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Ignacio Rubio Scola, Gildas Besancon, Didier Georges, Marcos Guillén, Jean-François Dulhoste, Rafael Santos

ON THE DESIGN OF A NONLINEAR STATE OBSERVER FOR THE LOCATION OF A BLOCKAGE IN A PIPELINE.

Ignacio Rubio Scola
Gildas Besançon*
Didier Georges
{*ignacio.rubioscola, gildas.besancon, didier.georges*}*@gipsa-lab.fr*
Département Automatique, GIPSA-lab. Institut polytechnique de Grenoble. Saint Martin d'Hères, France.
*Institut Universitaire de France.

Marcos L. Guillén
Jean-F. Dulhoste
Rafael Santos
{*marcosg, djean, rsantos*}*@ula.ve*
Grupo Termofluidodinámica. Dpto. De Ciencias Térmicas. Escuela de Ing. Mecánica.Universidad de Los Andes. Mérida, Venezuela.

Abstract. *This paper presents a design of a nonlinear state observer in order to locate a blockage in a pipeline. First, a mathematical model of the pipeline is designed using the water hammer equations and applying the finite difference method for the solution with n sections. Then, based on the experience obtained from the leak location with nonlinear observers, a second mathematical model is obtained from a discretization in only two sections and a nonlinear state observer is build. In this new model, the blockage is at the beginning of the second section of pipeline. By analytical analysis, it is found that to achieve satisfactory the location of the blockage, the observer requires certain specific inputs, called persistent inputs.These persistent inputs are found empirically.*

Then, numerical simulations are performed with the state observer and the mathematical model with two sections and applying both entries obtained above. A satisfactory location of the blockage is obtained. Also simulations where measurements are subject to white Gaussian noise are done.

Key words: Modeling, Pipeline, Blockage, Nonlinear Observer

1 INTRODUCTION

Nowadays, pipelines are a cheap way to transport liquids. But faults, as leaks or blockages, can cause big and expensive problems to solve, generating a loss in money and time repairing it. A lot of work have being done on leaks detection and location but not so much on blockages detection and location ([5] and [3]). Particularly, in this work, the problem is solve using model based techniques.

2 PIPELINE MODEL WITH PUNTUAL BLOCKAGE

The dynamics of the fluid in a pipeline could be described by the Water Hammer partial differential equations considering convective changes in velocity and constant liquid density [4]:

$$\frac{\partial H}{\partial t} + \frac{c^2}{gA}\frac{\partial Q}{\partial z} = 0, \quad \frac{\partial Q}{\partial t} + Ag\frac{\partial H}{\partial z} + \frac{fQ|Q|}{2DA} = 0 \tag{1}$$

where H means the pressure head (m) and Q the flow rate in the pipeline (m^3/s), varying in space z (m) and time t (s). Then, the constants parameters c, g, A, D, f and L represent the wave speed in the fluid (m/s), the gravitational acceleration (m/s^2), the cross-sectional area of the pipe (m^2), the diameter of the pipe (m), the friction coefficient, and the pipe length (m) respectively. The initial conditions for the fluid dynamics must be fixed at $Q(z,0)$, $H(z,0)$ for $z \in [0, L]$ and also the boundary conditions on Q, H, for $z \in \{0, L\}$. As showed in the previous figure, the pipeline input

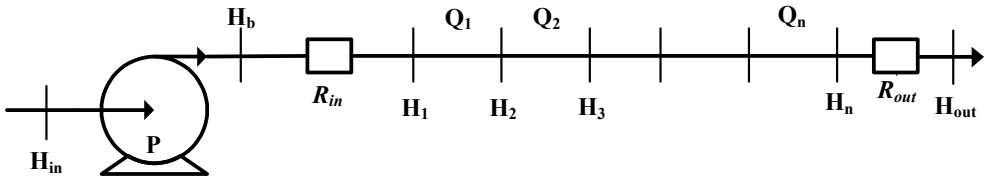

Figure 1: Schema of system Pump-Restriction-Pipeline-Restriction, the pipeline is dicretized in n sections.

is connected to a pump (P) with a restriction (R_{in}) and the pipeline output is only connected to a restriction (R_{out}). The dynamics of the pump and both restrictions are hundred time faster than the pipeline dynamics, so the static equations could be used to describe the behavior of these elements. The follows equations described the static relations between the pressures of the pump and both restrictions:

$$H_{in} - H_b = A_P Q_1^2 + B_P Q_1 + C_P; \quad H_b - H_1 = R_{in} Q_1^2; \quad H_n - H_{out} = R_{out} Q_n^2 \tag{2}$$

where H_{in} is the pump input pressure, H_b is the pressure between the pump and the first restriction, H_1 and H_n, Q_1 and Q_n are the input and output pipeline pressures and flows of the pipeline respectively, H_{out} is the output pressure of the second restriction and finally the constants A_P, B_P, C_P are the pump constants and R_{in} and R_{out} are the restrictions constants.

In order to solve numerically the partial differential equations Eq.(1) a dicretization in space must be done. Imposing as boundary conditions $H(0,t)$ and $H(L,t)$ the discretized equations for each section (i) are:

$$\dot{H}_i = -\frac{c^2}{gA}\frac{Q_{i+1} - Q_i}{z_{i+1} - z_i}; \quad \dot{Q}_i = -Ag\frac{H_i - H_{i-1}}{z_i - z_{i-1}} - \frac{f_i Q_i |Q_i|}{2DA} \tag{3}$$

where the H_i's and Q_i's refers to the approximate values of H, Q at z_i's.

The model accuracy will depend on the number of points taken for such discretization, but for a purpose of observer design based on this model, so in order to reduce the observer computational burden, the minimum number of sections are would be chosen. If a punctual blockage occurs at

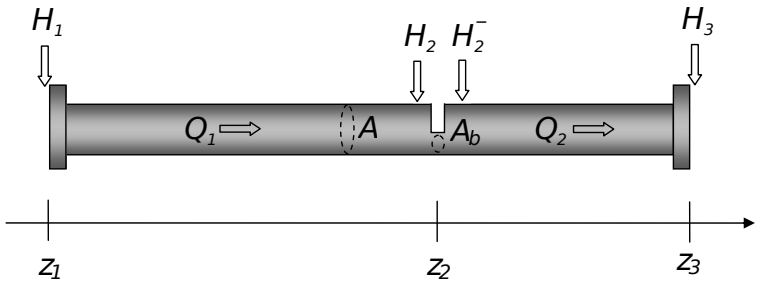

Figure 2: Pipeline discretized in 2 sections with a blockage at position z_2 (begin of the second section).

some position z_2, the pipe cross-sectional area A would be reduced into some A_b. Three sections are enough to include the blockage position z_b, as in Fig.2 ($z_1 = 0$, $z_2 = z_b$, and $z_3 = L$). Finite-difference model (3) in that case reduces to:

$$\begin{aligned}
\dot{Q}_1 &= -Ag\frac{H_2 - H_1}{z_2 - z_1} - \frac{f_2 Q_1 |Q_1|}{2DA} \\
\dot{H}_2 &= -\frac{c^2}{gA}\frac{Q_2 - Q_1}{z_3 - z_2} \\
\dot{Q}_2 &= -Ag\frac{H_3 - H_2^-}{z_3 - z_2} - \frac{f_3 Q_2 |Q_2|}{2DA}
\end{aligned} \tag{4}$$

As the blockage is considered punctual, the relation between the pressure before the blockage (H_2) and after (H_2^-) could be described as an orifice plate using Bernoulli's equation:

$$H_2^- = H_2 - \frac{Q_2^2}{2gA^2}\left[\left(\frac{A}{A_b}\right)^2 - 1\right] - h_L \tag{5}$$

In this equation h_L represents the pressure losses due to the blockage, as its geometry is unknown the worst case should be considerate, that is h_L equal to zero. In practice the blockage will be smaller than the represented by this equation.

Finally the connection between the pipeline model and the pump and both restriction is done by these two following equations:

$$\begin{aligned}
H_1 &= -(R_{in} + C_P)Q_1^2 - B_P Q_1 - A_P + H_{in} = F_P(H_{in}, Q_1) \\
H_3 &= R_{out}Q_2^2 + H_{out} = F_R(H_{out}, Q_2)
\end{aligned} \tag{6}$$

In the next section the equations of the state space model and the equations of the observer will be discussed.

3 STATE SPACE MODEL AND OBSERVER EQUATIONS

Considering the following state vector:

$$Obs = \frac{\left(\frac{A}{A_b}\right)^2 - 1}{2gA^2}; \quad x^T = [Q_1, H_2, Q_3, z_2, Obs]^T \tag{7}$$

and the equations from the previous section (Eq.4, Eq.5 and Eq.6) the state space model is build:

$$\begin{aligned}
\dot{x}_1 &= -Ag\frac{x_2 - F_P(u_1, x_1)}{x_4} - \frac{f_2 x_1|x_1|}{2DA} \\
\dot{x}_2 &= -\frac{c^2}{gA}\frac{x_3 - x_1}{x_4} \\
\dot{x}_3 &= -Ag\frac{F_R(u_2, x_3) - x_2 + x_3^2 x_5}{L - x_4} - \frac{f_3 x_3|x_3|}{2DA} \\
y &= [x_1, x_3]^T
\end{aligned} \tag{8}$$

Before trying to localize the blockage, first the blockage should be detected. The detection of the blockage could be solved if the pipeline is in steady state [3]. So, considering u_1 and u_2 in constant value for a long time enough, the dynamics of the pipeline model are zero ($\dot{x} = 0$) and also the friction losses are equal among the pipeline ($f_2 = f_3 = f$). The information of the blockage is found solving the following equations:

$$\begin{aligned}
x_5 &= \frac{F_R(u_2, y_2) - F_P(u_1, y_1)}{y_2^2} + \frac{fL}{2DA^2g}\frac{|y_2|}{y_2} \\
\dot{x}_{5F} &= \alpha(x_5 - x_{5F}); \quad \frac{A_b}{A} = \frac{1}{\sqrt{1 + 2gA^2 x_{5F}}}
\end{aligned} \tag{9}$$

The first part of the second equation is just a first order low-pass filter to make the detection algorithm robust respect to the output variables noise where α^{-1} is the time constant and x_{5F} is x_5 filtered.

Now that the blockage is detected in steady state, an observer for the location is build. The followed change of coordinates: $\zeta_1 = \frac{x_1}{Ag}$, $\zeta_2 = \frac{1}{x_4}$, $\zeta_3 = -\frac{x_2}{x_4}$ with the new inputs $u_{\zeta 1} = y_1$, $u_{\zeta 2} = y_2$ produced the follow system:

$$\dot{\zeta}_1 = F_P(u_1, u_{\zeta 1})\zeta_2 + \zeta_3 - \frac{f_2 u_{\zeta 1}|u_{\zeta 1}|}{2DA^2g}; \quad \dot{\zeta}_2 = 0; \quad \dot{\zeta}_3 = \frac{c^2}{Ag}(u_{\zeta 2} - u_{\zeta 1})\zeta_2^2 \tag{10}$$

which is a system of type:

$$\dot{\zeta} = A_\zeta(u)\zeta + \Phi_\zeta(\zeta, u); \quad y_\zeta = C_\zeta \zeta \tag{11}$$

with the matrices:

$$A_\zeta(u) = \begin{bmatrix} 0 & F_P(u_1, u_{\zeta 1}) & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \Phi_\zeta(u, \zeta) = \begin{bmatrix} -\frac{f_2}{2DA^2g} u_{\zeta 1}|u_{\zeta 1}| \\ 0 \\ \frac{c^2}{Ag}(u_{\zeta 2} - u_{\zeta 1})\zeta_2^2 \end{bmatrix}, \; C_\zeta = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{12}$$

and under a persistent input u_1 and γ large enough ,an exponential observer for such system is:

$$\begin{aligned}
&\dot{\hat{\zeta}} = A_\zeta(u)\hat{\zeta} + \Phi(u,\hat{\zeta}) - K(C_\zeta\hat{\zeta} - \zeta_1) \\
&K = PC_\zeta^T R^{-1}; \quad P(0) > 0 \\
&\dot{P} = \gamma P + A_\zeta P + PA_\zeta^T - PC_\zeta^T R^{-1} C_\zeta P + Q
\end{aligned} \tag{13}$$

where Q, R and γ are positive tuning parameters, for more details see [2]. Φ needs to be Lipschitz, assuming that the blockage position z_2 is to be found within some interval $[\varepsilon, L-\varepsilon]$ for some small $\epsilon > 0$, Φ can be extended to be Lipschitz over $\mathbb{R}^3$ for the observer design. The position of the blockage is $x_4 = \frac{1}{\hat{\zeta}_2}$.

With the persistent input applied the system is no more in steady state so a new way to measure the blockage is build. The change of coordinates $\xi_1 = \frac{x_3(L-x_4)}{Ag}$, $\xi_2 = x_5$ and the inputs $u_{\xi 1} = y_1$, $u_{\xi 2} = y_2$, $u_{\xi 3} = -\frac{\hat{\zeta}_3}{\hat{\zeta}_2}$, $u_{\xi 4} = \frac{1}{\hat{\zeta}_2}$ result in the system:

$$\dot{\xi}_1 = -F_R(u_2, u_{\xi 2}) + u_{\xi 3} - u_{\xi 2}^2 \xi_2 - \frac{f_3(L-u_{\xi 4})}{2DA^2 g} u_{\xi 2}|u_{\xi 2}|; \quad \dot{\xi}_2 = 0 \tag{14}$$

which is a system of type:

$$\dot{\xi} = A_\xi(u)\xi + B_\xi(u); \quad y_\xi = C_\xi \xi \tag{15}$$

and the matrices of such system are:

$$A_\xi(u) = \begin{bmatrix} 0 & -u_{\xi 2}^2 \\ 0 & 0 \end{bmatrix}, \; B_\xi(u) = \begin{bmatrix} u_{\xi 3} - F_R(u_2, u_{\xi 2}) - \frac{f_3(L-u_{\xi 4})}{2DA^2 g} u_{\xi 2}|u_{\xi 2}| \\ 0 \end{bmatrix}, \; C_\xi = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{16}$$

Using Eq.13 an exponential observer is build with parameters γ_O, R_O and Q_O. The relation $\frac{A_b}{A}$ is found in the same way as Eq.9 replacing x_{5F} by ξ_2

4 SIMULATIONS

In this section, the prototype described in [1] is presented as practical example where its parameters are listed in Table.1. The simulation consist on first on a steady state operation mode, with $u_1 = 6.5m$ and $u_2 = 1$. Suddenly, at time $t = 10s$, a blockage, $A_b = 0.9A$ appears at $z_2 = 25m$. When the blockage is detected, the persistent input is injected, $u_1 = 6.5 + 0.5sin(2\pi t) + 1.5sin(2.06\pi t)$, and the position is estimated. To show the second part of the observer, the blockage, at time $t = 500$, is moved to $z_2 = 80m$ and is augmented to $A_b = 0.5A$. There is a noise of some percent of the output value in the outputs of the system. The parameters of the algorithms are: $\alpha = 1$, $R = 1$, $Q = I_D$, $\gamma = 4$, $R_O = 1$, $Q_O = I_D$ and $\gamma_O = 1$. Figure.4 shows the simulation results, the observer recovers well the position and the blockage size. Also the simulation shows that the noise robustness augments as the obstruction augments.

$g\ (m/s^2)$	$c\ (m/s)$	$A\ (m^2)$	$f\ (s^{-2})$	$L\ (m)$
9.81	373	0.003	0.0189	85

Table 1: Pipeline parameters.

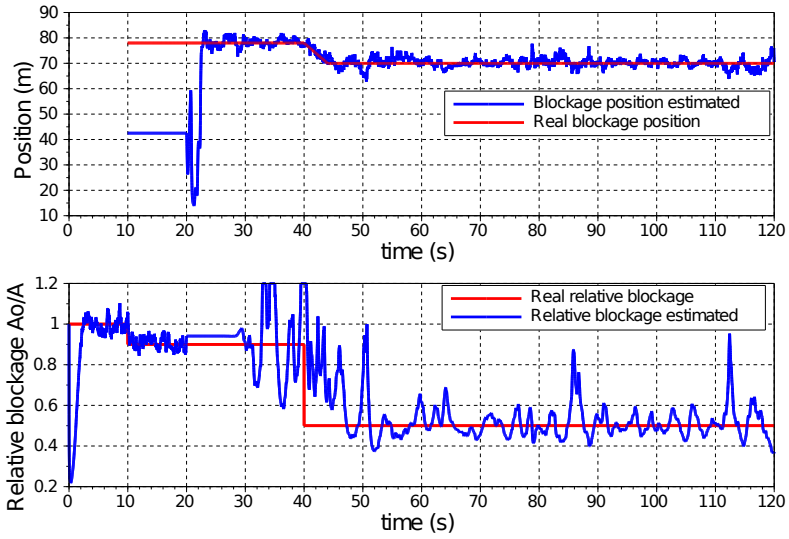

Figure 3: Simulation of one blockage in two different positions and different sizes.

5 CONCLUSIONS AND FURTHER WORK

In this paper, a technique to detect and locate blockage in a pipeline has being proposed. Based on a mathematical model an observer was build and the performance and accuracy were shown on some figures of numerical simulations.

In the future, the noise robustness will be improved and also the observer will be improved to be applied to real data.

Acknowledgements

This work was supported in part by a French-Venezuelan collaboration program ECOS-Nord V11M02/ PI-2010000303 Fonacit, Fundayacucho Venezuela.

REFERENCES

[1] Begovich, 0., Pizano, A., & Besançon, G., Online implementation of a leak isolation algorithm in a plastic pipeline prototype. *Latin American Applied Research*, vol. 42, no. 2, pp. 131-140, 2012.

[2] Besançon, G., Nonlinear Observers and applications. Springer, 2007.

[3] Besançon, G., Rubio Scola, I. Guillén, M., Dulhoste, J.F., Santos, R., & Georges, D., Observer-based detection and location of partial blockages in pipelines. *2nd International Conference on Control and Fault-Tolerant Systems.*, 2013.

[4] Chaudry, H., Applied Hydraulic Transients. Van, 1979.

[5] Wang,X. J., Lambert, M. F., & Simpson,A. R., Detection and location of a partial blockage in a pipeline using damping of fluid transients,*Journal of water resources planning and management,* vol.131, no. 3, pp. 244-249, 2005.