Low Complexity Tail-biting Trellises for Some Extremal Self-Dual Codes

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Abstract

We obtain low complexity tail-biting trellises for some extremal self-dual codes for various lengths and fields such as the [12,6,6] ternary Golay code and a [24,12,8] Hermitian self-dual code over GF(4). These codes are obtained from a particular family of cyclic Tanner graphs called necklace factor graphs.

Keywords : self-dual codes, tail-biting trellises, necklace factor graph.

1 Introduction

The representation of linear block codes by trellises is a very powerful description which allows an efficient soft decision decoding. We consider a family of codes introduced in [2] based on the use of short length codes and interleavers. From this family it is possible to extract a sub-family of codes adapted to iterative decoding. Indeed every code of this sub-family is associated with a necklace factor from which a tail-biting trellis can be deduced. Among this family, it is interesting to find codes with the best minimal distance as in [3]. Herein, we obtained some extremal self-dual codes over $GF(2)$ and $\mathbb{Z}_4$.

In this paper, we extend the construction [2] to several fields and we formalize the constraints on the necklace graph given in [3] to get codes with the best minimum distances. By this way, we have low complexity tail-biting trellises for several codes like the [12,6,6] ternary Golay code and a [24,12,8] Hermitian self-dual code over GF(4).

2 Necklace Factor Graph

For an introduction to factor graphs we refer the reader to [4]. We recall that a factor graph of a code C over $GF(q)$ consists of *check nodes* representing local constraints of C, and *variable nodes* which take values in an alphabet. We distinguish between two types of variable nodes: *symbol nodes* which are associated with the symbol of the codewords of C and *state nodes* which are used for computing the codewords of C but which are not transmitted. A variable node is *adjacent* to a check node if the corresponding variable is involved in the corresponding local constraint.

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We consider $[N, K, D]$ linear codes C over $GF(q)$ obtained from a $[n, k, d]$ linear base code B over $GF(q)$ (see [3]). We focus on codes having a *cyclic* factor graph $N_t(C)$ with t *necklaces* like the one given in Figure 1.

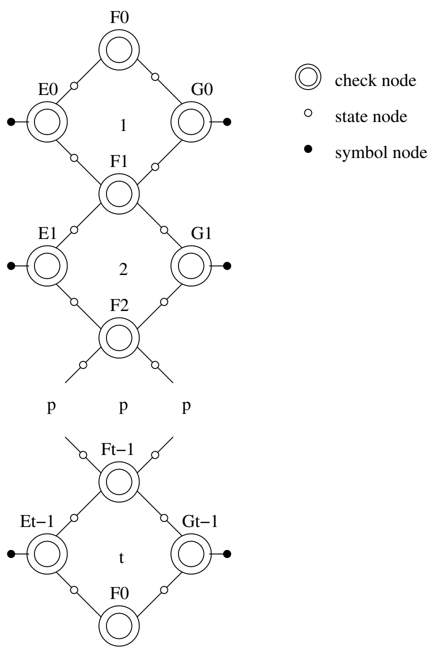

Figure 1: A necklace factor graph of order t.

Each check node represents the base code B. Symbol nodes take values in $GF(q)^k$. State nodes take values in $GF(q)^{\frac{k}{2}}$ if k is even and in $GF(q)^{\lfloor \frac{k}{2} \rfloor}$ or $GF(q)^{\lfloor \frac{k}{2} \rfloor + 1}$ if not. All the variable nodes adjacent to a check node form a codeword of B and all the symbol nodes of the $N_t(C)$ form a codeword of C.

Proposition 1 *If all the local constraints of a necklace factor graph representing a code C are self-dual codes over $GF(q)$ then C is also a linear self-dual code.*

Proof It is an application of Theorem 7.3 given in [4] since any necklace graph is a normal factor graph.

□

3 Constraints on the Necklace Graph

Among all the codes that have necklace graphs described in the Section 2, we are particularly interested in those that have the best minimum distances. We meet this requirement firstly by defining the properties that should satisfy a necklace graph, and then by searching exhaustively interleavers that check them.

Property of diffusion: for any check node which has degree four, if one of its adjacent state node has non-zero (Hamming) weight, then at least three of them has non-zero weights.

Property of expansion: for any check node which has degree three, if exactly one state node has non-zero weight, then the symbol node has always a weight greater than a certain constant $b \geq d - k/2$.

4 Low Complexity Tail-Biting Trellises

lemma 1 *Any necklace factor graph $N_t(C)$ of a code C can be put into the form of a t-section tail-biting trellis $T_t(C)$.*

Proof It is sufficient to group together variable nodes and check nodes of the same level (see Figure 2) to obtain a new factor graph which is basically a tail-biting trellis.

□

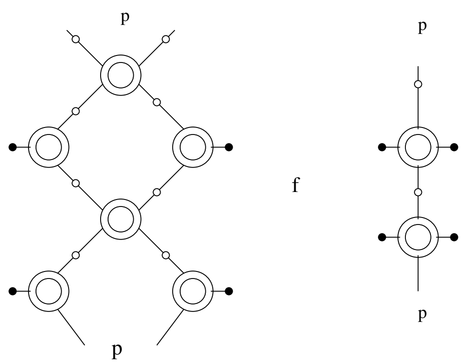

Figure 2: Transformation of a necklace graph into a tail-biting trellis.

There exist several types of complexity for a tail-biting trellis ([1]). We are only interested in the maximum state complexity.

Definition 1 *Let T be a t-section tail-biting trellis with state spaces $S_0, \ldots, S_{t-1}$. The* maximum state complexity $\mathcal{S}_{max}$ *of T is defined as*

$$S_{max} = max\{|S_0|, \ldots, |S_{t-1}|\}.$$

lemma 2 *Let C be a $[N, K, D]$ linear code over $GF(q)$ with necklace graph $N_t(C)$ obtained from a $[n, k, d]$ linear code B. Each states space of the tail-biting trellis $T_t(C)$ deduced from $N_t(C)$ is of size q^k and therefore S_{max} is equal to q^k states.*

The following table gathers the parameters of the obtained codes along with the complexities of their associated tail-biting trellises.

q	B	C	t	$S_{max}(T_t(C))$
3	[4,2,3]	[8,4,3]	2	3^2
3	[4,2,3]	[12,6,6]	3	3^2
3	[4,2,3]	[16,8,6]	4	3^2
3	[4,2,3]	[20,10,6]	4	3^2
3	[12,6,6]	[24,12,9]	2	3^6
4 Euclidean	[4,2,3]	[8,4,3]	2	4^2
4 Euclidean	[4,2,3]	[12,6,6]	3	4^2
4 Euclidean	[4,2,3]	[16,8,6]	4	4^2
4 Hermitian	[6,3,4]	[12,6,4]	2	4^3
4 Hermitian	[8,4,4]	[24,12,8]	3	4^4
5	[6,3,4]	[18,9,6]	3	5^3
5	[8,4,4]	[24,12,8]	3	5^4

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