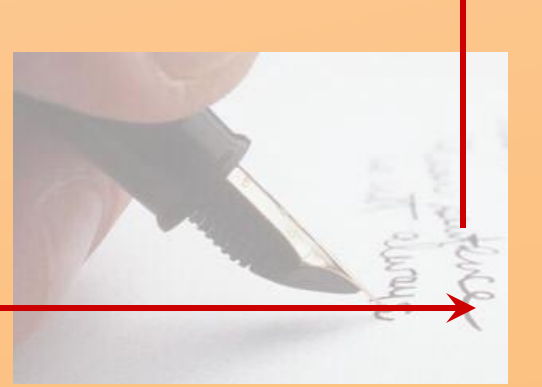


Stochastic Uncertainty Quantification of the Eddy Current in Human Body by using Polynomial Chaos Decomposition

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Problem

How to estimate the effect of the uncertainty on ELF induced currents?

Maxwell's equations

Quasi-static Maxwell's equation: $\text{curl} \mathbf{e} + \partial_t \mathbf{b} = 0$; $\text{curl} \mathbf{h} = \mathbf{j}$; $\text{div} \mathbf{b} = 0$

$$\mathbf{j} = \sigma \mathbf{e} ; \mathbf{b} = \mu \mathbf{h}$$

| | | | | | | | |
|--------|------|--------------------------|------|--------------------------|-----|---|----------------|
| ϕ | grad | \mathbf{e}, \mathbf{a} | curl | \mathbf{b} | div | 0 | Faraday's side |
| | | $\sigma \updownarrow$ | | $\mu \updownarrow$ | | | |
| 0 | div | \mathbf{j} | curl | \mathbf{h}, \mathbf{t} | | | Ampere's side |

Faraday's law is imposed in a strong sense, and Ampere's law has to be imposed in a weak sense by using the Finite Element method:

ϕ -a formulation: $(\sigma(\partial_t \mathbf{a} + \text{grad} \phi), \text{grad} \phi') = 0 \quad \forall \phi' \in H(\text{grad}, \Omega)$

Assumptions: displacement currents, reaction field are disregarded.

Quantification of the uncertainty

The random (= uncertain) variables are expanded as:

$$\sigma_G(\omega) \approx \sum_{i=0}^{P_{in}} \sigma_{G_i} \psi_i(\xi(\omega)) ; \quad \sigma_W(\omega) \approx \sum_{i=0}^{P_{in}} \sigma_{W_i} \psi_i(\xi(\omega))$$

known coefficients
Normal random variable
orthogonal Hermite polynomials

The quantity of interest are expanded in a similar way:

$$j_{Avg}(\omega) \approx \sum_{i=0}^{P_{out}} j_{Avg_i} \psi_i(\xi(\omega)) ; \dots$$

unknown coefficients

The unknown coefficients are computed by a Hermite-Gauss numerical quadrature:

$$j_{Avg_i} = \frac{E[j_{Avg}(\omega) \psi_i(\xi(\omega))]}{E[\psi_i(\xi(\omega))^2]}$$

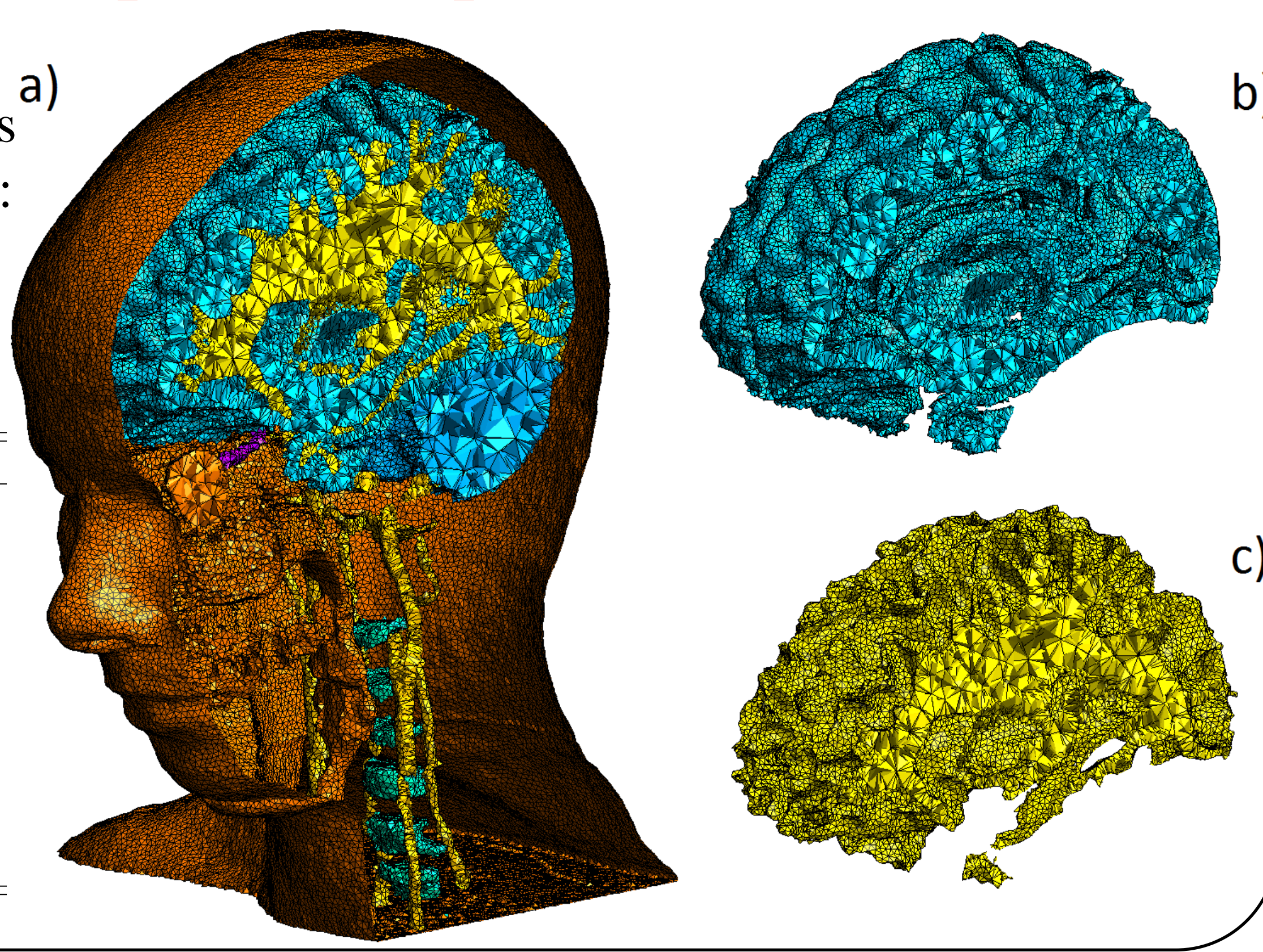
computed analytically

A deterministic Finite Element code (GetDP) is used as a "black box" for computing the values of j_{Avg} in the quadrature points.

The computational phantoms

ELLA (IT'IS foundation), has been meshed with Iso2mesh:
— 45 different tissues
— 1 million of nodes
— 5.8 millions of tetrahedra.

| Tissue | Conductivity (S/m) |
|--------------|--------------------|
| Muscle | 0.2330 |
| White matter | 0.0533 |
| Grey matter | 0.0753 - 0.5155 |
| Cerebellum | 0.0953 - 0.3020 |
| Eyes | 1.5000 |
| Spinal chord | 0.0274 |
| Lungs | 0.0684 |
| Heart | 0.0827 |
| Veins | 0.7000 |
| Liver | 0.0367 |
| Bones | 0.0504 |
| Cartilage | 0.1714 |
| Nerve | 0.0274 |
| CSF | 2.0000 |
| Midbrain | 0.0643 |



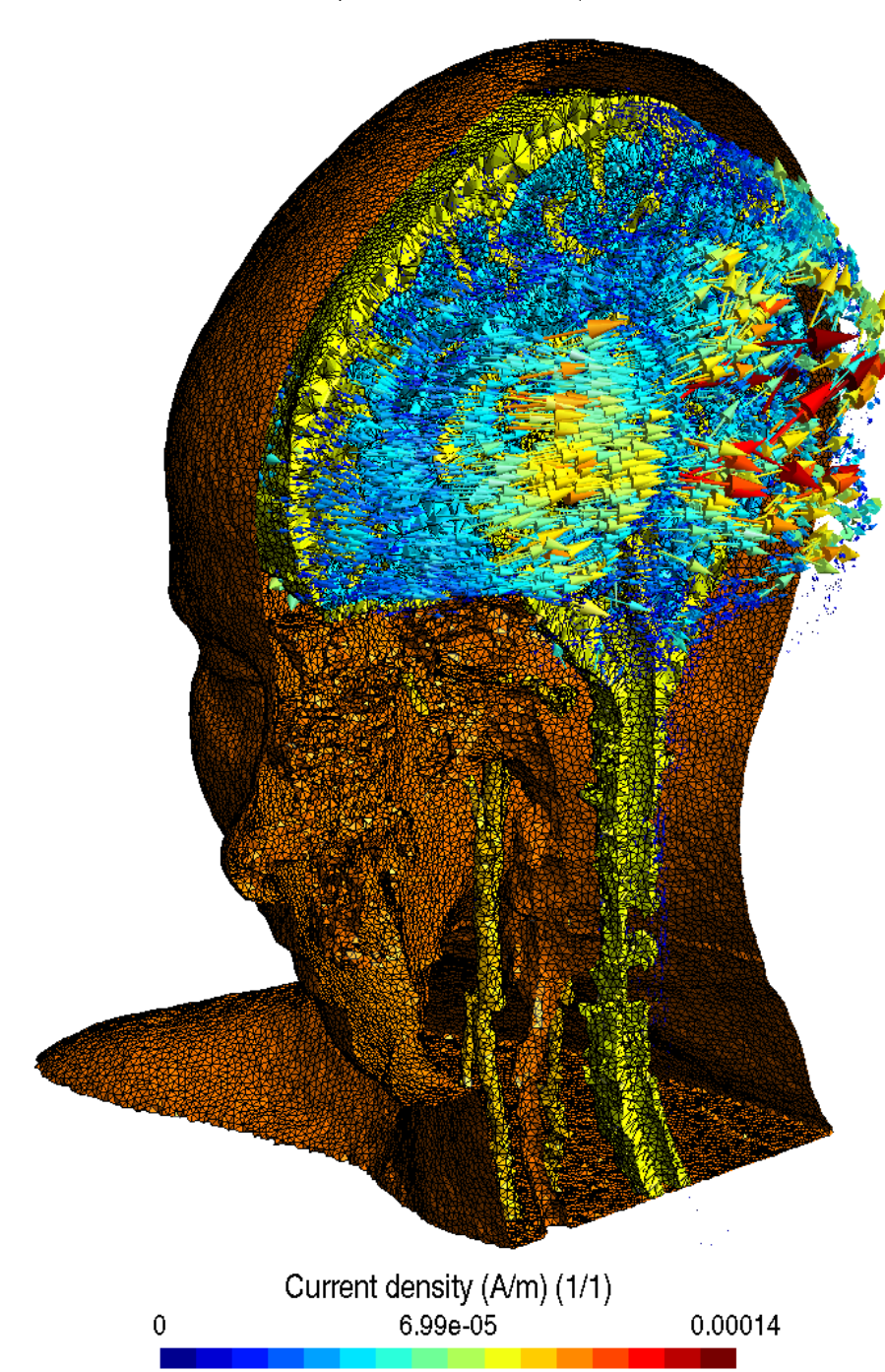
Exposure to the field generated by a wire (I=1kA, 50Hz)

Exposure conditions: the wire is placed close to the left side of the head (ELLA)

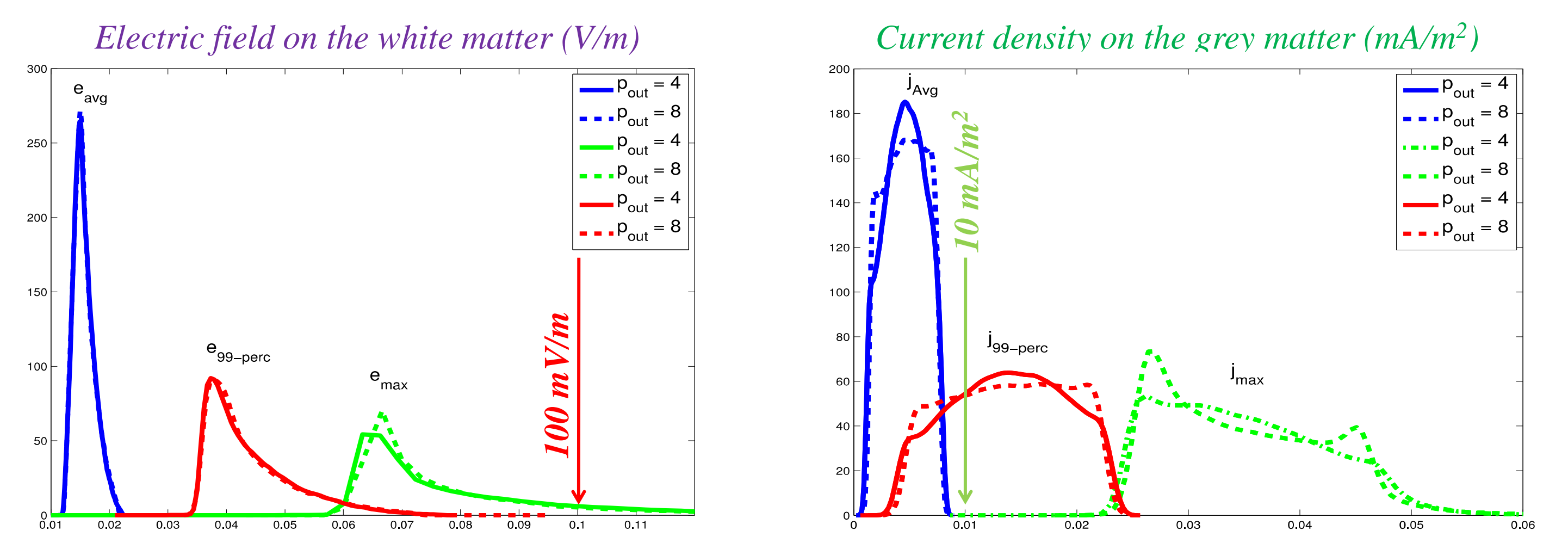
The probabilistic density of the average, maximum and 99% percentile are computed for the induced current density \mathbf{j} , and electric field \mathbf{e} .

Basic restrictions from ICNIRP 1998 : $\|\mathbf{j}_{max}\| < 10 \text{ mA/m}^2$ in the central nervous system (CNS)

Basic restrictions from ICNIRP 2010 : $\|\mathbf{e}_{99\%-perc}\| < 100 \text{ mV/m}$ in the CNS, 800 mV/m in other parts of the body

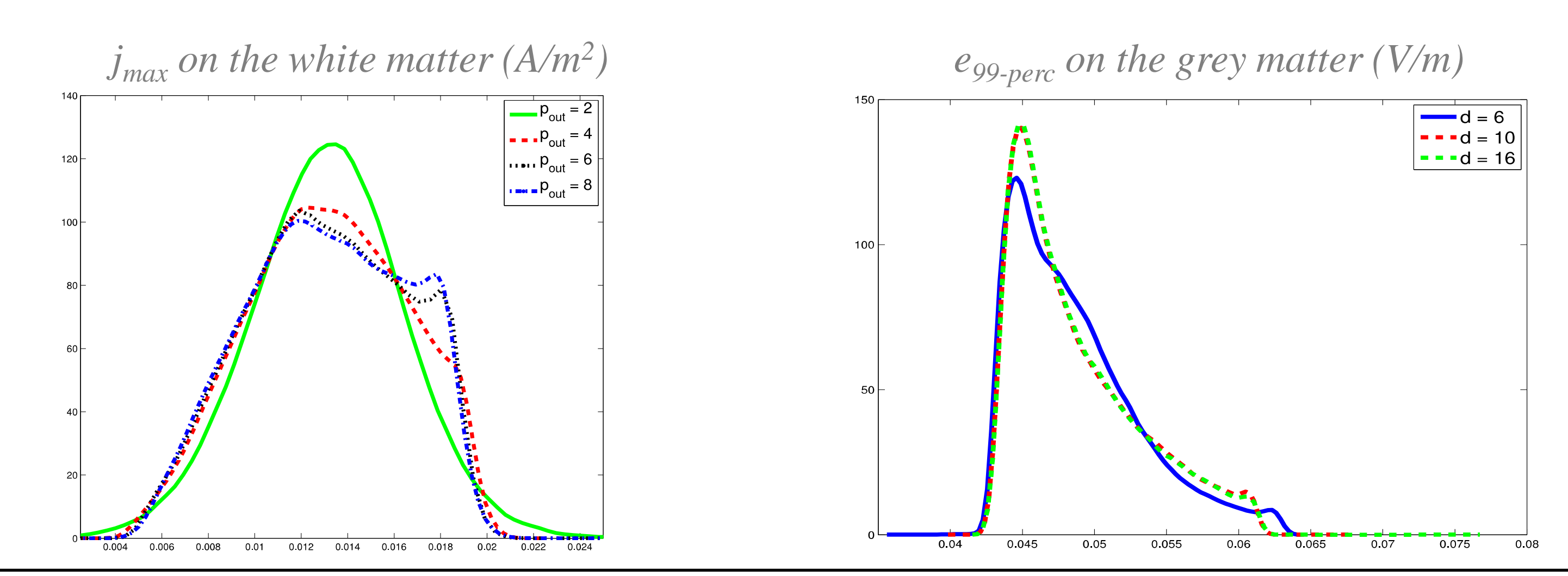


Which is the effect of the uncertainty on these values?

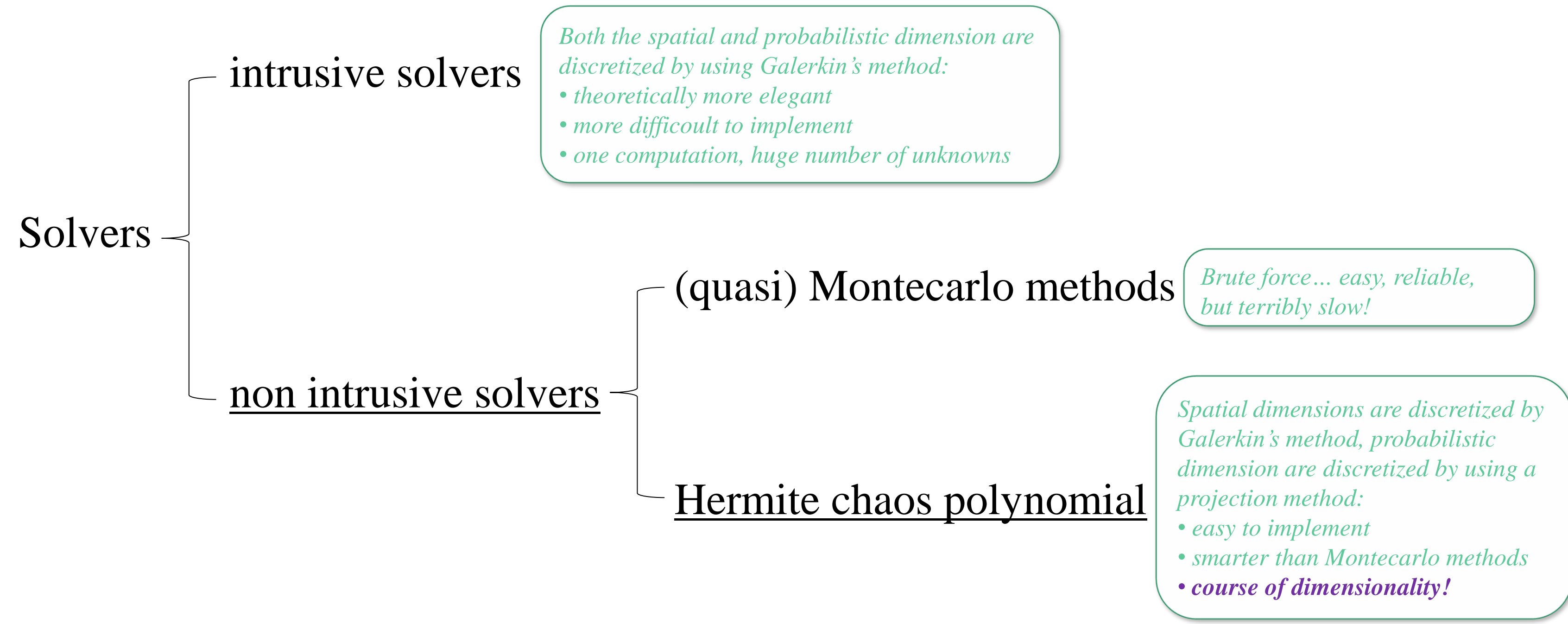


The basic restriction defined in the 2010 edition of ICNIRP recommendations are never exceeded. Conversely, the basic restriction defined in the 1998 edition would be certainly exceeded.

The computed probability distributions converges for $P_{out} > 6$ and $d > 8$



Probabilistic solvers



| | Number of uncertain variables | | | |
|------|-------------------------------|--------|--------|------------|
| | N=2 | N=3 | N=5 | N=8 |
| d=2 | 4 | 8 | 32 | 256 |
| d=5 | 25 | 125 | 3125 | 390625 |
| d=8 | 64 | 4096 | 32768 | 16777216 |
| d=12 | 144 | 248832 | 248832 | 5159780352 |

Number of quadrature points

Conclusion

- The proposed method allows to quantify the dosimetric uncertainty with respect of the conductivities of tissues.
- A characterization of the randomness of the conductivities is required.
- Not all the sources of uncertainty are taken into account.
- It is possible to increase the speed of computations by using sparse grid schemes.