

# Variational Shape Matching For Shape Classification and Retrieval

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## Abstract

In this paper we define a multi-scale distance between shapes based on geodesics in the shape space. The proposed distance, robust to outliers, uses shape matching to compare shapes locally. The multi-scale analysis is introduced in order to address local and global variabilities. The resulting similarity measure is invariant to translation, rotation and scaling independently of constraints or landmarks, but constraints can be added to the approach formulation when needed. An evaluation of the proposed approach is reported for shape classification and shape retrieval on the part *B* of the MPEG-7 shape database. The proposed approach is shown to significantly outperform previous works and reaches 89.05% of retrieval accuracy and 98.86% of correct classification rate.

*Key words:* Shape classification, shape retrieval, contour matching, shape geodesics, multi-scale analysis, robustness.

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## 1. Introduction and related work

This work addresses the definition of a robust distance between shapes based on shape geodesics. The proposed distance is applied to shape classification and shape retrieval. Recently, computer vision has extensively studied object recognition and known significant progress, but current techniques do not provide entirely significant solutions [Daliri and Torre, 2008; Veltkamp and Hagedoorn, 2001].

Regarding shape analysis and classification, similarity measures may be defined from information extracted from the whole area of the object (region-based techniques) [Kim and Kim, 2000], or from some features which describe only the object boundary (boundary-based techniques) [Costa and Cesar, 2001]. The latter category may also comprise skeleton description [Lin and Kung, 1997; Sebastian and Kimia, 2005]. Skeleton description of shapes has a lower sensitivity to articulation compared with boundary and region descriptions, but it is with the cost of higher degree of computational complexity due to tree or graph matching [Sebastian and Kimia, 2005; Sebastian et al., 2003]. On the other hand, boundary-based object description is considered more important than region-description because an object's shape is mainly discriminated by the boundary. In most cases, the central part of object contributes little to shape recognition.

The boundary-based approach described in this paper is established on a comparison between matched contours. Contour matching has been already widely applied to object recognition based on shape boundary [Diplaros and Milios, 2002]. Two major classes of techniques can be distinguished: those based on rigid transformations, and those based on non-rigid deformations

26 [Veltkamp and Hagedoorn, 2001]. Methods of the first type search optimal  
27 parameters which align feature points assuming that the transformation is  
28 composed of translation, rotation and scaling only. They may lack accuracy.  
29 Methods based on elastic deformations rely on the minimization of some ap-  
30 propriate matching criterion. They may present the drawback of asymmetric  
31 treatment of the two curves and in many cases lack of rotation and scaling in-  
32 variance [Veltkamp and Hagedoorn, 2001]. Existing techniques typically take  
33 advantage of constraints specific to the applications or use shape landmarks.  
34 These points are generally defined as minimal or maximal shape curvature  
35 [Del Bimbo and Pala, 1999; Super, 2006], as zero curvature [Mokhtarian and  
36 Bober, 2003], at a distance from specific points [Zhang et al., 2003], on con-  
37 vex or concave segments [Diplaros and Milios, 2002], or any other criteria  
38 suitable to involved shapes.

39 Shape analysis from geodesics in shape space has emerged as a powerful  
40 tool to develop geometrically invariant shape comparison methods [Younes,  
41 2000]. Using shape geodesics, we can state the contour matching as a varia-  
42 tional non rigid formulation ensuring a symmetric treatment of curves. The  
43 resulting similarity measure is invariant to translation, rotation and scaling  
44 independently on constraints or landmarks, but constraints can be added to  
45 the approach formulation when available. This paper is an extension of the  
46 work presented in [Younes, 2000] to the task of shape classification and the  
47 task of shape retrieval.

48 The following is a summary list of the contributions of our work:

- 49 – Geodesics in shape space have been introduced to develop efficient  
50 shape warping methods [Younes, 2000]. Recently, we have exploited

51 the corresponding similarity measure to define a new distance for shape  
52 classification and applied it to marine biological archives [Nasreddine  
53 et al., 2009a,b]. This distance takes advantage of local shape features  
54 while ensuring invariance to geometric transformations (e.g. transla-  
55 tion, rotation and scaling). To deal with local and global variabilities,  
56 we derive here a new multi-scale approach proposed for shape classifi-  
57 cation and shape retrieval.

58 – We establish the gain of the proposed method over state-of-art methods  
59 for shape classification and shape retrieval. The test is carried out on a  
60 complex shape database, the part  $B$  of the MPEG-7 Core Experiment  
61 CE-Shape-1 data set [Jeannin and Bober, 1999]. This database is the  
62 largest and the most widely tested among available test shape databases  
63 [Daliri and Torre, 2008].

64 The subsequent is organized as follows. In Section 2 is detailed the pro-  
65 posed framework for shape matching in the shape space, from where a robust  
66 similarity measure between two shapes is taken. We discuss in Section 3 the  
67 benefit of the proposed similarity measure on shape matching performances.  
68 Sections 4 and 5 derive a multi-scale distance proposed for shape classifi-  
69 cation and shape retrieval. In Section 6 we evaluate the proposed distance  
70 for shape classification and shape retrieval for part  $B$  of the MPEG-7 shape  
71 database and we compare results to other state-of-art schemes.

## 72 **2. Proposed contour matching**

73 In this paper a boundary-based approach is considered. The comparison  
74 between shapes is based on a similarity measure using shape geodesics. The

75 proposed similarity measure is applied to shape classification and retrieval.  
 76 A multi-scale analysis is performed to take into account both local and global  
 77 differences in the shapes.

78 *2.1. Shape geodesics*

79 There are various ways to solve for shape matching problem, and many  
 80 similarity measures have been proposed in the case of planar shapes [Veltkamp,  
 81 2001]. Shape geodesics have emerged as a powerful tool to develop geometri-  
 82 cally invariant shape comparison methods [Younes, 2000]. Shapes are consid-  
 83 ered as points on an infinite-dimensional Riemannian manifold and distances  
 84 between shapes as minimal length geodesic paths. Retrieving the geodesic  
 85 path between any two closed shapes resorts to a matching issue with respect  
 86 to the considered metric. Let us consider two shapes  $\Gamma$  and  $\tilde{\Gamma}$  locally char-  
 87 acterized by the angle between the tangent to the curve and the horizontal  
 88 axis ( $\theta$  and  $\tilde{\theta}$  respectively). Following [Younes, 2000], the matching issue is  
 89 stated as the minimization of a shape similarity measure given by :

$$SM^{\Gamma, \tilde{\Gamma}}(\phi) = 2 \arccos \int_{s \in [0,1]} \sqrt{\phi_s(s)} \left| \cos \frac{\theta(s) - \tilde{\theta}(\phi(s))}{2} \right| ds \quad (1)$$

90 where  $s$  refers to the normalized curvilinear abscissa defined on  $[0, 1]$ ,  $\phi$  is  
 91 a mapping function that maps the curvilinear abscissa on  $\Gamma$  to the curvi-  
 92 linear abscissa on  $\tilde{\Gamma}$  and  $\phi_s = \frac{d\phi}{ds}$ . The similarity measure considered here  
 93 includes a measure of the difference between the two orientations  $\theta$  and  $\tilde{\theta}$ ,  
 94  $\left( \cos \frac{\theta(s) - \tilde{\theta}(\phi(s))}{2} \right)$ , and a term that penalizes the torsion and stretching along  
 95 the curve,  $(\sqrt{\phi_s(s)})$ .

96 Curve parametrization via angle function  $\theta(s)$  naturally leads to a repre-  
 97 sentation which complies with the expected invariance properties (translation  
 98 and scaling). A translation of the curve has no effect on  $\theta$ , and an homo-  
 99 thety has no effect on the normalized parameter  $s$ . Thus curves modulo  
 100 translation and homothety will be represented by the same angle function  
 101  $\theta(s)$ . A rotation of angle  $c$  transforms the function  $\theta(s)$  into the function  
 102  $\theta(s) + c$  modulo  $2\pi$ . For rotation invariance, the minimization of  $SM^{\Gamma, \tilde{\Gamma}}(\phi)$   
 103 over all choices for the origins of the curve parameterizations is considered.

104 *2.2. Robust variational formulation*

105 Given two shapes  $\Gamma$  and  $\tilde{\Gamma}$  respectively encoded by  $\theta(s)$  and  $\tilde{\theta}(s)$ , the  
 106 matching problem comes to the registration of two 1D signals [Nasreddine  
 107 et al., 2009a,b]. The registration consists in retrieving the transformation  
 108 that best matches points of similar characteristics. Formally, it resorts to  
 109 determining the transformation function  $\phi(s)$  such that  $\theta(s) = \tilde{\theta}(\phi(s))$ . Here,  
 110 this issue is stated as the minimization of an energy  $E^{\Gamma, \tilde{\Gamma}}(\phi)$  involving a data-  
 111 driven term,  $E_D^{\Gamma, \tilde{\Gamma}}$ , that evaluates the similarity between the reference and  
 112 aligned signals and a regularization term<sup>1</sup>,  $E_R$ .

$$E^{\Gamma, \tilde{\Gamma}}(\phi) = (1 - \alpha)E_D^{\Gamma, \tilde{\Gamma}}(\phi) + \alpha E_R(\phi) \quad (2)$$

$$E_R(\phi(s)) = \int_{s \in [0,1]} |\phi_s(s)|^2 ds \quad (3)$$

113 where  $\alpha$  is a variable that controls the regularity. From time causality, the  
 114 minimization of  $E^{\Gamma, \tilde{\Gamma}}(\phi)$  has to be carried out under the constraint  $\phi_s > 0$ .

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<sup>1</sup>The regularization term is considered in order to obtain a smooth transformation function.

115 To ensure more robustness against outliers, we have introduced a ro-  
 116 bust criterion as a modification of the similarity measure issued from shape  
 117 geodesics [Nasreddine et al., 2009b]. Using a robust estimator  $\rho$ , the shape  
 118 registration issue resorts then to minimizing:

$$\begin{aligned}
 E^{\Gamma, \tilde{\Gamma}}(\phi) &= (1 - \alpha)E_D^{\Gamma, \tilde{\Gamma}}(\phi) + \alpha \int_{s \in [0,1]} |\phi_s(s)|^2 ds \\
 E_D^{\Gamma, \tilde{\Gamma}}(\phi) &= \arccos \int_{s \in [0,1]} \sqrt{\phi_s(s)} \left| \cos \frac{\rho(r(s))}{2} \right| ds
 \end{aligned} \tag{4}$$

119 where  $r(s) = \theta(s) - \tilde{\theta}(\phi(s))$ . Several forms of the robust estimator  $\rho$  were  
 120 proposed [Black and Rangarajan, 1996]. We will use the Leclerc estimator  
 121 given by:

$$\rho(r) = 1 - \exp(-r^2/(2\sigma^2)) \tag{5}$$

122 with  $\sigma$  is the standard deviation of data errors  $r$ .

### 123 2.3. Numerical implementation

124 To solve for the minimization of  $E^{\Gamma, \tilde{\Gamma}}(\phi)$ , two methods are considered:  
 125 dynamic programming and an incremental scheme.

126 A dynamic programming algorithm is applied as follows. Given a dis-  
 127 cretisation step and the discretized vectors  $\theta(s_i)_{i=1..N}$  and  $\tilde{\theta}(\tilde{s}_j)_{j=1..M}$ , the  
 128 algorithm considers in the plane  $[s_1, s_N] \times [\tilde{s}_1, \tilde{s}_M]$  the grid  $G$  which contains  
 129 the points  $p = (x, y)$  such that either  $x = s_i$  and  $y \in [\tilde{s}_1, \tilde{s}_M]$ , or  $y = \tilde{s}_j$  and  
 130  $x \in [s_1, s_N]$ . We fetch a continuous and increasing matching function that is  
 131 linear on each portion that does not cut the grid. The value of the energy  
 132  $E^{\Gamma, \tilde{\Gamma}}(\phi)$  is calculated at each point of the grid depending on the values at

133 previous points, and the minimum is chosen. This procedure is iterated over  
 134 all choices for the origins of the curves. This algorithm is more detailed in  
 135 [Trouvé and Younes, 2000].

136 As an alternative, we have proposed an incremental iterative minimiza-  
 137 tion [Nasreddine et al., 2009b], which is shown to be computationally more  
 138 efficient than the dynamic technique in the case of registration without land-  
 139 marks (see section 3 for comparison). At iteration  $k$ , given current esti-  
 140 mate  $\phi^k$  we solve for an incremental update:  $\phi^{k+1} = \phi^k + \delta\phi^k$  such that  
 141  $\delta\phi^k = \underset{\delta\phi}{\operatorname{argmin}} E^{\Gamma, \tilde{\Gamma}}(\phi^k + \delta\phi)$ . The initialization of the algorithm is given by  
 142 the identity function taken in turn for all choices for the origins of the curves.  
 143 For each of these initializations, the algorithm iterates two steps:

- 144 1. the computation of the robust weights  $\omega_i^k$  issued from the linearization  
 145 of the Leclerc estimator as  $\omega_i^k = \frac{2}{\sigma^2} \exp(\frac{-r^2(s_i)}{\sigma^2})$  [Black and Rangarajan,  
 146 1996],
- 147 2. the estimation of  $\delta\phi^k = \{\delta\phi^k(s_i)\}$  as successive solutions of the lin-  
 148 earized minimization  $\delta\phi^k = \underset{\delta\phi}{\operatorname{argmin}} \sum_i E_i^k$ . The key approximation of  
 149 this linearization is:  $\tilde{\theta}(\phi^{k+1}) = \tilde{\theta}(\phi^k + \delta\phi^k) \approx \tilde{\theta}(\phi^k) + \tilde{\theta}_s(\phi^k) \cdot \delta\phi^k$ . For  
 150  $\alpha = 0$ , the equation we obtain does not have a unique solution. The  
 151 resulting  $\delta\phi^k(s_i)$  for  $\alpha \neq 0$  is given by:



$$\begin{aligned}
\delta\phi^k(s_i) &= \frac{N(s_i)}{D(s_i)} & (6) \\
g(s_i) &= (1 - \alpha) \sin\left(\frac{\omega_i^k r(s_i)}{2}\right) [\tilde{\theta}(\phi^k(s_i)) - \tilde{\theta}(\phi^k(s_{i-1}))] \\
N(s_i) &= -\sqrt{\phi^k(s_{i+1}) - \phi^k(s_{i-1})} g(s_i) \cos\left(\frac{\omega_i^k r(s_i)}{2}\right) \\
&\quad + 2\alpha [2\phi^k(s_i) - \phi^k(s_{i-1}) - \phi^k(s_{i+1}) \\
&\quad - \delta\phi^k(s_{i-1}) - \delta\phi^{k-1}(s_{i+1})] \\
D(s_i) &= \frac{1}{2} \sqrt{\phi^k(s_{i+1}) - \phi^k(s_{i-1})} g^2(s_i) - 4\alpha
\end{aligned}$$

### 152 3. Shape matching performances

153 To study the influence of the robust criterion and of the regularization  
154 term, we evaluate here the matching process for synthetic contours (one con-  
155 tour is obtained by applying a known transformation to the other one). Some  
156 examples of these synthetic shapes are given in Figure 1 with a representation  
157 of the used transformation function  $\phi$ .

158 {Figure 1 goes here}

159 In Figure 2 we report the mean square error  $MSE_\theta = E\left(\left|\theta - \tilde{\theta}(\phi)\right|^2\right)$   
160 obtained for different values of  $\alpha \in [0, 1]$ . This result is issued from the dy-  
161 namic programming algorithm. For high values of  $\alpha$ , the regularity term  
162 is favored over the similarity measure and the alignment results in high  
163  $MSE_\theta$  values. For small values of  $\alpha$ , the robust algorithm ensures so-  
164 lutions with smaller errors ( $MSE_\theta = 0.085$ ) corresponding to  $MSE_\phi =$

165  $E(|\phi_{applied} - \phi_{estimated}|^2) \approx 0.001$ . The gain<sup>2</sup> due to the robust solution  
 166 is represented in Figure 2(b); this gain is optimum for  $\alpha = 0$  and reaches  
 167 90%. The aligned shapes given in Figures 2(c) and 2(d) show the superiority  
 168 of the robust solution. The consistency of this result has been verified by  
 169 testing many transformation functions with different shapes.

170 {Figure 2 goes here}

171 Using the incremental iterative scheme, the minimization leads to the  
 172 same optimum as the dynamic programming except for  $\alpha = 0$ . For the  
 173 iterative scheme the regularity term is necessary,  $\alpha$  should have a nonzero  
 174 value to lead to a unique solution. Experimentally, a value of  $\alpha$  in the range  
 175  $[0.1, 0.2]$  is optimal.

176 {Figure 3 goes here}

177 In Figure 3, we report another test for a synthetic shape obtained by  
 178 applying an occlusion on the shape given in Figure 1(c). The results of its  
 179 matching to the reference shape given in Figure 1(a) are reported in Figures 4  
 180 and 5. We see that the robust algorithm is more robust against the occlusion,  
 181 it is still able to align the curves and to retrieve the applied transformation  
 182 with minor errors. The transformation estimated by the non robust algorithm  
 183 (Figure 4(b)) is in contrast far from the real one (Figure 1(b)).

184 {Figure 4 goes here}

185 {Figure 5 goes here}

186 The relevance of the robust solution is even more visible when we analyze  
 187 the evolution of the incremental algorithm through the initializations in turn

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<sup>2</sup>defined as:  $\frac{MSE_{\theta}^{NonRobust} - MSE_{\theta}^{Robust}}{MSE_{\theta}^{NonRobust}} \times 100$

188 for all choices for the origins of the curves. We report in Table 1 matching  
 189 results for initialization far from the correct solution, we notice that with the  
 190 robust criterion  $MSE_\theta$  decreases through iterations to attain the optimum.  
 191 In contrast  $MSE_\theta$  values remain greater when the non robust criterion is  
 192 used and only a local minimum is reached. These experiments show that  
 193 this criterion is robust to the initialization of the choice of the origins of  
 194 the curves. Hence, only one arbitrary initialization may be considered in  
 195 practice.

196 Regarding computational complexity, the incremental method is also more  
 197 efficient when shape matching with no landmarks is addressed. The dynamic  
 198 programming needs a relatively longer time. For example, for the synthetic  
 199 contours considered in Figure 1, this time reaches 9.7 times that required by  
 200 the robust iterative scheme.

201 {Table 1 goes here}

#### 202 4. Distance-based shape classification

203 In this section, we exploit shape geodesics for shape classification. The  
 204 alignment cost used in Eq. 4 is taken as the similarity between any two shapes.  
 205 On the basis of a general algebraic and variational framework, [Younes, 2000]  
 206 has proved that the constructed cost function meets all the conditions nec-  
 207 essary for a true distance between planar curves.

Formally, the distance between two shapes  $S_1$  and  $S_2$  is defined as:

$$d(S_1, S_2) = E_D^{S_1, S_2}(\phi^*) \textbf{ where } \phi^* = \underset{\phi}{\operatorname{argmin}} E^{S_1, S_2}(\phi) \quad (7)$$

208 In this work, a multi-scale characterization is issued from the combination  
209 of shape matching costs at different scales. Here, the *scale* is defined as the  
210 resolution of shape sampling, as in [Attalla and Siy, 2005].

211 In order to exploit local and global variabilities, the distance used for  
212 shape comparison is a combination of distances measured at different scales.  
213 Formally, the distance between shapes  $S_1$  and  $S_2$  is defined as follows:

$$d(S_1, S_2) = \frac{1}{N} \sum_{k=1}^N d_k(S_1, S_2) \quad (8)$$

214 where  $d_k$  is the distance defined in Equation 7 between the same shapes at  
215 the  $k^{th}$  scale and  $N$  the number of considered scales.

216 Assuming we are provided with a set of categorized shapes,  $(S_l, C_l)$ , where  
217  $S_l$  is the shape of the  $l^{th}$  sample in the database and  $C_l$  its class, the classi-  
218 fication of a new shape  $S$  may be issued from a nearest neighbor criterion.

## 219 5. Distance-based shape retrieval

220 In addition to shape classification performance, we also address shape  
221 retrieval [Del Bimbo and Pala, 1999]. A retrieval problem consists in deter-  
222 mining which shapes in the considered database are the most similar to a  
223 query shape. The classification accuracy of a shape descriptor does not neces-  
224 sarily give a relevant guess of the retrieval efficiency [Kunttu et al., 2006]. As  
225 for classification, the distance used for shape retrieval is the distance defined  
226 in Equation 8.

227 **6. Comparison to other schemes**

228 To compare the proposed approach to the state-of-the-art shape recogni-  
229 tion approaches, we proceed to an evaluation of shape classification and re-  
230 trieval performances on the part *B* of the MPEG-7 shape database [Jeannin  
231 and Bober, 1999]. This database is composed of a large number of different  
232 types of shapes: 70 classes of shapes with 20 examples of each class, for a  
233 total of 1400 shapes. The classes include natural and artificial objects. The  
234 shape recognition on this database is not simple because elements present  
235 outliers so that some samples are visually dissimilar from other members of  
236 their own class (Figure 6). Furthermore, there are shapes that are highly  
237 similar to examples of other classes (Figure 7).

238 {Figure 6 goes here}

239 {Figure 7 goes here}

240 We do not discuss edge detection here; it is an obvious step in image anal-  
241 ysis. The dataset of shape outlines are issued from an automated extraction  
242 of the outlines using the *Matlab* image processing toolbox<sup>3</sup>.

243 With a view to being invariant to flip transformation, the optimal match-  
244 ing between two shapes results from Equation 4 where matching costs are  
245 computed between the first shape and the second one flipped or not.

246 Shape representation is given by points equally sampled along the bound-  
247 ary. Shape sampling at different scales with 32, 48, 64 and 192 points is  
248 considered.

249 Classification rates are issued from the *leaving one out* method where

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<sup>3</sup>Website: <http://www.mathworks.com/products/image/>

250 each shape in turn is left out of the training set and used as a query image.  
251 Retrieval accuracy is measured by the so-called *Bull's eye test* [Jeannin and  
252 Bober, 1999]: for every image in the database, the top 40 most similar shapes  
253 are retrieved. At most 20 of the 40 retrieved shapes are correct hits. The  
254 retrieval accuracy is measured as the ratio of the number of correct hits of  
255 all images to the highest possible number of hits which is  $20 \times 1400$ .

256 As mentioned in Section 3, the best shape matching in term of mean  
257 square error is obtained for  $\alpha = 0.1$ . The results of shape classification  
258 carried out on this database do not change significantly ( $\pm 0.01\%$ ) by taking  
259  $\alpha$  in the range  $[0.05, 0.2]$ . Note that the value of  $\alpha$  intervenes in the process of  
260 convergence of the shape matching and not in the expression of the distance  
261 of Equation 8. In Figure 8 we report the variation of the correct shape  
262 classification rate with respect to  $\alpha$ .

263 {Figure 8 goes here}

264 {Table 2 goes here}

265 The proposed approach based on shape geodesics has been compared to  
266 state-of-the-art schemes for part *B* of the MPEG-7 dataset as reported in  
267 Table 2. Methods are categorized according to single-scale versus multi-scale  
268 and local versus global approaches. By global, we refer here to methods such  
269 that the shape descriptors hold information from all points along the shape  
270 (e.g., Fourier methods, Zernike moments) in contrast to techniques exploiting  
271 local shape features such as matching-based or wavelet-based schemes.

272 The proposed multi-scale approach outperforms reported schemes with a  
273 correct classification rate of 98.86% corresponding to a gain in term of correct  
274 classification rate between 0.3% and 17%. Regarding the *bull's eye*, a score

275 of 89.05% is reached. This is greater by 1.35% than the best result reported  
276 previously. The highest scores of previous works are those of methods based  
277 on shape matching and/or with hierarchical analysis (shape tree, hierarchi-  
278 cal procruste matching, string of symbols, IDSC, fixed correspondance with  
279 chance probability functions); this fact justifies the choices operated to de-  
280 velop the proposed approach which relies on shape matching coupled with a  
281 multi-scale analysis.

282 From the results reported in Table 2, one may analyze the performances  
283 of the different categories of techniques. Performances comparison between  
284 the single-scale and the multi-scale approaches shows clearly that multi-scale  
285 analysis is very relevant. The single-scale approches reach an average rate of  
286 correct classification of 94.04% and an average retrieval rate of 77.62% to be  
287 compared respectively to 97.16% and 81.91% for the multi-scale approaches.  
288 The performances of the method presented in this paper are improved by  
289 3.81% in correct classification rate and by 3.35% in retrieval score when  
290 considering a multi-scale analysis instead of its single-scale form. The gain  
291 both in classification and retrieval performances clearly state the relevance  
292 of the multi-scale approach for shape analysis.

293 Global methods are greatly outperformed by local schemes: for instance  
294 for a single-scale analysis, 86% versus 96.73% and 66.85% versus 80.85%  
295 for the mean correct classification and retrieval rates respectively for the  
296 global techniques and local ones. The later can be argued to provide more  
297 flexibility to exploit local shape differences. As expected, a similar conclusion  
298 holds when comparing multi-scale global and local schemes. It may also be  
299 noted that matching-based schemes also depict greater performances than

300 other local approaches (e.g., for multi-scale ones, 97.1% and 87.26% versus  
301 95.5% and 74.77% for the mean correct classification rate and mean retrieval  
302 rate respectively).

303 Compared to the other matching-based approaches, the gain reported for  
304 our approach may be associated with two main features. Before all, these  
305 results stress the relevance of the chosen shape similarity measure encoding  
306 geometric invariance to translation, rotation and scaling. The second impor-  
307 tant property, often not fulfilled by matching-based schemes, is the symmetry  
308 of the similarity measure, i.e. the measure of the similarity between *shape 1*  
309 and *shape 2* is the same than between *shape 2* and *shape 1*. This property is  
310 guaranteed by the fact that the matching is stated as a minimal path issue  
311 in the shape space. Regarding our multi-scale strategy, we proceed similarly  
312 to [Daliri and Torre, 2008], the multi-scale similarity measure is a mean over  
313 several scales. In previous works [Felzenszwalb and Schwartz, 2007; McNeill  
314 and Vijayakumar, 2006], the multi-scale analysis comes up through the shape  
315 matching process where the shape matching at a given resolution depends  
316 on all matchings performed at lower resolutions.

317 We further analyze the proposed multi-scale matching-based scheme for  
318 object classes depicted in Figure 9 for which a lower retrieval accuracy is  
319 reported. These shapes within these classes are highly similar, the local  
320 curvature differs in a small number of points only. Experimentally we notice  
321 that the use of the robust criterion leads to consider these data points as  
322 outliers. For example, if we focus on the nearest 20 neighbors of the samples  
323 of the class *spoon*, more than 50% are elements of the classes *watch*, *pencil*,  
324 *key* and *bottle*; if we use the similarity measure without the robust weights,



325 95% of the nearest 20 neighbors are of the same class, *spoon*. Using robust  
326 weights, the average retrieval accuracy is penalized due to the low accuracies  
327 obtained for these 6 classes, but overall it remains greater than without the  
328 use of the robust weights.

329 {Figure 9 goes here}

330 Future work will explore the combination of the proposed approach to  
331 kernel-based statistical-learning. Recently, in [Yang et al., 2008] authors  
332 propose to combine classical metrics to learning through graph transduc-  
333 tion. It has been shown that this approach yields significant improvements  
334 on retrieval accuracies. For example, the retrieval rate using the *IDSC*  
335 [Ling and Jacobs, 2007] is improved by 5.6% when combined to the learning  
336 graph transduction. We will focus on the combination of machine learn-  
337 ing techniques such as random forest and SVMs to the proposed multi-scale  
338 matching-based similarity measure.

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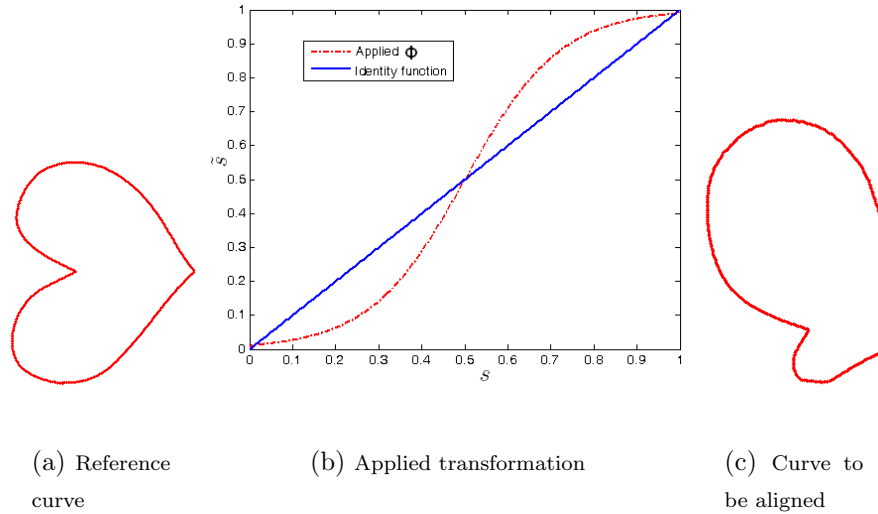
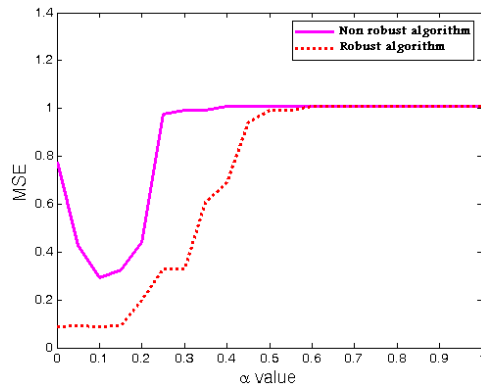


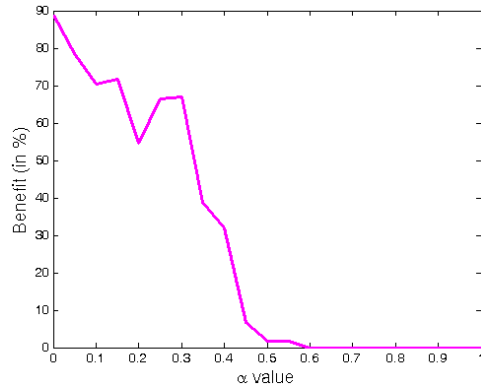
Figure 1: Test on synthetic shapes. We have applied a known transformation (1(b)) on the shape of 1(a) to get the shape 1(c).  $s$  and  $\tilde{s}$  are the normalized curvilinear abscissas on the curves.

Table 1: Optima  $MSE_{\theta}$  obtained by the robust and the non robust algorithms with the gain due to the robust solution for initializations of  $\phi$  at points which are far from the correct solution from different angles. This experiment is carried out on synthetic shapes given in Figure 1.

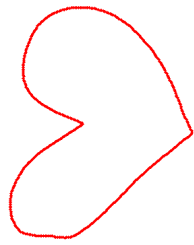
Angle	$MSE_{\theta}^{NonRobust}$	$MSE_{\theta}^{Robust}$	Gain = $\frac{MSE_{\theta}^{NonRobust} - MSE_{\theta}^{Robust}}{MSE_{\theta}^{NonRobust}} \times 100$
35°	<b>0.293</b>	<b>0.087</b>	<b>70.30%</b>
45°	<b>8.66</b>	<b>0.089</b>	<b>98.97%</b>
90°	<b>0.296</b>	<b>0.085</b>	<b>71.28%</b>
135°	<b>1.78</b>	<b>0.086</b>	<b>95.17%</b>



(a)  $MSE_\theta$  ( $rad^2$ ) versus  $\alpha$  values



(b) Gain due to the robust algorithm



(c) Aligned curve with the robust algorithm for  $\alpha = 0.1$



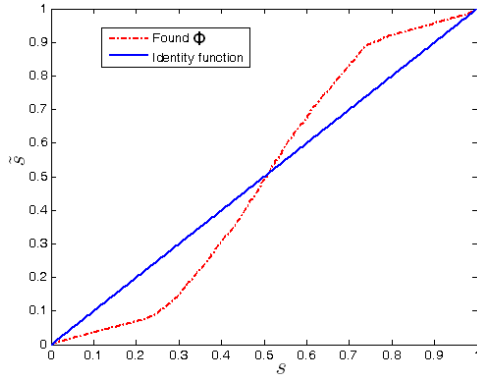
(d) Aligned curve with the non robust algorithm for  $\alpha = 0.1$

Figure 2: Results of shape matching on synthetic contours depicted in Figure 1 using the dynamic programming for different values of  $\alpha \in [0, 1]$ .

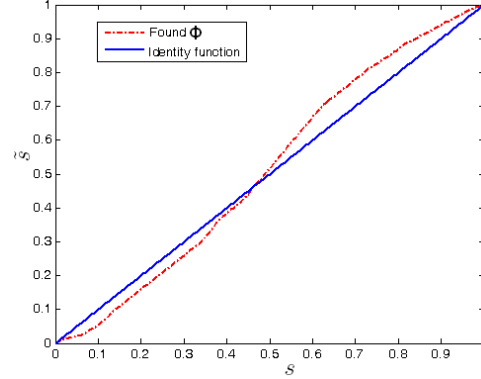


Figure 3: Test on synthetic shapes. Occluded shape obtained from the shape 1(c).

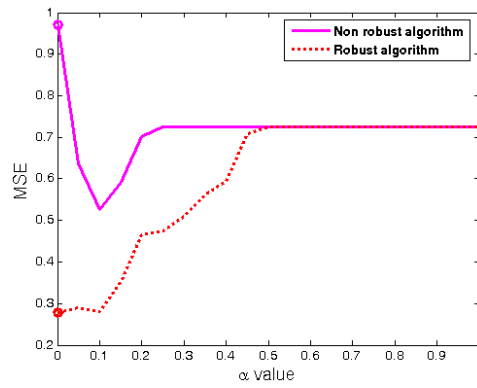




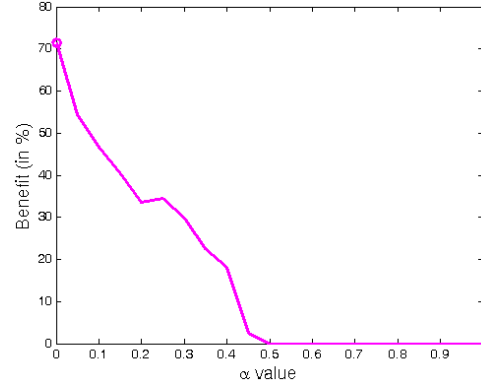
(a) Transformation found with the robust algorithm for  $\alpha = 0.1$



(b) Transformation found with the non-robust algorithm for  $\alpha = 0.1$

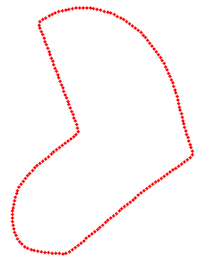


(c)  $MSE_{\theta}$  versus  $\alpha$  values



(d) Gain due to the robust algorithm

Figure 4: Results of shape matching using the iterative scheme for different values of  $\alpha \in ]0, 1]$ . We register here the occluded shape of Figure 3 with respect to the reference 1(a).  $s$  and  $\tilde{s}$  are the normalized curvilinear abscissas on the curves.



(a) Aligned  
curve with  
the robust  
algorithm for  
 $\alpha = 0.1$



(b) Aligned  
curve with  
the non  
robust al-  
gorithm for  
 $\alpha = 0.1$

Figure 5: Results of shape matching. Aligned shapes by the robust and non robust algorithms; the reference shape is given in Figure 1(a) and the shape to be aligned in Figure 3.

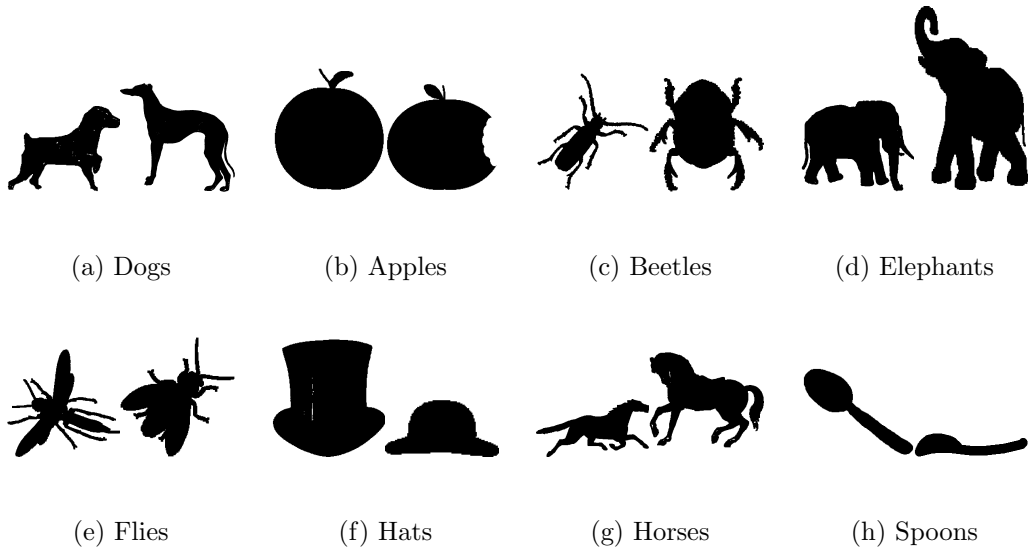


Figure 6: Examples of shapes that are visually dissimilar from other samples of their own class.

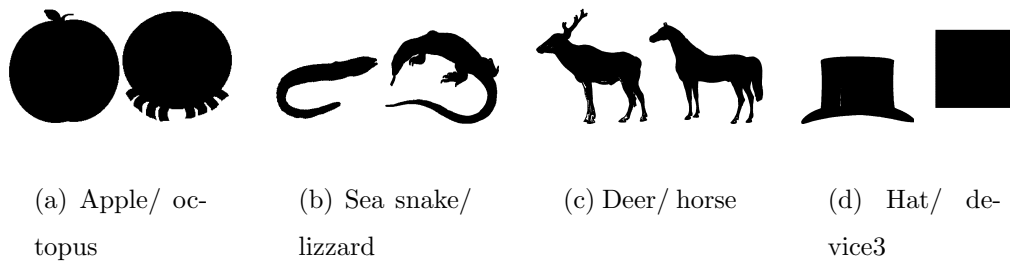


Figure 7: Examples of pair of shapes issued from different classes but highly similar.

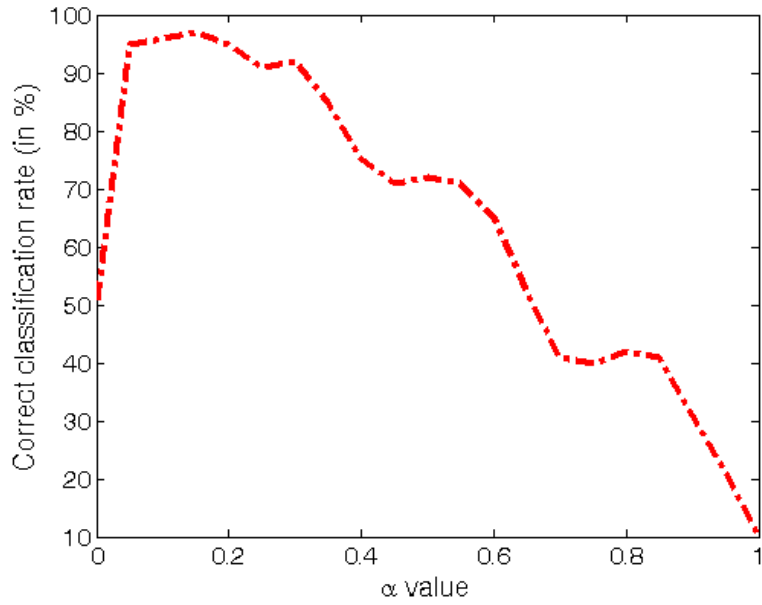


Figure 8: The correct classification rate (in %) on the MPEG-7 shape database versus the values of  $\alpha$  ( $\alpha$  is the coefficient that controls the regularity of the solution).

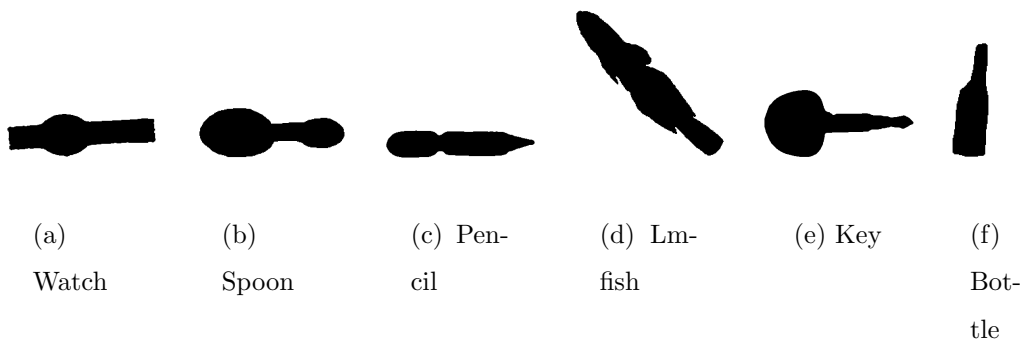


Figure 9: Examples of shapes from different classes with high similar curvature.

Table 2: Recognition accuracy measured as nearest neighbor classification rate and retrieval accuracy measured by the bull’s eye test on the MPEG-7 shape database.

Aspect		Method	Retrieval accuracy	Classification rate	
single-scale approaches	Global schemes	<i>Skeleton DAG</i> [Lin and Kung, 1997]	60%	NA	
		<i>Multilayer eigenvectors</i> [Super, 2006]	70.33%	NA	
		<i>Elliptic FD</i> [Nixon and Aguado, 2007]	NA	82%	
		<i>Zernike moments</i> [Kim and Kim, 2000]	70.22%	90%	
	Local schemes	Matching based	<i>Shape context</i> [Belongie et al., 2002]	76.51%	NA
			<i>Parts correspondence</i> [Latecki, 2002]	76.45%	NA
			<i>Curve edit distance</i> [Sebastian et al., 2003]	78.17%	NA
			<i>Inner-distance shape context (IDSC)</i> [Ling and Jacobs, 2007]	85.40%	NA
			<i>Racer</i> [Super, 2003]	79.09%	96.8%
			<i>Normalized squared distance</i> [Super, 2003]	79.36%	96.9%
			<i>Fixed correspondence</i> [Super, 2006]	80.78%	97%
			<i>Fixed correspondence + Chance probability functions</i> [Super, 2006]	83.04%	97.2%
			<i>Fixed correspondence + aggregated-pose chance probability functions</i> [Super, 2006]	84%	97.4%
			<b>Proposed scheme (64 points)</b>	<b>85.7%</b>	<b>95.05%</b>
Multi-scale approaches	Global schemes	<i>Multi-scale Fourier Descriptors 2D</i> [Direkoglu and Nixon, 2008]	NA	95.5%	
	Local schemes	Other criteria	<i>Wavelet</i> [Chuang and Kuo, 1996]	67.76%	NA
			<i>Curvature Scale Space</i> [Mokhtarian et al., 1996]	75.44%	NA
			<i>Optimized CSS</i> [Mokhtarian and Bober, 2003]	81.12%	NA
	Local schemes	Matching based	<i>Shape tree</i> [Felzenszwalb and Schwartz, 2007]	87.7%	NA
			<i>Hierarchical procruste matching</i> [McNeill and Vijayakumar, 2006]	86.35%	95.71%
			<i>String of symbols</i> [Daliri and Torre, 2008]	85.92%	98.57%
			<b>Proposed scheme</b>	<b>89.05%</b>	<b>98.86%</b>