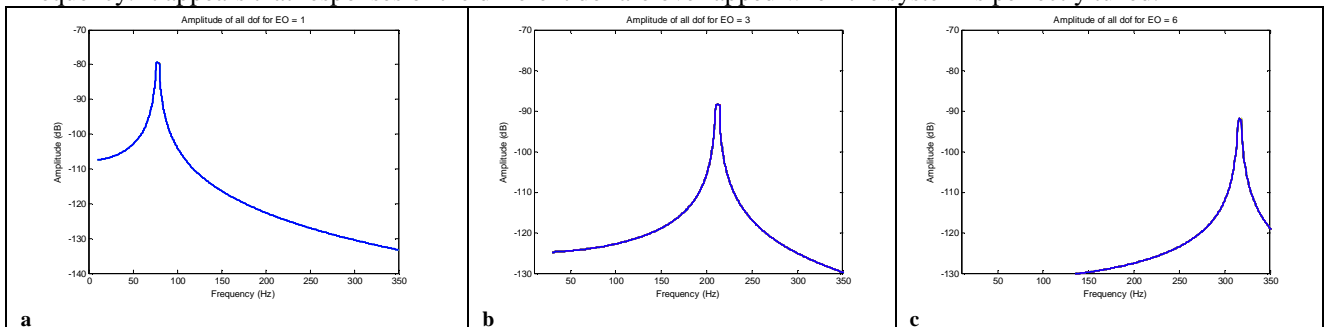


Figure 8: FRF of the tuned system calculated by mono point (a) and Engine Order (b) excitation

For $EO = 1$, using this distributed excitation, the second mode of vibration at one nodal diameter is highlighted (Fig. 8a). The frequency obtained also corresponds to the second frequency given by the punctual excitation (Fig. 6). The same type of behaviour happens for higher values of EO .

Parametric studies

The objective of this section is to show the influence of the characteristics of contact parameters on the response amplitudes of systems linked by dry friction devices.

Effect of contact stiffness k_{di}

Values considered in this analysis are: $f_0=1N$, $m_i=1\text{ Kg}$, $k_i=1e4\text{ N/m}$, $c_i=0.5\text{ N/m/s}$, $\mu=0.5$ and $F_n=10\text{ N}$. Different values are considered for the contact stiffness k_{di} : $1e4\text{ N/m}$, $1e5\text{ N/m}$ and $1e6\text{ N/m}$. The evolution of the nonlinear response of each dof as a function of the Engine Order excitation is calculated for several value of engine order excitation. For $EO=0$, the value of the tangential stiffness k_{di} has no influence on the first resonant frequency located at 15.9 Hz as well as on the amplitude of its associated mode. This result is obvious as, for the first mode of vibration of the system, there is no relative movement between the dof and therefore all the dof have the same magnitude of motion. But, for $EO > 0$, whatever the engine order considered, an increase in tangential stiffness k_{di} causes resonance at higher frequencies and peaks flattening (transition from one sliding state to a majority stick/slip state). The same behaviour occurs for the other modes. For $k_{di} = 1e4\text{ N/m}$, the amplitude peaks appear quite sharp, highlighting a stuck contact behaviour. For $k_{di} = 1e5\text{ N/m}$ and $k_{di} = 1e6\text{ N/m}$, peaks are clearly truncated, and amplitudes are much lower, showing the apparition of stick/slip phenomena. Following Fig. 9 illustrates these comments.

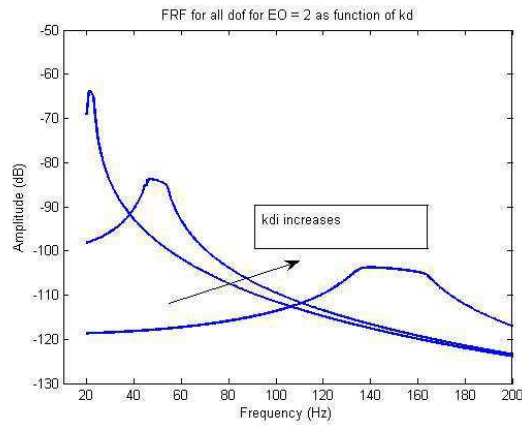


Figure 9: FRF associated for the tuned structure for engine order ($EO=2$) as function of contact stiffness.

Effect of friction coefficient μ_i

Values considered in this analysis are: $f_0=1N$, $m_i=1\text{ Kg}$, $k_i=1e4\text{ N/m}$, $c_i=0.5\text{ N/m/s}$, $k_{di} = 1e5\text{ N/m}$, $F_n = 10\text{ N}$. The range of coefficient of friction μ is now varied from 0.2 to 0.8 . The influence of the friction coefficient μ is easily identifiable when examining the responses obtained with different Engine Order EO (here $EO = 3$). While frequency does not change as a function of μ , it appears that the larger the friction coefficient is, the larger the percentage of stick will be (sliding threshold $\mu \cdot F_n$ increases when μ increases). An optimal value in terms of peaks flattening may be found. Following Fig. 10 illustrates graphically these comments.

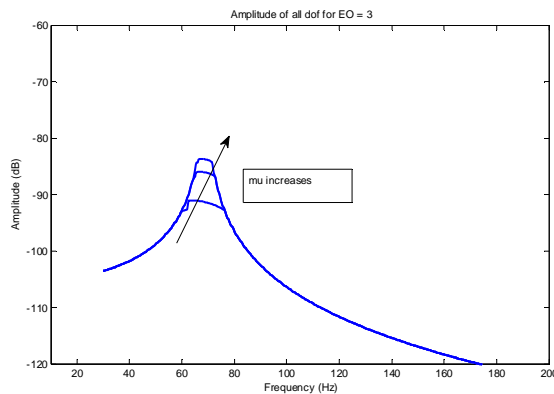


Figure 10: FRF associated for the tuned structure for engine order ($EO=3$) as function of friction coefficients.

Effect of normal force F_n

Values considered in this analysis are: $f_0=1\text{N}$, $m_i=1\text{ Kg}$, $k_i=1\text{e}4\text{ N/m}$, $c_i=0.5\text{ N/m/s}$, $\mu=0.5$, $k_d=1\text{e}5\text{ N/m}$. If F_n increases (10N, 40N and 100N), the percent of slip decreases (and even disappears for $F_n = 100\text{ N}$) and maximum amplitudes increase. The resonant frequencies are insensitive to changes in F_n . Note that it is quite obvious that parameters μ and F_n have the same influence on shifts and amplitudes as the product $\mu \cdot F_n$ defines the threshold of sliding. Thus, the higher this product is, the longer the system remains stuck, with large peak amplitudes. When product decreases, slip becomes more important, causing a decrease in level for the maximum amplitudes.

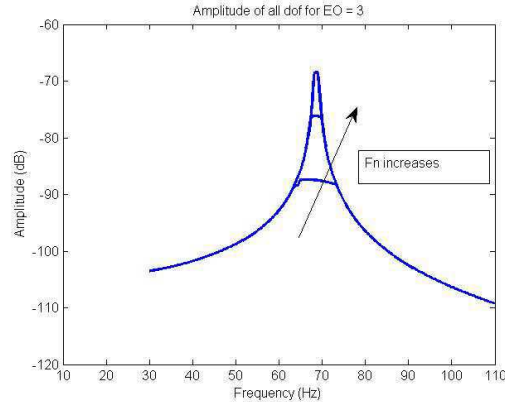


Figure 11: FRF associated for the tuned structure for different engine order (EO=6) as function of forces F_n .

This qualitative study of the effect of contact parameters μ , F_n and k_d can lead to a better knowledge of the system's sensitivity to these parameters and then helps for optimization. Parameters μ and F_n allows lowering or increasing amplitudes between the limits given by the two linear cases (totally slipping and totally stuck). The tangential stiffness k_d leads to a spread of the curve with a majority stick/slip states.

DYNAMICS OF THE MISTUNED CYCLIC SYMMETRICAL N DOF SYSTEM

A mistuned system happens when all the dof are not characterized by the same parameters m , k , c , and / or contact parameters. Mistuning leads to mode splitting: double modes with nodal diameters split at two different frequencies. Resultant frequencies are close but the associated modes are distinct and characterized by a richer harmonic content than that associated with the corresponding tuned mode. The following study is based on the mistuning of the system through different values of contact parameters as friction coefficient μ , tangential stiffness k_d and normal force F_n for each of the 13 DOF. The objective is to evaluate the influence of variations of contact parameters on the global dynamics of the system.

The simulation parameters are as follows: $f_0 = 1\text{N}$, $m_i = 1\text{ kg}$, $k_i = 1\text{e}4\text{ N/m}$, $c_i = 0.5\text{ N/m/s}$, $\mu_i = 0.5$ and $k_{di} = 1\text{e}6\text{ N/m}$. From preliminary results with this set of data, the efficiency of friction devices (in terms of peaks flattening) is obtained for a value of normal force F_n of about 10 N. This optimal stick/slip configuration is chosen to analyze the influence of the variation of contact parameters. The nominal contact parameters are: $F_n = 10\text{ N}$, $\mu = 0.5$, $k_d = 1\text{E}6\text{ N/m}$ and 25 standard uniform distribution of these parameters around their nominal values will be considered with physically acceptable variations of 40% for friction coefficient, 20% for tangential stiffness and 30% for normal force. Figure 12 superimposes the calculated responses of the 13 DOF for an Engine Order excitation EO=3 for an arbitrary set of these parameters as well as the response for the tuned system.

As shown, mistuning due to variations of contact parameters has several effects:

The first one is that other modes (than the mode normally selected by the shape of the Engine Order excitation) are excited. Here for EO=3, we can observe peak flattening around 220 hertz corresponding to the third mode of resonance, but peaks associated to other modes are also present (in particular, EO=1, 2, 4, 5 and 6).

The second concerns the splitting of these modes. Except for the third mode, we can see that modes separate in two peaks around the frequency of the tuned system; but splitting is not associated to peak flattening.

The third effect concerns the amplitude of response of these modes. Figure 12 demonstrates that localization of vibrations appears on different DOF of the system, with levels exceeding the level associated to the natural mode (here EO=3). Here the first and second modes show higher amplitudes.

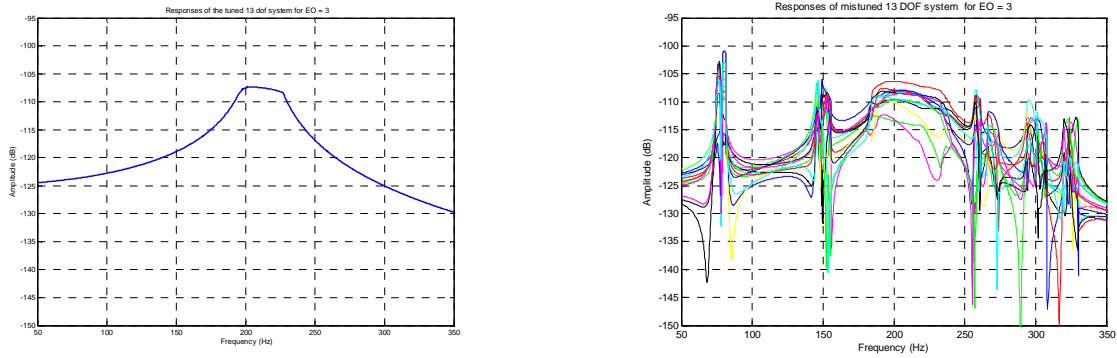


Figure 12: Responses for the 13 blades for an arbitrary set of random data

In order to have a better representation of these few results, the lower and upper envelope curves are extracted for the 25 set of random data. Figures 13 presents the results obtained on the blades 2, 6, 10 as well as the maximal amplification factor for all dof at the worst configuration. The amplification factor is defined by the ratio between the maximum amplitude response due to the engine order excitation of mistuned system and the maximum amplitude response of the tuned system as function of the frequency. As we can see, the amplification factor is equal to 16.6 for the fifth dof. Even if the engine order excitation is equal to 3 (near 220 hertz), the maximal amplitude is observed on the first mode of resonance, near 77 hertz.

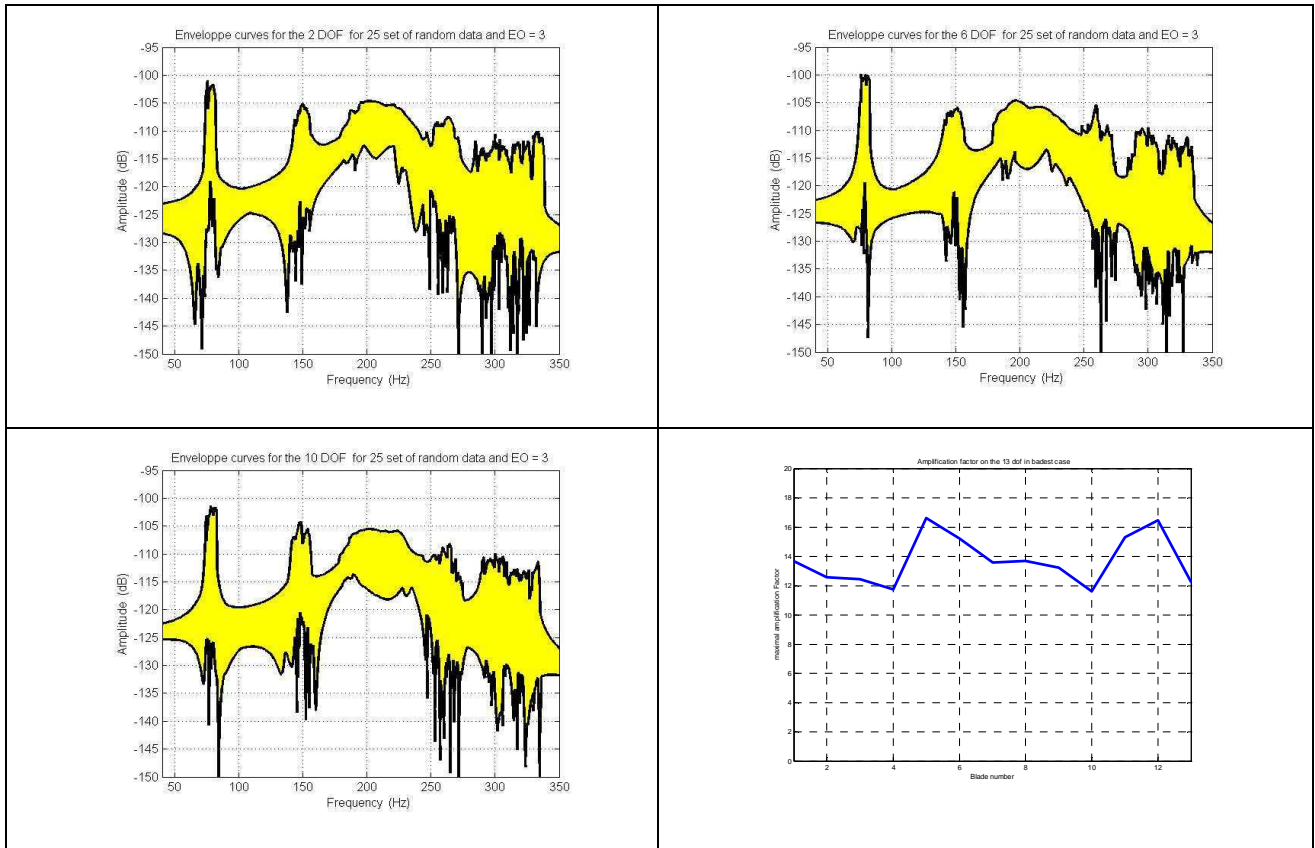


Figure 13: Envelope curves for 25 arbitrary set of random data

These results demonstrate that it becomes very difficult to calculate with accuracy the response of such system because of the randomness of contact parameters. Even if the friction device seems to be efficient in terms of peak flattening relative to the principal engine order, the structure exhibits high levels of amplitude for other engine orders, associated to high levels of stresses.

CONCLUSIONS AND PERSPECTIVES

In this study, the Multi Harmonic Balance Method restricted to a single harmonic has been validated on a mass-spring system with one degree of freedom. It was then adapted to a 13 DOF mass spring system satisfying the property of cyclic symmetry. Dry friction in each contact of an assembly may vary due to tolerances, non uniformities or wear.

In order to control the behaviour of such system, parametric studies were conducted to determine the influence of contact parameters (friction coefficient μ , normal force F_n and tangential stiffness k_d) on the response of the system. The analysis of the behaviour of a mistuned system generated from the single variation of contact parameters has been illustrated and discussed. Phenomena associated to mistuning are highlighted. The prediction of the dynamic behaviour of bladed assemblies, including the combined effects of mistuning and nonlinearity associated with dry friction appears complex. The effect of such variations cannot be accounted for by deterministic approaches... More realistic modelling associated to specific numerical methods have to be developed and probabilistic investigations need to be conducted in order to obtain more precise prediction of responses of the structure. Considering stochastic analyses of bladed assemblies, many approaches have been developed but most are limited to linear systems. Dealing with non-linear systems, the Monte Carlo technique is easy to apply but becomes very heavy when considering actual structures modelled using a high number of degrees of freedom and then should be associated with efficient reduction techniques

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