

High-Dimensional Range profile geometrical visualization and performance estimation of classification of radar targets via a Gaussian mixture model

Thomas Boulay^{1,2}, Ali Mohammad-Djafari¹, Nicolas Gac¹, Julien Lagoutte²

¹ Laboratoire des Signaux et Systèmes (L2S, UMR 8506 CNRS - SUPELEC - Univ Paris Sud 11)
Supélec, Plateau de Moulon, F-91192 Gif-sur-Yvette, FRANCE.

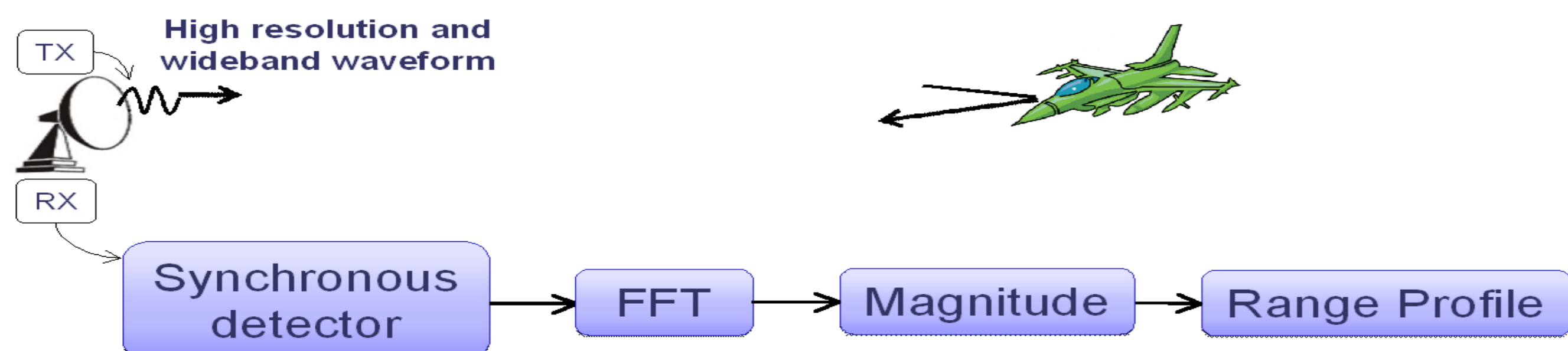
² THALES AIR SYSTEMS, voie Pierre Gilles de Gennes, 91470 Limours en Hurepoix, FRANCE

Emails: thomas.boulay@lss.supelec.fr, djafari@lss.supelec.fr, nicolas.gac@lss.supelec.fr, julien.lagoutte@thalesgroup.com

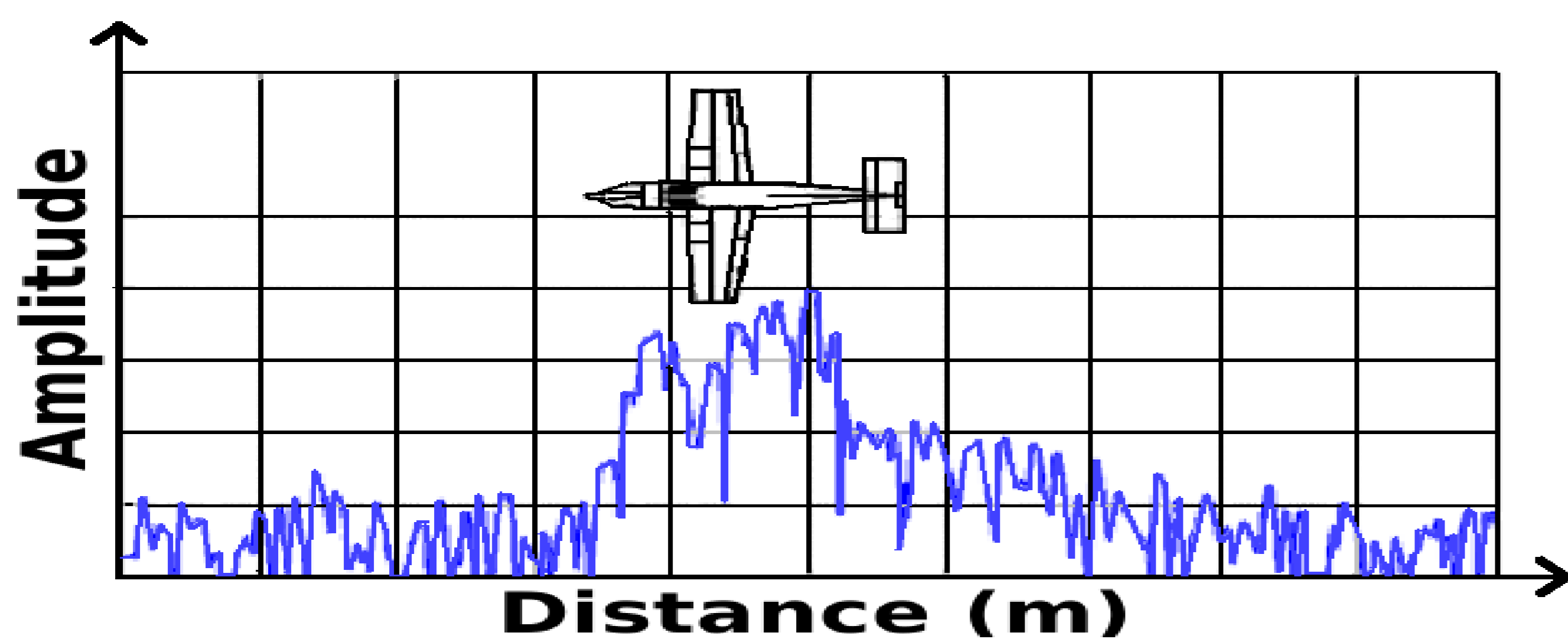
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Non Cooperative Target Recognition (NCTR) ①

Formation of range profile



Example of range profile



Mixture Models for NCTR application ④

$$p(\mathbf{z}_{T,i} | \mathbf{Z}_T) = \sum_{k=1}^K \pi_T^k p(\mathbf{z}_{T,i} | \theta_T^k)$$

In low-dimensional space $\mathbf{z}_{T,i} = \mathbf{y}_{T,i}$, in high-dimensional space $\mathbf{z}_{T,i} = \mathbf{x}_{T,i}$

Gaussian Mixture Models (GMM)

$$p(\mathbf{z}_{T,i} | \theta_T^k) = \mathcal{N}(\mathbf{z}_{T,i} | \theta_T^k), \theta_T^k = \{\mathbf{m}_T^k, \mathbf{C}_T^k\}$$

$$\mathbf{m}_T^k = \frac{1}{N_T^k} \sum_{i=1}^{N_T^k} \mathbf{z}_{T,i}^k$$

$$\mathbf{C}_T^k(p, q) = \frac{1}{N_T^k - 1} \sum_{i=1}^{N_T^k} (z_{T,i}^k(p) - m_T^k(p)) (z_{T,i}^k(q) - m_T^k(q))$$

Decision Rule

- if $p_i \geq TG$, the class i is granted \rightarrow Granted Matrix (GrM)
- if $TD \leq p_i < TG$, the class i is dubious \rightarrow Dubious Matrix (DuM)
- if $p_i < TD$, the class i is denied \rightarrow Denied Matrix (DeM)

Objectives ②

- Visualize high-dimensional range profiles in 2D space
- Estimate graphically and analytically classification performances

Non Linear Dimension Reduction (NLDR) ③

From high-dimensional space to low-dimensional space

$$\mathbf{x}_{T,i} \rightarrow \mathbf{y}_{T,i}$$

$$\mathbf{x}_{T,i} \in \mathbb{R}^M \rightarrow \mathbf{y}_{T,i} \in \mathbb{R}^2$$

Stochastic Neighbor Embedding (SNE)

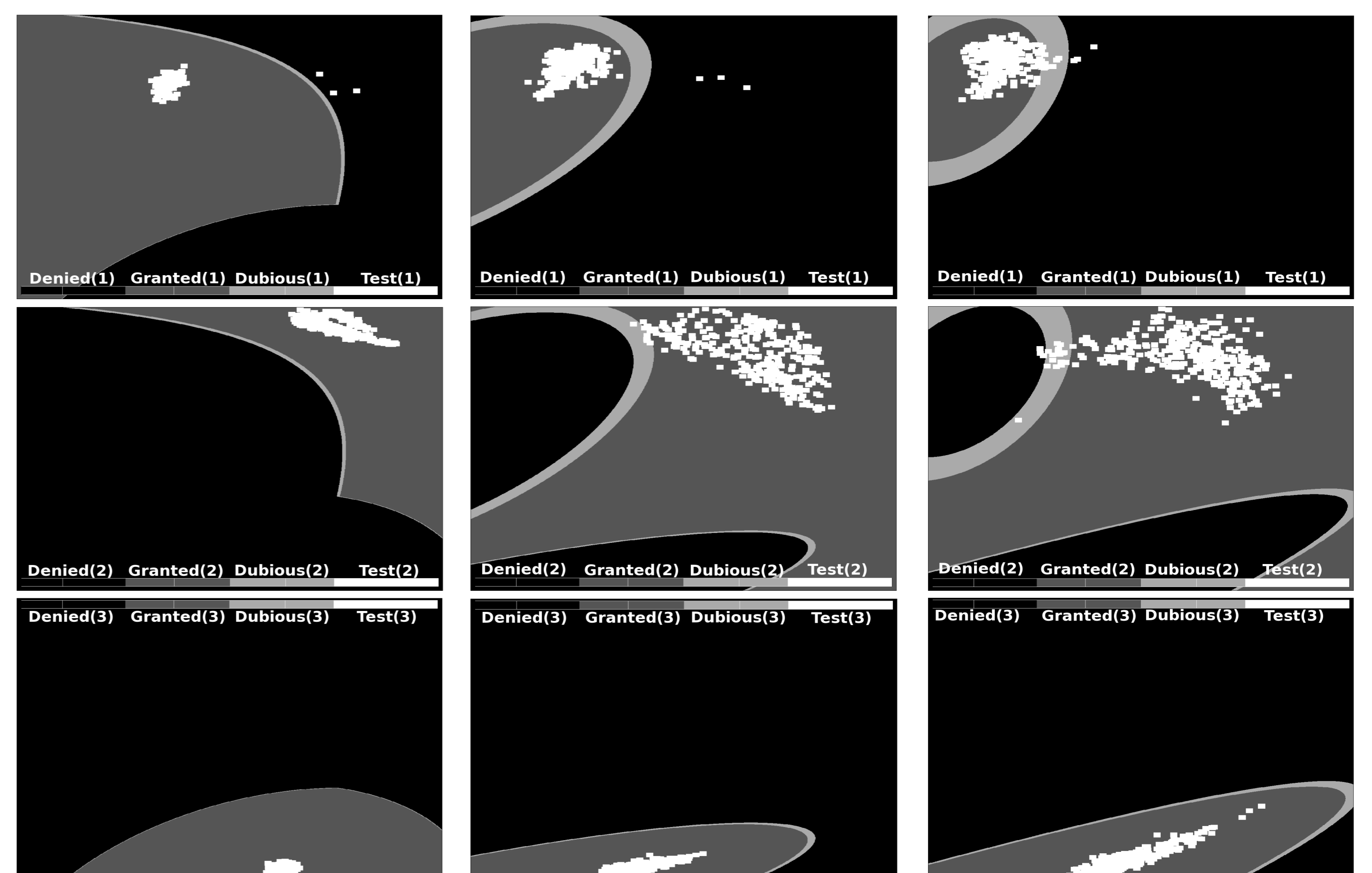
$$p_{ji} = \frac{\exp(-\|\mathbf{x}_{T,i} - \mathbf{x}_{T,j}\|^2 / (2\sigma_i^2))}{\sum_{k \neq i} \exp(-\|\mathbf{x}_{T,i} - \mathbf{x}_{T,k}\|^2 / (2\sigma_i^2))}$$

$$q_{ji} = \frac{\exp(-\|\mathbf{y}_{T,i} - \mathbf{y}_{T,j}\|^2)}{\sum_{k \neq i} \exp(-\|\mathbf{y}_{T,i} - \mathbf{y}_{T,k}\|^2)}$$

t-Distributed Stochastic Neighbor Embedding (t-SNE)

$$q_{ij} = \frac{(1 + \|\mathbf{y}_{T,i} - \mathbf{y}_{T,j}\|^2)^{-1}}{\sum_{k,l,k \neq l} (1 + \|\mathbf{y}_{T,k} - \mathbf{y}_{T,l}\|^2)^{-1}}$$

Probability map in low-dimensional space ⑤



t-SNE 2D visualization of data for class 1, 2 and 3 (row), for SNR = 30dB, SNR = 20dB and SNR = 15dB (column).

Decision matrix in low-dimensional space ⑥

SNR	Class	GrM			DuM			DeM		
		1	2	3	1	2	3	1	2	3
SNR=30dB	Class 1	99.1	0.9	0	0	0	0	0.9	99.1	100
	Class 2	0	100	0	0	0	0	100	0	100
	Class 3	0	0	100	0	0	0	100	100	0
SNR=20dB	Class 1	99.1	0.9	0	0	0	0	0.9	99.1	100
	Class 2	0	98.5	0	0.9	1.5	0	98.5	0	100
	Class 3	0	0	100	0	0	0	100	100	0
SNR=15dB	Class 1	95.6	0.9	0	3.5	1.1	0	0.9	98	100
	Class 2	0.6	94.4	0	5	4.7	0	94.4	0.9	100
	Class 3	0	0	100	0	0	0	100	100	0

Decision matrix in high-dimensional space ⑦

SNR	Class	GrM			DuM			DeM		
		1	2	3	1	2	3	1	2	3
SNR=30dB	Class 1	100	0	0	0	0	0	0	100	100
	Class 2	0	100	0	0	0	0	100	0	100
	Class 3	0	0	100	0	0	0	100	100	0
SNR=20dB	Class 1	99.4	0.6	0	0	0.3	0	0.6	99.1	100
	Class 2	0.3	99.7	0	0	0.3	0	99.7	0	100
	Class 3	0	0	100	0	0	0	100	100	0
SNR=15dB	Class 1	97.7	2.3	0	0.3	0	0	2	97.7	100
	Class 2	0.3	99.7	0	0	0	0	99.7	0.3	100
	Class 3	0	0	100	0	0	0	100	100	0

Perspectives ⑧

- Generalize this method for any kind of algorithm
- Evaluate for each kind of algorithm, the critical dimension from which the performances begin to degrade
- Determine the best appropriate algorithm from data