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# An Interpretation of Secondary Consolidation for the Batiscan Clay

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March 2, 2014

## Abstract

This paper presents an alternative interpretation of the phenomena of secondary consolidation for the sensitive Batiscan clay, a Champlain sea deposit from eastern Canada. Predictions were carried-out based on the framework model proposed by Martins (1992) and are consistent with concepts developed by Terzaghi (1941), Taylor (1942, 1948), Bjerrum (1967), Schmertmann (2012) and others. A simple numerical procedure for predicting secondary consolidation, as well as the assessment of the parameters and the predictions are presented in this paper. It is shown that satisfactory predictions were achieved both qualitatively and quantitatively for most of the 9 secondary consolidation tests. In addition an assessment of the variation of  $K_0$  over time during secondary consolidation is included in this paper.

## 1 Introduction

Secondary consolidation is a research topic that still attracts attention today as no agreement exist on the mechanism or its prediction. However, irrespective of its limitations, the  $C_\alpha/C_c$  approach is the most used.

Another research line that is gaining attention is the Isotach approach. In this approach, the models intended for predicting secondary consolidation, as well as undrained creep and other rate effects, relate the stresses to strains by means of lines of same strain rate (isotach meaning same rate/speed). Taylor (1942) was the first to propose such a model, although nowadays many others have been proposed. Examples of Isotach models are the one proposed by Leroueil et al. (1985) and the one by Martins (1992).

This paper deals with these two Isotach models and intends on presenting an alternative method for assessing secondary consolidation. Before discussing into detail the proposed method the two aforementioned isotach models will be briefly presented.

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## 2 The model proposed by Leroueil et al (1985)

Leroueil et al. (1985) proposed a model for natural clays which was successfully applied to Champlain Sea clays. The model can be described by the two equations presented below:

$$\sigma'_p = f(\dot{\varepsilon}_v) \quad (1)$$

$$\dot{\sigma}'_v / \sigma'_p = g(\varepsilon_v) \quad (2)$$

Where:

$\sigma'_p$  is the preconsolidation pressure;  
 $\dot{\varepsilon}_v$  is the vertical strain rate;  
 $\sigma'_v$  is the vertical effective stress; and  
 $\varepsilon_v$  is the vertical strain

These equations were reached after Leroueil et al. (1985) demonstrated based on their experimental findings that the influence of  $\dot{\sigma}'_v$  is not significant.

By combining Equations (1) and (2) it can be seen that the vertical effective stress is the product of a function of the vertical strain rate by a function of the vertical strain. More importantly, with Equations (1) and (2), it is possible to obtain a relationship between the vertical strain rate and the vertical strain for a given vertical effective stress. This relationship can then be used to predict the evolution of strain over time under a constant vertical effective stress. That is, this relationship can be used for predicting secondary consolidation.

Watabe and Leroueil (2012) develop further Equation (1) by considering that the function  $f(\dot{\varepsilon}_v)$  can be represented by a power law function which reaches a limit, called  $\sigma'_{pL}$ , at infinitesimal strain rate ( $\dot{\varepsilon} = 0$ ).

An isotach model that can provide an alternative interpretation of secondary consolidation is the one proposed by Martins (1992). Although this model was originally developed for explaining rate effects such as creep and stress relaxation under undrained conditions in the triaxial apparatus, the modification proposed by Martins, which was confirmed by Thomasi (2000), as well as the one proposed by Alexandre (2006) allow the treatment of secondary consolidation. The model proposed by Martins (1992), as it was originally conceived, will be briefly presented in the next section.

## 3 The model proposed by Martins (1992)

According to Terzaghi (1941), the contact between clay particles would be of two types. Terzaghi called the contact types “solid bonds” and “film bonds”. In his view, both contacts would be able to transmit effective stresses and would result from the adsorbed water layers that surround the clay particles. The

“solid bonds” would result from the contact between the adsorbed water layers in the immediate vicinity of the clay particle, which, according to Terzaghi, would be in the solid state. The “film bonds” would result from the contact between adsorbed water layers which would not be in the solid state but which would possess a viscosity higher than of the viscosity of the free water (by free water it should be understood the water that flows out of the pores of soil during seepage or consolidation).

Having this picture in mind, Martins (1992) assumed as a hypothesis that the shear strength of saturated normally consolidated clays has two components, the frictional resistance and the viscous resistance.

The frictional resistance would develop between Terzaghi’s “solid bonds” and it would be a function of the shear strain. The viscous resistance would develop between Terzaghi’s “film bonds” and it would be a function of the strain rate. The equation for the shear strength would then be:

$$\tau = \tau_f + \tau_v = \sigma' \cdot \tan \phi'_{mob} + \eta(e) \cdot \dot{\epsilon} \quad (3)$$

Where:

$\tau_f$  is the frictional resistance (the solid component of the shear stresses);

$\tau_v$  is the viscous resistance (the viscous component of the shear stresses);

$\phi'_{mob}$  is the mobilized effective angle of internal friction;

$\eta(e)$  is the coefficient of viscosity of the adsorbed water layer that surrounds the clay particles (a function of the void ratio for a normally consolidated clay); and

$\dot{\epsilon}$  is the strain rate.

The normal effective stress,  $\sigma'$ , is taken as the difference between the normal total stress,  $\sigma$ , and pore-pressure,  $u$ .

One problem in the model is that, because Equation (3) is written in terms of shear stresses, rate/time effects related to the normal effective stress such as the hydrostatic relaxation observed by Arulanandan et al. (1971), secondary consolidation under hydrostatic conditions and such cannot be macroscopically explained.

To overcome this problem it was suggested by Martins that Equation (3) could be generalized for the normal effective stresses. In this regard, Thomasi (2000) carried out similar tests as Arulanandan et al. (1971), confirming the possibility of generalizing the normal effective stress equation as proposed by Martins. The generalized (3) Equation is:

$$\sigma' = \sigma'_s(\epsilon) + \sigma'_v(e, \dot{\epsilon}) \quad (4)$$

Where:

$\sigma'$  is the normal effective stress;

$\sigma'_s$  is the normal effective stress due to the “solid” bonds; and

$\sigma'_v$  is the normal effective stress due to the “film” bonds.

Equations similar to Equations (3) and (4) were proposed by Taylor (1942) and (1948).

A state of stress that combines Equations (3) and (4) was proposed by Alexandre (2006) for the case of axial-symmetry and is reproduced below:

$$\sigma'_s = \left( \frac{\sigma'_{1s} + \sigma'_{3s}}{2} \right) + \left( \frac{\sigma'_{1s} - \sigma'_{3s}}{2} \right) \cdot \cos(2 \cdot \alpha) \quad (5)$$

$$\tau_s = \left( \frac{\sigma'_{1s} - \sigma'_{3s}}{2} \right) \cdot \sin(2 \cdot \alpha) \quad (6)$$

$$\sigma'_v = \left( \frac{\sigma'_{1v} + \sigma'_{3v}}{2} \right) + \left( \frac{\sigma'_{1v} - \sigma'_{3v}}{2} \right) \cdot \cos(2 \cdot \alpha) \quad (7)$$

$$\tau_v = \left( \frac{\sigma'_{1v} - \sigma'_{3v}}{2} \right) \cdot \sin(2 \cdot \alpha) \quad (8)$$

Where the indexes “s” and “v” represent the “solid” and the “viscous” components of the effective stresses and the indexes “1” and “3” represent the major and minor principle stresses. In the above equations and for a given plane,  $\alpha$  is the angle between the direction perpendicular to the plane considered and the plane where  $\sigma'_1$  acts.

It can be seen that Equations (5) and (6) describe a circle for the solid effective stress state as well as Equations (7) and (8) constitute a circle for the viscous effective stress state. These two circles combined give the effective stress state at a point.

An important remark about this model is that when the strain rate is made equal to zero, the effective stress state decreases to a lower stationary value greater than zero, which is the solid effective stress state. If a real laboratory test could be carried-out with a strain rate equal to zero then the results of such a test would comprise only the solid component of the effective stress state. However, as a test with a strain rate equal to zero is impossible to perform there will always be an additional component of the effective stress state, which is the viscous effective stress component.

The concepts introduced in this section were intended to provide the minimum information for the development of the study of the secondary consolidation mechanism, which will be presented in the next section. For a better understanding of this model the reader is referred to Martins (1992), Martins et al. (2001), Alexandre (2006) and Santa Maria et al. (2010).

## 4 The mechanism of Secondary Consolidation

According to Equation (3) and (4), for the same strain, the higher the strain rate the higher the shear stress and the higher the effective normal stress. An example of this behavior, which is characteristic of Isotach models, can be seen in Vaid and Campanella (1977) and another in Leroueil et al. (1985) as reproduced below.

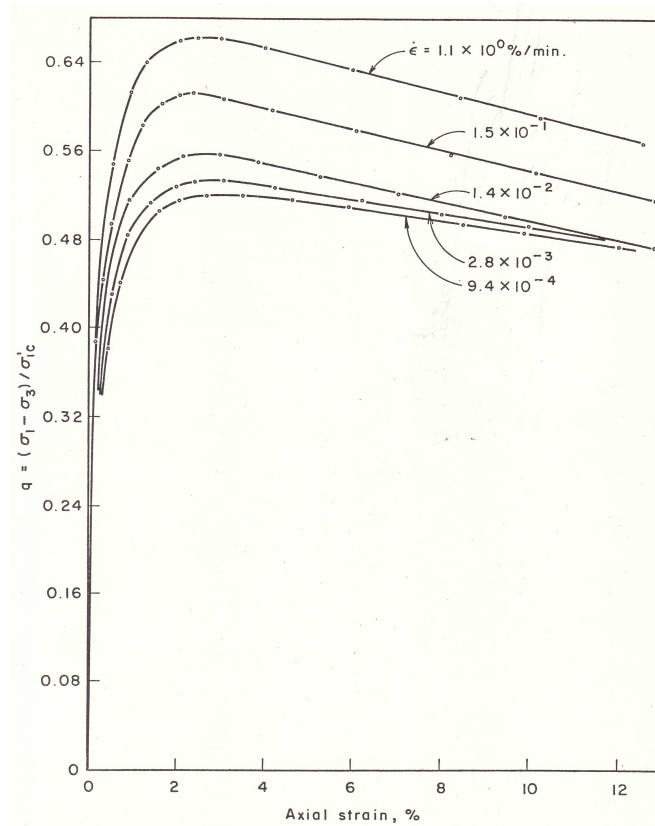


Figure 1: Vaid and Campanella (1977)

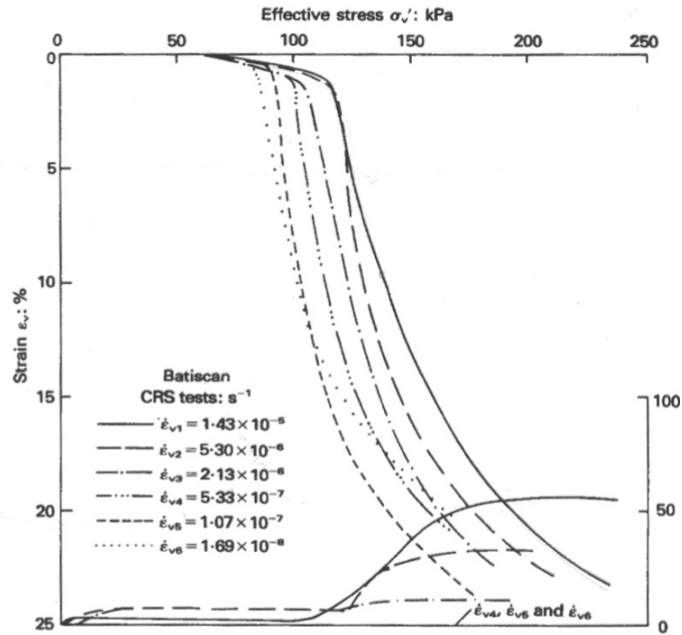


Figure 2: Leroueil et al. (1985)

Another way of understanding Equation (4) is thinking about a vertical line through a given effective stress, that is, to think what occurs during secondary consolidation. According to Equation (4) and considering that the strain rate decreases as the strain increases, then, the viscous effective stress should decrease and the solid effective stress should increase as the soil undergoes secondary consolidation. Therefore secondary consolidation is the transference of the viscous component of the effective stress to the solid component under constant effective stress. Physically, the transference occurs between the bonds, with the film bonds becoming solid bonds with time. When the last film bond becomes a solid bond, the strain rate drops to zero and secondary consolidation comes to an end. This mechanism of transference of effective stress components between bonds was originally conceived by Terzaghi (1941).

If the relationship between strain rate and strain is known for a given constant effective stress, then, it is possible to assess settlement caused by secondary consolidation, at least between two known strain rate curves. To this point, the calculation of secondary consolidation using Martins's model is similar to the calculation using the model proposed by Leroueil et al. (1985). However, according to Martins's model there will be a zero strain rate line where all the rate effects come to an end and all the effective stress is of a solid component nature. In this way the model proposed by Martins (1992) allows for the assessment of the total secondary consolidation settlement, as long as it is possible to assess the zero strain rate line, or in other words, the End-of-Secondary (EOS) con-

solidation line. The assessment of the EOS consolidation line will be addressed in the next section.

## 5 Experimental Determination of the zero strain rate consolidation line

The first experimental method for assessing the EOS consolidation line is to carry out long-term consolidation tests where the consolidation (primary + secondary) process comes to an end. Martins (1997) carried out 3 tests for a soil fabricated in the laboratory, a mix of caulin (80%) and bentonite (20%) that lasted to about 2 years and one test in an organic clay from Rio de Janeiro that lasted for about 5 years as shown in Andrade (2009). Figures 3 and 4 reproduced below present these results.

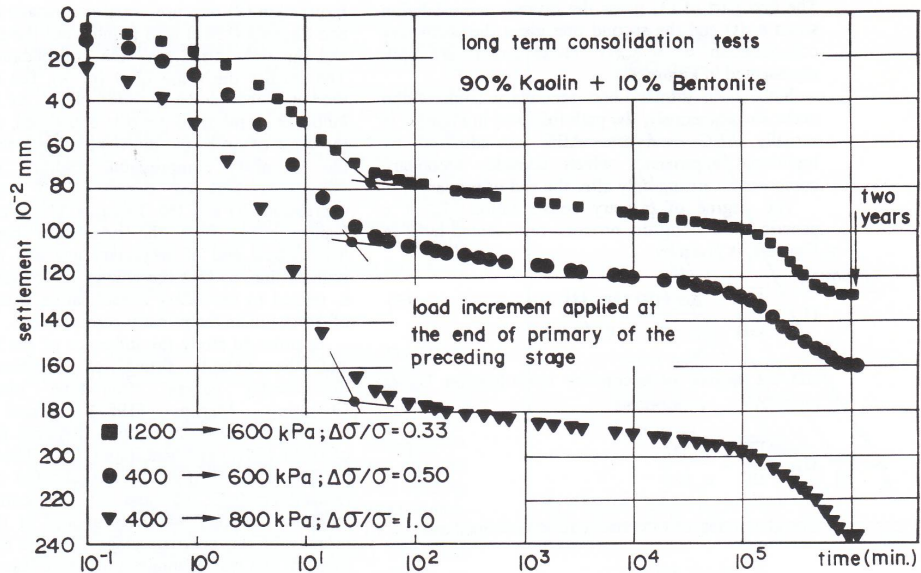


Figure 3: Martins (1997)



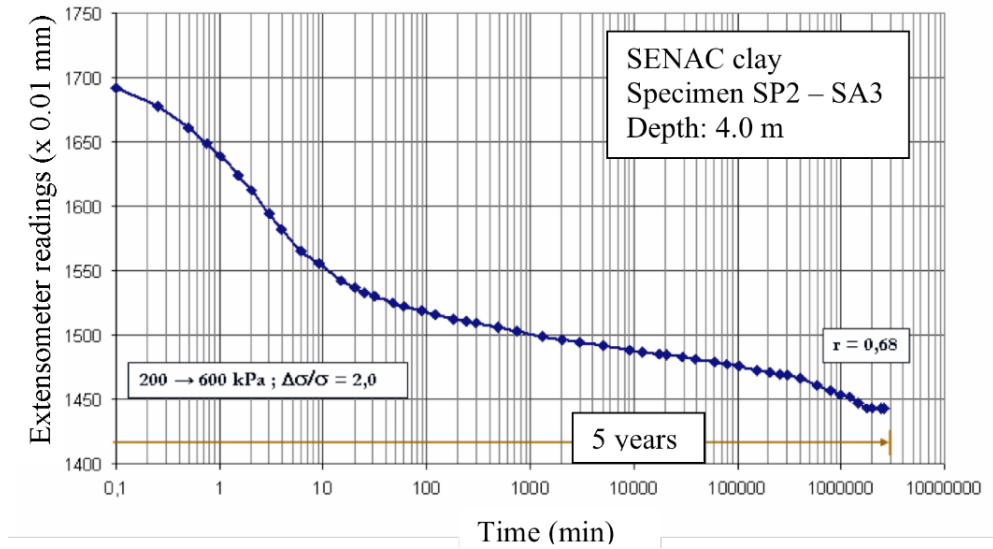


Figure 4: Andrade (2009)

The second method would be carrying out stress relaxation tests in the edometer apparatus once the soil reaches the end-of-primary consolidation, where the excess pore-pressure is practically zero. In order to allow for a stress relaxation test in the consolidation apparatus, a load cell or a proving ring can be positioned in the edometric frame in a place that prevents further movements of the loading arm as the consolidation process proceeds. Some deformation that results from the loading of the load cell/proving ring always occurs and therefore these tests cannot be considered “perfect”. Therefore they are called “imperfect” stress relaxation tests.

Garcia (1996) carried out such tests on the same organic clay from Rio de Janeiro which Martins carried out the long term consolidation test mentioned above. The nine tests carried out by Garcia (1996) lasted from 10,000 to 80,000 minutes. However, none of them have reached the EOS consolidation line, therefore “imperfect” stress relaxation tests are also very time consuming. Figure 5 below shows the stress relaxation test that lasted for 80,000 minutes, together with a prediction from Alexandre and Martins (2012).

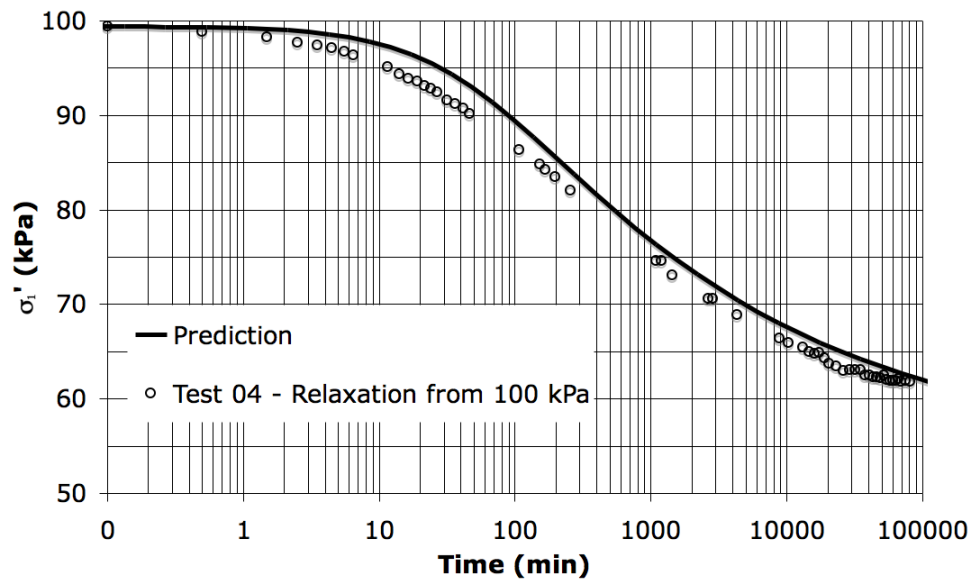


Figure 5: Edometric stress relaxation test result from Garcia (1996) and a prediction from Alexandre and Martins (2012)

A third method consist in generating unloadings with different OCR's and observing the evolution of the rebound strain with time. This was carried out by Feijó (1991) in the Sarapui river clay and is reproduced below.

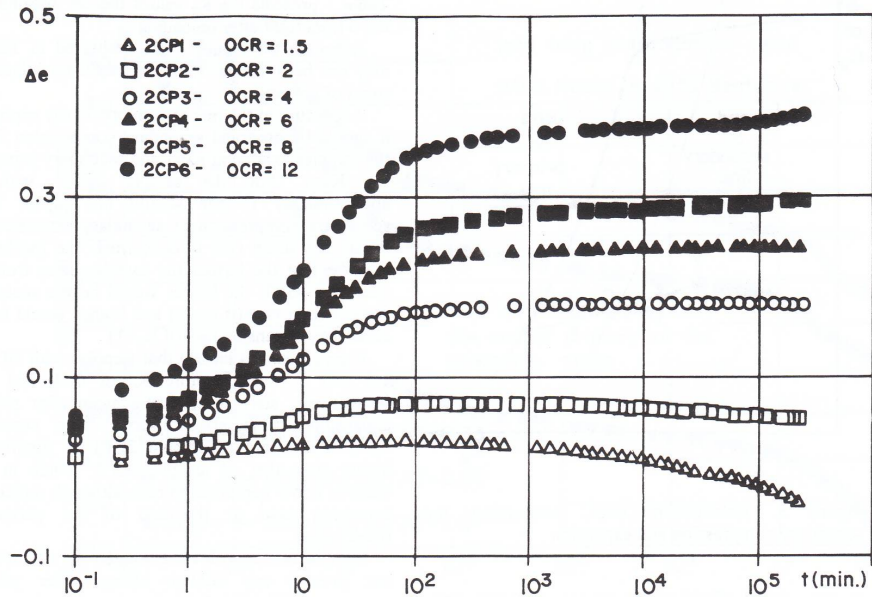


Figure 6: Feijó (1991)

As can be seen in the figure above, the tests lasted to about 200,000 minutes. The test carried-out with an  $OCR = 2$  experienced a slight compression after the “end” of primary rebound and the test with  $OCR = 4$  did not present any sign of compression after the “end” of primary rebound. Therefore, the EOS consolidation line should correspond to an OCR in between these two values. According to his experience with other clays, Martins found that the EOS line is close to a line which corresponds to an OCR between 1.6 and 2.2 for some clays from Rio de Janeiro, Brazil.

Considering the time required for carrying out any of these tests, it is not economically feasible their execution on a routine basis. Alternatively, a mixed experimental-analytical method can be used.

Considering again Equation (4), for a given strain and time during secondary consolidation, what it is known is that the viscous component of the effective stress is been transferred to the solid component. However, their actual values are not known. In terms of measured quantities, all is known is the strain, the strain rate and the vertical effective stress which the soil is undergoing during secondary consolidation at a given time. Not even the shape of the viscous and solid curves are known. Therefore the unknowns at a given strain during secondary consolidation are the values of the solid effective stress,  $\sigma'_s$  and the viscous effective stress, which is a function of the strain rate. Assuming a particular function for representing the viscous component, characterized by  $n$  constants, the number of unknowns is  $n + 1$ . To solve for the  $n + 1$  variables it is then required equal number of equations. These equations can be written

if tests are carried out that allow the determination of  $n + 1$  compression lines of same strain rate, or  $n + 1$  isotachs. These  $n + 1$  isotachs can be determined by any of the tests mentioned above, however, *it is not required that these tests reach the EOS line*, saving therefore considerable time and resources.

As the viscous function is not known it is necessary to search for the best fitting one. Therefore the  $n + 1$  variables will correspond to the viscous function with the higher number of constants,  $n$ . As this method applies to any rate effects under any drainage and stress condition, an example will be provided below using the tests results of conventional CIU tests with different strain rates carried out by Vaid and Campanella (1977) shown in Figure 1.

According to Alexandre (2006), a power law function of the strain rate was found to represent the viscous behavior with a correlation coefficient of about  $R^2 = 0.99$ . Therefore, for assessing the zero strain rate deviatoric stress x strain curve, a power law function will be used. Considering Equations (5) to (8), the following equation can be written for the deviatoric stress:

$$\sigma_d = \sigma_{ds} + \sigma_{dv} \quad (9)$$

Where  $\sigma_{ds} = \sigma'_{1s} - \sigma'_{3s}$  and  $\sigma_{dv} = \sigma'_{1v} - \sigma'_{3v}$ . Assuming a power law function for the viscous component it can be written that  $\sigma_{dv} = K \cdot \dot{\varepsilon}^n$  and therefore Equation (9) can be re-written as:

$$\sigma_d = \sigma_{ds} + K \cdot \dot{\varepsilon}^n \quad (10)$$

In this case, the variables to be determined are  $\sigma_{ds}$ ,  $K$  and  $n$ . Therefore, the number of required equations are 3. Using the axial strain of 2.5% as the strain for this determination and the curves of the strain rates of 1.1,  $1.4 \times 10^{-2}$  and  $9.4 \times 10^{-4}$  %/min, one gets for their respective deviatoric stresses 0.66, 0.55 and 0.52 (normalized in respect to the consolidation stress). The non-linear system of equations is therefore:

$$0.66 = \sigma_{ds}/\sigma'_{1c} + K \cdot \left(1.1 \frac{\%}{min}\right)^n \quad (11)$$

$$0.55 = \sigma_{ds}/\sigma'_{1c} + K \cdot \left(1.4 \times 10^{-2} \frac{\%}{min}\right)^n \quad (12)$$

$$0.52 = \sigma_{ds}/\sigma'_{1c} + K \cdot \left(9.4 \times 10^{-4} \frac{\%}{min}\right)^n \quad (13)$$

The solution for the system presented above is  $\sigma_{ds}/\sigma'_{1c} = 0.485$ ,  $K = 0.171$  (in general  $[K] = FL^{-2}T^n$  but in this case due to the normalization,  $[K] = T^n$ ) and  $n = 0.225$ . As there are 2 more curves with different strain rates, it

is possible to set 2 more systems using these curves and the curves with strain rate equal to  $1.1\%/min$  and  $9.4 \times 10^{-4}\%/min$  (the “lower” and “upper” bound strain rate curves). The solutions for these curves are  $\sigma_{ds}/\sigma'_{1c} = 0.484$ ,  $K = 0.172$ ,  $n = 0.224$  and  $\sigma_{ds}/\sigma'_{1c} = 0.385$ ,  $K = 0.272$  and  $n = 0.101$ . The first two sets of solutions agree well and the third seems too low. Regardless, by this method it is possible to assess lower and upper bounds for the unknowns. The value assessed by Alexandre (2006) for  $\sigma_{ds}/\sigma'_{1c}$ , based on a graphical method and on the results of the undrained creep and undrained constant load tests was  $\sigma_{ds}/\sigma'_{1c} = 0.46$ . For Vaid and Campanella (1977), this value would be between 0.5, which is the creep test that did not fail within 3 weeks, and 0.518, which is the creep test that did fail.

## 6 Batiscan Clay

According to Leroueil et al. (1985) “Batiscan is situated on the north shore of the St. Lawrence river, about 110 km west of Quebec city. The clay tested was taken at a depth of 7.25-7.46 m. The water content is about 80 %, the plasticity index 21 % and the liquidity index 2.7. The preconsolidation pressure, as determined from conventional oedometer test, is 88 kPa...”. Also according to the authors, the liquidity limit is 43 %, the clay fraction is 81 %, the undrained strength, as determined by field vane, is 25 kPa and the normal effective vertical stress at a depth of 7.3 m is 65 kPa.

## 7 Assessment of parameters

The method outlined in Section 5 was applied to the Bastican clay for vertical strains of 1, 5, 10, 15 and 20 % using the data provided in Figures 5 and 7 from Leroueil et al. (1985). As the power law function was successfully applied to the Haney clay, as shown in Alexandre (2006), and on an organic clay from Rio de Janeiro-Brazil, as shown in Alexandre and Martins (2012), the logical choice for this clay is also a power law function. For this determination 4 strain rates were chosen:  $10^{-5}s^{-1}$ ,  $10^{-6}s^{-1}$ ,  $10^{-7}s^{-1}$  and  $10^{-8}s^{-1}$ . As the number of variables requiring determination is 3, two systems of 3 equations were solved for each strain in order to provide a lower and upper bound range for the zero strain rate line. The first consisted of the strain rates  $10^{-5}s^{-1}$ ,  $10^{-6}s^{-1}$ , and  $10^{-8}s^{-1}$  while the second system consisted of the strain rates  $10^{-5}s^{-1}$ ,  $10^{-7}s^{-1}$  and  $10^{-8}s^{-1}$ . Table 1 presents these determinations.

$\varepsilon$ (%)	1 <sup>st</sup> System			2 <sup>nd</sup> System		
	$\sigma'_s$ (kPa)	$K$	$n$	$\sigma'_s$ (kPa)	$K$	$n$
1	63.5	218.1	0.13	74.3	374.13	0.19
5	80.8	589.1	0.23	77.7	431.62	0.19
10	89.1	539.2	0.21	84.9	400.8	0.18
15	107.3	1786.6	0.32	104.8	1048.8	0.27
20 (average curve)	136.6	1753.8	0.31	134.7	1221.7	0.27
20 (lower bound curve)	123.0	3140.9	0.36	127.4	1520.1	0.30

Table 1 - First assessment of parameters.

Note:  $[K] = FL^{-2}T^n$

In fact, the procedure for determining the three unknowns,  $\sigma'_s$ ,  $K$  and  $n$ , was applied as explained above but only the solid effective vertical normal stress,  $\sigma'_s$  was kept. A figure was then made with these points and a smooth curve was drawn by hand similar in shape to the curves from the CRS tests. The figure below presents these points as well as the assessed curve and the CRS test points.

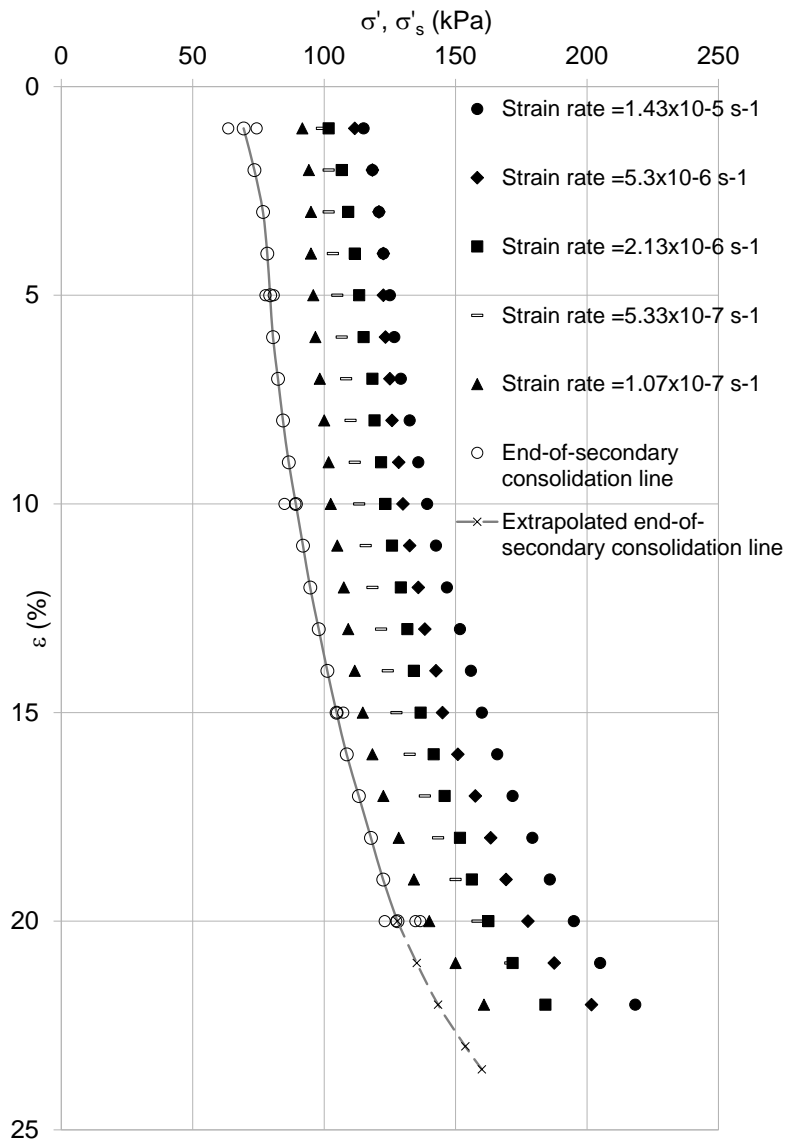


Figure 7: Assessment of the end-of-secondary consolidation line (the zero strain rate line).

The lower bound curve for  $\varepsilon = 20\%$  was assessed because of the scatter shown in Figure 5 of Leroueil et al. (1985) and also because the pairs of values  $(\varepsilon, \sigma'_s) = (20\%, 136.6kPa)$  and  $(\varepsilon, \sigma'_s) = (20\%, 134.7kPa)$  are inconsistent with Figure 4 from Leroueil et al. (1985). In this case only the lower bound solid effective stresses were used. Then,  $\sigma'_s$  was determined graphically for strains between 1 and 22 % at 1 % intervals using the smooth curve and each individual power law function was then determined using the CRS curves from Figure 1 of Leroueil et al. (1985). The obtained values of  $\sigma'_s$ ,  $K$ ,  $n$  and the correlation coefficient for each power law function are presented in the table below.

$\varepsilon$ (%)	$\sigma'_s$ (kPa)	$K$	$n$	$R^2$
1	69.4	237.0	0.15	0.96
2	73.5	307.0	0.17	0.96
3	76.7	414.2	0.19	0.97
4	78.4	517.4	0.21	0.96
5	79.4	550.0	0.22	0.97
6	80.5	575.3	0.22	0.97
7	82.4	624.4	0.22	0.97
8	84.3	661.5	0.23	0.98
9	86.5	772.4	0.24	0.98
10	89.2	1050.5	0.27	0.98
11	92.0	1116.1	0.27	0.98
12	94.7	1277.5	0.28	0.98
13	97.9	1772.9	0.31	0.97
14	101.2	2245.2	0.33	0.97
15	104.7	2479.2	0.34	0.97
16	108.5	2894.5	0.35	0.96
17	113.2	3429.2	0.36	0.95
18	117.8	3031.7	0.34	0.97
19	122.4	2510.3	0.33	0.96
20	127.9	3101.9	0.34	0.92
21	135.2	1822.5	0.29	0.92
22	143.4	2156.8	0.30	1.00
23	153.7	Not assessed		
23.55	160	Not assessed		

Table 2 - Final assessment of parameters.

A check regarding the consistency of the zero strain rate line was carried out using the power law functions presented in Table 2. In this regard, the viscous effective vertical normal stress were assessed based on the individual functions for the strain rate of the specific CRS test and then subtracted from the CRS curves. These curves were plotted together in the figure below:



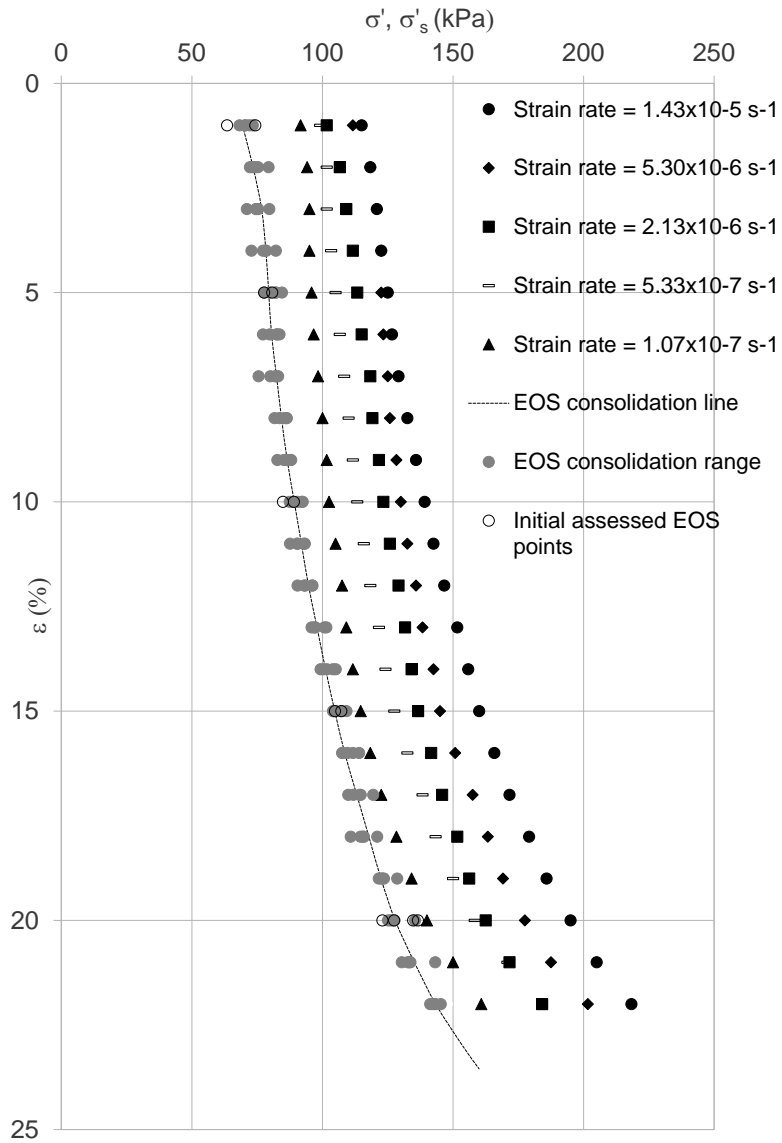


Figure 8: Assessment of the zero strain rate “band”/“range”

As can be seen on Figure 8, the zero strain rate line is in fact a “band” or a “range”. If it is assumed an average curve at the center of the band interval, then the difference in  $\sigma'_s$  for the outer limits of the band and the average curve varies from about 3.5 to 5.5 kPa. Despite of the scatter, this band can be considered narrow, at least in comparison to the differences between the CRS curves. Moreover, this narrow band shows the consistency of zero strain rate line assumption.

One point that must be clarified is that the procedure for assessing the zero strain rate line does not always give the correct estimate. Depending on the chosen  $(\dot{\epsilon}, \sigma')$  points, the  $(\epsilon, \sigma'_s)$  values for the zero strain rate line are simply too low. In the present work the chosen points had a difference in strain rate of one order of magnitude (10x faster or slower). Choosing points closer than one order of magnitude in strain rate led to values that the present authors believe not to be representative of the zero strain rate line. According to Brown and Heywood (1984), even non-linear least squares regression techniques present problems. In addition it is important to understand the geological history of the deposit and apply engineering judgment. In the case of the Batiscan clay, the creep tests presented in Leroueil et al. (1985) give an indication that the zero strain rate line assessment is appropriate as comparisons of strains when the tests were interrupted are somehow consistent with the zero strain rate band.

Another important remark is that the test with a strain rate  $\dot{\epsilon} = 1.69 \times 10^{-8} \text{ s}^{-1}$  was not used for the assessment of the parameters as it crosses the other test curves. The reason for this behavior is not clear but Leroueil et al. (1985) suspect of natural variability or thixotropic hardening.

A small note is appropriate at this point. Figure 8 can be seen to resemble the picture drawn by Bjerrum (1973) about secondary consolidation. The difference is that instead of lines of equal times, in the present model these are lines of equal strain rates. In addition, in this model, the zero strain rate line represents the end of secondary consolidation.

## 8 Predictions

For a given test at and after the “end” of primary consolidation, when the excess pore-pressure is approximately negligible, it is possible to assess the strain rate for a given strain as below:

$$\dot{\epsilon} = \left[ \frac{\sigma' - \sigma'_s(\epsilon)}{K} \right]^{\frac{1}{n}} \quad (14)$$

With the pairs  $(\epsilon, \dot{\epsilon})$  is then possible by numerical integration to assess the time for a given strain with the following equation:

$$t(\epsilon) = \int \left( \frac{d\epsilon}{dt} \right)^{-1} \cdot d\epsilon \quad (15)$$

Equations (14) and (15) were used in combination with the parameters assessed and shown in Table 2 to predict secondary consolidation. The results of the predictions as well as the experimental data are presented in the following figures. The dashed lines are the predictions and the circles are the experimental points. Predictions were made for  $\dot{\epsilon} \leq 10^{-6} s^{-1}$  as, according to Leroueil et al. (1985), based on their experience the effective stress is approximately equal to the applied stress for strain rates at or below this value.

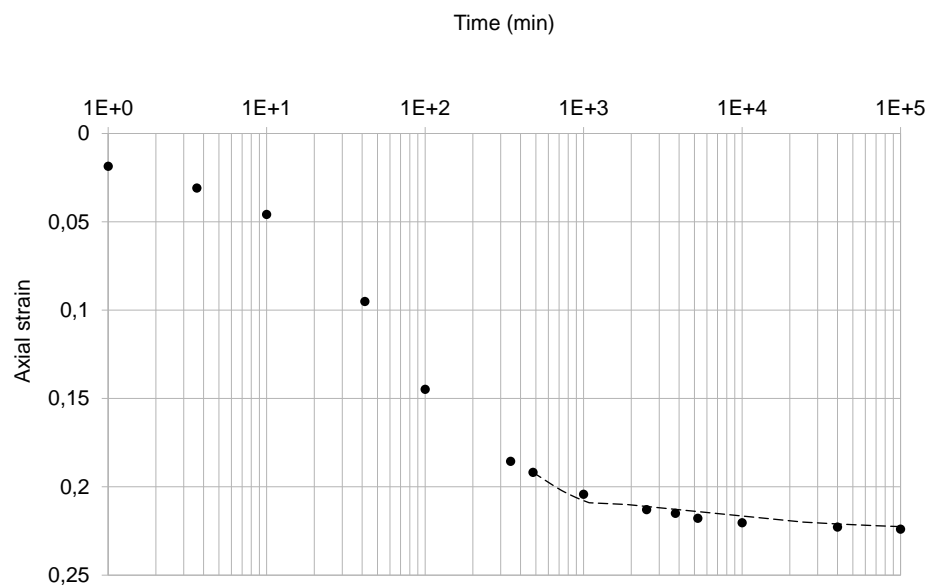
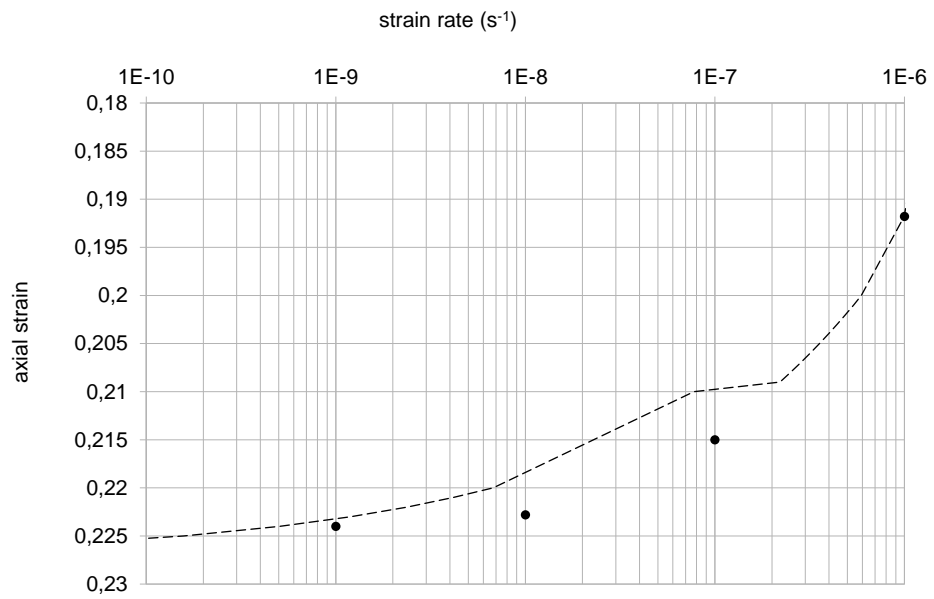


Figure 9:  $\sigma = 151kPa$

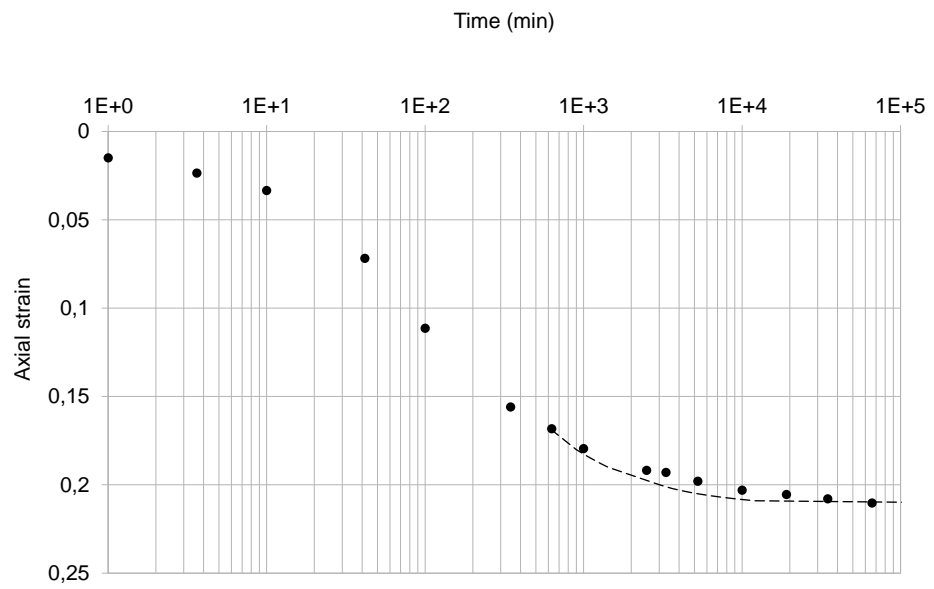
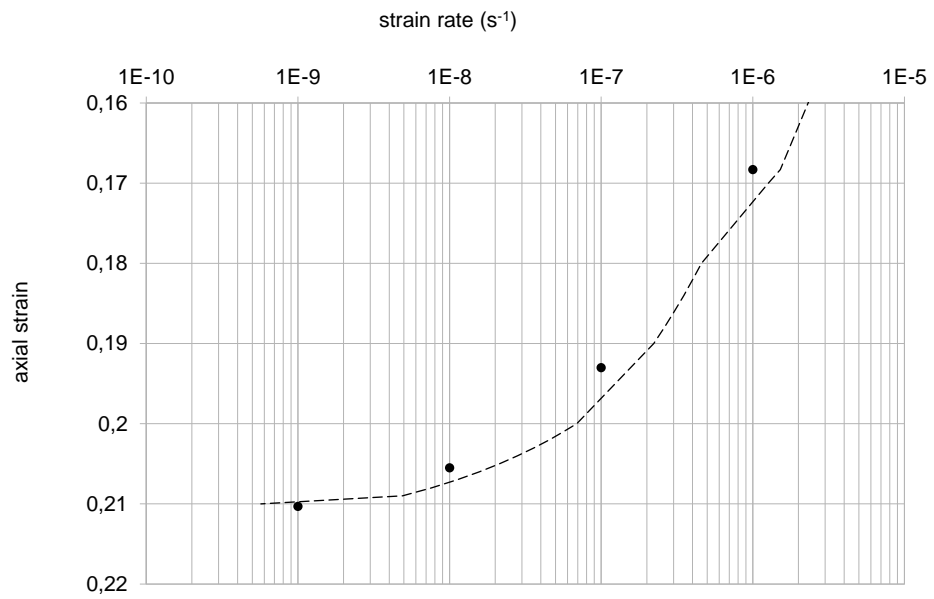


Figure 10:  $\sigma = 139kPa$

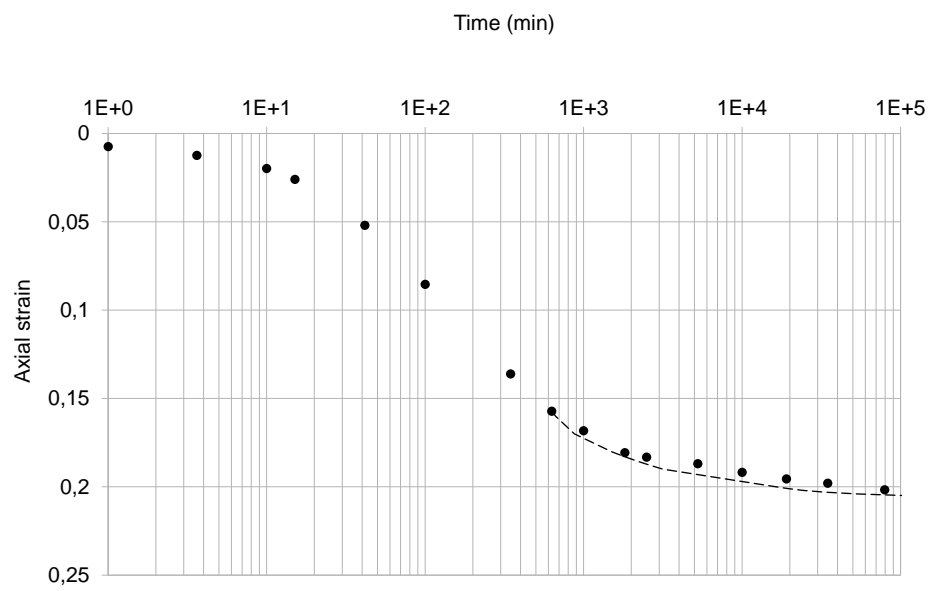
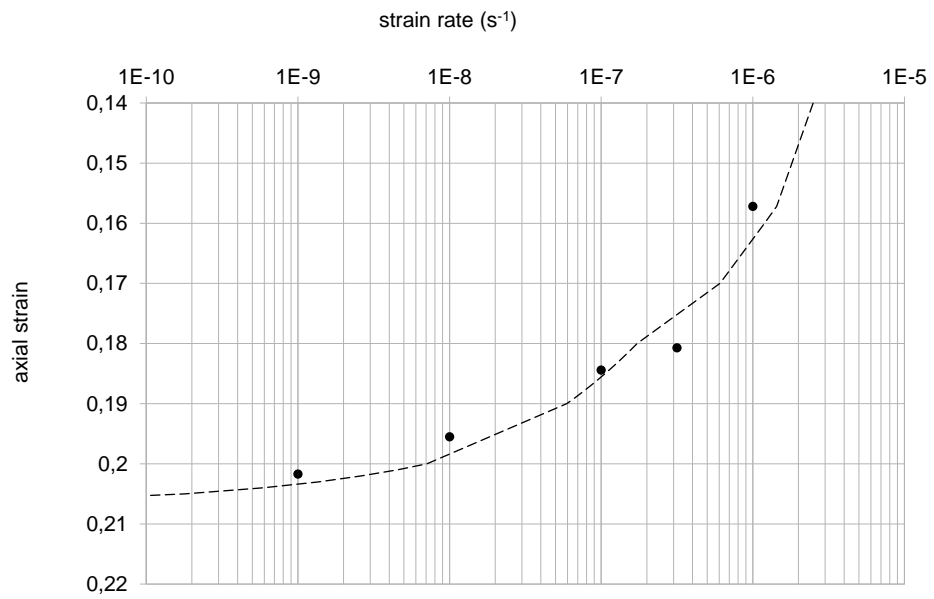


Figure 11:  $\sigma = 133kPa$

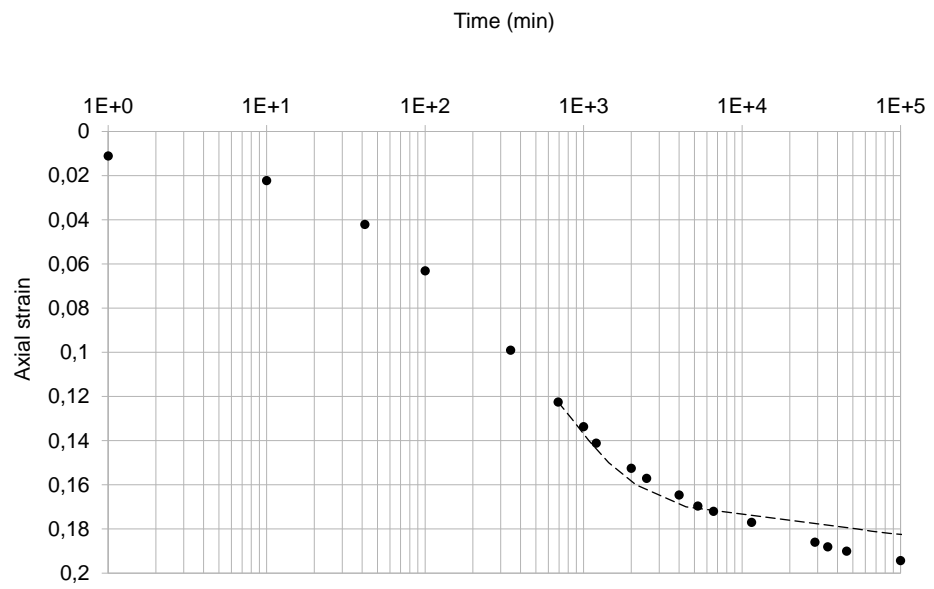
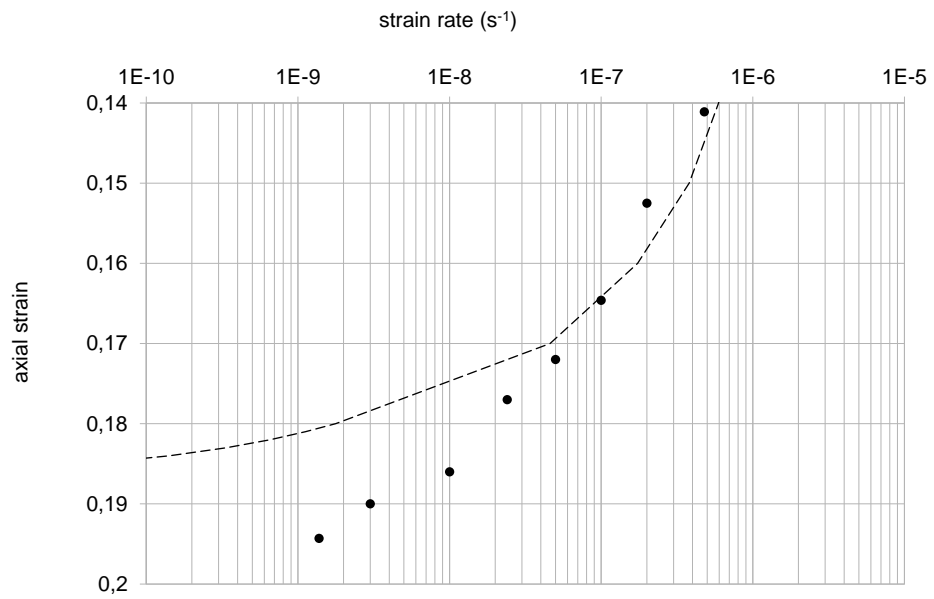


Figure 12:  $\sigma = 121kPa$

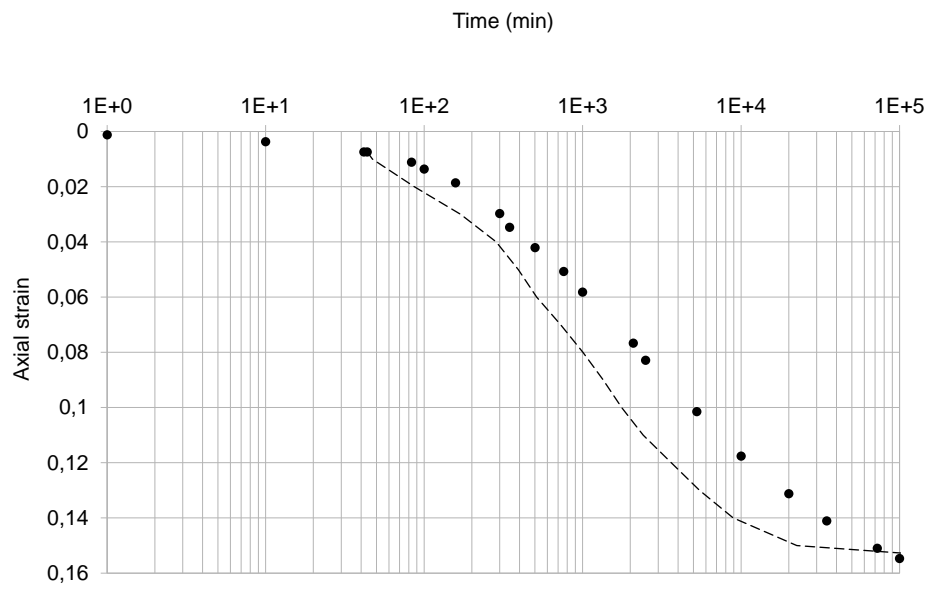
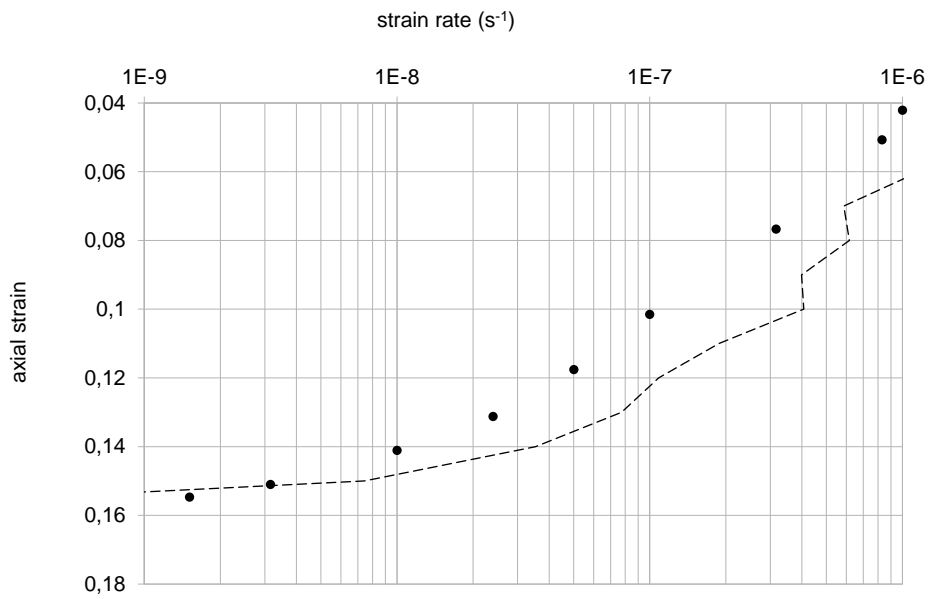


Figure 13:  $\sigma = 109kPa$



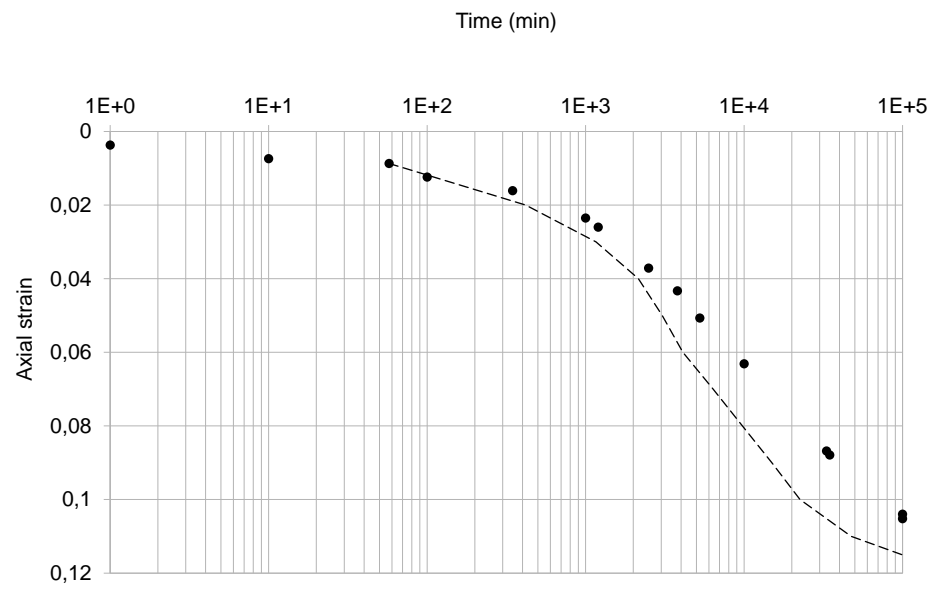
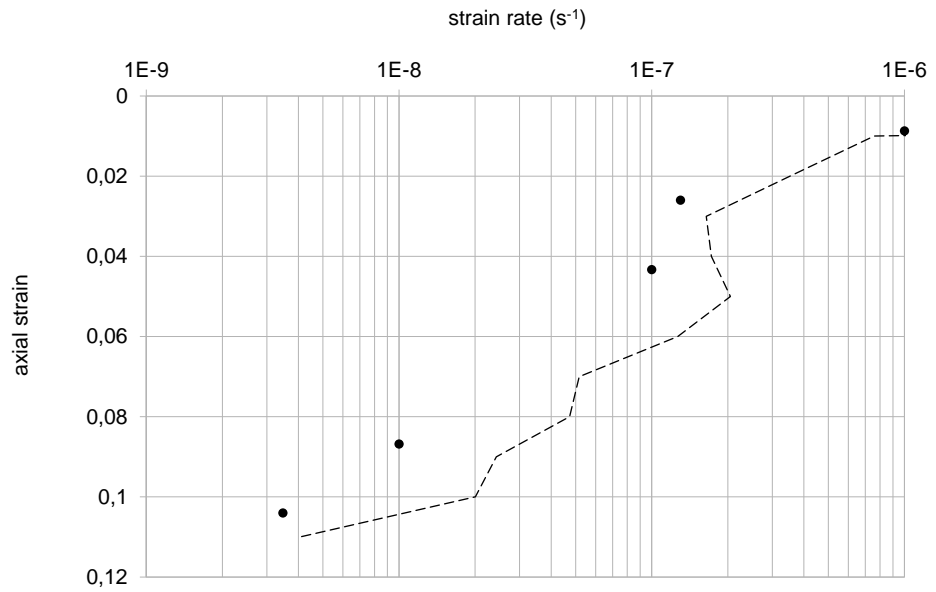


Figure 14:  $\sigma = 98kPa$

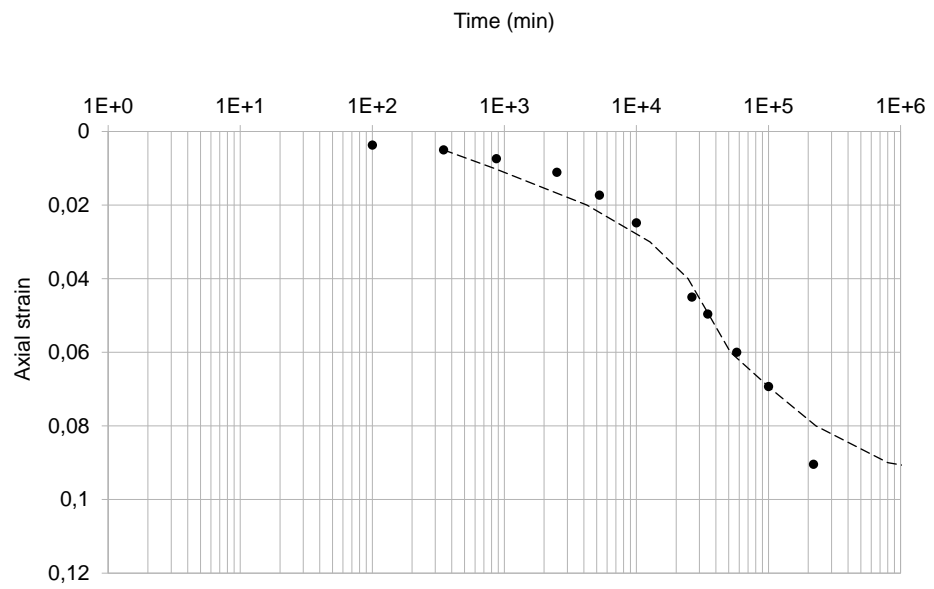
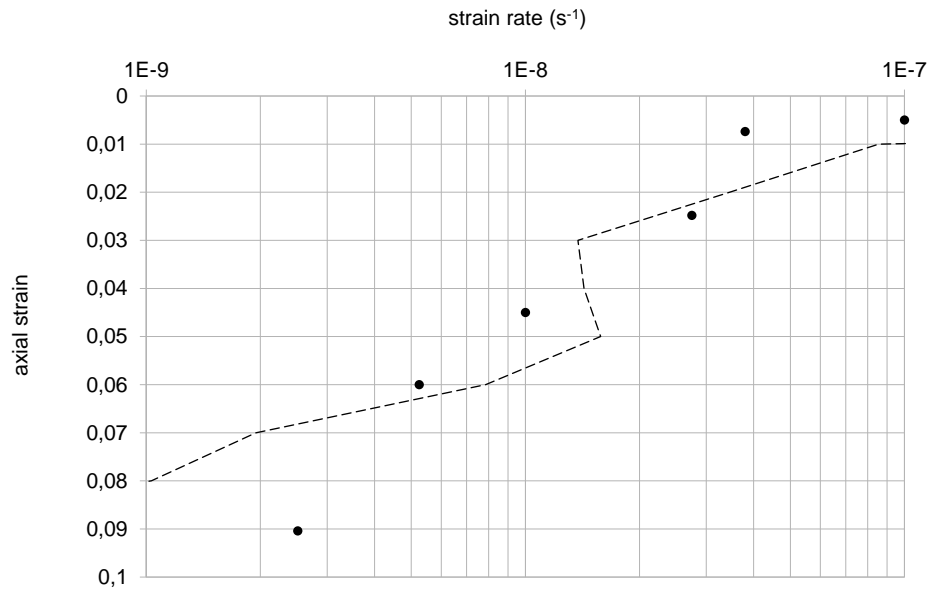


Figure 15:  $\sigma = 90kPa$

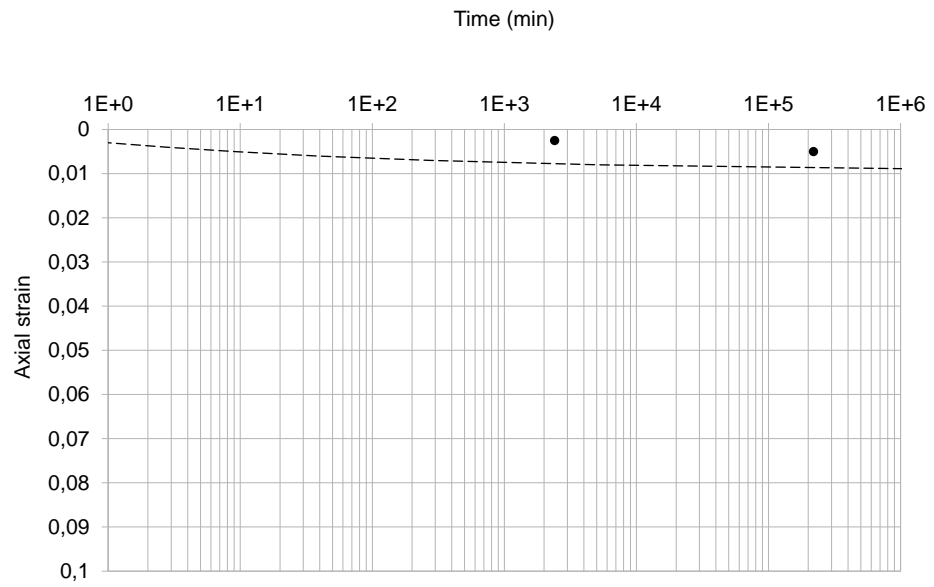
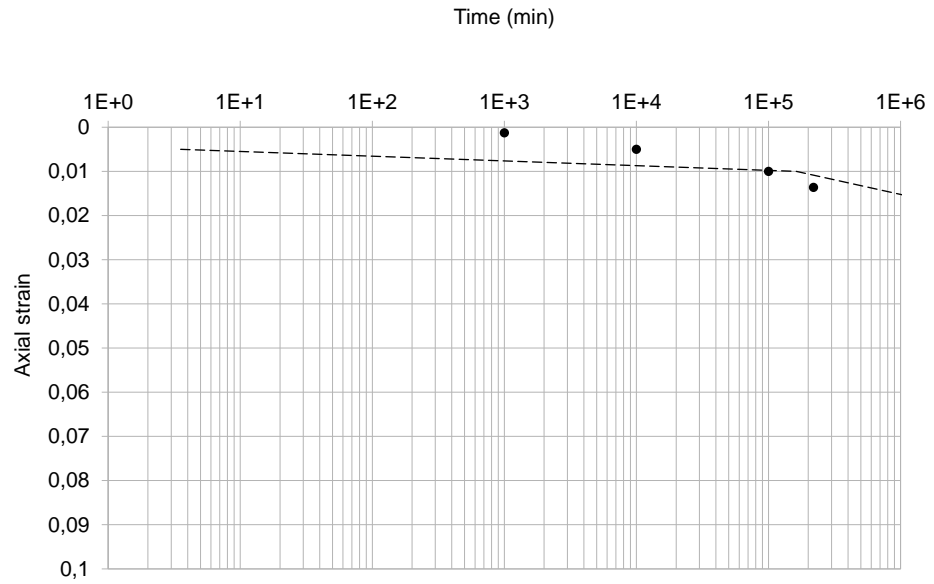


Figure 16: At the top of the page  $\sigma = 78kPa$  and at the bottom of the page  $\sigma = 67kPa$

## 9 Discussions

### Secondary Consolidation Predictions

As can be seen, the predictions are in general agreement with the experimental data both qualitatively and quantitatively for most of the tests which corroborates the decision of dividing the effective stresses in two components as suggested by Terzaghi himself. The predictions presented herein could have been obtained graphically, provided one could assess the zero strain rate curve and the remaining strain rate curves in between it and the experimentally determined ones. To this end, the proposed method is useful. However it does not go without mentioning that more research and analysis of field cases, such as the one by McRostie and Crawford (2001) are paramount in validating the proposed model and method.

Regarding the geological history of the deposit, according to the data in Leroueil et al (1985), the *OCR* of the Batiscan clay at a depth of 7.3 m is about 1.35. However, assuming an average solid pre-consolidation stress,  $\sigma'_{ps}$ , of about 69.4 kPa, then the “solid” *OCR*,  $OCR_s = \sigma'_{ps}/\sigma'_v$ , is about 1.07. Therefore in the present interpretation the clay from Batiscan is slightly over consolidated, being the difference between these two definitions of *OCR* the aging of the deposit. When considering the limits of the solid curve band,  $OCR_s$  will be a little less or more than the indicated value, between about 1 (normally consolidated) and 1.14.

At this point it is important to discuss the applicability of the method for assessing secondary consolidation and its consequences.

As proposed, the equations for assessing secondary consolidation are only valid when the excess pore-pressure is small enough so it can be disregarded. In the case of Batiscan clay, according to Leroueil et al (1985), this happens when the strain rate is equal or smaller than  $10^{-6}s^{-1}$ . However, that is not the same as to say that secondary consolidation comes only after primary consolidation. The authors believe that during primary consolidation there is an interaction between excess pore-pressure and viscous effective stress that is difficult to assess however there will be a contribution of the viscous effective stress in the development of strains during this phase. Therefore, consolidation settlements should not, in a strict sense, be assessed as the sum of primary consolidation as given by Terzaghi’s classical theory plus the proposed method outlined in this paper.

Despite of the limitations of the proposed method, it can be used for assessing secondary consolidation in the field where the excess pore-pressure becomes small.

Regarding the models proposed by Leroueil et al. (1985) and by Martins (1992) it can be shown mathematically their similarity. In the model proposed by Leroueil et al. (1985), the effective stress is a product of a function  $f(\dot{\varepsilon})$  and a function  $g(\varepsilon)$ . On the other hand, in the model proposed by Martins (1992)  $\sigma' = \sigma'_s(\varepsilon) + \sigma'_v(e, \dot{\varepsilon})$  where  $e$  is the void ratio. As the function  $f(\dot{\varepsilon})$  cannot be zero when the strain rate is zero (otherwise the soil would behave like a fluid),

then it can be written that  $f(\dot{\varepsilon}) = [1 + j(\dot{\varepsilon})]$  and therefore  $\sigma' = f(\dot{\varepsilon}) \cdot g(\varepsilon) = [1 + j(\dot{\varepsilon})] \cdot g(\varepsilon) = g(\varepsilon) + g(\varepsilon) \cdot j(\dot{\varepsilon})$  in the model proposed by Leroueil et al. (1985). Comparison of this last expression with the one proposed by Martins (1992) reveals that  $\sigma'_s(\varepsilon) = g(\varepsilon)$  and that  $\sigma'_v(e, \dot{\varepsilon}) = g(\varepsilon) \cdot j(\dot{\varepsilon})$  leading to the conclusion that both models are indeed mathematically similar.

When it comes to predicting the entire phenomena there are at least two know theories that can be somehow used. The first is the one proposed by Taylor (1942), known as Theory B, and the second is the one proposed by Barden (1965). Both theories divide the effective stress in two components, one dependent on strain and the other dependent on strain rate but their functions are quite different from each other. To represent the viscous component Taylor suggested a linear function (a Newtonian viscous function) and was able to find an analytical solution whereas Barden suggested a power law function just like the authors of this paper as a better approximation and solved the problem numerically. The problem with the theory proposed by Taylor is that the viscous function is highly non-linear as can be seen for example in Tables 1 and 2 and therefore that will lead, most likely, to significant differences between predictions and the actual behavior. On the other hand, although Barden's theory incorporates a better function, the problem is solved only numerically. In addition, he did not consider how the viscous function changes during secondary consolidation as can be seen once again in Tables 1 and 2. That been said, the authors do not know of better alternatives for predicting (primary and secondary) consolidation.

Despite of its complexity, a possibility is the numerical solution of a differential equation similar to the one proposed by Barden however using a power law function (or any other function) that changes with strain (void ratio). That is, considering that  $\sigma'_v = f(\varepsilon, \dot{\varepsilon})$ .

Finally, the authors wonder if the process of structuration/destructuration is related to the viscous component of the effective stress as noted by the ever changing character of the viscous function with strain development.

## $K_0$

Schmertmann (1983) proposed what he called a simple question about secondary consolidation and more than 30 years later no definitive answer was reached. However, the majority of the experimental results point in the direction of the increase of  $K_0$  over time during secondary consolidation.

One interpretation for the increase of  $K_0$  over time comes from Schmertmann himself. In Schmertmann (2012), he wrote "*As previously discussed in 12.2, the secondary part of the consolidation process of a clay's structure adjusting to an increased load involves a time transfer from an unstable  $\Phi'a$  to a stable  $\Phi'\beta$ . As developed herein,  $\Phi'a$  results from particle/AWL sliding and does not depend on particle interference effects, while  $\Phi'\beta$  does so depend. Despite the one dimensional constraint for the  $K_0$  condition,  $\Phi'a$  can dissipate with time because of its viscosity. But, a matching increase in  $\Phi'\beta$  cannot fully develop with time because particles, on average, cannot move across, nor can shear strain*

*develop to increase interference along the lateral boundary minor principal plane. This reducing  $\Phi'$  to  $\Phi'\beta$  transfer ability reduces the clay's ability to sustain an increased shear loading and  $K_0$  increases with time to maintain equilibrium."*

Another interpretation comes when using the model proposed by Martins (1992) or this model as modified by Alexandre (2006). Consider that in an inclined plane in a consolidation test specimen the shear stress is the sum of a frictional and a viscous component resisting compression. Writing the force equilibrium equation along the inclined plane will relate  $\sigma_3$  (the lateral stress),  $\sigma_1$  (the applied vertical stress) and the frictional and viscous components of the shear stresses. As the viscous shear stress resists compression, the effect of the viscous component will be to alleviate somehow the lateral stress. As at the end of secondary consolidation the strain rate and the viscous component have to drop to zero, then the lateral stress will have to increase. Therefore,  $K_0$  should increase during secondary consolidation.

Following Alexandre and Martins (2012), Equations (5) to (8) can be combined to assess the time dependent behavior of  $K_0$  over time during secondary consolidation. Considering that  $K_0$  is defined as:

$$K_0 = \frac{\sigma'_3}{\sigma'_1} \quad (16)$$

And that, on the other hand,  $\sigma'_3 = \sigma'_{3s} + \sigma'_{3v}$ ,  $\sigma'_1 = \sigma'_{1s} + \sigma'_{1v}$ , it follows that:

$$K_0 = \frac{\sigma'_{3s} + \sigma'_{3v}}{\sigma'_{1s} + \sigma'_{1v}} \quad (17)$$

In addition, defining a "solid" coefficient of earth pressure at rest as:

$$K_{0s} = \frac{\sigma'_{3s}}{\sigma'_{1s}} \quad (18)$$

Assuming that macroscopically there is no strain rate in the horizontal direction, then, the following can be written:

$$K_0 = \frac{K_{0s} \cdot \sigma'_{1s}}{\sigma'_{1s} + \sigma'_{1v}} \quad (19)$$

As during secondary consolidation the viscous component is transferred to the solid component, the denominator of Equation (19) remains unaltered while the numerator increases, therefore  $K_0$  should increase over time. Assessing  $\sigma'_{1v}$  and  $\sigma'_{1s}$  at each strain allows for the assessment of the pattern of increase of  $K_0$  over time. The figure below shows a sketch of this pattern.

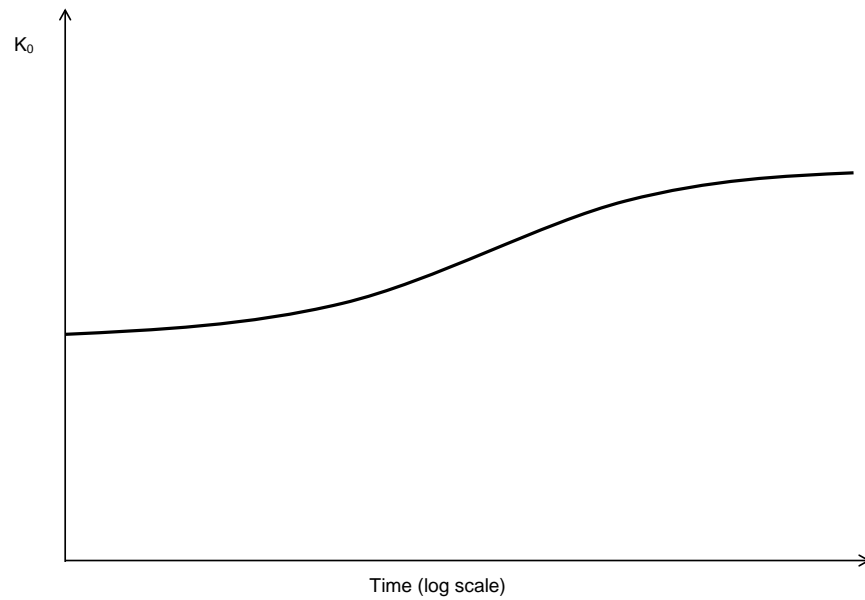


Figure 17: Pattern of the increase of  $K_0$  during secondary consolidation, after Alexandre and Martins (2012)

Although this pattern of increase in  $K_0$  can be seen in Lacerda (1976), it can be said that the test carried out by Lacerda is not a true  $K_0$  test as it was done in the triaxial apparatus. Regarding the Batiscan clay, a Figure adapted from Mesri & Castro (1987), which shows the variation of  $K_0$  for the Batiscan clay can be seen below:

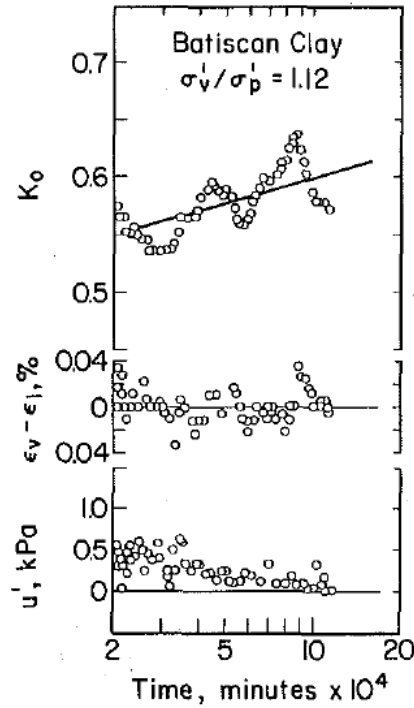


Figure 18:  $K_0$  increase during secondary consolidation adapted from Mesri & Castro (1987)

Although the pattern of variation of  $K_0$  as shown in the figure above is not the same as the predicted by this model,  $K_0$  does increase over time. Using a value of  $K_{0s} = 0.65$ , the equations mentioned above and the upper and lower bounds of the experimental data, the following figure can be made. The points represent the experimental data curve and the dashed line is the fitting curve.



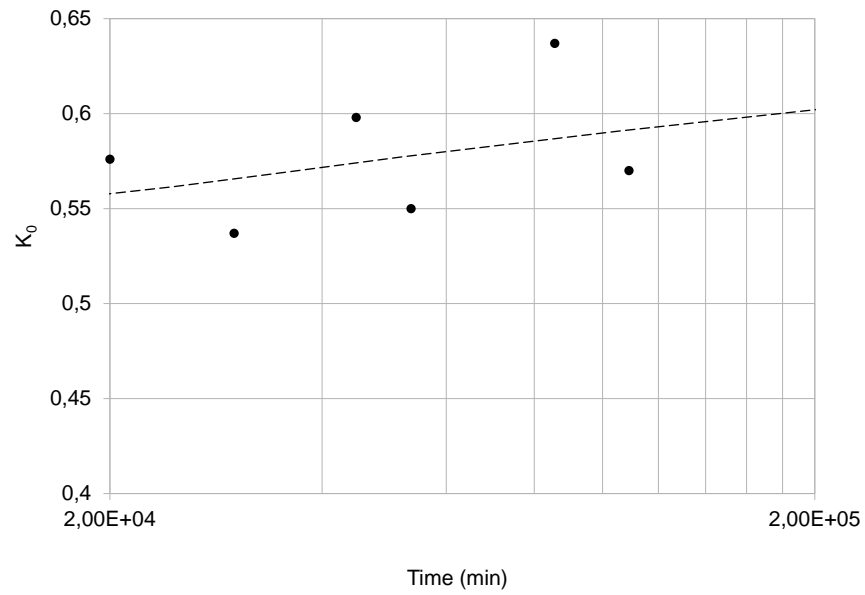


Figure 19: Fitting of the increase of  $K_0$  over time during secondary consolidation.

A figure showing the effective stress path, the Mohr circles of the solid and viscous components as well as the Mohr circle of the effective stresses (sum of the solid and viscous component) of a hypothetical secondary consolidation test is presented below.

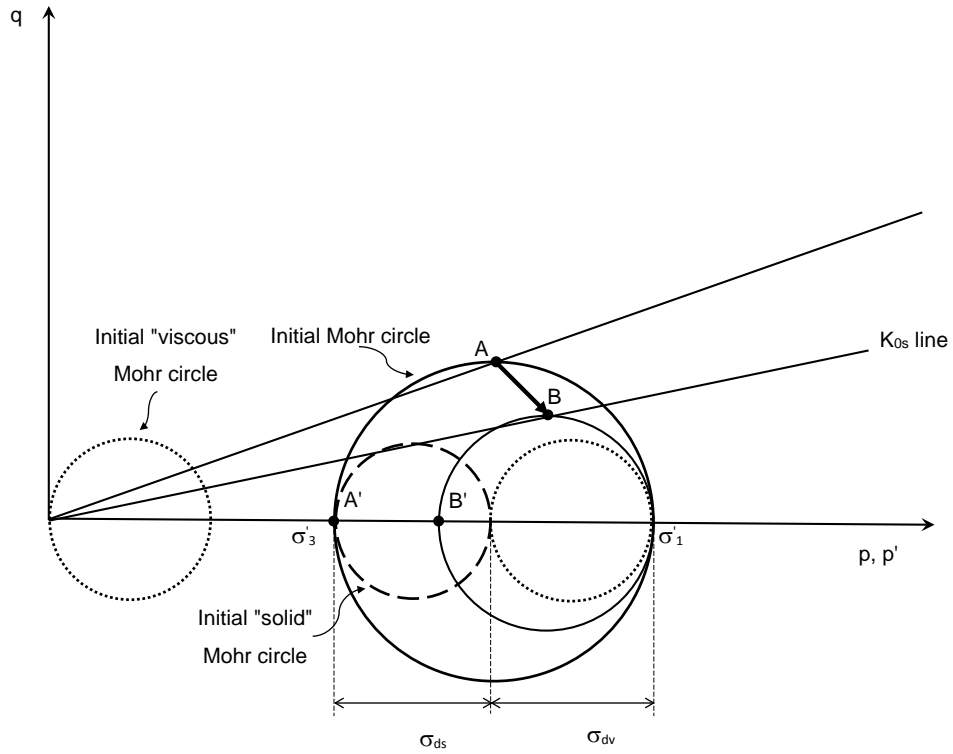


Figure 20: Hypothetical secondary consolidation stress path.

According to the figure, at the beginning of the process, the Mohr circle of the solid component is represented by the dashed circle that passes through point A' and the Mohr circle of the viscous component is represented by the finely dashed circle passing through  $\sigma'_1$  (in fact the viscous circle is at the origin as  $\sigma_{3v} = 0$ , however for considering the deviatoric and shear stresses it can be seen side-by-side with the solid circle). The sum of these circles gives the Mohr circle of effective stresses. With time, the viscous circle decreases in size until it vanishes while the solid circle increases and is displaced to the right until it reaches  $\sigma'_1$  along the  $K_{0s}$  line. At the same time the strain increases and the strain rate decreases as the film bonds are progressively transferred to the solid bonds and the EOS consolidation line is reached. At this point, secondary consolidation comes to an end and the final solid circle is the one passing through point B' and  $\sigma'_1$ . As can be seen in the figure, the effective stress path goes from point A to B while  $\sigma'_3$  increases and the shear stress is reduced. Secondary consolidation therefore can be seen as the combination of creep in the vertical direction and stress relaxation in the horizontal direction. In addition, this mechanism is consistent with the possibility of the occurrence of stress relaxation during secondary consolidation as foreseen by Taylor (1942)

more than 70 years ago.

## 10 Acknowledgments

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## 11 Conclusions

A new method for assessing secondary consolidation was proposed and its validity was assessed. Comparison between predictions and experimental results support the tentative conclusion of the validity of both model and method. Significant savings in terms of cost and time can be achieved in making predictions of secondary consolidation by means of the present method. The limitations and applicability to field cases are also presented in this paper.

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