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Estimates $L^r - L^s$ for solutions of the $\bar{\partial}$ equation in strictly pseudo convex domains in \mathbb{C}^n .

Eric Amar

Abstract

We prove some new estimates for solutions of the $\bar{\partial}u = \omega$ equation in a strictly pseudo convex domain Ω in \mathbb{C}^n . For instance if the (p, q) current ω has its coefficients in $L^r(\Omega)$ with $1 \leq r < 2(n+1)$ then there is a solution u in $L^s(\Omega)$ with $\frac{1}{s} = \frac{1}{r} - \frac{1}{2(n+1)}$. These results are based on Carleson measures of order α introduced and studied in [4] and on the subordination lemma [5].

1 Introduction.

Let Ω be a bounded strictly pseudo convex domain with smooth C^∞ boundary. We shall denote these domains as s.p.c. domains in the sequel.

Ovrelid [10] proved that if we have a (p, q) current ω , $\bar{\partial}$ closed in Ω and such that its coefficients are in $L^r(\Omega)$ then there is a $(p, q-1)$ current u solution of the equation $\bar{\partial}u = \omega$ and with coefficients still in $L^r(\Omega)$. Let us define a norm on these currents :

$$\omega \in L^r_{(p,q)}(\Omega), \omega = \sum_{|I|=p, |J|=q} \omega_{I,J} dz^I \wedge d\bar{z}^J \Rightarrow \|\omega\|_r^r := \sum_{|I|=p, |J|=q} \|\omega_{I,J}\|_r^r.$$

Then Ovrelid proved that $\|u\|_r < C\|\omega\|_r$, where the constant C does not depend on ω .

In the case of $r = \infty$, this was done before by Lieb [9] and Romanov and Henkin [11] proved that still for $r = \infty$, there is a solution u in the space Lipschitz $1/2$. In the book of Henkin and Leiterer [8] we can find precise references for these topics.

The aim of this work is to improve the L^r estimates of Ovrelid in the direction of Romanov and Henkin but with a different approach.

We already got this kind of results in [2] by use of Skoda's kernels [12] but we were dealing with boundary values instead of inside ones. Nevertheless using Skoda results we shall prove the following theorem, where $A \lesssim B$ means that there is a constant $C > 0$ independent of A and B such that $A \leq CB$.

Theorem 1.1 *Let Ω be a s.p.c. domain in \mathbb{C}^n then for $1 < r < 2n + 2$ we have*

$$\forall \omega \in L^r_{(p,q)}(\Omega), \bar{\partial}\omega = 0, \exists u \in L^s_{(p,q-1)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{L^s(\Omega)} \lesssim \|\omega\|_{L^r(\Omega)},$$
for any s such that $\frac{1}{s} > \frac{1}{r} - \frac{1}{2(n+1)}$.

We shall also show :

Theorem 1.2 *Let Ω be a s.p.c. domain in \mathbb{C}^n then for $1 < r < 2n + 2$ we have*

- $\forall \omega \in L^r_{(p,q)}(\Omega), \bar{\partial}\omega = 0, \exists u \in L^s_{(p,q-1)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{L^s(\Omega)} \lesssim \|\omega\|_{L^r(\Omega)},$

with $\frac{1}{s} = \frac{1}{r} - \frac{1}{2(n+1)}$.

- *For $r = 2n + 2$ we have*

$$\exists u \in BMO_{(p,q)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{BMO(\Omega)} \lesssim \|\omega\|_{L^{2n+2}(\Omega)}.$$

If ω is a $(p, 1)$ form we have also :

- *for $r = 1,$*

$$\exists u \in L^{s,\infty}_{(p,0)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{L^{s,\infty}(\Omega)} \lesssim \|\omega\|_{L^1(\Omega)}$$

with $\frac{1}{s} = 1 - \frac{1}{2(n+1)}$.

- *for $r > 2n + 2,$*

$$\exists u \in \Gamma^{\beta}_{(p,0)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{\Gamma^{\beta}(\Omega)} \lesssim \|\omega\|_{L^r(\Omega)},$$

where $\beta = 1 - \frac{2n+2}{r}$ and Γ^{β} is an anisotropic Lipschitz class of functions.

Moreover the solution u is linear on the data ω .

The classes $BMO(\Omega)$ and $\Gamma^{\beta}(\Omega)$ will be defined later. The space $L^{s,\infty}_{(p,0)}(\Omega)$ is the Lorentz space [7].

This theorem is stronger than theorem 1.1 because here, in the case $1 \leq r < 2(n+1)$ we get the result for the end point s such that $\frac{1}{s} = \frac{1}{r} - \frac{1}{2(n+1)}$.

Of course if $u \in L^s_{(p,q-1)}(\Omega)$ for $s > r$ then $u \in L^r_{(p,q-1)}(\Omega)$ hence we also have an strong improvement to Ovrelid's theorem.

Because the class Lipschitz 1/2 is contained in $\Gamma^1(\Omega)$ we see that we recover the Romanov-Henkin result when $r = \infty$ in the case of $(p, 1)$ forms.

Even if they do not appear in the statement, the Carleson measures of order α , A. Bonami and I introduced in [4], are at the heart of the proof.

2 Proof of the first theorem.

Let Ω be a s.p.c. in \mathbb{C}^n , defined by the function $\rho \in \mathcal{C}^{\infty}(\mathbb{C}^n)$, i.e. $\Omega := \{z \in \mathbb{C}^n :: \rho(z) < 0\}$ and $\forall z \in \partial\Omega, \partial\rho(z) \neq 0$.

Let $\Omega' := \{(z, w) \in \mathbb{C}^n \times \mathbb{C} :: \rho'(z, w) := \rho(z) + |w|^2 < 0\}$ and lift a current ω to Ω' this way : $\omega'(z, w) := \omega(z)$.

Lemma 2.1 *Let Ω be a s.p.c. domain in \mathbb{C}^n , with the above notations we have*

$$\omega \in L^r_{(p,q)}(\Omega) \Rightarrow \omega'(z, w) \in L^r_{(p,q)}(\partial\Omega').$$

Proof.

This is an instance of the subordination principle [1], [5]. Let $f(z) \in L^r(\Omega)$ and set $f'(z, w) := f(z)$ in Ω' , then, by the main lemma in [5], p. 6,

$$\|f'\|_{L^r(\partial\Omega')}^r := \int_{\partial\Omega'} |f'(z, w)|^r d\sigma(z, w) = \int_{\Omega} |f(z)|^r \sqrt{-\rho(z) + \frac{|\text{grad}\rho(z)|^2}{4}} \left\{ \int_{|w|^2 = -\rho(z)} d|w| \right\} dm(z),$$

where $d|w|$ is the normalized Lebesgue measure [5] on the circle $|w|^2 = -\rho(z)$. Because $\bar{\Omega}$ is compact, we have $\forall z \in \bar{\Omega}, \sqrt{-\rho(z) + \frac{|\text{grad}\rho(z)|^2}{4}} \leq C(\rho) < \infty$ hence we have

$$\|f'\|_{L^r(\partial\Omega')}^r \leq C(\rho) \int_{\Omega} |f(z)|^r dm(z) = C(\rho) \|f\|_{L^r(\Omega)}^r.$$

It remains to apply this taking for f any coefficient of ω . ■

Proof of theorem 1.1.

Since Ω is a s.p.c. domain so is Ω' by the subordination lemma [5]. By use of lemma 2.1 we have that $\omega' \in L^r_{(p,q)}(\partial\Omega')$ and still $\bar{\partial}\omega' = 0$, hence we can apply Skoda's theorem 2 in [12] to get that there is a solution u' of $\bar{\partial}_b u' = \omega'$ such that

$$u' \in L^s_{(p,q-1)}(\partial\Omega') \text{ with } \frac{1}{s} > \frac{1}{r} - \frac{1}{2(n+1)}.$$

We have

$$u'(z, w) = \sum_{I,J} a'_{I,J}(z, w) dz^I \wedge d\bar{z}^J.$$

Because ω' does not depend on w we have that the coefficients of u' are holomorphic in w , hence we can set (recall that u' is defined on $\partial\Omega'$)

$$\forall z \in \Omega, a_{I,J}(z) := \int_{|w|^2 = -\rho(z)} a'_{I,J}(z, w) d|w|$$

and

$$u(z) := \sum_{I,J} a_{I,J}(z) dz^I \wedge d\bar{z}^J,$$

then exactly as in [3] we still have

$$\bar{\partial}u = \omega \text{ in } \Omega.$$

Moreover the subordination lemma [5] gives again $u \in L^s_{(p,q-1)}(\Omega)$, because $u' \in L^s_{(p,q-1)}(\partial\Omega')$. ■

3 Carleson measures of order α .

For Ω a s.p.c. domain in \mathbb{C}^n , let $V^0(\Omega)$ be the space of bounded measures in Ω , and $V^1(\Omega)$ the space of Carleson measures in Ω as defined for instance in [4]. We know that these spaces form a interpolating scale for the real method [4], and we set

$$V^\alpha(\Omega) := (V^0, V^1)_{(\alpha, \infty)} ; W^\alpha(\Omega) := (V^0, V^1)_{(\alpha, p)} \text{ with } p = \frac{1}{1-\alpha}.$$

Recall that a (p, q) form ω is in $W^\alpha_{(p,q)}(\Omega)$ (resp. $V^\alpha_{(p,q)}(\Omega)$) if its coefficients and the coefficients of $\frac{\omega \wedge \bar{\partial}\rho}{\sqrt{-\rho}}$ are measures in $W^\alpha(\Omega)$ (resp. $V^\alpha(\Omega)$) see [4] and [6].

A (p, q) form is in $L^r_{(p,q)}(\Omega)$ if just its coefficients are in $L^r(\Omega)$.

Let $\Omega' := \{(z, w) \in \mathbb{C}^n \times \mathbb{C} :: \rho'(z, w) := \rho(z) + |w|^2 < 0\}$ and lift a current ω to Ω' as before : $\omega'(z, w) := \omega(z)$.

Our first result links L^r estimates to Carleson α ones.

Theorem 3.1 *Let Ω be a s.p.c. domain in \mathbb{C}^n then we have*

$$\omega \in L^r_{(p,q)}(\Omega) \Rightarrow \omega'(z, w) := \omega(z) \in W^{\alpha}_{(p,q)}(\Omega')$$

$$\text{with } \alpha = \frac{1}{r'} + \frac{1}{2(n+1)}.$$

Proof.

Let $U' := \bigcup_{j=1}^N Q'(\zeta'_j, h_j) \cap \partial\Omega'$ be an open set in $\partial\Omega'$ and $T(U') = \bigcup_{j=1}^N Q'(\zeta'_j, h_j)$ be its associated "tent" set inside [4] ; in order to see that a measure $d\mu = f dm$, with m the Lebesgue measure in \mathbb{C}^n , belongs to $V^\alpha(\Omega')$ we have to show, see [4],

$$\int_{T(U')} |f(z')| dm(z') \leq C |U'|^\alpha$$

where $|U'| := \sigma(U')$ is the Lebesgue measure of U' on $\partial\Omega$, and with a constant C independent of U' .

Because we are dealing with (p, q) currents here, this means that we have to estimate

$$A := \int_{T(U')} \frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}} dm(z, w)$$

with $\rho'(z, w) := \rho(z) + |w|^2$ is equivalent to the distance of $(z, w) \in \Omega'$ to the boundary $\partial\Omega'$.

Back to A ,

$$A := \int_{T(U')} \frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}} dm(z, w) \leq \sum_{j=1}^N \int_{Q'_j} \frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}} dm(z, w).$$

The Carleson window Q'_j is equivalent to the product $(Q'_j \cap \partial\Omega') \times [h_j]_{\nu_j}$ with $[h_j]_{\nu_j}$ the real segment of length h_j supported by the real normal ν_j to $\partial\Omega'$ at ζ'_j . Set $h := \max_{j=1, \dots, N} h_j$, we shall replace Q'_j by $Q''_j := (Q'_j \cap \partial\Omega') \times [h]_{\nu_j}$.

So we have

$$A \leq \sum_{j=1}^N \int_{Q'_j} \frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}} dm(z, w) \leq \sum_{j=1}^N \int_{Q''_j} \frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}} dm(z, w),$$

where now all the depths have the same value h . Hence by Fubini we have

$$A \leq \int_0^h \frac{1}{\sqrt{t}} \left\{ \int_{U'_t} |\omega(z)| d\sigma(z, w) \right\} dt$$

with $U'_t := \bigcup_{j=1}^N Q''_j \cap \partial\Omega'_t$ and $\partial\Omega'_t := \{(z, w) \in \Omega' :: \rho(z) + |w|^2 = -t\}$.

We can estimate the inner integral by Hölder

$$\int_{U'_t} |\omega(z)| d\sigma(z, w) \leq \left(\int_{U'_t} |\omega(z)|^r d\sigma(z, w) \right)^{1/r} \left(\int_{U'_t} d\sigma(z, w) \right)^{1/r'} \quad (3.1)$$

but

$$\int_{U'_t} |\omega(z)|^r d\sigma(z, w) \leq \int_{\partial\Omega_t} |\omega(z)|^r d\sigma(z, w) \leq C(\rho) \int_{\Omega_t} |\omega(z)|^r \left\{ \int_{|w|^2 = -\rho(z) - t} d|w| \right\} dm(z)$$

where $d|w|$ is the normalized Lebesgue measure on the circle $|w|^2 = -\rho(z) - t$. Hence, with $\Omega_t := \{z \in \Omega : \rho(z) < -t\}$,

$$\int_{U'_t} |\omega(z)|^r d\sigma(z, w) \leq C(\rho) \int_{\Omega_t} |\omega(z)|^r dm(z) = C(\rho) \|\omega\|_{L^r(\Omega)}^r.$$

For the last factor of (3.1) we have

$$\int_{U'_t} d\sigma(z, w) = \sigma(U'_t) \lesssim \sigma(U'),$$

so

$$A \leq \int_0^h \frac{1}{\sqrt{t}} \left\{ \int_{U'_t} |\omega(z)| d\sigma(z, w) \right\} dt \lesssim \|\omega\|_{L^r(\Omega)} (\sigma(U'))^{1/r'} \int_0^h \frac{dt}{\sqrt{t}} = \frac{1}{2} \|\omega\|_{L^r(\Omega)} \sqrt{h} \sigma(U')^{1/r'}.$$

Recall that $\sigma(Q'_j) \simeq h_j^{(n+1)}$ then we have

$$\sqrt{h} = \sqrt{\max_j h_j} \lesssim \max \sigma(Q'_j)^{1/2(n+1)} \leq \sigma(\bigcup_{j=1}^N Q'_j \cap \partial\Omega')^{1/2(n+1)},$$

so finally we get

$$A := \int_{T(U')} \frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}} dm(z, w) \lesssim \|\omega\|_{L^r(\Omega)} \sigma(U')^{\frac{1}{r'} + \frac{1}{2(n+1)}}.$$

This means that $\frac{|\omega(z)|}{\sqrt{-\rho'(z, w)}}$ is a Carleson measure in Ω' of order α with

$$\alpha = \frac{1}{r'} + \frac{1}{2(n+1)}.$$

To get a usual Carleson measure, we need $\alpha = 1$ hence

$$\frac{1}{r'} + \frac{1}{2(n+1)} = 1 \iff r = 2(n+1).$$

We have by theorem 1 in [4], written in our situation, that if $\mu \in V^\alpha(\Omega')$ then $P^{0*}(\mu) \in L^{r, \infty}(\partial\Omega')$, where $P^{0*}(\mu)$ is the "balayage" of μ by the Hardy Littlewood kernel P_t^0 . Hence we have that the linear operator P^{0*} sends $V^{\alpha_0}(\Omega')$ to $L^{r_0, \infty}(\partial\Omega')$, and $V^{\alpha_1}(\Omega')$ to $L^{r_1, \infty}(\partial\Omega')$ with, as usual, $\alpha_j = 1 - \frac{1}{r_j}$. This means that

$$f \in L^r(\Omega) \Rightarrow \mu := f/\sqrt{-\rho'} dm \in V^\alpha(\Omega') \Rightarrow P^{0*}(\mu) \in L^{s, \infty}(\partial\Omega')$$

with control of the norms.

So we have a linear operator T such that, with $r_0 < r_1$,

$$T : L^{r_0}(\Omega) \rightarrow L^{s_0, \infty}(\partial\Omega'), \text{ with } \frac{1}{s_0} = \frac{1}{r_0} - \frac{1}{2(n+1)};$$

$$T : L^{r_1}(\Omega) \rightarrow L^{s_1, \infty}(\partial\Omega'), \text{ with } \frac{1}{s_1} = \frac{1}{r_1} - \frac{1}{2(n+1)};$$

hence we can apply Marcinkiewich interpolation theorem between these two values of $r \in]1, 2(n+1)[$ i.e.

$$T : L^r(\Omega) \rightarrow L^s(\partial\Omega'), \text{ with } \frac{1}{s} = \frac{1}{r} - \frac{1}{2(n+1)} \text{ and } r \leq s$$

which is needed to apply Marcinkiewich theorem, with control of norms.

But this implies by theorem 2 in [4], that $\mu := f/\sqrt{-\rho'} dm \in W^\alpha(\Omega')$. ■

4 The main result.

Let Ω be a domain in \mathbb{C}^n defined by the function ρ as above ; define $\Omega' \subset \mathbb{C}^{n+1}$ the lifted domain : we shall define the anisotropic class $\Gamma^\beta(\partial\Omega')$ as in [4] ; we say that a vector field X on $\partial\Omega'$ is *admissible* if X is of class \mathcal{C}^k and at any point of $\zeta \in \partial\Omega'$, $X(\zeta)$ belongs to the complex tangent space of $\partial\Omega'$ at ζ .

We say that $u \in \Gamma^\beta(\partial\Omega')$ if u is bounded on $\partial\Omega'$ and u belongs to the usual Lipschitz $\Lambda^{\beta/2}(\partial\Omega')$, where $\partial\Omega'$ is viewed as a real manifold, and on any integral curve of an admissible vector field, $t \in [0, 1] \rightarrow \gamma(t) \in \partial\Omega'$, the function $u \circ \gamma$ belongs to $\Lambda^\beta(0, 1)$.

We can now define the class $\Gamma^\beta(\Omega)$: take a function u defined in Ω and lift it as $u'(z, w) := u(z)$ in Ω' ; then $u \in \Gamma^\beta(\Omega)$ if $u' \in \Gamma^\beta(\partial\Omega')$. We have that $u \in \Gamma^\beta(\Omega)$ implies that $u \in L^\infty(\Omega)$ and $u \in \Lambda^{\beta/2}(\Omega)$ with a Lipschitz constant uniform in Ω .

The same way we define function $u \in BMO(\Omega)$ if $u' \in BMO(\partial\Omega')$. We have that $u \in BMO(\Omega)$ implies that $u \in \bigcap_{r \geq 1} L^r(\Omega)$.

Now we are in position to prove our main result.

Theorem 4.1 *Let Ω be a s.p.c. domain in \mathbb{C}^n then for $1 < r < 2n + 2$ we have*

$$\forall \omega \in L^r_{(p,q)}(\Omega), \bar{\partial}\omega = 0, \exists u \in L^s_{(p,q-1)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{L^s(\Omega)} \lesssim \|\omega\|_{L^r(\Omega)},$$

$$\text{with } \frac{1}{s} = \frac{1}{r} - \frac{1}{2(n+1)}.$$

For $r = 2n + 2$ we have

$$\exists u \in BMO_{(p,q)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{BMO(\Omega)} \lesssim \|\omega\|_{L^{2n+2}(\Omega)}.$$

If ω is a $(p, 1)$ form we have also :

for $r = 1$, we have

$$\exists u \in L^{s,\infty}_{(p,0)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{L^{s,\infty}(\Omega)} \lesssim \|\omega\|_{L^1(\Omega)}$$

$$\text{with } \frac{1}{s} = 1 - \frac{1}{2(n+1)}.$$

for $r > 2n + 2$ we have

$$\exists u \in \Gamma^\beta_{(p,0)}(\Omega) :: \bar{\partial}u = \omega, \|u\|_{\Gamma^\beta(\Omega)} \lesssim \|\omega\|_{L^r(\Omega)},$$

where $\beta = 1 - \frac{2(n+1)}{r}$ and Γ^β is an anisotropic Lipschitz class of functions.

Moreover the solution u is linear on the data ω .

Proof.

By use of theorem 3.1 we have that $\omega' \in W^\alpha_{(p,q)}(\Omega')$ with $\alpha = \frac{1}{r'} + \frac{1}{2(n+1)}$ where Ω' is still s.p.c. [5],

hence we can apply the theorem 7 in [4] if ω is a $(p, 1)$ current or the generalisation to (p, q) current done in theorem 4.1 in [6] to get that there is a solution u' of $\bar{\partial}_b u' = \omega'$ such that

$$u' \in L^s_{(p,q-1)}(\partial\Omega') \text{ with } \frac{1}{s} = 1 - \alpha = \frac{1}{r} - \frac{1}{2(n+1)}.$$

Because ω' does not depend on w we have that the coefficients of u' are holomorphic in w hence with

$$u'(z, w) = \sum_{I,J} a'_{I,J}(z, w) dz^I \wedge d\bar{z}^J$$

we can set (recall that u' is defined on $\partial\Omega'$)

$$\forall z \in \Omega, a_{I,J}(z) := \int_{|w|^2 = -\rho(z)} a'_{I,J}(z, w) d|w|$$

and we set also

$$u(z) := \sum_{I,J} a_{I,J}(z, w) dz^I \wedge d\bar{z}^J,$$

then exactly as in [3] we still have

$$\bar{\partial}u = \omega \text{ in } \Omega.$$

Moreover the subordination lemma [5], gives us $u \in L^s_{(p,q-1)}(\Omega)$.

The last two results came directly from [4], theorem 7 and theorem 8 with the fact that we apply them in $\Omega' \subset \mathbb{C}^{n+1}$ so we have from theorem 8 that $\beta = 2(n+1)(\alpha-1)$. ■

Remark 4.2 *In the range $1 < r < 2n+2$ theorem 1.2 is stronger than theorem 1.1 because we get the result with $\frac{1}{s} = \frac{1}{r} - \frac{1}{2(n+1)}$ and not only for $\frac{1}{s} > \frac{1}{r} - \frac{1}{2(n+1)}$.*

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