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Stochastic uncertainty quantification of the eddy current in human body by using polynomial chaos decomposition

Roman Gaignaire♣, Riccardo Scorretti♡, Ruth V. Sabariego♣ and Christophe Geuzaine♣

The finite element method can be used to compute the electromagnetic fields induced in the human body by environmental extremely low frequency (ELF) fields. However, the electric properties of tissues are not precisely known and may vary depending on the individual, his/her age and other physiological parameters. In this paper, we account for the uncertainties on the conductivities of the brain tissues and spread them out to the induced fields by means of a non-intrusive approach based on the chaos Hermite polynomial with the finite element method as a black box [1], [2]. After showing the convergence of the method, we compute the probability to be over the thresholds define by the norms.

Index Terms— **Non-intrusive methods, polynomial chaos decomposition, stochastic methods**

I. Introduction

The uncertainty in electrical parameters, as well as the variability of the human body between different individuals, is a major problem in numerical dosimetry. Determining the conductivities of the human tissues is since years subject of research [3]. Tissues are highly heterogeneous "materials"and possibly anisotropic [4]. The properties of tissues change rather quickly after death [5], so that measurements performed *in vitro* on excised tissues may not be representative. Besides for ethical reasons, most of *in vivo* measurements have been performed on animals. Moreover, the age [6] and the physiological condition [7] may also significantly alter these properties. Obtaining data for characterising a foetus is even harder [8]. At ELF frequencies, the measurements are performed either by identifying an equivalent RC circuit, or by a four-point measurement. At those frequencies, electrode polarization is (another) major source of errors. Some tentatives have been performed [7], [9] for numerically estimating the conductivity and permittivity by modeling tissues as porous media, with a limit success. Gabriel *et al* have done a huge work [10]–[12] by collecting most of the existing data, and building a tissue database, which is currently the reference for dosimetric computations [13]. In a recent work [14], new measurements have shown large differences with respect to previously published data: the new values of conductivities are often higher and muscle-type tissues are found to be much less anisotropic. More recently, several techniques based on the magnetic resonance (MR) have been proposed. MR electric impedance tomography (MREIT) methods are based on the detection of the shielding effect of electrical currents on the magnetic field; these electrical currents may be either induced by the MR device itself, by a third external coil [15], or injected into the body through electrodes [16]. Diffusion tensor MREIT methods are based on the proportionality between water diffusion tensor and electrical conductivity. These last techniques are particularly interesting when dealing with brain [17], because the diffusion signal in this organ is strong enough, while other MREIT techniques are not applicable due to the high resistivity of the skull. Moreover, this technique provides information about the anisotropy of tissues. Using this technique, Sekino *et al* found higher conductivity values [18] than those reported by Gabriel in [10]. All MRI based techniques share the enormous advantage to be applicable *in vivo* in humans beings. In spite of the amount of work accomplished, the dielectric properties of human tissues are still poorly known: e.g. at 50 Hz the conductivities of the white and grey matter in [11] and [19] span, respectively, within $[0.0753\ ;\ 0.5155]$ and $[0.0533\ ;\ 0.302]$ S/m. Therefore, it is crucial to quantify the effect of this uncertainty on the electromagnetic fields computed in the human body. One could use a extremely time-consuming Monte-Carlo (MC) simulation to statistically characterise the induced fields (some weeks in our model). In this paper, we use a chaos polynomial (CP) approach, the so-called *non intrusive probabilistic algorithms*, which assumes that the variances of conductivities are finite [1], [2] and allows to completely characterise the induced field in the probabilistic dimension with a much lower computational cost (a few hours).

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II. Incorporation of stochastic uncertainty

A. Deterministic framework

The fields induced in the human body are computed by the finite element (FE) method using a $\phi - \mathbf{a}$ formulation. Details of this formulation and the phantom used (Fig. 1) can be found in [20]. We simulate the exposure to the field generated by an infinite cable (current $I = 1000\,\mathrm{A}$ at $50\,\mathrm{Hz}$) placed at a few centimeters from the left side of the head. In the literature on protection against ELF fields, three global quantities are generally computed for each organ from the current density $\mathbf{j}$: the (spatial) average j_{Avg}, the maximum value j_{max}, and the 99% percentile $j_{99-perc}$ [21]. Similar definitions exist for the electric field $\mathbf{e}$. The basic restriction of the 1998 edition of ICNIRP's recommendations [22] is based on j_{max}, while the 2010 edition [23] focus on $e_{99-perc}$. In particular, for a 50 Hz occupational exposure the threshold values in the central nervous system (CNS) recommended by the ICNIRP are $j_{max} < 10\,\mathrm{mA/m^2}$ and $e_{99-perc} < 100\,\mathrm{mV/m}$.

B. Uncertainties

In this paper, we focus on the fields induced in the brain. The conductivities of the white matter $\sigma_W(\omega)$ and of the grey matter $\sigma_G(\omega)$ are modeled within a probabilistic framework[1],

[1]The notation: $x(\omega)$ means that x is a random variable.

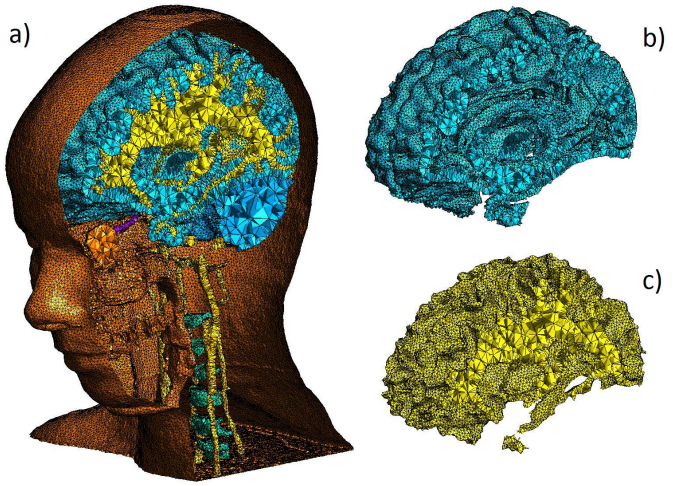


Fig. 1. a) Mesh of whole head; b) grey matter; c) and white matter.

by assuming that they are random variables defined on a probabilistic space Θ, F, P. Therefore j_{Avg}, j_{max}, $j_{99-perc}$, e_{Avg}, e_{max} and $e_{99-perc}$ turn out to be random as well. In particular, by using the maximum entropy principle [24] we model (arbitrarily) $\sigma_G(\omega)$ and $\sigma_W(\omega)$ as independent random variables, uniformly distributed:

$$\sigma_G(\omega) \sim \mathrm{U}([0.0753\ ;\ 0.5155])\ (\mathrm{S/m}) \quad (1)$$

$$\sigma_W(\omega) \sim \mathrm{U}([0.0533\ ;\ 0.3020])\ (\mathrm{S/m}) \quad (2)$$

C. The non intrusive approach

As the conductivities of the brain and the cerebellum are two independant random variables of finite variance, we can expand them as a truncated series of order p_{in} in the bi-dimensional Hermite polynomials of a random gaussian vector $\xi(\omega) = (\xi_1(\omega), \xi_2(\omega))$, known as *Hermite chaos polynomials* [2]:

$$\sigma_G(\omega) \approx \sum_{i=0}^{P_{in}} \sigma_{Gi}\psi_i(\xi(\omega)) \quad (3)$$

$$\sigma_W(\omega) \approx \sum_{i=0}^{P_{in}} \sigma_{Wi}\psi_i(\xi(\omega)) \quad (4)$$

where σ_{Gi} and σ_{Wi} are scalar values that depend on the probabilistic law of the conductivities, $P_{in} = C_{2+p_{in}}^{p_{in}}$ is the number of bi-dimensional polynomials of order less than p_{in}, and ψ_i is the i^{th} bi-dimensional polynomial of Hermite. To solve the stochastic problem, we use an approach based on a CP decomposition of both the conductivity and the induced fields [2]. We assume the conductivities to be of finite variance, with no assumption on the shape of the probabilistic distribution.

The values of those induced fields –the average current density in the brain $\mathbf{j_{Avg}}(\omega) = \mathbf{j_{Avg}}(\xi(\omega))$– are computed by the FEM from any couple of values $(\sigma_G(\xi(\omega)), \sigma_W(\xi(\omega)))$. The average density belongs to a space that can be spanned by the polynomials $\psi(\xi(\omega))$ and thus written as a truncated series to an order p_{out}:

$$\mathbf{j_{Avg}}(\omega) = \sum_{m=0}^{P_{out}} \mathbf{j_{Avg}}_m \psi_m(\xi(\omega)). \quad (5)$$

To compute the value of the unknown real coefficients $\mathbf{j_{Avg}}_m$, we use the orthogonality properties of the Hermite polynomials:

$$\mathbf{j_{Avg}}_m = \frac{\mathrm{E}[\mathbf{j_{Avg}}(\omega)\psi_m(\xi(\omega))]}{\mathrm{E}[\psi_m(\xi(\omega))^2]}, \quad (6)$$

where $\mathrm{E}[\cdot]$ is the mathematical expectation. The denominator can be computed analytically. The integral in the numerator is computed by means of a Hermite Gauss integration scheme with d integration points [2]:

$$\mathrm{E}[\mathbf{j_{Avg}}(\omega)\psi_m(\xi(\omega))] \approx \sum_{i=1}^{d} \dots \sum_{j=1}^{d} w_{i,j}(\mathbf{j_{Avg}}((t_1,t_2)_{i,j}))\psi_m((t_1,t_2)_{i,j}), \quad (7)$$

with $(t_1, t_2)_{i,j}$ the i,j-th Gauss point and $w_{i,j}$ the associated weight in the bi-dimensional Cartesian rule. The deterministic problem must thus be computed d^2 times, with the conductivity evaluated through (3) and $(\xi_1(\omega), \xi_2(\omega)) = (t_1, = t_2)_{i,j}$, $i, j = 1, \dots, d$.

III. Results and Discussion

The non intrusive method is governed by three parameters: p_{in}, p_{out} and d; p_{in} is linked to the precision on the approximation made on the input random variables $\sigma_G(\omega)$ and $\sigma_W(\omega)$, p_{out} is the order of truncation of the studied global quantities (j_{Avg}, j_{max}, $j_{99-perc}$ and the corresponding quantities for the electric field) and d is the number of quadrature points. Herein, we have chosen $p_{in} = 16$, while p_{out} and d vary. For the sake of conciseness, we deal with the white and grey matter (though the method could handle other tissues during the same computation as well).

A. Influence of the input parameters

The probabilistic density (PD) of $e_{99-perc}$ in the grey matter obtained with $p_{out} = 8$, $p_{in} = 16$ and different values of d is shown in Fig. 2. The curves of the PD obtained with $d = 10$ and $d = 16$ are nearly superposed, what proofs the convergence of the method with increasing values of d. Concerning dispersion parameters as the mean and the standard deviation, the convergence is reached as soon as $d \geq 4$ (mean: $0.0487V/M$, standard deviation: 0.07 when $d = 2$ to 0.0044 as soon as $d = 6$).

The PD of j_{max} in the white matter obtained with $d = 16$, $p_{in} = 16$ and different values of p_{out} are plotted in Fig. 3. Again, one observes that convergence is achieved as p_{out} increases. The value of p_{out} has a minor influence on the central dispersion parameters (mean and variance): the mean is constant and equal to $13.1\ \mathrm{mA/m}$, and the standard deviation converges with $p_{out} \geq 3$ to $3.4\ \mathrm{mA/m}$. It can be observed that the support of the PD is bounded to $21\,\mathrm{mA/m}$ for $p_{out} \geq 4$ (that is, most likely $j_{max} < 21\,\mathrm{mA/m}$): conversely, $p_{out} = 2$ would lead to the wrong conclusion that j_{max} may exceed $21\,\mathrm{mA/m}$ with a non negligible probability.

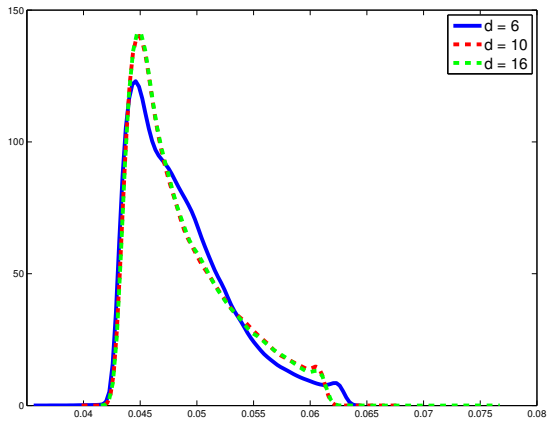


Fig. 2. Probabilistic density of $e_{99-perc}$ in the grey matter for $p_{out} = 8$, $p_{in} = 16$ and $d = 6$ (blue), $d = 10$ (red), $d = 16$ (green).

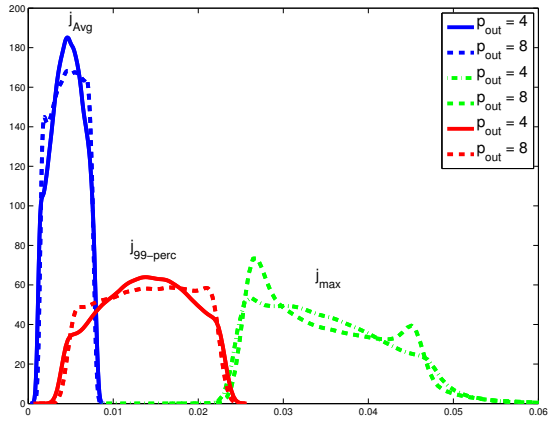


Fig. 4. Density of probability (in $\mathrm{A/m^2}$) of j_{Avg} (blue), $j_{99-perc}$ (red) and j_{max} (green) in the grey matter ($p_{in} = 16$, $d = 16$).

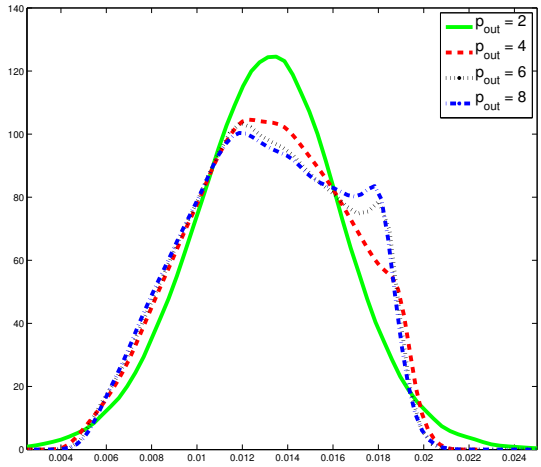


Fig. 3. Density of probability of j_{max} $(\mathrm{A/m^2})$ in the white matter for $d = 16$, $p_{in} = 16$ and $p_{out} = 2$ (black), $p_{out} = 4$ (red), $p_{out} = 6$ (green), $p_{out} = 8$ (blue).

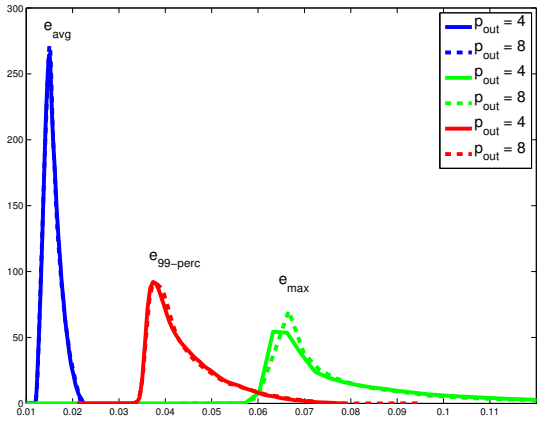


Fig. 5. Density of probability (in $\mathrm{V/m}$) of e_{Avg} (blue), $e_{99-perc}$ (red) and e_{max} (green) in the whie matter ($p_{in} = 16$, $d = 16$).

B. Analysis of the results

The PD of j_{Avg}, $j_{99-perc}$ and j_{max} linked to the induced current density in the grey matter are represented in Fig. 4 for $p_{out} = 4$ and $p_{out} = 8$. For $p_{out} = 6$, the curves are very close to those for $p_{out} = 8$ so the method has converged. One observes that while j_{Avg} is always under to $10\,\mathrm{mA/m^2}$, this does not hold for $j_{99-perc}$ and $\mathbf{J}_{max}$, which are likely to exceed this hazardous limit — the exact probability is computed hereafter.

The PD of e_{Avg}, $e_{99-perc}$ and e_{max} related to the electric field in the white matter are depicted in Fig. 5 for $p_{out} = 4$ and $p_{out} = 8$. The curves for $p_{out} = 4$ and $p_{out} = 8$ are much more similar than in the previous case. These PDs are more peaked than those corresponding to the current density, i.e. they are less dispersed around their means. Moreover, the area under these curves for $||\mathbf{e}|| > 100\,\mathrm{mV/m}$ equals 0 for e_{Avg} and $e_{99-perc}$ and nearly 0 for e_{max}.

In order to avoid health hazards, ICNIRP recommends that in the central nervous system $j_{max} \leq 10\,\mathrm{mA/m^2}$ [22] or $e_{99-perc} \leq 100\,\mathrm{mV/m}$ [23]. As these global quantities are available as a polynomial expansion like (5), we can estimate the probability -p- that these recommendations are not fulfilled –thus, in the case of [22], p is: $\mathcal{P}\{j_{max} \leq 10\,\mathrm{mA/m^2}\}$. To this aim, a large number n of couples of independent values following a normal variable $(\xi_{1i}, \xi_{2i})_{1 \leq i \leq 10^4}$ are sampled. The polynomial expansion (5) is evaluated for each pair of values and the number of occurrences (i.e. the number of pairs) for which the basic restriction is exceeded are counted. The probability p and the confident interval CI are estimated by means of the central limit theorem as:

$$p = \frac{\text{Number of occurences}}{n} \tag{8}$$

$$CI = z_{.05/2}\sqrt{\frac{p(1-p)}{n}} \tag{9}$$

where $z_{.05/2}$ is the confidence interval of 97.5% of a normal random variable. The cost of this computation is negligible

with regard to the coefficient evaluation via (6).

The probability that each of the global quantities exceeds the basic restriction for the simulated exposure has been computed with $n = 10^5$ and the obtained values are reported in Table I. For instance, the third line in Table I reads: the probability for $e_{max} > 100\,\mathrm{mV/m}$ is of $80\% \pm 0.27\%$ with a risk of 5%[2]. According to most recent recommendations [23], the basic

TABLE I

PROBABILITY TO EXCEED THE THRESHOLD DEFINED BY THE NORMS.

Field	Probability in Grey matter	in white matter
e_{avg}	0 ± 0	0 ± 0
$e_{99-perc}$	0 ± 0	0 ± 0
e_{max}	$0.8 \pm 2.7 \times 10^{-3}$	$0.14 \pm 2.2 \times 10^{-3}$
j_{avg}	0 ± 0	0 ± 0
$j_{99-perc}$	$0.73 \pm 2.8 \times 10^{-3}$	$0.16 \pm 2.3 \times 10^{-3}$
j_{max}	1 ± 0	$0.80 \pm 2.5 \times 10^{-3}$

restriction $e_{99-perc} \leq 100\,\mathrm{mV/m}$ is most likely fulfilled – it would not have been the case if we consider e_{max} instead of $e_{99-perc}$. Concerning the 1998 recommendations [22], the restriction $j_{max} \leq 10\,\mathrm{mA/m^2}$ is clearly not fulfilled for the grey matter (see also Fig. 4), while for the white matter the probability for j_{max} to exceed $10\,\mathrm{mA/m^2}$ is of 80%. These probabilities decrease respectively to 73% and 16% when dealing with $j_{99-perc}$ instead of j_{max}.

IV. CONCLUSION

The proposed recommendations for avoiding health issues due to over-exposure to ELF radiations may require to evaluate the induced fields in the human body by dosimetric methods.

Unfortunately, these computations are largely affected by the uncertainty on the conductivities of human tissues. Indeed, an arbitrary security factor 3 is considered in [23] in order to account for "dosimetric uncertainties".

We propose an effective method for quantifying the uncertainty on some relevant quantities (notably j_{max} and $e_{99-perc}$), provided that a characterisation of the statistical distribution of the conductivities is available. Our simulations suggest that latest 2010 ICNIRP recommendations are more permissive than those in the former edition. However, it has to be pointed out that in our computations we use a quite pessimistic statistical law for the conductivities, and also that other sources of uncertainty (posture, physiognomy...) are disregarded.

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[2] i.e. the risk that the real probability is inferior to 79.23% or superior to 80.27% equals 5%.

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