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Predicting the growth and yield of *Pinus radiata* in Bolivia

Gustavo Guzmán · Timo Pukkala · Marc Palahí · Sergio de-Miguel

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Abstract

• **Context** *Pinus radiata* D. Don is the most widely planted conifer in the Inter-Andean Valleys of Bolivia. However, Bolivia lacks knowledge on stand dynamics and yield prediction of radiata pine, and the particular Bolivian conditions prevent a straightforward application of equations developed elsewhere.

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• **Aims** This study developed site index models, individual tree diameter increment models, individual tree height models, and self-thinning models for Bolivian *P. radiata* plantations.
• **Methods** Dominant height was modeled using the algebraic difference approach. Diameter increment was modeled as a function of tree size, site index, and competition. Individual tree height was modeled as a function of tree diameter, dominant height, and dominant diameter. Self-thinning was modeled as a function of quadratic mean tree diameter.
• **Results** By using the models, it can be calculated that the mean annual increment in volume with the optimal rotation length is less than $10 \text{ m}^3\text{ha}^{-1}$ on poor sites and $25 \text{ m}^3\text{ha}^{-1}$ on the best sites. Rotation lengths that maximize wood production are 25–30 years for very good sites and 35–40 years for poor sites.
• **Conclusion** The models were assembled into a growth and yield simulator, allowing the simulation of stand development and making it possible to predict future yields and to compare alternative management schedules.

Keywords Pine plantations · Site index model · Individual tree modeling · Self-thinning model · Simulation · Forest management

1 Introduction

Pinus radiata D. Don is a fast-growing softwood conifer that has been intensively studied since the start of commercial tree plantations at the end of the nineteenth century, especially in New Zealand, Chile, Australia, and Spain, where it has been planted on a large scale (Richardson 1998). The studies have analyzed, among other things, its growth and developed models for even-aged plantation dynamics.

In New Zealand and Australia, much effort has been devoted to growth and mortality modeling and simulation (e.g., Woollons and Hayward 1985; O’Hehir et al. 2000). In Chile, *P. radiata* modeling has constituted a priority within the forest research agenda during the last decades, and simulation systems have been developed based on both stand and individual tree equations (e.g., Garcia 1970, 1974; Trincado and Leal 2006). More recently, several studies have proposed new equations to model the growth and mortality of local radiata pine plantations in Spain (López-Sánchez et al. 2003; Álvarez-González et al. 2004; Diéguez-Aranda et al. 2005; Castedo-Dorado et al. 2006, 2007; Crecente-Campo et al. 2009).

The history of *P. radiata* in Bolivia is much more recent than in the above-mentioned regions. Exotic species plantations were promoted mainly since the 1960s until the 1990s within international cooperation initiatives dealing with community forestry. *P. radiata* is locally interesting for multi-purpose forestry in Bolivia as radiata pine forests are considered one of the main sources of sawn-wood, firewood, and mushrooms in the Inter-Andean Valleys. The plantations in Cochabamba represent 71% of the total planted area in the country, and *P. radiata* represents one of the most utilized species together with *Eucalyptus globulus*. Tree plantations are generally found in small- or medium-sized holdings, and they are of significant economic importance in impoverished rural areas. *P. radiata* has been criticized as an exotic species that has been promoted to the detriment of native ones. However, it currently constitutes the only forest cover in many large deforested areas of the region and, therefore, plays an important role in providing environmental services such as reducing erosion and providing habitats of significance to biodiversity.

At present, the oldest plantations are reaching the end of the rotation. Few *P. radiata* stands are properly thinned or

otherwise managed, partly due to the lack of science-based information. Although much research has been done worldwide, Bolivia lacks knowledge on the stand dynamics and yield prediction of radiata pine. The particular Bolivian conditions, when compared to New Zealand, Australia, Chile, or Spain, namely drier climate, higher altitude, poorer soils, and smaller influence of the ocean, prevent a straightforward application of equations developed elsewhere. Therefore, in view of the importance of this species in the area, there is a need for a system that enables reliable growth and yield predictions of *P. radiata* to support plantation management and planning. Among the existing modeling approaches, the empirical individual tree growth models have higher resolution than, for instance, the stand-level ones (Pretzsch et al. 2002) and enable flexible and detailed simulations for even-aged stand dynamics.

The aim of the research reported in this article was to develop a system of models, which allow the simulation of *P. radiata* stand dynamics in the Inter-Andean Valleys of Bolivia. The model set consisted of site index, diameter increment, tree height, and self-thinning models. The models can be used for decision support to improve forestry decision making and to find optimal management schedules for radiata pine plantations in Bolivia.

2 Materials and methods

2.1 Data sets

Two different datasets from 45 temporary and 51 permanent plots, covering a wide altitudinal gradient at high-elevation plantations, were used. The permanent plot data were used to fit a site index model. Both datasets (96 plots) were used in diameter increment, height, and self-thinning modeling (Table 1).

Table 1 Summary of the data gathered from the temporary and permanent plots

Variable	Min	Max	Mean		
			All plots	Temporary	Permanent
Number of trees/ha	295	3,825	1,325	1,188	1,580
Dominant height, m	1.6	25.4	7.6	15.7	7.1
Stand age, years	1	34	7.9	21.2	7.1
Dominant diameter, cm	3.6	47.1	12.2	25.0	11.4
Site index, m	10.5	30.8	21.0	21.4	20.2
Tree height, m	1.4	27.0	6.4	14.2	5.8
Diameter at breast height, cm	0.1	54.0	13.6	16.6	7.9
Stand basal area, m ² /ha	0.3	65.4	20.0	26.2	8.5
Diameter increment, cm/year	0.05	4.90	0.95	0.6	1.5
Altitude, m	2,180	3,840	3,399	3,502	3,209

Temporary plots were of variable size and contained approximately 75 trees each. Diameter at breast height (d , centimeters) for every tree and the total tree height (h , meters) and bark thickness of 12 trees of different sizes were measured. The annual growth of the past 5 years was measured for every tree from increment cores. The stand age (T , years) of each plot was computed by adding 1 year to the number of annual rings counted on cores taken from three to five trees bored at 30-cm height. Every permanent plot was 400 m² in area and was measured two to eight times: d was measured from every tree and h either from all trees or from at least three sampled trees. The measurement intervals ranged from 1 to 7 years.

2.2 Data preparation

To calculate the height of trees for which height was not measured, the first step of data preparation was to fit a plot-wise model for tree height. In the permanent plot data, a separate model was fitted for every measurement. The model was as follows:

$$h = 1.3 + d^2 / (a + b \times d)^2 \tag{1}$$

where a and b are the parameters to be estimated for every plot or measurement.

Stand dominant height was calculated as the mean height of 100 largest trees/ha. In a very few cases, in the permanent plot data, there were less than five height measurements. In these cases, plot- and measurement-specific height model was not used, but the missing heights were calculated from all height measurements of the permanent plots using the following model, which was based on all height measurements on the permanent sample plots:

$$h = 1.3 + T^{0.367} \left[\frac{d^2}{(2.504 + 0.340d)^2} \right] \tag{2}$$

In the temporary plot data, another model was fitted between single bark thickness ($bark$, millimeters) and diameter at breast height (d , centimeters). This model was used to convert the measured under-bark radial increments into over-bark diameter growth. The method is a simplification but with negligible consequences. The model was common for all plots. The bark model was as follows:

$$bark = 7.008 + 0.138 \times d \tag{3}$$

The bark model and the measured radial increments were used to backtrack the tree and stand development of the temporary plots. Diameter under- and over-bark as well as stand variables were calculated backward for 5 years using 1-year steps. In permanent plots, diameter increment was obtained as the difference in diameter between two measure-

ments (d at the next measurement – d now). If the measurement interval was longer than 1 year, the diameter difference was converted into 1-year growth by dividing by the length of the measurement interval (in years).

2.3 Dominant height modeling

Temporary plots were not used to model dominant height development since only the current dominant height was available for these plots. The permanent plot data consisted of 221 pairs of stand age and dominant height. Eleven different models derived from Hossfeld (cited by Peschel 1938), Korf (1939), Schumacher (1939), Chapman-Richards (Richards 1959), Sloboda (1971), and McDill and Amateis (1992) were tested. Equations were derived using the algebraic difference approach (ADA) (Bailey and Clutter 1974), by equating subdefined ratios of base equations (Amaro et al. 1998), or utilizing the generalized algebraic difference approach (Cieszewski and Bailey 2000). The parameters were estimated using the Levenberg–Marquardt algorithm (Moré 1977) in nonlinear regression analysis using nonlinear least squares. Five best models were selected based on the statistical performance and were visually analyzed for biological consistency. Finally, one model was selected. To further test it, the selected model was visually compared to the measured dominant height developments of the plots (Fig. 1). The model selected for dominant height was used to calculate the site index of every plot. Since, according to previous research and forestry practice elsewhere, a 30-year rotation length might be suitable for this species, the base age was taken as 30 years, which means that site index is equal to the dominant height at that age. Site index was used as a potential predictor in diameter increment modeling.

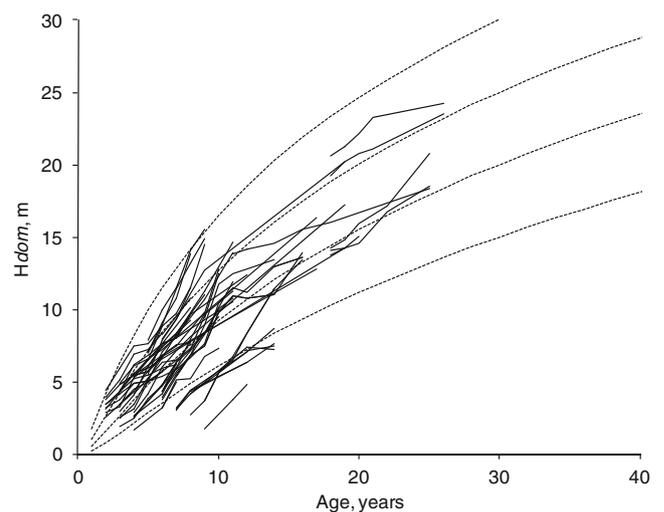


Fig. 1 Comparison between the measured dominant height series and the predictions of the selected Schumacher model for site indices 15, 20, 25, and 30 m

2.4 Diameter increment modeling

The total number of observations for diameter increment modeling was 25,439 (16,526 from temporary plots and 8,913 from permanent plots). The purpose was to develop the following type of model for the future 1-year diameter increment: $i_d = f(\text{tree size, site quality, competition})$, where i_d is future 1-year diameter increment (centimeters).

Each type of predictor (tree size, site quality, competition) was described based on several variables. The site variables available for every plot and measurement were site index and elevation (Table 1). Several variables which describe competition were calculated for each plot and tree. One of them was stand basal area (G), which describes the overall stocking in the stand. Another tested competition variable was the basal area of larger trees (BAL), i.e., the total basal area (square meters per hectare) of all trees of the plot which were larger than the subject tree (Wykoff 1990). An individual tree variable called dominance (DOM) was also tested. This variable yields values between 0 and 1 and was calculated as: $DOM = 1 - BAL/G$ (Schróder and Gadow 1999). Low values of DOM represent trees at a competitive disadvantage, whereas high values represent dominant trees. Tree size was described by means of diameter at breast height. Transformations of predictors were tested. Preliminary models were first fitted separately for permanent and temporary plot data. Since the increments and the relationships were fairly similar in both sets, they were combined for the final model. Both fixed- and mixed-effects modeling approaches were tested using nonlinear least squares and maximum likelihood, respectively, in nonlinear regression analysis. Fixed-effects models may perform better when the aim is prediction in forestry practice (e.g., Temesgen et al. 2008; Pukkala et al. 2009). On the other hand, mixed effects can take into account the hierarchical structure given by the fact that trees of the same plot and same measurement occasion are correlated observations (Pinheiro and Bates 2000).

2.5 Height–diameter modeling

All measured heights were used to develop an individual tree height–diameter model. The total number of observations was 7,076 (434 from temporary plots and 6,642 from permanent plots). Preliminary height models were first fitted separately to both sets. The comparison of the two models showed that the relationships were rather similar in both sets. Therefore, the two data sets were combined. Different height–diameter models based on Stoffels and Van Soest (1953) power equation modified by Tomé (1989) were tested by using nonlinear regression analysis.

2.6 Self-thinning modeling

A self-thinning model was developed by first selecting the plots which were assumed to be on the self-thinning limit and then fitting a model between number of trees and mean diameter. The model for the self-thinning limit was fitted as follows using nonlinear regression analysis:

$$N_{\max} = b_0 \times D_q^{b_1} \quad (4)$$

where N_{\max} is the maximum possible number of trees per hectare and D_q is quadratic mean diameter of trees (centimeters). Moreover, several additional predictors as suggested by Schütz and Zingg (2010) were tried but without success. The self-thinning frontier was modified following the method proposed by Vanclay and Sands (2009) resulting in a model that predicts some mortality already below the self-thinning line. The statistically fitted self-thinning line was used as a reference in the manual fine-tuning of the Vanclay and Sands model; the latter model had to join the self-thinning line. An attempt was also made to model individual tree survival using the permanent plot data and the logistic model type. However, the model predicted unrealistically low survival rates. This is because some trees which were actually cut had been recorded as dead trees. In addition, there was high mortality in some plots, which may have been caused by factors other than competition.

2.7 Model evaluation

The criteria used in model evaluation were (a) agreement with current biological knowledge, (b) logical behavior in extrapolations and long-term simulations, (c) simplicity and robustness, (d) statistical significance (p value < 0.05) and non-biasness, and (e) homoscedasticity and normal distribution of residuals. The statistical analyses were carried out using SPSS software (SPSS Inc. 2008).

2.8 Simulation procedure

When combined, the models can be used to simulate the stand dynamics of *P. radiata* in Bolivia. The input data consist, for instance, of diameters of N trees measured within a plot or empirical diameter distribution of a stand. Dominant height (H_{dom}) and age (T) of the stand are needed. The diameter (d) and frequency are required for every tree or diameter class. The site index (SI) is calculated from H_{dom} and T . A 1-year time step can be simulated as follows:

1. Increment stand age by 1 year, calculate new H_{dom} ($H_{dom\text{new}} = f(T_{\text{new}}, SI)$)
2. Increment tree diameters, calculate the quadratic mean diameter (D_q)

3. Calculate self-thinning limit as a function of the initial stocking (N_0) and D_q ($N_{\max}=f(N_0, D_q)$), reduce the frequencies of trees if required (if $N > N_{\max}$)
4. Calculate dominant diameter (D_{dom}) and tree heights ($h=f(d, D_{dom}, H_{dom})$)
5. Calculate the remaining tree-level attributes and stand characteristics

In simulation, trees having lower growth rates had a higher mortality rate (Vanclay 1994). As there were no available volume data, the taper equation fitted by Castedo-Dorado et al. (2007) for *P. radiata* was used in tree and stand-level volume estimation.

3 Results

3.1 Dominant height model

The selected model for dominant height development, based on Schumacher’s (1939) equation, was as follows:

$$H_2 = \exp \left[4.778 + \left(\frac{T_1}{T_2} \right)^{0.329} \times (\ln(H_1) - 4.778) \right] \quad (5)$$

where T_1 and T_2 are the stand ages in measurement occasions 1 and 2 and H_1 and H_2 are the dominant heights in measurement occasions 1 and 2.

The R^2 of the model was 0.962 and the mean square error was 0.768. From this equation, site index can be obtained by using measured stand age and dominant height. If site index is known, dominant height at any projection age can be also obtained. The predictions of the dominant height model were visually compared to the measured dominant height developments of several plots (Fig. 1). The comparison shows that there are clear disagreements in some plots and very good agreements in other plots. Since the measured H_{dom} series are quite different and sometimes cross each other, the disagreements shown in Fig. 1 cannot be solved by any anamorphic or polymorphic model. Therefore, Schumacher’s equation was not rejected on the basis of the visual analysis.

3.2 Diameter increment model

After testing many combinations of predictors, the following model was selected to describe the future 1-year diameter increment:

$$i_d = \exp(-2.569 - 0.031 \text{ BAL} - 0.006 d^{1.41} - 0.199 \ln(G) + 1.134 \ln(SI) + 0.147 \text{ DOM}) \quad (6)$$

The R^2 is 0.427 and the residual standard error is 0.371 cm. The relative root mean square error (RMSE) in percent of the mean was 64.1%. According to the model, increasing stand basal area and *BAL* decrease diameter increment (Fig. 2). Trees at a competitive advantage (higher *DOM*) have higher predicted growth than dominated trees. With a given stand basal area and *BAL*, increasing diameter decreases diameter increment. Improving site index greatly increases growth rate.

3.3 Height–diameter model

Of the tested alternatives for tree height models, all gave rather similar predictions, but the following model was judged the best on the basis of fitting statistics, simplicity, and visual examination of the residuals and model shape:

$$h = H_{dom} \times (d/D_{dom})^{0.304 - 0.203 \ln(d/D_{dom})} \quad (7)$$

The R^2 of this model is 0.945 and the residual standard error is 0.942 m. The relative RMSE was 14.7%. The model shows, for example, that the height of a tree of certain diameter is greater in an old stand than in a young stand, i.e., the $d-h$ curve moves up when the stand develops. This height model guarantees that, in simulations, the height development of individual trees follows the dominant height development of the stand; for a tree with d equal to dominant diameter, the predicted height equals stand dominant height.

3.4 Self-thinning model

The following model was fitted for the maximum number of trees per hectare (self-thinning limit):

$$N_{\max} = 728,599.182 \times D_q^{-2.038} \quad (8)$$

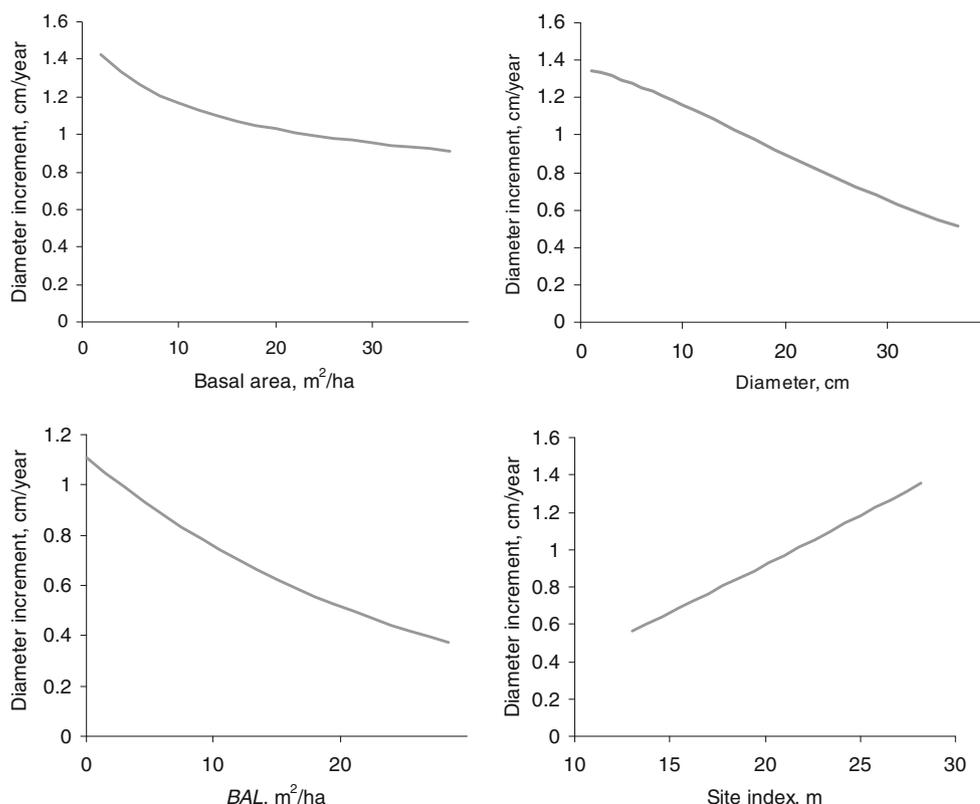
Based on Eq. 8, the following model was adjusted for stand-specific application of the self-thinning limit (Vanclay and Sands 2009):

$$N_{\max} = \frac{N_0}{\left(1 + \left(\left(\frac{\pi N_0}{40,000} \right) \frac{D_q^2}{55} \right)^{3.5} \right)^{\frac{1}{3.5}}} \quad (9)$$

where N_0 is the initial stocking (number of trees per hectare).

This model agrees with the self-thinning limit based on the densest plots (Eq. 8) but is more realistic since it predicts some mortality already before reaching the self-

Fig. 2 Modeled dependence of diameter increment on basal area, BAL , d , and site index according to the diameter increment model



thinning limit (Fig. 3). Therefore, this latter model was finally used in simulation.

3.5 Simulation examples

The models were first used to simulate the development without any thinning treatments of three young plantations

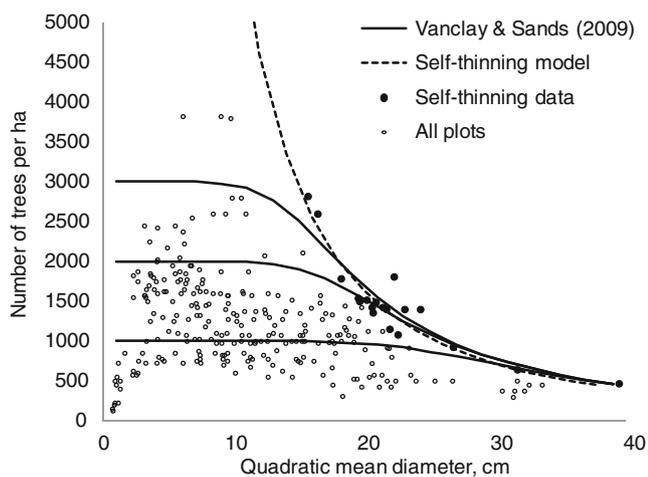


Fig. 3 Self-thinning limit using Eq. 8 (self-thinning model) and using the method proposed by Vanclay and Sands (2009) to apply the self-thinning model (Eq. 9) for three different initial stockings ($N_0=3000$, $N_0=2000$, and $N_0=1000$). The filled points represent the observations used to fit Eq. 8

(Fig. 4), the first representing site index 26.6 m (very good site), the second 22.3 (rather good site), and the third 16.5 m (rather poor site). On the best site, stand volume reached 1,000 m³ ha⁻¹ in 60 years, and 600 m³ ha⁻¹ was reached on the poorest site. Self-thinning began at 20 years on the best site and at 25 years on the poorest site. The mean annual increment (MAI) with the optimal rotation length was 24.4 m³ ha⁻¹ on the best site and 11.7 m³ ha⁻¹ on the poorest site. The rotation length that maximized wood production (maximum sustainable yield) was 27.5 years for the best site and 35.5 years for the poorest site. Therefore, the models indicate that suitable rotation lengths for *P. radiata* in wood production are 25–30 years for very good sites, 30–35 years for medium sites, and 35–40 years for poor sites in the Inter-Andean valleys of Bolivia.

The model set was also used to simulate the optimal stand development that maximizes wood production under three different forest management scenarios: (a) absence of thinning, (b) one thinning, and (c) two thinnings (Fig. 5). The mean harvested volume was the highest for the scenario with two thinnings (25.8 m³ ha⁻¹ year⁻¹ and a 39-year optimal rotation length), followed by the one-thinning scenario (25.1 m³ ha⁻¹ year⁻¹ and a 35-year optimal rotation length) and by the no-thinning scenario (23.5 m³ ha⁻¹ year⁻¹ and a 30-year optimal rotation length).

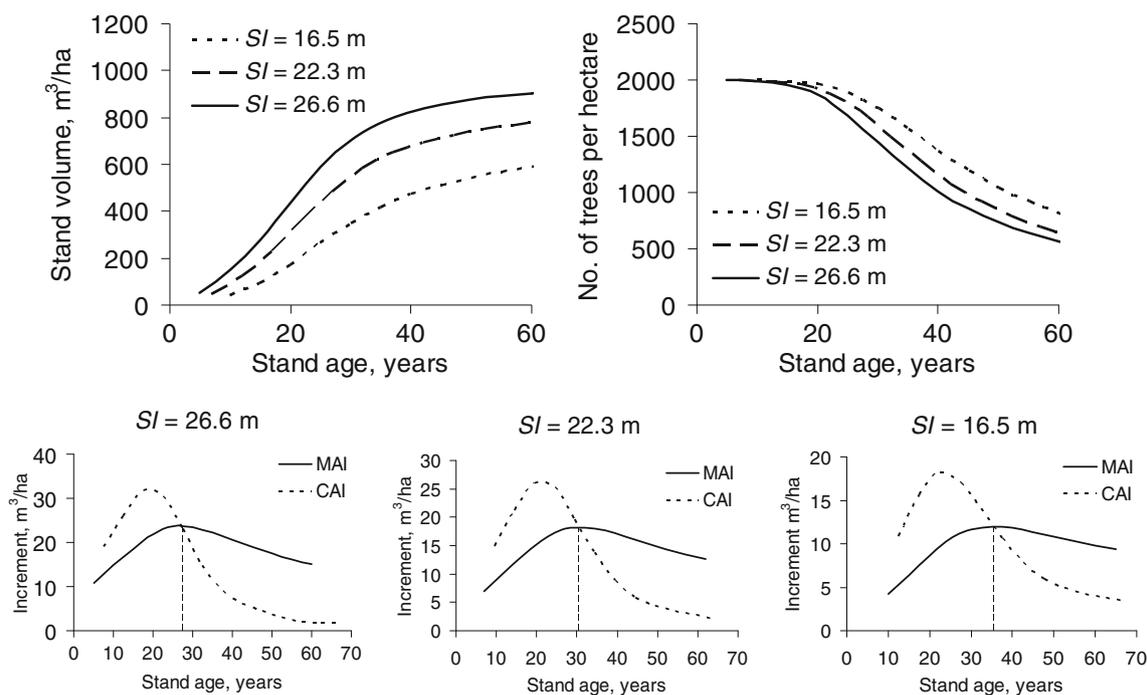


Fig. 4 Model-based simulations for *P. radiata* in Bolivia: yield, survival, and MAI and CAI curves in a very good site ($SI=26.6$ m), in a rather good site ($SI=22.3$ m) and in a rather poor site ($SI=16.5$ m)

4 Discussion

This study presented a site index model, a self-thinning model, and individual tree models for diameter increment and tree height for *P. radiata* stands in the Inter-Andean Valleys of Bolivia. All modeling data represented stands younger than 35 years. Therefore, the set of models should be used with caution beyond that age.

The Schumacher polymorphic-disjoint dynamic equation, which was selected as the dominant height model, is path invariant and computes predictions directly from any age–dominant height pair without compromising

consistency of the predictions. The selected site index model was derived using the ADA (Bailey and Clutter 1974), which results in mathematically reliable equations that always yield coherent results. This model does not require a preselected index age; whatever the base age, height will equal site index when age equals base age. The selected model was compared with other site index equations previously developed for this species in Spain (Diéguez-Aranda et al. 2005) and in Australia (Haywood 2009) by examining whether their predictions properly followed the measured dominant height growth. None of those models was suitable for Bolivia. The Spanish model clearly underestimated the dominant height development of young stands. Both alternative models overestimated the growth in older stands and good sites.

The diameter increment model was fitted as a fixed model and as a mixed model that included a random plot factor. Both approaches resulted in similar models with the same predictors. However, similarly to previous research (e.g., Temesgen et al. 2008; Shater et al. 2011), the RMSE was smaller for the fixed-effects model than for the fixed part of the mixed model. Therefore, when plot factors are not estimated, the fixed-effects model is recommended for predicting diameter increment in *P. radiata* stands in Cochabamba (Bolivia). Theoretically, mixed models are able to incorporate the stochastic between-plot and between-tree structure alleviating problems associated with the presence of nested structure, which otherwise may

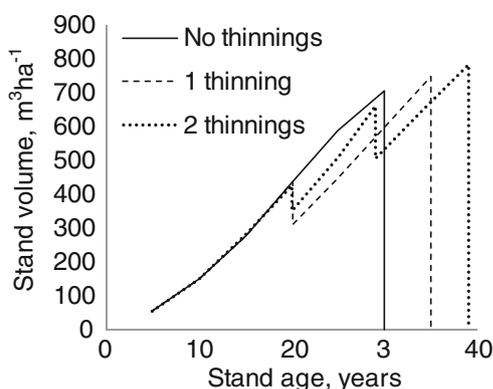


Fig. 5 Simulation of the stand volume development in optimal schedules which maximize wood production under three different management scenarios (zero, one, or two thinnings) in a 5-year-old stand growing in a very good site ($SI=26.6$ m)

result in biased estimates of standard errors of parameter estimates.

The height–diameter model developed in this study was compared with other equations developed for *P. radiata* in Chile (Trincado and Leal 2006) and in Spain (Castedo-Dorado et al. 2006; Canga-Libano et al. 2007) all providing similar height–diameter relationships for *P. radiata* in Bolivia in good, average, and poor sites. This model is useful, besides in growth simulations, also to predict individual tree heights in inventory situations when H_{dom} and d are measured but not all trees are measured by height.

To account for mortality, a model of the self-thinning limit was developed. Schütz and Zingg (2010) suggested using SI as well as different transformations of D_q to improve the model for maximal stand density. In this study, the self-thinning limit was not found to vary according to site index, and transformations of D_q were not significant additional predictors. On the other hand, it fairly well followed the Reineke (1933) law but with a slightly higher average slope of the maximum density. According to Garcia (2009), a model that complies either with Reineke or with the $-3/2$ power law can be trusted to behave reasonably when extrapolated. Furthermore, the model based on Vanclay and Sands (2009) was also developed to account for the slight mortality that often occurs in a stand before it reaches the self-thinning limit (Fig. 3).

Simulations were used to demonstrate how the models work together in medium and long-term growth and yield prediction. The models suggest that the mean annual volume increment, with the optimal rotation length, is less than $10 \text{ m}^3 \text{ ha}^{-1}$ on the poorest sites of Cochabamba and more than $25 \text{ m}^3 \text{ ha}^{-1}$ on the best sites. Toro and Stanley (1999) reported that the productivity of radiata pine ranges from 18 to $35 \text{ m}^3 \text{ ha}^{-1} \text{ year}^{-1}$ in Chile, and in New Zealand, Shula (1989) estimated a maximum productivity of $43 \text{ m}^3 \text{ ha}^{-1} \text{ year}^{-1}$ for this species. Such differences, together with the differences in dominant height development and the lack of previous suitable individual tree diameter increment models, justify our study as they confirm the need for specific growth and yield models for the Bolivian conditions. Suitable rotation lengths for *P. radiata* in wood production are 30 to 40 years in the Inter-Andean valleys of Bolivia. Similar rotation lengths (from 28 to 44 years) have been proposed in northwest Spain (Sánchez et al. 2003). However, when economic profitability is maximized and thinnings are used, the optimal rotation lengths would be different since the economically optimal rotation length depends on the stand establishment costs, prices and dimensions of different timber assortments, and discounting rate.

This study is the first one providing a complete set of growth models on an individual tree basis for *P. radiata* in

Bolivia, and to our knowledge, very few attempts have been done elsewhere. *P. radiata* in Bolivia is of great economic and social importance, and the models presented in this study provide valuable information for further studies on optimizing the management and evaluating alternative management regimes for the species.

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