Computation method for the short-term stability of quartz crystal resonators obtained from passive phase noise measures

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Computation Method for the Short-Term Stability of Quartz Crystal Resonators Obtained From Passive Phase Noise Measures

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***Abstract*—The amplitude–frequency effect is a well-known phenomenon in quartz crystal resonators. It can distort the results of short-term stability measurements. In our case, results are computed from phase noise measurements in passive bridge systems. This article presents a method to correct computation of short-term stability from passive measurements.**

Since 1975, researchers have tried to discriminate the resonator's noise in crystal oscillators by means of passive systems [1]–[4]. The technique of carrier suppression to characterize the inherent phase stability about the ultrastable resonators was demonstrated at the beginning of the 21st century [5], [6]. Recently, a more advanced version of this instrument has been implemented [7]–[9]. The principle of this passive method, presented in Fig. 1, is to reduce the noise of the source as much as possible.

An oscillator is always noisier than the resonator alone because the random noises of the resonator and of the sustaining electronics are added in the output signal. Obviously, the noise of the best oscillator used as the source is always higher than the noise of the best resonator alone. Thus, the direct feeding of the driving source signal through only one resonator does not permit the extraction of information about the resonator noise from the total output noise. To perform the resonator noise measurement, the source signal can be subtracted when passing through two identical arms with two resonators considered to be quasi-identical. Then, the contribution of the source is reduced while the characteristic noise of both resonators is preserved. This is due to the noncorrelation of the intrinsic noise of each resonator. When the carrier suppression is achieved, the resulting signal is free of the source noise. Hence, the sum of both resonators noises is measured. First, this signal is then strongly amplified. Next, the signal is mixed with the source signal to be transposed to the low-frequency domain. Finally, it is processed by a fast Fourier spectrum analyzer (FFT). With this method, the measured noise for the two resonators is above the noise level of the driving source. Calibration of the measurement system is obtained by injecting a known sideband on one of the arms of the bridge. The result of the measurement bench is corrected using the calibration factor determined from this sideband.

Usually, the flicker floor of the short-term stability of a resonator is given by the Allan standard deviation $\sigma_y(\tau)$ [5], [6], [10], which corresponds to an oscillator containing a tested resonator in which the only source of flicker frequency noise is the resonator inside the oscillator. In the case of flicker frequency noise (which is the resonator's noise), the relationship between the floor of the Allan standard deviation, σ_{y_floor}, and the power spectral density (PSD) of relative frequency fluctuations $S_y(f)$, is given by [10]

$$\sigma_{y\_\text{floor}} = \sqrt{2 \cdot (\ln 2) \cdot S_y(1\ \text{Hz})}, \tag{1}$$

where ln is the natural logarithm function. $S_y(f)$ is given by the measurement of the PSD of the phase fluctuations, $S_\phi(1\ \text{Hz})$, and the half bandwidth, also called the cut-off frequency F_L [5]:

$$S_y(1\ \text{Hz}) = \left[\frac{F_\text{L}^2 + 1}{f_\text{res}^2}\right] S_\phi(1\ \text{Hz}) \quad \text{with } F_\text{L} = \frac{f_\text{res}}{2Q_\text{L}}, \tag{2}$$

where f_res is the resonant frequency of the resonator and Q_L is the loaded quality factor of the resonator.

The standard measure for characterizing frequency and phase instabilities in the frequency domain is $\mathcal{L}(f)$. It is defined as one half of $S_\phi(f)$ [10]. The measured $\mathcal{L}(f)$ of both 5-MHz resonators is shown in Fig. 2. Different drive level powers dissipated by the resonators are shown. The measured resonators are SC-cut quartz crystals. Drive power level P_Xtal of the resonators varies from 20 to 200 µW. The resonator is considered as a low-pass filter. The cut-off frequency can be determined by the $\mathcal{L}(f)$ curve at the intersection of the f^{-1} and f^{-3} asymptotes. A variation of F_L is shown in Fig. 2 for both these extreme cases.

The $\mathcal{L}(f)$ measurements are linked to Table I, in which the σ_{y_floor} was obtained with the method previously described. The total noise measured is related to the two resonators. If the two resonators are considered to be iden-

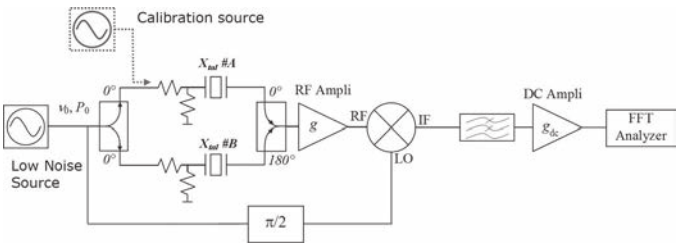

Fig. 1. Principle of the passive measurement bench.

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TABLE I. σ_{y_FLOOR} ACCORDING TO THE RESONATOR'S DISSIPATED POWER, P_{XTAL}.

P_{Xtal} (μW)	$\mathcal{L}$(1 Hz) (dBc/Hz)	F_{L} (Hz)	σ_{y_floor}	Q_{L} (10^6)
20	−121.5	1.4	$3.4 \cdot 10^{-13}$	1.8
200	−130	0.8	$9.5 \cdot 10^{-14}$	3.1

The resonators are 5-MHz SC-cut quartz crystals and are considered to be identical.

tical, then half of this noise is attributed to one resonator. In that case, it is equivalent to get $S_\phi(1\text{ Hz})$ equal to the $\mathcal{L}$(1 Hz) measured.

The short-term stability of the resonator seems to be better at 200 μW than at 20 μW. Unfortunately, a simple calculation from the loaded Q shows that the result of σ_{y_floor} is not realistic. Typically, a 5-MHz ultra-stable resonator exhibits an unloaded quality factor, Q_{X}, of about $2.5 \cdot 10^6$. By means of (2), the loaded quality factor goes from $1.8 \cdot 10^6$ to $3.1 \cdot 10^6$; however, it is not possible to obtain a loaded quality factor higher than the unloaded quality factor. Moreover, identical results are obtained when F_{L} is measured by injecting a white noise instead of the calibration source. In this case, we can directly observe the transfer function of the resonator inside the bench [5].

To analyze the phenomenon, the transfer function of the resonator must be observed near the carrier frequency with a network analyzer. Resonators have been measured in their impedance matching (PI) network by means of a 4195A network analyzer (Agilent Technologies Inc., Santa Clara, CA). Fig. 3 shows the phase of the transfer function of the resonator inserted in the PI network for different values of power dissipations. It is given for three resonator's power dissipations. The amplitude–frequency effect of the quartz crystal is clearly shown. The low-pass filter behavior of the resonator can be characterized by the cut-off frequency defined by the ±45° bandwidth:

$$\Delta F = |F_- - F_+| = 2F_{\text{L}}. \tag{3}$$

F_- and F_+ are the frequencies which correspond to a phase equal to ±45°.

In this kind of test setup, the measured noise or the transfer function of the resonator are obtained at the Fourier sideband frequencies. Thus, the noise spectrum is folded up around the carrier frequency. The spectrum folding implies that the asymmetry of the positive half bandwidth F_+ induced a reduction of F_{L} measured in the curves of resonator's noise in a high-power situation. In this case, F_{L} does not represent the actual value of the loaded quality factor. Thus, the great improvement of the short-term stability is not apparent. The classical calculus of the short-term stability must be done taking into account the resonator's drive level power and its amplitude–frequency effect. The previous method of computing the short-term stability using the filter behavior remains available for low-power drive levels. Therefore, Fig. 3 shows that the phase of the transfer function is not really affected by the drive power near the null phase. These curves are used in the conversion of the resonator's flicker frequency noise into the measured phase noise. F_{L} can be defined by inverting the slope of the relationship between $S_y(f)$ and $S_\phi(f)$ according to

$$S_y(f) = \frac{1}{\left[f_{\text{res}} \frac{\text{d}\phi}{\text{d}f}\right]^2} S_\phi(f) \quad \text{then } F_{\text{L}} = \frac{1}{\frac{\Delta\phi}{\Delta f}\Big|_{\pm 1^\circ}}, \tag{4}$$

where f_{res} is the resonant frequency of the resonator and $\text{d}\phi/\text{d}f$ is the derivative of the phase according to the frequency near zero phase.

We chose to compute the slope between ±1° (Fig. 3). The measurement of this slope should be done in the same framework as in the noise measurement. With this new method, the incorrect computation of the short-term sta-

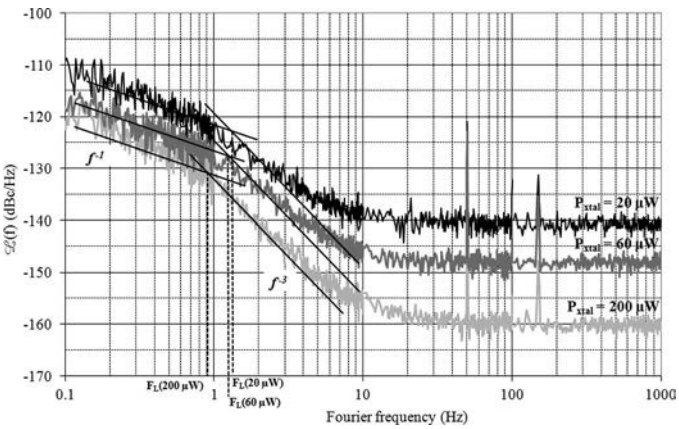

Fig. 2. The $\mathcal{L}(f)$ of 5-MHz SC-cut resonators according to the dissipated power in the crystal. For $P_{\text{xtal}} = 20$ μW, the cut-off frequency F_{L} is 1.4 Hz. For $P_{\text{xtal}} = 200$ μW, F_{L} is 0.8 Hz.

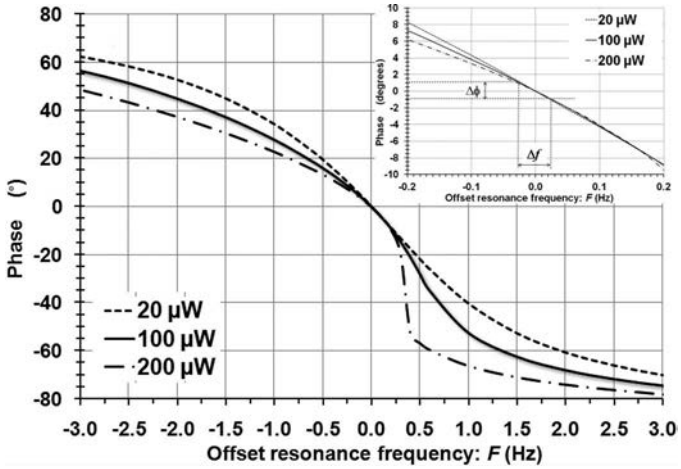

Fig. 3. Phase of 5-MHz SC-cut resonator, transfer functions obtained near its resonant frequency and according to P_{Xtal}.

TABLE II. Comparison of the Short-Term Stability Calculus in the Worst Case With $P_{\mathrm{XTAL}} = 200$ µW.

$\mathcal{L}$(1 Hz) (dBc/Hz) −130	Slope (±1°)	Bandwidth (±45°)	Noise curve
F_{L} (Hz)	1.55	1.5	0.8
$\sigma_{y\_\mathrm{floor}}$	$1.37 \cdot 10^{-13}$	$1.34 \cdot 10^{-13}$	$9.5 \cdot 10^{-14}$
Q_{L} (10^6)	1.61	1.67	3.12
$Q_{\mathrm{L}}/Q_{\mathrm{X}}$	63%	66%	125%

Resonators are considered to be identical.

bility and the loaded Q-factor is not possible. Table II shows a comparison of both methods. The noise source results are compared with the bandwidth methods of the first-order filter and with the slope method. Better results are obtained with the slope method. Moreover, the method can be easily applied inside the measurement bench when an OCXO is used as a source. F_{L} can be obtained by small variations of the source frequency near the null phase instead of long measurements with a network analyzer.

Table III presents the resonator noise according to the dissipated power. In this kind of measurement, the error in the determination of $\mathcal{L}(f)$ is usually ±2 dB because of the FFT analyzer precision. Error in the determination of F_{L} can be reduced to 0.1 Hz with the proposed method. Even with the correcting computation, the noise of the resonator seems to be power dependent. Indeed, about half the difference in the calculated $\sigma_y(\tau)$ values is from the difference in $\mathcal{L}(f)$. In our case, the decrease of $\mathcal{L}(f)$ is not due to a permanent change, as can be observed in [11] after a burn-in process. Variation of the acoustic resonator noise according to the dissipated power has been already observed in BAW resonators (see [5], [12]) and more recently in film bulk acoustic resonators (FBARs) [13].

A passive measurement system of a resonator's phase noise has been presented. Measurements have been correlated to the amplitude–frequency effect, a well-known phenomenon in quartz crystal resonators. These investigations have led to consideration of an alternative way of determining the loaded Q-factor for the resonator to obtain more reliable short-term stability of quartz crystal resonators. After correction of the measurements, we find that the short-term stability of the observed resonators evolves according to the dissipated power. A dependence of the noise on the dissipated power is now clearly shown.

TABLE III. Short-Term Stability According to the Power Dissipated by the Resonators.

P_{Xtal} (µW)	$\mathcal{L}$(1 Hz) (dBc/Hz)	$\sigma_{y\_\mathrm{floor}}$
20	-121.5 ± 2	$3.65 \cdot 10^{-13} \pm 1.2 \cdot 10^{-13}$
60	-126.7 ± 2	$2.01 \cdot 10^{-13} \pm 0.6 \cdot 10^{-13}$
100	-128.5 ± 2	$1.63 \cdot 10^{-13} \pm 0.5 \cdot 10^{-13}$
200	-130 ± 2	$1.37 \cdot 10^{-13} \pm 0.4 \cdot 10^{-13}$

Resonators are considered to be identical. With slope method, $F_{\mathrm{L}} = 1.55 \pm 0.1$ Hz. Errors in the measured $\mathcal{L}$(1 Hz) are ±2 dB.

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