HAL open science

Fagacées: a tree-centered growth and yield model for sessile oak (Quercus petraea L.) and common beech (Fagus sylvatica L.)

Le Moguédec, Dhôte

▶ To cite this version:

Le Moguédec, Dhôte. Fagacées: a tree-centered growth and yield model for sessile oak (Quercus petraea L.) and common beech (Fagus sylvatica L.). Annals of Forest Science, 2012, 69 (2), pp.257-269. 10.1007/s13595-011-0157-0 . hal-00930728

HAL Id: hal-00930728

https://hal.science/hal-00930728v1

Submitted on 11 May 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution 4.0 International License

Annals of Forest Science (2012) 69:257–269
DOI 10.1007/s13595-011-0157-0

ORIGINAL PAPER

Fagacées: a tree-centered growth and yield model for sessile oak (*Quercus petraea* L.) and common beech (*Fagus sylvatica* L.)

Gilles Le Moguédec · Jean-François Dhôte

Received: 21 December 2010 / Accepted: 7 July 2011 / Published online: 20 December 2011
© The Author(s) 2011. This article is published with open access at Springerlink.com

Abstract
• ***Introduction*** Fagacées is a growth model that has been developed for pure Oak or Beech stand management in even-aged high forests and coppice with standards forests. It has been calibrated for the plain forests of northern France.
• ***Objectives*** This paper presents all the characteristics of this model: general structure, equations, and parameters. It can be classified as a distance-independent tree-centered model. Its main originality is that tree growth is organised according to a top–down organisation.
• ***Result*** At first, the growth is computed at the stand level. Then it is allocated between the individual trees. The paper presents and discusses successively these two parts. In addition, the model delivers the stem profile of each tree: rings and compartments within the stem. Some other additional modelling that have been defined for batch mode purposes are also briefly presented.

Handling Editor: Daniel Auclair

G. Le Moguédec · J.-F. Dhôte
INRA, UMR 1092, Laboratoire d'Etude des Ressources Forêt-Bois (LERFoB), Centre INRA de Nancy, 54280 Champenoux, France

J.-F. Dhôte
e-mail: jean-francois.dhote@onf.fr

G. Le Moguédec (✉)
INRA, UMR AMAP, Boulevard de la Lironde, TA A-51/PS2, 34398 Montpellier cedex 5, France
e-mail: gilles.moguedec@cirad.fr

Present Address:
J.-F. Dhôte
ONF, Direction Technique et Commerciale Bois, Boulevard de Constance, 77300 Fontainebleau, France

Keywords Growth and yield model · Stand growth · Tree-centered · Sessile oak · Common beech

1 Introduction

Many forest dynamics models have been developed in the last decades. They generally present various kinds of organisation and structure that allow some classifications (see Porté and Bartelink 2002 for a review of forest model classifications). The particular organisation and structure of a given model depend on the purpose for which it has been developed.

Among these models, the Fagacées growth and yield model can be classified as a distance-independent tree-centered model. It has been mainly developed during the 1990s, first for the management of pure Beech stands (Dhôte 1991), then expanded to pure Oak stands (Dhôte 1999). Its development has been motivated by the lack of growth and yield model adapted for managing these species in France. Indeed, despite the fact that Sessile Oak (*Quercus petraea* Liebl.) and Beech (*Fagus sylvatica* L.) are respectively the first and third forest species in France (they represent respectively 12.3% and 9.3% of the productive forest area; Inventaire Forestier National 2007), no growth and yield model were developed for these species in France before. The main reference for Oak was a yield table published in 1962 (Pardé 1962) and for Beech a yield table published in 1973 (Décourt 1973). These tables have then been republished in 1984 (ENGREF et al. 1984). They contain reference trajectories for Oak and Beech silviculture and can be considered as ancestors of growth models for these species in France. The Fagacées growth model has been developed to provide the foresters a more flexible tool than growth and yield tables to manage even-

aged high forest stands and coppice with standards stands, the forest structures that are the most common in France for Oak and Beech.

The main originality of the Fagacées model lies in its top–down organisation (Fig. 1): once the global growth of the stand has been computed, it is allocated among the trees within the stand. The model thus presents at the same time some properties of stand models and some properties of individual models.

Unlike other classical tree-centered forest growth models used for European common species like SILVA (Pretzsch et al. 2002) or BWINPRO (Shröder et al. 2007), Fagacées is not a spatialised model. As for other distance-independent growth models like PROGNAUS (Monserud and Sterba 1996), competition between individuals is not explicitly computed but is assessed through global relations such as absolute or relative position within a histogram.

Although most parts of this model have already been published (generally in French, sometimes only in poorly accessible documents), it has never been fully presented. We present here the general structure of this model, the whole set of equations of the growth part and the corresponding parameters. We present also some additional procedures used to obtain a full simulator so that the growth simulator can be entirely rebuilt from the information presented in this paper.

Since Fagacées is a tree-centered model, individual trees are described by individual characteristics such as age, diameter at breast height (d_{130}) and height (h). This information can be further presented as a whole stem profile that will be used to compute volumes and wood qualities.

This paper contains many notations and formulae. In order to help the reader, Table 1 presents the main notations used.

2 Equations of the growth model

2.1 Origin of the data

Various samples have been used to establish or calibrate the equations presented in this paper.

The main data used were obtained from permanent stands that were established in plain forest of the northern part of France covering a climatic gradient from oceanic (Normandy region) to semi-continental (Alsace region). All of them were established between 1883 and 1935 and most of them were issued from natural regeneration. They are currently maintained by the French National Institute for Agronomical Research (INRA). They were established in order to study the influence of thinning intensity and frequency on wood production. Several plots were kept as control plots, with no silvicultural treatment. This point will be crucial for the adjustment of a self-thinning relation. Details on the permanent stands can be found in papers published by Pardé (1962), Oswald (1981) and Dhôte (1999).

For Oak, 11 sites contain 32 plots (several plots per site) between 0.2 and 1 ha and 339 forest inventories. The age classes vary from 27 to 277 years, but main data concern plots between 40 and 160 years old. The chosen sites mainly correspond to areas devoted to high-quality oak

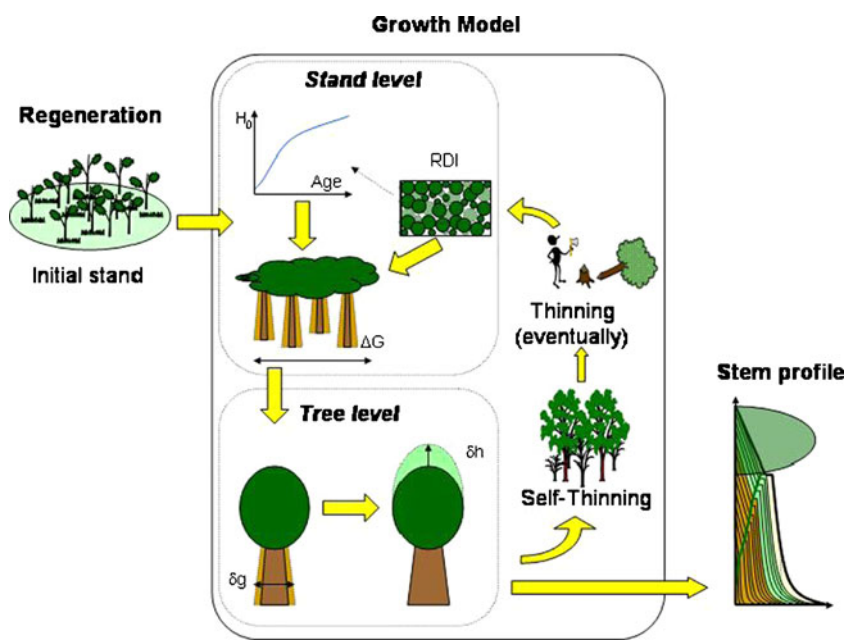

Fig. 1 General structure of the "Fagacées" growth model

Table 1 Main notations used in the paper

Symbol	Signification	Unit
N	Number of trees per hectare in the stand	Number/ha
A	Age of the stand (≃age of the trees within the stand)	Years
RDI	Relative Density Index	–
H_0	Top height of the stand	m
H	Average height of the stand	m
H_{100}	Site Fertility Index (top height at age 100)	m
h	Individual height of a tree	m
D_{130} or DBH	Diameter at 130 cm (breast height)	cm
D_g	Average quadratic diameter of the stand	cm
d_{130}	Individual diameter at 130 cm	cm
$d(z)$	Individual diameter at height z	cm
G	Basal area of the stand	m²/ha
g	Basal area of the tree	cm²
σ	Threshold circumference for individual growth	cm
γ	Maximum efficiency of growth	cm/year
μ_1 to μ_7	Parameters of the individual diameter profile	
c_r	Crown ratio of a tree	–
r_h	Crown radius of a tree	m
φ	Parameter to control batch thinning	–

timber production and at least average site fertility index (top height at 100 years between 21 and 29 m).

For Beech, the main data represents five sites and 33 plots, with areas from 0.2 to 1 ha. Age classes vary from 35 to 130 years. As for Oak, the plots were chosen mainly within good fertility conditions (Dhôte 1997).

The top height curve for Oak was established from a sampling of 50 pure Oak plots (0.06 ha each) from five regions of the northern part of France. These plots were chosen so that they cover a large range of site fertility index. In each plot, the first, third and fifth largest dominant trees were sampled for a detailed stem analysis. This sampling was conducted at the French Forest Service (ONF) by Duplat and Tran-Ha (1997).

Relations for individual trees are issued partly from 82 mature sessile Oak trees measured in detail in five areas from Northern France sampled during a convention between INRA and ONF (Nepveu and Dhôte 1998) and partly from a sample of 3,000 individual conventional data collected in the permanent plots already mentioned (Dhôte et al. 2000).

Self-thinning curves, that are used to compute the Relative Density Index, have also been adjusted on data issued from the latter permanent plots (Dhôte 1997). Except for the self-thinning curves (see below), all the relations presented in this paper have been fitted using least squares methods.

2.2 The Relative Density Index

The notion of Relative Density Index (RDI) is based on Reineke's observations (Reineke 1933) in very dense monospecific stands for several species from North America. Reineke defined a self-thinning line which represents the maximum number of stems a forest stand at a given diameter could support before self-thinning occurs. He defined a Stand Density Index to convert the number of trees in a stand to the number of trees the same stand would have for the same density with an average diameter of 10 in. (25 cm).

However, the values of this index can vary within a very large interval and the values obtained for different species or regions cannot be directly compared. The RDI has then been defined in order to be scaled within the [0, 1] interval. It is computed as the ratio of the actual number of stems in a stand and the theoretical number of stems this stand could support with the same DBH (Fig. 2):

$$\mathrm{RDI} = \frac{N}{N_{\max}} \tag{1}$$

where, N is the current number of trees and $N_{\max}$ the theoretical maximum number of trees obtained from the self-thinning equation.

For both sessile Oak and Beech, the self-thinning line is given by:

$$\ln(N_{\max}) = a - b \cdot \ln(\pi \cdot D_g) \tag{2}$$

where, D_g is the average quadratic diameter expressed in centimetres. However, this self-thinning relation is not

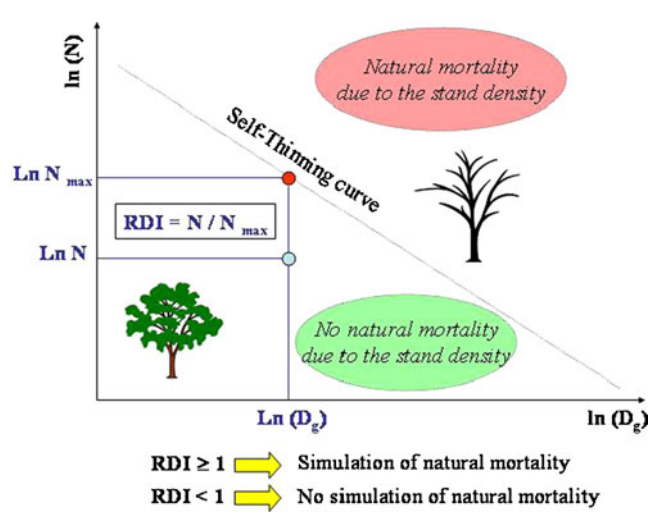

Fig. 2 Construction of the Relative Density Index (RDI) and introduction of self-thinning in the Fagacées model

always a straight line. In particular, the slope parameter b can increase as the forest grows (Zeide 2005). Nevertheless, this formulation is considered as a good approximation among a wide range of ages and fertility conditions.

Since the theoretical maximum value of the RDI is 1, Eq. 2 can be used in Eq. 1 to obtain a new computation of RDI:

$$\mathrm{RDI} = \frac{N \cdot (\pi \cdot D_g)^b}{\exp(a)} \tag{3}$$

The values of parameters a and b used in Fagacées are given in Table 2. They have been obtained by visually adjusting a straight line on the upper limit of the points [ln(D_g); ln(N)] from the data of the permanent plots (Dhôte 1997).

Two main criticisms of this approach can be formulated. First, the chosen adjustment method is known to be highly dependent on the data quality. In particular, it is necessary that part of the stands used to adjust the relation is close from the self-thinning situation. Since there are some stands with no sivicultural treatment for decades in the data used, we consider that this condition is verified. Other methods exist to fit self-thinning relations such as the stochastic frontier analysis but they need additional assumptions to be properly applied (Zhang et al. 2005). Second, the parameters do not change according to the site fertility index. The fact that these parameters should change according to species or according to site fertility index is discussed (Jack and Long 1996; Bi et al. 2000; Morris 2003). In our case, if two different parameter estimations have been conducted for Oak and Beech, no significant relation with the site fertility index H_{100} has been highlighted. However, it should be remarked that the values of the b parameter found here differ from those found by Pretzsch (2006) for the same species in central and southern Germany. It is higher (in absolute value) for Oak: the value used for the b parameter is 1.701 whereas it ranges from 1.222 to 1.628 according to site in Pretzsch et al. For Beech, the value used in Fagacées is lower: 1.574 against 1.723–1.873 in Pretzsch et al.

2.3 Stand initialisation

Natural regeneration is simulated to generate an initial stand. The word "recruitment" would actually be more appropriate than "regeneration" since the trees appear in the model only once they have reached a given state (circumference at 130 cm greater or equal than 1 cm). Several studies (Sano 1997; Fortin et al. 2006) showed that tree diameters are generally distributed according to an inverse j-shaped distribution. Among such distributions, the power distribution, the Weibull distribution and the exponential distribution are the simplest and are commonly used (Nord-Larsen and Johannsen 2007). Here, the initial individual diameters are randomly generated from an exponential distribution, which has been chosen for simplicity. Its unique parameter is chosen so that the average quadratic diameter of the stems equals a given value D_g:

If the random variable D follows an exponential distribution with expected value equal to θ, then:

$$E(D^2) = 2 \cdot \theta^2 \tag{4}$$

The parameter θ is chosen so that $\sqrt{E(D^2)} = D_g$ hence:

$$\theta = \frac{D_g}{\sqrt{2}} \tag{5}$$

Simulations begin only when the growth model becomes valid, around 15 years old. Since the model is used to compare silvicultural scenarios, the same starting point is chosen for all of them. This starting point is defined by the initial RDI, RDI_0, from which the initial number of stems N_0 to be generated is obtained according to Eq. 2:

$$\ln(N_0) = \ln(\mathrm{RDI}_0) + a - b \cdot \ln(\pi \cdot D_{g_0}) \tag{6}$$

Since a "do-nothing" scenario (corresponding to unmanaged forest from regeneration) belongs to the set of scenarios to be studied, the value of RDI_0 has been fixed to 1.

Data analysis from forest inventories in several areas of the northern half of France showed the following fact. When the trees are about 15 years old, an oak stand in natural regeneration conditions has an average quadratic diameter D_g approximately equal to 1 cm, regardless of the stand fertility index. So the reference D_{g0}=1 cm at age A_0= 15 years is taken as a starting point for the for regeneration simulations. The individual heights h are then obtained from the individual diameters and the stand top height using the allometric Eq. 24 that will be presented later in this paper.

2.4 The stand level of the growth model

2.4.1 Top height curves

For Sessile oak, the top height depends only on stand age and on stand fertility index. The curve equation (Eq. 7 and Fig. 3), based on data from even-aged high forests available

Table 2 Parameters used for the computation of RDI

Parameter	Sessile oak	Beech
a	14.000	13.688
b	1.701	1.574

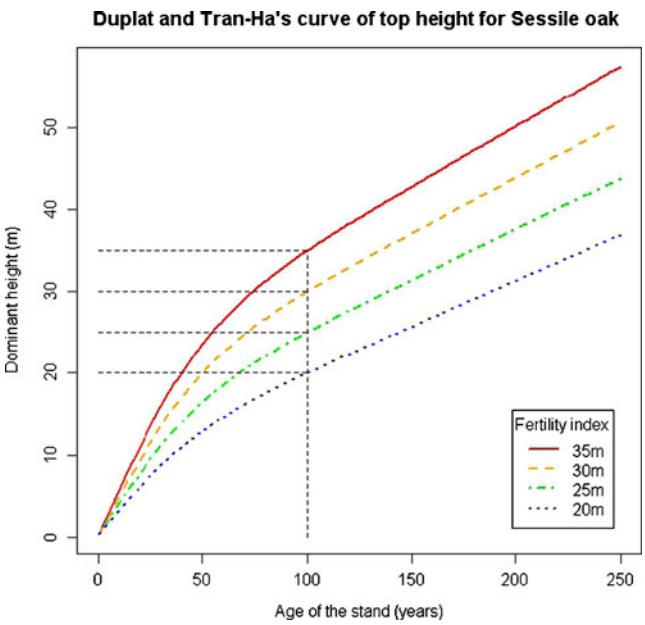

Fig. 3 Top height curve for Sessile oak (after Duplat and Tran-Ha 1997)

at the ONF, has been established by Duplat and Tran-Ha (1997). It is written as:

$$\begin{aligned} H_0 &= f(A, H_{100}) \\ &= m_1 + (m_2 \cdot A + m_3 \cdot H_{100} + m_4)[1 - \exp(-m_5 \cdot A^{m_6})]^{m_7} \\ &\quad + (m_8 + m_9 \cdot H_{100}) \cdot A \end{aligned} \tag{7}$$

where,

- H_0 is the top height of the stand (metres);
- A is its age (years);
- H_{100} is the fertility index (metres).

Since the model has been designed for even-aged stands, the age of the stand is also the age of its trees. The site fertility index H_{100} is defined by the top height of the stand at 100 years. The values of the parameters m_1 to m_9 are given in Table 3.

This curve presents an oblique asymptote. Usually, growth equations present a horizontal asymptote. But even the oldest stands (around 220 years old) used to establish Eq. 7 were still growing. Since no high standard forest older than 250 years was available in the data, this curve is generally considered as acceptable until 250 years and may be extrapolated until 300 years, not after, due to the oblique asymptote.

Table 3 Parameters of Duplat and Tran-Ha's curve for Sessile oak top height

m_1=0.30 m	m_4=−7.10703 m	m_7=0.71192
m_2=−0.039339 m/year	m_5=0.0024168	m_8=0.10445 m
m_3=0.78595	m_6=1.56660	m_9=0.0023513 year^{-1}

This equation is mainly used in the model to compute the dynamics of stand height between two successive steps of simulation. The stand height is generally supposed to be independent from the silviculture for many timber species. However, this is not completely true. Hence in several forest models, for example the one from Nord-Larsen and Johannsen (2007), the height is estimated as an increasing function of the stand density. In the Oak version of the Fagacées model, the increment of stand height ΔH_0 between the ages A_t and A_{t+1} is computed as:

$$\Delta H_0 = \begin{cases} H_0(A_{t+1}) - H_0(A_t) & \text{if RDI} \geq 0.5 \\ 2 \cdot \text{RDI} \cdot [H_0(A_{t+1}) - H_0(A_t)] & \text{if RDI} < 0.5 \quad \text{if RDI} \geq 0.5 \end{cases} \tag{8}$$

The growth reduction factor due to low density can appear strong. It has been built so that the growth tends to 0 for very low density and to lead to continuous growth around RDI= 0.5. This reduction factor has been introduced mainly to give the model a behaviour for low density consistent with the theory. But, due to the lack of available data for very low stand densities, it has not been validated in this area.

For Beech, the current version of top height equation has been fitted by Bontemps (2006) on the Lundqvist–Matérn equation:

$$H_0(A) = K \cdot \exp\left(-\left[\left(\ln \frac{K}{H_{A_0}} \right)^{-m} + \frac{R \cdot m \cdot C_m}{K} (A - A_0) \right]^{-\frac{1}{m}} \right) \tag{9}$$

which is a classical sigmoid growth curve with horizontal asymptote. In this parameterisation, A is the current stand age, A_0 a reference age, H_{A0} the top height at the reference age, K the value of the horizontal asymptote and m a shape parameter. The quantity C_m is a function of m adjusted to give to the parameter R the meaning of the growth speed at the inflexion point (taken as fertility indicator). For the Lundqvist–Matérn equation it can be written as:

$$C_m = \exp\{(1 + m)[1 - \ln(1 + m)]\} \tag{10}$$

However, contrary to the Oak case, it has not been possible to give to Eq. 9 a single set of parameters for the whole northern part of France (Table 4).

Table 4 Parameters of Bontemps' curve for Beech top height

Parameter	Unit	Northwestern France	Northeastern France
A_0	Year	5	5
H_{A0}	m	1.3	1.3
K	m	44	67.3
R	m/year	0.4	0.4
m	–	0.517	1.218

2.4.2 Basal area increment

The basal area increment between two steps of simulation is given by a potential-reductor equation:

$$\Delta G = f_1(H_0, \Delta H_0) \cdot f_2(\text{stand density}) \tag{11}$$

where f_1 is a function that expresses the potential stand growth, that depends only on the current top height H_0 and its variation ΔH_0 between two steps of simulation and f_2 is a function that expresses how the potential growth is reduced by the stand density.

The potential stand growth f_1 has the following expression:

$$\begin{cases} \text{Oak :} & f_1(H_0, \Delta H_0) = p_1 + p_2 \cdot \Delta H_0 - p_3 H_0 \\ \text{Beech :} & f_1(H_0, \Delta H_0) = p_1 + p_2 \cdot \Delta H_0 \end{cases} \tag{12}$$

The reductor function takes a different form for Oak and Beech. For Oak, the stand density is expressed by the RDI index:

$$f_2(\text{RDI}) = \frac{(1+r) \cdot RDI}{r + RDI} \tag{13}$$

whereas for Beech, it uses the sum of diameters exceeding a given threshold Σ_σ:

$$f_2(\Sigma_\sigma) = 1 - \exp[-(r_1 + r_3) \cdot \Sigma_\sigma] \tag{14}$$

The stand density index Σ_σ used for Beech is computed as

$$\Sigma_\sigma = \left(\frac{1}{\text{stand area}}\right) \cdot \sum_{d_{130} \geq \sigma} d_{130} \tag{15}$$

The diameter threshold σ cannot be determined at this step. It will be numerically adjusted to equilibrate stand growth and individual growth later in this paper (Eq. 16).

For both Oak and Beech, the values of the parameters used for functions f_1 and f_2 can be found in Table 5. They have been empirically established and adjusted on existing datasets (Dhôte 1995; Nepveu and Dhôte 1998).

In the global Eq. 11, the growth has two independent components (for Oak at least if RDI≥0.5): the stand height curve that depends only on age and fertility index and the stand density that depends on silviculture.

The first component (Eq. 12) has the general shape of the derivative of a classical growth curve: first increasing, reaching a maximum then decreasing. However, it should not be used after 300 years which is the limit of validity for the model.

The second component (Eq. 13 for Oak, Eq. 14 for Beech) expresses how the increment of basal area given by 12 is depressed if the stand density is not at its maximum value. This is the only way by which the silviculture acts on growth in the model. It is an increasing and concave function of stand density with values increasing from 0 to 1. It expresses that the speed of growth increases with stand density, but relatively more for low density values than for high densities.

Table 5 Parameters used for the increment of basal areal

Parameter	Sessile oak	Beech	Unit
p_1	0.2719	0.1512	m²/ha/year
p_2	2.8018	2.0525	m/ha
p_3	5.9684×10^{-3}	–	m/ha/year
r	0.21852	–	–
r_1	–	3.6586×10^{-4}	ha/cm
r_2	–	1.6706×10^{-4}	ha/cm
r_3	–	1.4476×10^{-4}	ha/cm

2.5 From stand level to tree level

Fagacées is a tree-centered model. Within the stand, each tree growth is individually computed as follows. Once the global growth is computed, it is allocated among all the individuals (Fig. 1). The individual growth is first computed for the increment of the individual basal area. The individual height is then adjusted by allometry.

The transition from stand level to tree level for the basal area increment is given by Eq. 16:

$$\sum_{\text{trees}} \delta g(\sigma) = \Delta G \cdot S \tag{16}$$

where, δg is the individual basal area increment and S is the total stand area (ha). The parameter σ is an intermediate quantity used in the computation of individual basal area increment (Eq. 17 below). It is adjusted to make individual basal area growth sum up to stand growth as determined by the stand growth compartment of the model.

2.6 Growth of an individual tree

2.6.1 Increment of individual basal area

Dhôte (1999) has shown that in oak stands, individual tree growth is favoured for trees belonging to the biggest trees of the stand. At the opposite, the competition for a small tree can be so strong that its growth is almost stopped. For that reason, in the Fagacées model, the increment of basal area for a given tree is given by:

$$\delta g = \gamma \cdot \max(d_{130} - \sigma; 0) \tag{17}$$

where,

- δg is the individual increment of basal area (square centimetre per year);

- γ is a maximal efficiency for growth (centimetre per year), see below for explanations;
- d_{130} is the current tree diameter at 130 cm (centimetre);
- σ is a threshold which defines a minimum diameter for radial growth to occur (centimetre).

Only the trees with a diameter exceeding σ will grow (Fig. 4). This threshold σ has no fixed value but is numerically adjusted at each step of the simulation according to Eq. 16.

In this model, the growth is expressed in terms of basal area which is linked to the diameter at 130 cm (d_{130}) by the relation:

$$g = \pi \cdot d_{130}^2/4 \tag{18}$$

If expressed in terms of diameter, the increment can be rewritten as:

$$\delta d_{130} = \sqrt{d_{130}^2 + \frac{4}{\pi}\delta g} - d_{130} = d_{130} \cdot \left(\sqrt{1 + \frac{4}{\pi} \cdot \frac{\delta g}{d_{130}^2}} - 1\right) \tag{19}$$

To interpret the γ parameter of Eq. 17, suppose that the σ parameter is neglectable compared to the biggest diameters in the stand. In this case, from the fact that $2\frac{\gamma}{d_{130}} << 1$, the first-order Taylor series of relation (19) gives for the high values of d_{130}:

$$\begin{aligned}\delta d_{130} &= d_{130} \cdot \left(\sqrt{1 + \frac{4}{\pi} \cdot \frac{\gamma(\pi \cdot d_{130} - \sigma)}{d_{130}^2}} - 1\right) \\ &\approx d_{130} \cdot \left(1 + 2\frac{\gamma}{d_{130}} - 1\right) = 2\gamma\end{aligned} \tag{20}$$

The γ term represents the theoretical maximum increment of tree radius, so that it is called "maximum efficiency" of the growth.

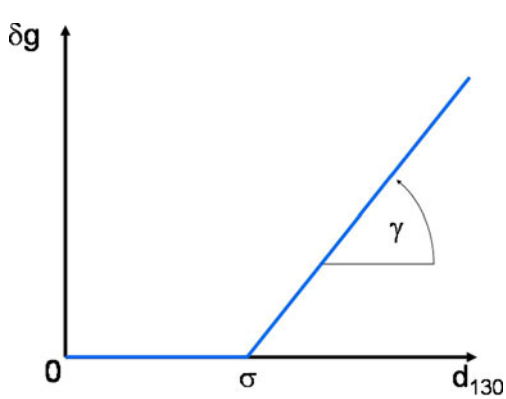

Fig. 4 Individual growth of basal area

For sessile Oak, it has been found to be linked to the fertility index of the stand H_{100} by the relation:

$$\gamma = 0.2007 + 0.0314 \cdot H_{100} \tag{21}$$

where, γ is expressed in centimetres per year and H_{100} is expressed in metres.

For Beech, this coefficient depends on the variation of top height ΔH_0 but also on the stand density expressed with the index Σ_σ defined in Eq. 15:

$$\gamma = f_1(\Delta H_0) \cdot [r_3 + r_1 \cdot \exp(-r_2 \cdot \Sigma_\sigma)] \tag{22}$$

The parameters of this equation are given in Table 5. The diameter threshold σ is determined by numerically solving the equation

$$(\Sigma_\sigma - \sigma \cdot N_\sigma) \cdot [r_3 + r_1 \cdot \exp(-r_2 \cdot \Sigma_\sigma)] = f_2(\Sigma_\sigma) \tag{23}$$

where, $N_\sigma$is the number of trees per hectare with diameter exceeding σ, so that global growth given by (16) equals total individual growth given by (17).

The γ coefficient depends only on fertility index for Oak and on both fertility index and stand density index for Beech. This can be related to the fact that Beech is more shade-tolerant than Oak. Hence, it reacts more progressively than Oak to stand density variations.

2.6.2 Increment of individual height

Once the tree diameter d_{130} (cm) is computed, the tree height h (m) is computed using an allometric relation (Dhôte and de Hercé 1994):

$$h = 1.30 + \frac{\alpha(d_{130}) - \sqrt{\alpha^2(d_{130}) - 4 \cdot \pi \cdot q_1 \cdot q_2 \cdot (H_0 - 1.30) \cdot d_{130}}}{2 \cdot q_2} \tag{24}$$

where,

$$\alpha(d_{130}) = H_0 - 1.30 + \pi \cdot q_1 \cdot d_{130} \tag{25}$$

This is a hyperbolic curve, that crosses the point (d_{130}= 0 cm, h=1.30 m) and presents a horizontal asymptote at h= H_0. The values of parameters q_1 and q_2 are different for sessile Oak and Beech (Table 6). This relation is considered as robust within a wide range of ages and site conditions

Table 6 Parameters of the allometric relation for individual height

Parameter	Sessile oak	Beech	Unit
q_1	0.373	0.412	m/cm
q_2	0.98812	0.98764	–

(Dhôte and de Hercé 1994). But since it is only an allometry, the individual height consistency depends on those of the top height growth and of the individual basal area growth.

2.7 Robustness and sensitivity analysis

The quality of the model can be studied at three levels: the general organisation of the different model components, the general form of the equations that describe these components and the parameters associated to these equations.

The Fagacées model can be considered as robust due to its top–down organisation. The main equations have been established at the stand scale, that is known to be generally more stable and reliable than the tree scale (Peng 2000).

The form of these equations has been chosen so that they respect theoretical considerations regarding the phenomenons they are supposed to represent. For example, the RDI Eq. 3 is derived from the linear relationship between the maximum number of stems and the stand diameter in a log–log scale as expressed by Reineke's self-thinning rule. Due to these constrained forms, the model cannot have an aberrant behaviour. However, these forms may be appropriate only within a limited domain. For example the top hight curve used for Oak that presents an oblique asymptote cannot be extrapolated to very old stands. It is however often difficult to precisely define such domain limits. There is in general no difficulty to establish the validity of a relation in parts of the domain where observations are available. The question is how far from the observed area these relations can be extrapolated. An answer may be given by expert's advice, but this answer cannot be very precise.

Except for self-thinning, the values of the parameters associated to the different relations have been obtained from observations using least squares methods. In most cases, the stability of the estimations has been studied by cross-validation, bootstrap or other resampling methods. These results have then been used to study the sensitivity of the response to variations of the parameters, at the scale of the different relations. In all cases, the numerical outputs obtained from variations of the parameters were comparable to the observed data.

In a general way, all available data have been used. The robustness of the behaviour at the model scale has always been privileged by the choice of the form of the equations and the way they have been arranged. In several cases, it would have been possible to use equations more adapted to the available data. But these equations were more sensitive to variations of parameters or to the bounds of the validity domain. With the chosen formulation, there is a slight loss of precision in terms of numerical outputs, but a more stable behaviour of the equations.

3 Complementary modelling for the evaluation of a silvicultural scenario

3.1 Stem profile

3.1.1 Stem taper curve

In Fagacées the trees are individualized. So each stem of the stand has its own characteristics. At the end of a growth step, each stem is allocated a diameter (d_{130}) and a total height h. These two variables are then used along with allometric relations to deliver a stem profile. The stem profile is a function of the height z in the tree at which the diameter is computed. For both Beech and Oak, this function has the same general form (Eq. 26 and Fig. 5).

The stem profile can be assimilated to a succession of two cones, one for the stem above the crown basis, the other for the stem under the crown basis (Hatsch 1997). The latter has to be corrected at the bottom of the stem to take the butt influence into consideration.

For $0 \le z \le h$, the diameter of the stem above bark (in centimetre) at height z (in metre) is given by:

$$d(z) = \begin{cases} \mu_2 \cdot (\mu_1 - z) & \text{if } \mu_3 \le z \le \mu_1 \text{(within the crown)} \\ [\mu_4 \cdot (\mu_3 - z) + \mu_5 \cdot \mu_2 \cdot (\mu_1 - \mu_3)] \dfrac{1+\lambda(z)}{1+\lambda(\mu_3)} & \text{if } 0 \le z \le \mu_3 \text{(under the crown)} \end{cases} \tag{26}$$

where, λ (z) is a function to represent the variation of diameter at the butt:

$$\lambda(z) = \mu_6 \cdot \exp\left[-\frac{z}{\mu_7} \ln(100 \cdot \mu_6)\right] \tag{27}$$

with:

- μ_1 is the height of the apex, it is then equal to the total height h (metre);
- μ_2 is the decrease of diameter per metre in the crown (centimetre per metre);

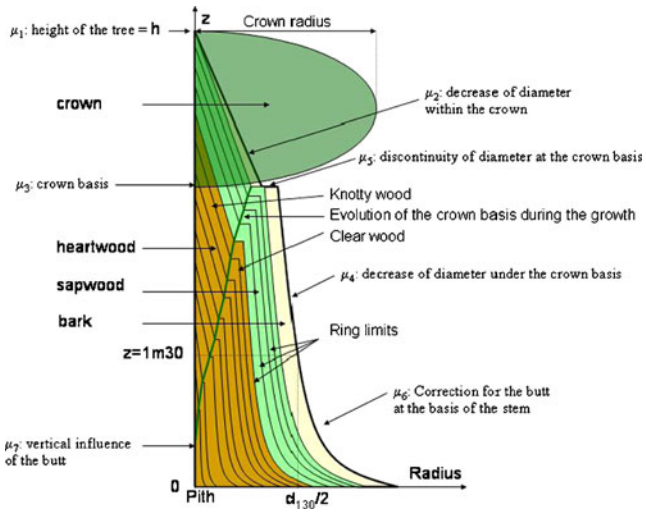

Fig. 5 Stem profile in the Fagacées model: compartments of the stem and parameters of the stem taper curve

- μ_3 is the height of the crown basis (metre);
- μ_4 is the decrease of diameter per metre under the crown (centimetre per metre);
- μ_5 is the discontinuity of the diameter at the crown limit (no dimension, $\mu_5 \geq 1$);
- μ_6 represents the correction for the butt at the basis of the stem (no dimension);
- μ_7 represents the height at which the influence of the butt falls to 1% of the stem diameter (metre).

All the parameters from μ_1 to μ_7 are computed from the individual diameter d_{130}, the individual height h and top height of the stand H_0 using allometric relations. Details on these computations can be found in Hatsch (1997) and Dhôte et al. (2000). These relations do not correspond to dynamic models: the parameter values at a given iteration of the simulation do not depend on their values at the previous iterations.

3.1.2 Volume within the stem

The main application of the stem taper Eq. 26 is the computation of volume between two heights. Since the modelled stem is a solid of revolution, the volume between two heights is then computed as:

$$V(z_1, z_2) = \frac{\pi}{40000} \int_{z_1}^{z_2} d^2(z)dz \tag{28}$$

Again, the coefficient used in this formula is due to the fact that the diameter is expressed in centimetres whereas the height is expressed in metres. The volume is then expressed in m^3. This computation can be conducted either numerically or analytically. In order to be adapted to further modifications of the stem taper Eq. 26, the numerical computation has been chosen in the current version of the model.

3.1.3 Compartments within the stem

From the bark to the pith, three compartments are considered in the stem: bark, sapwood and heartwood. In addition, the evolution of the crown basis during growth helps to define two parts in the sapwood and in the heartwood: knotty and clear wood (see Fig. 5). The limits between all these compartments are based on allometric relations that can be found in Dhôte et al. (2000).

They provide detailed results on the geometry of each stem: volume of sapwood and heartwood, ring width. These characteristics can then be used to predict various properties such as wood density that are crucial to estimate some characteristics of the stem such as its wood quality or its carbon content. Various indicators can then be derived to process a multicriteria evaluation of the simulated stand (Bucket et al. 2005).

3.2 Thinning and self-thinning

Self-thinning occurs after the growth step, but only when RDI exceeds 1 (Fig. 2). In this case, the smallest trees systematically die until RDI falls to 1. Thinning is the main intervention included in Fagacées. It starts when the stand reaches a given stage (i.e. age, average diameter, RDI). In the original version of the model, it was manually conducted. Its technical characteristics (number of stems or basal area to harvest in each diameter class) were a consequence of the user's decision. In the current version, another possibility using a parameterised rule has been defined. This rule allows the programme to run in "batch mode" without user intervention. It is used for studies that need a lot of simulations.

It has been designed so that the thinned trees are preferably, but not systematically, the smallest of the stand. It could however be modified to allow thinning from above. But in that case, the top height growth would be modified, so the current Eq. 8 should be replaced by another one, more adapted to this situation.

The thinning procedure depends on two user-defined parameters. The first one is the thinning intensity (expressed in terms of basal area of RDI to reach), the second one defines the kind of thinning. It takes its value in the [0;1] interval. The 0 value corresponds to the case where the thinning can occur until the maximum diameter of the stand, the 1 value to a thinning strictly from below.

The thinning procedure is a two-step one: the first defines the classes of diameter that may be thinned, the second one is the thinning itself.

3.2.1 First step of batch mode for thinning: determination of an upper diameter for thinning

When the thinning occurs, the stand can be described in terms of number of stems (N), average diameter at (D), average quadratic diameter (D_g) or density index (RDI). The lowest and the upper diameters (respectively $D_{\min}$ and $D_{\sup}$) can also be defined. A thinning will be realised by removing stems until a given state is reached. Very often, it is formulated in terms of a value of basal area to remove or a given targeted RDI (lower than the starting one!). The simplest way to reach this goal would be to systematically remove the smallest trees until the objective is achieved, but it does not correspond to the real silvicultural practice. A procedure has then been designed to define an upper diameter D_U below which thinning can occur.

A thinning more intensive than the aimed one is used. If the goal is to decrease to the value $\mathrm{RDI}_{\mathrm{obj}}$, a thinning to the value $\varphi \cdot \mathrm{RDI}_{\mathrm{obj}}$ will be considered (alternatively, if the goal is expressed in terms of basal area to remove, so that the basal area of the stand after thinning is G_{obj}, the value $\varphi \cdot G_{\mathrm{obj}}$ will be considered), where φ is an additional parameter chosen within the [0, 1] interval. The value of D_U is the diameter below which a systematic thinning of the smallest trees should occur to reach the $\varphi \cdot \mathrm{RDI}_{\mathrm{obj}}$ value.

In practice, a function called "partial RDI" is used to compute D_U: Let Ω(D) be the set of trees with diameter (d_{130}) greater than or equal to D, N_Ω(D) and D_Ω(D), respectively, the number of stems and the average quadratic diameter of Ω(D). The partial RDI RDI_Ω(D) is the value of RDI computed from N_Ω(D) and D_Ω(D) using Eq. 3.

The partial RDI is a decreasing function of diameter, with values varying from the RDI of the whole stand when D is less than or equal to D_{min}, to 0 when D is greater than or equal to D_{sup}. The value D_U is the lowest diameter for which $\mathrm{RDI}_\Omega(\mathrm{D}) \leq \varphi \cdot \mathrm{RDI}_{\mathrm{obj}}$ (Fig. 6). For clarity of illustration, the data used for Fig. 6 have been obtained by simulating a stand with exaggerated range of considered diameters.

3.2.2 Second step of batch mode for thinning: the thinning itself

Once D_U has been defined, each stem of the stand is associated to a probability of being thinned, which is a function of the stem diameter d_{130}. This probability is taken as proportional to:

$$\tau = \max\left(\frac{D_U - d_{130}}{D - D_{\inf}}; 0\right) \tag{29}$$

so that the small trees have a greater probability to be thinned than the big ones and no tree with $d_{130} \geq D_U$ will be thinned.

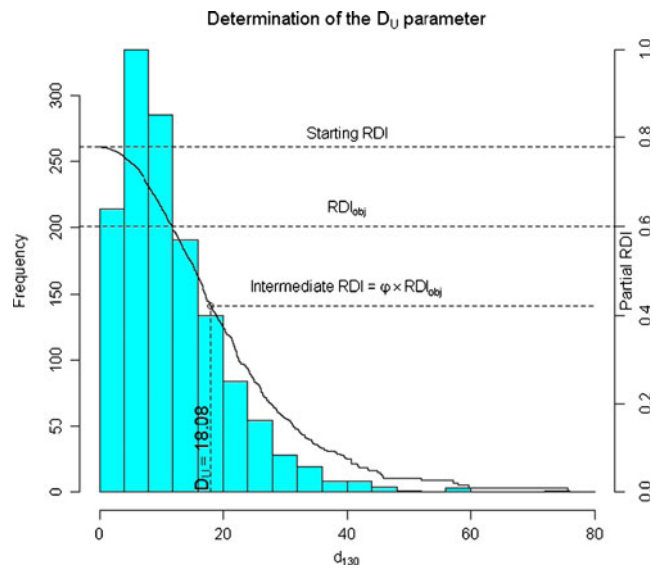

Fig. 6 The partial RDI function and the determination of the D_U parameter used for batch thinning

In practice, each tree is then associated to a random variable X defined as:

$$X = \tau \cdot U \tag{30}$$

where, U is randomly taken in an Uniform law [0, 1].

The trees are then ordered by decreasing X value. They are thinned from the beginning of the list until the targeted RDI or basal area is reached. Figure 7 presents an example of result. The chosen value of φ in this example is 0.7.

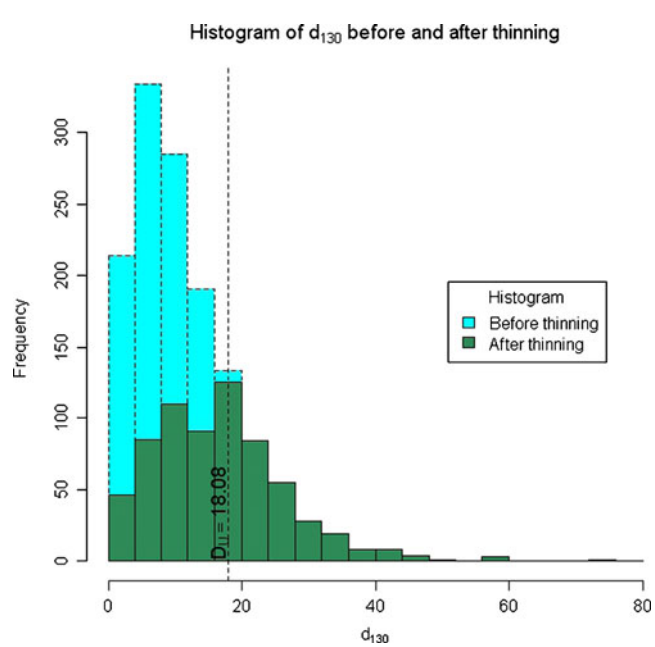

Fig. 7 Example of result of the batch rule for thinning

3.3 Batch scenarios

The first versions of the Fagacées model were designed to be manually run by the user who had to take the silvicultural decisions at each step. This approach still exists in the current version of Fagacées available through the CAPSIS platform (de Coligny 2005). However, since some studies based on this model need intensive simulations, a batch mode has also been developed. This mode needs batch silvicultural scenarios to be defined.

These batch scenarios are defined by four elements:

- a reference trajectory to follow in the age×RDI space;
- a tolerance around this reference trajectory;
- a rule for the kind of thinning;
- a stop condition for the simulation.

The principles of the batch scenarios are the following.

At the beginning, nothing special happens after regeneration. The trees grow with self-thinning as the only cause of mortality. Due to the fact that the Fagacées model has not been calibrated on very young stands, it is not worth to define precise instructions in the model for early stages.

The scenario itself begins at a given age (more than 15 years old). The silvicultural interventions will aim to make the stand follow a pre-defined trajectory in the age× RDI space. Of course, if this trajectory were to be exactly followed, an intervention would be needed at each step of the simulation (3-year steps in general). It is not realistic. So a tolerance envelope has been defined around the reference trajectory. Each time the stand reaches the upper limit of the envelope, an intervention is carried out to bring the stand RDI down to the lower limit. An additional constraint can be defined to forbid a too short interval between two consecutive thinnings. In that case, the second thinning will be delayed until this constraint is respected.

The previous procedure defines the date and the intensity of the thinnings. The thinnings themselves are conducted according to the thinning procedure detailed in Section 3.2. The φ parameter value to use is given by a function of the stand current state.

The scenario is run until a defined stop condition is encountered. This stop condition can be any logical rule based on the stand state (current age, average diameter).

Figure 8 illustrates an example of scenario for Oak where the main trajectory is defined by three straight segments to define three phases of the silvicultural schedule (preparation of the stand, designation of the crop trees and maturation). During the first two phases, the reference RDI varies linearly with the age of the stand. These two phases have pre-defined lengths. During the last phase, the reference RDI is maintained constant. This phase continues until the stop condition is encountered, so its length is not in general defined in advance. Here the final age has been set to 250 years just for illustration. The width of the tolerance envelope in this example has been fixed to a constant value (0.05 above and under the reference). The φ parameter used for thinning cannot be shown on this figure. It has also been set to a constant value (φ=0.7). At the end of the scenario, all the remaining trees are harvested so the RDI falls to 0.

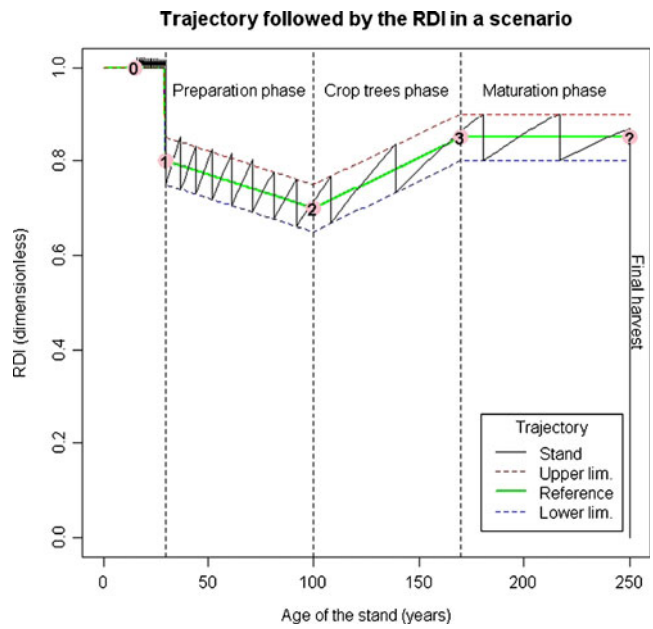

Fig. 8 An example of trajectory followed by a stand using the batch scenario

Since the functions for the reference trajectory, the tolerance envelope width and the φ parameter are not constrained to a particular form and the stop condition can be any logical rule based on the stand state, the procedure for batch scenarios was designed to study a very large set of silvicultural schedules. The only condition for the functions and the stop condition are to be consistent with what they are supposed to represent.

4 Discussion

Even if the equations can differ for Oak and Beech, the general approach used in the Fagacées growth model for these two species is the same. The model has been designed for high forest and coppice with standards forest management at a relatively large scale (a whole forest, a little region). According to Passioura (1973) cited by Battaglia and Sands (1998): "An appropriate model structure for management application is a transparent one: that it has few enough parameters so that each can be measured, directly or indirectly, and is composed of submodels corresponding to readily identifiable and experimentally accessible systems.". With its top–down organisation and the privileged

choice of robust equations, Fagacées does respect these general principles.

This top–down approach is not so common in forest modelling despite the fact it could lead to models useful for forest managers (Zeide 2003). It has been adopted by Corona et al. (2002) for the same reasons as Fagacées but in a much simpler situation: clonal plantation of poplar.

Although it is a tree-centered model, its main growth equation (Eq. 11) has been calibrated at the stand level. Indeed, the models adjusted at the scale of the stand are known to be generally more reliable for growth and yield than the models adjusted at the scale of the tree (Peng 2000).

The growth is mainly directed by the top height curve that is independent from silviculture as long as the stand density remains high enough. This result is generally admitted for a large range of initial spacing and thinning intensities as long as no thinning from above is performed (Skovsgaard and Vanclay 2008). Hence, in the current version, the model cannot be used to simulate silvicultures with thinnings from above.

If the stand density becomes very low, the stand height growth of Oak is depressed. However, due to the lack of available data for such stand densities, only an empirical correction has been introduced in the model. The model should then not be used to simulate silviculture with very low densities.

As a stand growth model, Fagacées is a classical one since the growth equations regulate stand height and basal area (Eqs. 7, 9 and 11). It becomes a tree-centered model by the way the global growth of basal area is allocated among individual trees (Eqs. 16 and 17). After this step, the individual heights and the whole stem profiles are simply rebuilt from allometries.

Both global and individual growths are expressed in terms of variation of basal area. They are consistent with differential equations that are classically used in process-based models. These equations also have a process-based interpretation whereas the individual height and the stem profile equations are purely descriptive relations (Dhôte 1996).

None of the growth equations contains random components. The Fagacées growth model is a purely deterministic one. As a consequence, due to Eq. 17, the hierarchy of trees will never change during stand development. If a given tree is smaller than another one at a given step of the simulation, it will remain smaller in all further steps, as long as none of them disappear due to thinning or self-thinning.

Another consequence is that the variability of diameters within a stand at a given age depends on the variability within the initial stand. It can only be modulated by the thinning rules the user may apply. However, the initial variability (expressed as diameter variance for example) is not enough to predict the evolution of diameter variability through growth. The whole diameter distribution is needed. Fagacées is a tree-individualised model and cannot be reduced to a population model nor to a stand model without loss of information.

If the growth equations have been calibrated so that they lead to unbiased predictions of basal area and global wood volume, the weakness of the variability representation may lead to biased prediction of wood quality. Indeed, wood quality clusters are based on conventional dimension limits for log diameter and ring width. Best qualities are defined by highest or lowest values for these dimensions. If the dimension variability is badly represented, the proportion of wood that verifies the wood quality conditions will be biased. In that case, alternative scenarios can be compared to predict which one produces the best quality in average, but not to predict precisely the wood quantities in each quality cluster. Hence the representation of the inter-tree variability is a point that could be improved in further versions of the model. This could be done for example by including random components for individual growth in the Eqs. 17 and 24.

5 Conclusion

The Fagacées model was built to simulate the consequences of a wide variety of silvicultures for even-aged high forests of Oak or Beech. Due to its top–down organisation, it combines the properties of a stand growth model (robustness and reliability) and some of tree-centered models (individual stem profiles). Only a very limited set of equations defines the growth model part and each of these equations reflects very simple principles. Thus the model can easily be improved if necessary. Several directions for that are considered: improvement of the growth equations, improvement of the stem taper profile, and designing of a model for Oak and Beech mixed stands based on the same concepts (Hein and Dhôte 2006).

The outputs of the growth model are used as inputs for modelling stem taper in order to obtain an estimation of the wood properties. However, the individual variability is badly represented. In addition, some defaults of Oak wood such as cracks or epicormic branches, that have a strong influence on wood quality, are not or insufficiently modelled. This can be a source of bias for some applications, related to wood quality. In this case, the model outputs have to be used only for comparisons of silvicultural scenarios. The under-representation of wood defects is due to a lack of knowledge on the deterministic mechanisms related to them. So it is still too early to include them in the model.

Despite these relative insufficiencies, this model is largely used by French forest managers and in forest research. It is still the only one available for Oak or Beech pure stands in the pedo-climatic conditions of northern

France. It has been distributed to forest modellers and to forest managers, mainly through the CAPSIS project (de Coligny 2005) available at http://www-capsis.cirad.fr/home. It has contributed to the development of new standards of silviculture for the French Oak forest by the French National Forest Service (Jarret 2004; Sardin 2008).

Acknowledgements The authors thank the editors and two anonymous referees for their valuable comments.

Open Access This article is distributed under the terms of the Creative Commons Attribution Noncommercial License which permits any noncommercial use, distribution, and reproduction in any medium, provided the original author(s) and source are credited.

References

Battaglia M, Sands P (1998) Process-based forest productivity models and their application in forest management. For Ecol Manage 102:13–32

Bi H, Wan G, Turvey N (2000) Estimating the self-thinning boundary line as a density-dependent stochastic frontier. Ecol 81:1477–1483

Bontemps JD (2006) Evolution de la productivité des peuplements réguliers et monospécifiques de Hêtre (Fagus sylvatica L.) et de Chêne sessile (Quercus petraea L.) dans la moitié nord de la France au cours du XXe siècle. Ph.D. thesis, Ecole Nationale du Génie Rural et des Eaux et Forêts, Nancy, France. http://pastel.paristech.org. Accessed 28 May 2011

Corona P, Marziliano P, Scotti R (2002) Top–down growth modelling: a prototype for poplar plantations in Italy. For Ecol Manage 161:65–73

de Coligny F (2005) Capsis: computer-aided projection for strategies in silviculture, a software platform for forestry modellers, in Workshop on Information Science for Agriculture and Environment (ISAE). Edited by Workshop on Information Science for Agriculture and Environment (ISAE), GuiZhou Normal University, GuiYang, P.R. China. 3–4 June 2005

Décourt N (1973) Tables de production pour les forêts françaises. Ecole Nationale du Génie Rural, des Eaux et des Forêts, Nancy

Dhôte JF (1991) Modélisation de la croissance des peuplements réguliers de Hêtre: dynamique des hiérarchies sociales et facteurs de production. Ann For Sci 48:389–416

Dhôte JF (1995) Définition de scénarios d'éclaircie pour le Hêtre et le Chêne. Rev For Fr 48:106–110

Dhôte JF (1996) A model of even-aged beech stands productivity with process-based interpretations. Ann For Sci 53:1–20

Dhôte JF (1997) Effet des éclaircies sur le diamètre dominant dans des futaies régulières de Chêne sessile. Rev For Fr 49:557–578

Dhôte JF (1999) Compétition entre classes sociales chez le Chêne sessile et le Hêtre. Rev For Fr 51:309–325

Dhôte JF, de Hercé E (1994) Un modèle hyperbolique pour l'ajustement de faisceaux de courbes hauteur-diamètre. Can J For Res 24:1782–1790

Dhôte JF, Hatsch E, Rittié F (2000) Forme de la tige, tarifs de cubage et ventilation de la production en volume chez le Chêne sessile. Ann For Sci 57:121–142

Duplat P, Tran-Ha M (1997) Modélisation de la croissance en hauteur dominante du Chêne sessile (*Quercus petraea* Liebl.) en France. variabilité inter-régionale et effet de la période récente (1959–1993). Ann For Sci 54:611–634

ENGREF, INRA, ONF (1984) In: Vannière B (ed) Tables de production pour les forêts françaises, 2nd edn. Ecole Nationale du Génie Rural, des Eaux et des Forêts, Nancy

Fortin M, Ung CH, Archambault L, Begin J (2006) Calibrating a generalized diameter distribution model with mixed effects. For Sci 52:650–658

Hatsch E (1997) Répartition de l'aubier et acquisition de la forme de la tige chez le Chêne sessile (*Quercus petraea* (matt) L.). Ph. D. thesis, Ecole Nationale du Génie Rural et des Eaux et Forêts, Nancy (France).

Hein S, Dhôte JF (2006) Effect of specied composition, stand density and site index on the basal area increment of oak trees (*Quercus* sp.) in mixed stands with beech (*Fagus sylvatica* L.) in northern France. Ann For Sci 63:457–467

Inventaire Forestier National (2007) Data and results of national inventories 2005, 2006, 2007. http://www.ifn.fr. Accessed 28 June 2011.

Jack S, Long J (1996) Linkages between silviculture and ecology: an analysis of density management diagrams. For Ecol Manage 86:205–220

Jarret P (2004) Guide des sylvicultures—Chênaie atlantique. ONF/Lavoisier, Paris

Bucket E, Le Moguédec G, Mothe F, Nepveu G (2005) Une modélisation des bilans "environnement" et "produits" de sylvicultures contrastées: cas du chêne sessile. Rev For Fr 57:311–330

Monserud R, Sterba H (1996) A basal areal increment model for individual trees growing in even- and uneven-aged forest stands in Austria. For Ecol Manage 80:57–80

Morris E (2003) How does fertility of the substrate affect intraspecific competition? Evidence and synthesis from self-thinning. Ecol Res 18:287–305

Nepveu G, Dhôte JF (1998) Sylviculture et qualité du bois du Chêne sessile. Rapport final du contrat INRA-ONF 1992–1996, Tech. rep. INRA, Champenoux

Nord-Larsen T, Johannsen V (2007) A state-space approach to stand growth modelling of european Beech. Ann For Sci 64:365–374

Oswald H (1981) Résultats principaux des places d'expérience du Chêne du Centre National de Recherches Forestières. Rev For Fr 33:65–85

Pardé J (1962) Table de production pour les forêts de Chêne rouvre de qualité tranchage du secteur ligérien. Tech. Rep. 11. Station de Recherches Forestière, Inra, Nancy

Passioura JB (1973) Sense and nonsense in crop simulation. J Aust Inst Agr Sci 39:181–183

Peng C (2000) Growth and yield models for uneven-aged stands: past, present and future. For Eco Manage 132:259–279

Porté A, Bartelink HH (2002) Modelling mixed forest growth: a review of models for forest management. Ecol Model 150:141–188

Pretzsch H (2006) Species-specific allometric scaling under self-thinning: evidence from long-term plots in forest stands. Oecologia 146:572–583

Pretzsch H, Biber P, Dursky J (2002) The single tree-based stand simulator SILVA: construction, application and evaluation. For Ecol Manage 162:3–21

Reineke LH (1933) Perfecting a stand-density index for even-aged forests. J Agric Res 46:627–638

Sano J (1997) Age and size distribution in a long-term forest dynamics. For Ecol Manage 92:39–44

Sardin T (2008) Guide des sylvicultures—Chênaies continentales. ONF/Lavoisier, Paris

Shröder J, Röhle H, Gerold D, Münder K (2007) Modeling individual-tree growth in stands under forest conversion in east Germany. Eur J Forest Res 126:459–472

Skovsgaard JP, Vanclay JK (2008) Forest site productivity: a review of the evolution of dendrometric concepts for even-aged stands. Forestry 81:13–31

Zeide B (2003) The U-approach to forest modeling. Can J For Res 33:480–489

Zeide B (2005) How to measure stand density. Trees 19:1–14

Zhang L, Bi H, Gove J, Heath LS (2005) A comparison of alternative methods for estimating the self-thinning boundary line. Can J For Res 35:1507–1514