Towards Domain Decomposition with Balanced Halo

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Nested Dissection has been introduced by A. George in 1973 ([3]) and is a well-known and very popular heuristic for sparse matrix ordering to reduce both the fill-in and the operation count during the numerical factorization. The basic standard idea is to build a "small separator C" of the graph associated with the original matrix in order to split the remaining vertices in two parts A and B of "almost equal size". The vertices of the separator C are ordered with the largest indices, and then the same method is applied recursively on the two subgraphs induced by A and B. This method has been implemented by graph partitioners such as METIS (http://glaros.dtc.umn.edu/gkhome/metis/parmetis/overview) or SCOTCH (http://gforge.inria.fr/projects/scotch/) whose main objectives are to minimize the size of the separator and to equilibrate the size of the two separated subgraphs. However, if we examine precisely the complexity analysis for the estimation of asymptotic bounds for fill-in or operation count when using Nested Dissection ordering, we can notice that the size of the halo of the separated subgraphs (set of external vertices adjacent to the subgraphs and previously ordered) play a crucial role in the asymptotic behavior achieved.

Moreover, this method based on a divide and conquer approach is also very well suited to maximize the number of independent computation tasks for parallel implementations. Then, by using the block data structure induced by the partition of separators of the original graph, very efficient parallel block solvers have been designed and implemented according to supernodal or multifrontal approaches. To name a few, one can cite MUMPS (http://graal.ens-lyon.fr/MUMPS/), PaStiX (http://pastix.gforge.inria.fr/) and SuperLU (http://crd-legacy.lbl.gov/ xiaoye/SuperLU/).

Considering now hybrid methods mixing both direct and iterative solvers such as HIPS [4], MaPHYS [1, 2, 5] and PDSLIN [8], obtaining a domain decomposition leading to a good balancing of both the size of domain interiors and the size of interfaces is a key point for load balancing and efficiency in a parallel context.

For this purpose, we revisit the algorithm introduced by Lipton, Rose and Tarjan ([6]) in 1977 which performed the recursion in a different manner: at each level, we apply recursively the method to the subgraphs induced by $A \cup C$ on one hand, and $B \cup C$ on the other hand. In these subgraphs, vertices already ordered (and belonging to the previous separators) are the halo vertices and the partition of these overlapping subgraphs is performed with three objectives: balancing of the two new parts A' and B', balancing of the halo vertices in the parts A' and B' and minimizing the size of the separator C'.

We worked specifically on the separation algorithm named greedy-graph-growing. In our implementation, we start with two seeds belonging to the halo, one for each part to generate and picked as far as possible from each other. Then, both parts are grown simultaneously. At each step, an adjacent vertex is added to the current smallest part; the choice halo/non-halo is oriented by current halo balance. It ends whenever there are no vertex left. If during the process, one part inhibits the progression of the other, the algorithm is restarted using this bad experiment to make a better choice. This strategy based on two starting seeds has led to better results than the classical method with a single seed.

We implemented our new algorithm in the Scotch partitioner [7]. Figure 1 shows our results on matrices `audi` and `Mario002`. This work is in progress and these preliminary results are promising. More details will be given during the presentation. Note that for the present, we only investigated the greedy-graph-growing algorithm; this work has to be completely integrated in a more general framework and in particular within a multilevel method.

In addition to this algorithmic work, and especially for the MaPHYS solver, a complexity analysis is currently carried on regarding the good asymptotic size for the subdomains (in which a sparse direct factorization is performed) in order to have the same asymptotic complexity bound for the computation of the preconditioner and for the iterative part. This is done by using the same theoretical framework used in [6] for general families of graph associated with 3D irregular problems for elliptic PDEs where the maximum number of iterations depends of the number of subdomains. The result is achieved by using balancing properties for the domain interior sizes and for the halo sizes as expected in the first work presented above.

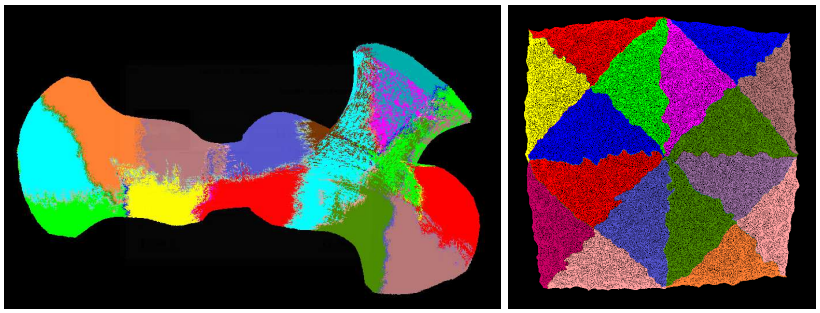

Figure 1: Results on Audi and Mario matrices.

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