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## **BIFURCATION-BASED MICRO/NANO-ELECTROMECHANICAL MASS DETECTION**

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### **ABSTRACT**

*When operating M/NEMS mass resonant sensors in the nonlinear regime, most of detection is based on the shift in the resonant frequency due to mass adsorption, as implemented in linear regime. This paper investigates an alternative mass detection based on the hysteretic behavior of the nonlinear frequency responses that takes advantage of bi-stability and bifurcations. A finite-degree-of-freedom reduced-order model of an electrostatically-actuated clamped-clamped microbeam is considered. Numerical results show that sudden jumps in amplitude make the detection of very small mass possible. Another interesting feature lies in the fact that the limit of detection can be set with the value of the operating frequency. However, it appears that this bifurcation-based mass detection does not exhibit the expected robustness. A possible improvement is proposed, based on the reinitialization of the system by a forced jump-down on the hysteretic response curve.*

## 1 INTRODUCTION

Mass detection via resonant micro/nano electromechanical systems (M/NEMS) has gained a lot of interest during the last decade, and single-protein mass detection is now the state-of-the-art in terms of experimental results [1]. This higher sensitivity is achieved mostly by scaling down the devices. At micro and nano scales, devices often exhibit large amplitudes of vibration and thus operate in the nonlinear regime. The detection is traditionally based on the shift in the resonant frequency due to mass adsorption, as implemented in linear regime. Several improvements or alternatives exploiting the nonlinear behavior of the resonator have been proposed, such as the increase of the frequency shift through parametric instability [2] or pull-in phenomenon [3]. More recently, Kumar *et al.* [4] proposed a bifurcation-based detection that exploits the bistability of a piezoelectrically actuated microcantilever. This alternative detection principle will be computationally analysed in this paper in the case of an electrostatically-actuated clamped-clamped microbeam.

## 2 NONLINEAR ELECTROMECHANICAL MODEL

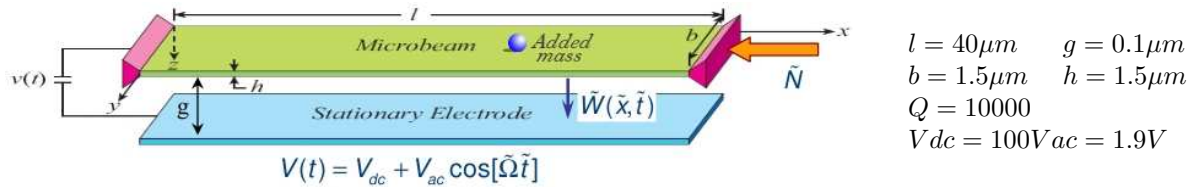


Figure 1. Geometry and data of the resonant microbeam with added mass.

The model for the nonlinear electrostatically-actuated clamped-clamped microbeam with added mass of Figure 1 is based on the model developed by Kacem *et al.* for inertial resonant sensors [5]. The nondimensional equation of motion is given by

$$\frac{\partial^4 w}{\partial x^4} dx + \frac{\partial^2 w}{\partial t^2} dx + \delta_{x_0}(x) m \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} dx - \left( N + \alpha_1 \int_0^1 \left[ \frac{\partial w}{\partial x} \right]^2 dx \right) \frac{\partial^2 w}{\partial x^2} dx = \alpha_2 \frac{[V_{dc} + V_{ac} \cos(\Omega t)]^2}{(1-w)^2} dx \quad (1)$$

where  $\tilde{w}(\tilde{x}, \tilde{t})$  is the nondimensional bending displacement of the beam,  $m$  is the ratio between the added mass and the microbeam mass, i.e.  $m = m_{added}/m_{beam}$  and  $\delta_{x_0}(x)$  is the Dirac function used to locate the added mass. The left-hand side of Equation (1) represents the inertial, damping, elastic and geometrically nonlinear mechanical forces, whereas the right-hand side stands for the nonlinear electrostatic forces with  $V_{dc}$ ,  $V_{ac}$ ,  $\Omega$  being the DC polarization voltage, the amplitude of the AC voltage, and the excitation frequency respectively.

A reduced-order model is generated by modal decomposition with the undamped linear mode shapes of the straight microbeam as basis functions in the Galerkin procedure. The solution of the resulting finite-degree-of-freedom nonlinear differential system is then computed by the Harmonic Balance combined with the Asymptotic Numerical Method for continuation analysis by means of the ManLab software [6].

### 3 DYNAMICS OF THE RESONANT SENSOR AND BIFURCATION-BASED DETECTION

The nonlinear microbeam exhibits a Duffing-like frequency response behavior, either hardening or softening depending on the dominating nonlinearity in the model (mechanical or electrostatic respectively). As shown in Figure 2a, the small amount of added mass downshifts the resonant frequency of the electromechanical resonator. This shift in frequency is commonly used to detect and measure the added mass [1]. However, the shift tends to zero as the added mass becomes smaller, thus limiting the amount of added mass which can be experimentally detected. Even if scaling down the device permits to increase the frequency shift and therefore improve the sensibility, shift-based detection shows some limitation when dealing with very small added mass such as molecules at atomic resolution.

Another approach consists in utilizing the dynamic jumps that occur in the Duffing hysteretic response. For this purpose, the resonator without added mass is operated at a constant excitation frequency  $\Omega_{op}$  slightly lower than the frequency  $\Omega_{lim}$  of the fold bifurcation point or limit point  $A_{lim}$ . At  $\Omega_{op}$ , the frequency response curve is multi-valued (red curve of Figure 2a) with two stable solutions  $A_1$  and  $A_2$ . In such a configuration, when a mass  $m$  is added, the response curve is shifted to the left (blue curve). At the operating frequency  $\Omega_{op}$ , this curve is single-valued. As a consequence, the solution jumps from point  $A_1$  to point  $B$ , resulting in a sudden increase in amplitude. Moreover, the smaller the added mass, the larger the increase of the response.

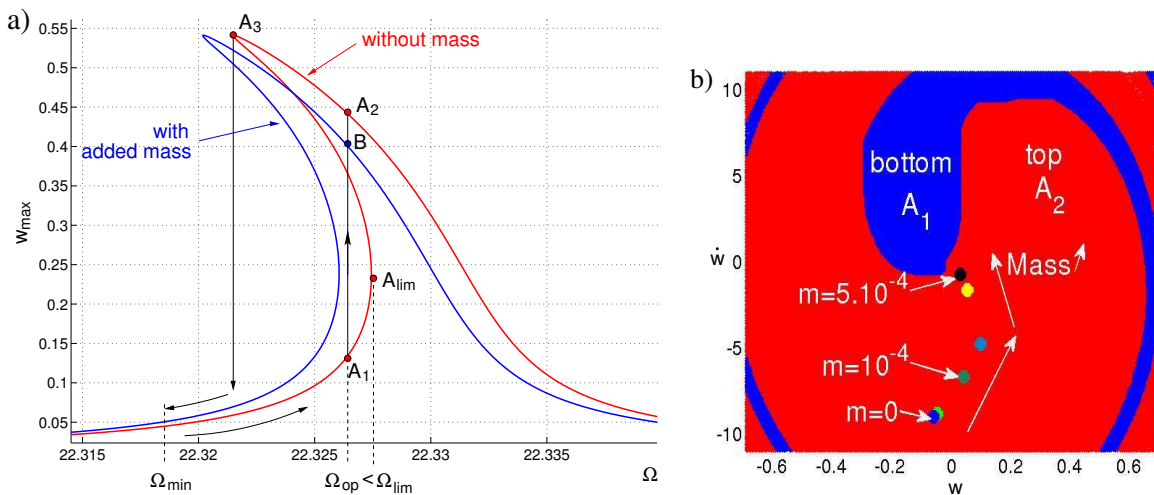


Figure 2: Softening behavior of the resonator. a) Shift in frequency of the due to added mass  $m = 5 \cdot 10^{-5}$  and jump phenomenon from  $A_1$  to  $B$ . b) Basin of attraction without added mass at  $\Omega_{op} = 22.3274$ .

Another interesting feature is that the limit of detection can be set with the value of the operating frequency  $\Omega_{op}$ . This can be useful if one wants to detect masses only above a given size. For instance, for the microbeam of Figure 1, operating at  $\Omega_{op} = 22.3274$  permits the detection of masses larger than  $m = 5 \cdot 10^{-5}$ , i.e. such that  $m_{added} \geq 10^{-17} kg$ , whereas operating at  $\Omega_{op} = 22.325$  permits the detection of masses larger than  $m \simeq 10^{-4}$ , i.e. such that  $m_{added} \geq 2 \cdot 10^{-17} kg$ . This limit of detection tends to zero as  $\Omega_{op}$  approaches  $\Omega_{lim}$ .

Although very promising, this bifurcation-based detection can be considered as robust only if the solution jumps back from point  $B$  to point  $A_1$  when the added mass leaves the microbeam. On the opposite, if the solution jumps to the upper-point  $A_2$  instead of  $A_1$ , then the next added mass will cause a jump from  $A_2$  to  $B$ , which can be tiny and difficult to detect if the mass is very small. The value of the added mass and the moment when the added mass

leaves the microbeam lead to different initial conditions at point  $B$ , which in turn lead to a jump towards  $A_1$  or  $A_2$ . The study of the basins of attraction of the microbeam without added mass (see Figure 2b) permits to conclude whether  $A_1$  or  $A_2$  will be reached. From the computations at  $\Omega_{op} = 22.3274$ , it turns out that for mass ratios equal or smaller than  $m = 5 \cdot 10^{-4}$ , i.e. for physical values  $m_{added} \leq 10^{-16} kg$ , the jump always occurs towards the upper solution  $A_2$  and consequently the bifurcation-based detection does not work anymore. When operating at  $\Omega_{op} = 22.325$ , the basin of attraction of the bottom stable solution is larger and  $A_1$  is reached when  $m > 1.5 \cdot 10^{-4}$ , i.e.  $m_{added} > 3 \cdot 10^{-17} kg$ . Since the masses of interest are much lower than this value, it can be concluded that the system never returns to its initial stable position, i.e. it is not reinitialized, and thus the bifurcation-based detection only works once.

A solution for forcing the reinitialization after the added mass has left the resonator consists in decreasing the operating frequency until the point  $A_3$  is reached and a jump-down takes place and then increasing the frequency again up to the initial operating frequency (point  $A_1$ ) as depicted in Figure 2a.

#### 4 CONCLUSIONS

An alternative mass detection based on nonlinear resonant sensors has been numerically investigated. This detection takes advantage of bistability and bifurcations of the hysteretic nonlinear responses. Contrary to the classical detection based on the shift in frequency, sudden jumps in amplitude make the detection of very small mass possible. Another interesting feature lies in the fact that the limit of detection can be set with the value of the operating frequency. However, it appears that this bifurcation-based mass detection does not exhibit the expected robustness. A possible improvement has been proposed, based on the reinitialization of the system by a forced jump-down on the hysteretic response curve.

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