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# **3WCC** temperature integration in a gasoline-HEV optimal energy management strategy

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**Abstract** – For a gasoline-Hybrid Electric Vehicle (HEV), the Energy Management Strategy (EMS) is the computation of the distribution between electric and gasoline propulsion. Until recently, the EMS objective was to minimize fuel consumption. However, decreasing fuel consumption does not directly minimize the pollutant emissions, and the 3 Way Catalytic Converter (3WCC) must be taken into account. This paper proposes to consider the pollutant emissions in the EMS, by minimizing, with the Pontryagin Minimum Principle, a tradeoff between pollution and fuel consumption. The integration of the 3WCC temperature in the EMS is discussed and finally a simplification is proposed.

**Keywords** – Hybrid Electric Vehicle, Energy Management Strategy, Pollutant Emissions, Optimal control, PMP

#### 1. Introduction

A gasoline-electric Hybrid Electric Vehicle (HEV) has two power sources (fuel and electricity) and two associated converters to ensure propulsion (a gasoline engine and an electrical machine), allowing Stop & Start and Zero Emission Vehicle operating modes. In this context, the Energy Management Strategy (EMS), which consists in finding the best power distribution to meet a driver request, provides the possibility to reduce the fuel consumption [1].

For this reduction, different optimal off-line strategies were proposed, based on the Pontryagin Minimum Principle (PMP) [2] or Dynamic Programming derived from Bellman's principle of optimality [3]. Some suboptimal online strategies were adapted from the PMP method such as the Equivalent Consumption Minimization Strategy (ECMS) [4] or Adaptive-ECMS (A-ECMS) [5].

However, decreasing the fuel consumption does not directly ensure the reduction of the pollutant emissions. In order to minimize both fuel consumption and pollutant emissions, three off-line EMS have been proposed.

- *Strategy A*. This strategy minimizes a tradeoff between *engine* pollutant emission and fuel consumption masses with the PMP method in the same way as for fuel consumption minimization. It has been applied in diesel-HEV [6][7][8] and gasoline-HEV [9] contexts.
- *Strategy C*. This strategy now minimizes a tradeoff between post-3WCC or *vehicle* pollutant emissions and fuel consumption masses. The PMP is applied by considering, as a second state, the 3 Way Catalytic Converter (3WCC) temperature [11][12][13], because of its key role in converting the engine pollution emissions.
- *Strategy B*. This strategy is a simplification of strategy C, without the 3WCC temperature constraint.

The paper compares these three strategies with a reference strategy minimizing only the fuel consumption. It highlights the better results of strategy B compared to those of strategy C. Integrating the 3WCC temperature dynamics, as in strategy C, reduces pollutant emissions with a relatively small increase of fuel consumption. Nevertheless, better reductions with smaller increase in fuel consumption can be found with a simpler method by changing the tradeoff between engine pollution and fuel consumption and without 3WCC temperature constraint (Strategy B).

The next section describes a 4-dynamics gasoline-HEV model, determined with the aim of testing different strategies. Section 3 formalizes the reference fuel consumption minimization strategy with the PMP method and introduces strategies A, B and C. For these strategies, the next section presents some simulation results for a first tradeoff between fuel consumption and NO<sub>x</sub> emissions. A second tradeoff between fuel consumption, CO and NO<sub>x</sub> emissions is proposed. This compromise shows also good results to decrease each pollutant species emissions including HC. Finally a conclusion is given and the simplification of strategy C into the strategy B, where the 3WCC temperature dynamics is not considered explicitly in the PMP optimization method, is discussed.

#### 2. Gasoline-HEV model

The HEV is a parallel mild-hybrid vehicle with the electrical machine connected to the gasoline engine by a belt. The HEV is modeled with a 4-states model represented on [*Figure 1*]. The four dynamical states are the battery State Of Charge *SOC*, 3WCC temperature  $\theta_{cata}$ , engine block temperature  $\theta_i$ , and engine water temperature  $\theta_w$ . Other variables are static and depend directly on the driving speed. From the driving cycle speed, with the vehicle and gearbox models, the rotation speeds of the thermal engine  $\omega_i$  and electrical machine  $\omega_e$  and the requested torque  $T_0$  can be deduced, see [*Figure 1*]. These computations take into account the gearbox ratio and the different transmission ratios and efficiencies.

#### 2.1. Engine model

The engine temperature  $\theta_i$  and the water temperature  $\theta_w$ dynamics are deduced from a simple 2-states zerodimensional thermal model derived from the Heat Equation. A look-up table gives the used fuel mass flow rate  $\dot{m}_{fuel}$ from the engine speed  $\omega_i$  and torque  $T_i$ . The engine pollutant emissions  $\dot{m}_{eng X}$ ,  $X \in \{CO, HC, NO_X\}$  are also deduced from  $\omega_i$  and  $T_i$  with three maps, then penalized with respect of  $\theta_i$  and engine restarting. Soot emissions, considered only recently for gasoline engines, are out of the scope of the paper.

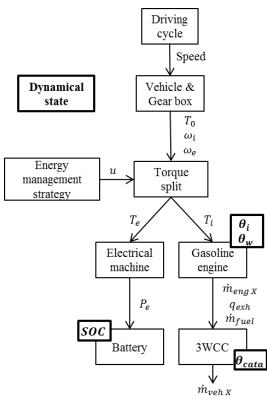


Figure 1: 4-states HEV model

#### 2.2. 3WCC model

In the gasoline-HEV context, the 3WCC is the only current technology that ensures that vehicles based on a spark-ignition engine comply with the CO, HC and  $NO_x$  emission standards. The operation of a 3WCC can be expressed by its pollutant conversion efficiency, defined as

$$\eta_X = 1 - \frac{m_{veh X}}{\dot{m}_{eng X}}, X \in \{CO, HC, NO_X\},$$
(2)

where  $\dot{m}_{eng X}$  and  $\dot{m}_{veh X}$  is the mass flow rate of pollutant species X, respectively at the input (the *engine* emissions) and output (the *vehicle* emissions) of the 3WCC.

The conversion is influenced by the following variables:

- temperature of the 3WCC monolith,  $\theta_{cata}$ , deduced from a simple 1-state zero-dimensional model,
- flow rate of exhaust gas through the monolith,  $q_{exh}$ , deduced from  $\omega_i$  and  $T_i$  with a map,
- Air-Fuel Ratio (*AFR*) of the mixture in the spark-ignition engine.

The dependence of the conversion efficiencies to AFR is neglected here, insofar as the air/fuel mixture is considered at the stoichiometry.

Eq. (2) can be rewritten as

$$\dot{m}_{veh X} = (1 - \eta_X) \, \dot{m}_{eng X}. \tag{3}$$

For each pollutant, the conversion efficiency is computed from the 3WCC temperature  $\theta_{cata}$  and exhaust gas flow rate  $q_{exh}$  with two maps:

$$\eta_X = \eta_{X\theta_{cata}} \left( \theta_{cata} \right) \eta_{Xq_{exh}} \left( q_{exh} \right). \tag{4}$$

#### 2.3. Battery model

At each time t, the power delivered by the electric machine  $P_e$  is computed from the speed  $\omega_e$  and torque  $T_e$ :

$$P_e = \omega_e T_e. \tag{5}$$

Then, the electrochemical battery power  $P_{\chi}$  is written from the power balance:

$$P_{\chi} = -P_e - P_{los} - P_{aux},\tag{6}$$

where the power losses  $P_{los}$  are deduced from the speed  $\omega_e$ and torque  $T_e$  by a look-up table and the power used by the auxiliaries  $P_{aux}$  is supposed here constant.

From (6), using an internal resistance model for the battery, the battery voltage  $U_{bat}$  can be deduced as:

$$U_{bat} = \frac{U_0}{2} + \sqrt{\frac{U_0^2}{4} - \frac{P_{\chi}}{R_{int}}},$$
(7)

where  $U_0$  is the open circuit voltage and  $R_{int}$  the internal resistance deduced from *SOC* by two look-up tables. Finally, by using (6) and (7), the battery current intensity

$$I_{bat} = \frac{P_{\chi}}{U_{bat}}$$
(8)

leads to the SOC dynamics:

$$S\dot{O}C = c \frac{I_{bat}}{Q_{max}},\tag{9}$$

where  $Q_{max}$  is the battery capacity and *c* a constant allowing to obtain a dimensionless expression of *SOC* in %.

#### 2.4. Control model

A parallel HEV has two propulsion systems and the requested torque at the entrance of the gearbox  $T_0$  is simply:  $T_0 = T_i + T_e$ , (10) where the thermal engine torque  $T_i$  and electrical machine torque  $T_e$  take into account the different transmission ratios to be expressed in the same referential, the entrance of the gear-box.

A torque split variable u is introduced as the ratio between the electrical machine torque and the requested torque:

$$u = \frac{T_e}{T_0}.$$
 (11)

Note that many variables can be now noted with respect on torque split control variable u, as for example the engine  $\dot{m}_{eng X}(u)$  or vehicle  $\dot{m}_{veh X}(u)$  emissions.

The goal of the EMS is to find the control u that fulfills different objectives. While minimizing HEV fuel consumption is the main objective, other secondary objectives can be considered such as oil temperature maximization [10] (to reduce fuel consumption), drivability [14], limitation of battery aging [15] or, in this paper, reduction of pollutant emissions [11] [12] [13].

#### 3. <u>Optimal strategies</u>

Some recalls of the optimization framework are first given, before presenting the strategies for fuel consumption minimization, and next for pollution / fuel consumption joint minimization, where a simplification is proposed.

#### 3.1. Pontryagin Minimum Principle

Consider a problem  $P_0$  where the goal is to minimize a discrete-time cost function  $J(\mathbf{x}(t), \mathbf{u}(t))$ , where  $\mathbf{x}(t)$  is the state vector and  $\mathbf{u}(t)$  the control vector. The system is expressed by the state vector dynamics:

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t))$$
(12)  
and the criterion to be minimized by:

$$J(\mathbf{x}(t), \mathbf{u}(t)) = \Phi\left(\mathbf{x}(t_f)\right) + \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t)) dt,$$
(13)

where  $\Phi(\mathbf{x}(t_f))$  is the final state constraint at final time  $t_f$ and  $L(\mathbf{x}(t), \mathbf{u}(t))$  a cost function. With an initial constraint (14) $\boldsymbol{x}(t_0) = x_0,$ 

 $P_0$  can be written, with U the admissible control space, as:  $(\min I(\mathbf{x}(t), \mathbf{u}(t)), t \in [t_0, t_f],$ 

$$P_{0}:\begin{cases} u \in U \\ \dot{x}(t) = f(x(t), u(t)), \\ x(t_{0}) = x_{0}. \end{cases}$$
(15)

Introducing the Hamiltonian:

$$H(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\lambda}(t))$$

$$= L(\mathbf{x}(t), \mathbf{u}(t))$$
(16)  
+  $\lambda^{T}(t) f(\mathbf{x}(t), \mathbf{u}(t)),$ 

with the Lagrange parameter vector (or co-state)  $\lambda(t)$ ,  $P_0$ can be rewritten as a dual problem  $P'_{O}$ :

$$P'_{O} \begin{cases} \dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t)), \\ \mathbf{x}(t_{0}) = x_{0}, \\ \dot{\boldsymbol{\lambda}}^{T}(t) = -\frac{\partial H(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\lambda}(t), t)}{\partial \mathbf{x}}, \\ \dot{\boldsymbol{\lambda}}^{T}(t_{f}) = \frac{\partial \phi\left(\mathbf{x}(t_{f})\right)}{\partial \mathbf{x}(t_{f})}, \\ \mathbf{u}^{*}(t) = \operatorname{argmin}_{u \in U} H(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\lambda}(t)). \end{cases}$$
(17)

The last equation, where  $\boldsymbol{u}^{*}(t)$  is the optimal control minimizing (13), corresponds to the Pontryagin Minimum Principle (PMP)[2].

#### 3.2. Fuel consumption minimization strategy

Optimal (off-line) strategies assume the knowledge of the full driving horizon, from time  $t_0$  to time  $t_f$ . Then, for fuel consumption, the following performance index has to be minimized:

$$J_{fuel} = \Phi\left(SOC(t_f)\right) + \int_{t_0}^{t_f} \dot{m}_{fuel}(u)dt, \tag{18}$$

where  $\dot{m}_{fuel}(u)$  is the engine fuel mass flow rate and  $\Phi(SOC(t_f))$  is a final state constraint ensuring charge sustaining:

$$\Phi\left(SOC(t_f)\right) = \begin{cases} 0, \text{ if } SOC(t_f) = SOC(t_0), \\ \infty, \text{ else.} \end{cases}$$
(19)

The performance index (18) is minimized with PMP, as described above, considering the one-state SOC dynamics (8):

$$\dot{SOC} = f(SOC, u). \tag{20}$$

To this end, the Hamiltonian

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$$(SOC, u, \lambda, t) = \dot{m}_{fuel}(u) + \lambda S\dot{O}C$$
(21)

is defined, where  $\lambda$  is the co-state associated to the SOC dynamics, respecting:

$$\dot{\lambda} = -\frac{\partial H(SOC, u, \lambda, t)}{\partial SOC}.$$
(22)

The optimal control  $u^*$  is obtained by minimizing (21), at each time *t*:

$$u^* = \underset{u \in U}{\operatorname{argmin}} H(SOC, u, \lambda, t).$$
(23)

In the case of HEV, considering

$$\frac{\partial H(SOC, u, \lambda, t)}{\partial SOC} = 0 \tag{24}$$

has a very little influence on fuel consumption, since the SOC dependence on  $R_{int}$  and  $U_0$  is low. Then  $\dot{\lambda} = 0$ ,  $\lambda$  is considered constant and a simple binary search can find the value ensuring the HEV fuel consumption minimization and

charge sustaining. Note, however, that this value strongly depends on the considered cycle [1].

#### 3.3. Pollution constrained fuel consumption minimization strategies

Strategy A. A first approach, when considering pollution in the EMS, is to define a tradeoff  $\dot{m}_{eng}(u)$  between fuel consumption and engine pollutant emissions:

$$\dot{m}_{eng}(u) = \left(1 - \sum_{X} \alpha_{X}\right) \dot{m}_{fuel}(u) + \sum_{X} \alpha_{X} \dot{m}_{eng X}(u),$$
(25)

where  $\dot{m}_{eng X}$  are the mass flow rates of engine pollutant species X,  $X \in \{CO, HC, NO_X\}$ , and  $\alpha_X$  the corresponding weighting factors, and derive a new performance index

$$J_{eng} = \Phi\left(SOC(t_f)\right) + \int_{t_0}^{t_f} \dot{m}_{eng}(u) dt$$
(26)

that can be minimized as (18).

Strategy C. The second approach is based on the tradeoff  $\dot{m}_{veh}(u)$  between fuel consumption and vehicle pollutant emissions:

$$\dot{m}_{veh}(u) = \left(1 - \sum_{X} \alpha_{X}\right) \dot{m}_{fuel}(u) + \sum_{X} \alpha_{X} \dot{m}_{veh X}(u).$$
(27)

In (4), for the minimization, the 3WCC conversion efficiencies are simplified as

$$\eta_X = f(\theta_{cata}). \tag{28}$$

A new performance index

$$J_{veh} = \Phi\left(SOC(t_f)\right) + \int_{t_0}^{t_f} \dot{m}_{veh}(u)dt$$
(29)

is then defined. If the  $\theta_{cata}$  dynamics is considered during minimization, the Hamiltonian becomes:

$$H(SOC, \theta_{cata}, u, \lambda, t) = \dot{m}_{veh}(u) + \lambda_1 SOC + \lambda_2 \dot{\theta}_{cata}.$$
(30)

Using (24), the first co-state  $\lambda_1$  can be found constant as in the fuel consumption minimization strategy. The second costate  $\lambda_2$  associated to the 3WCC temperature dynamics is obtained by solving

$$\dot{\lambda}_2 = -\frac{\partial H(SOC, \theta_{cata}, u, \lambda, t)}{\partial \theta_{cata}},$$
(31)

yielding to the exponential form:  $\lambda_2$ 

$$(t) = \lambda_{20} e^{at - b(u,t)}, \tag{32}$$

where a is a constant and b a function, which can be found from (27) and (28), and  $\lambda_{20}$  is the second co-state initial condition.

Strategy B. This strategy is a simplification of strategy C, and considers a zero 3WCC temperature co-state in (32):  $\lambda_{2}$ 

$$\lambda_0 = \lambda_2(t) = 0.$$
 (33)

The idea is to take into account in the minimization strategy the 3WCC temperature dynamics only through the vehicle pollutant emissions (27), and not in (30).

#### 4. Results

This section presents some simulations results obtained on Worldwide harmonized Light vehicles Test Cycles (WLTC). Tradeoffs between fuel consumption and NO<sub>X</sub> emissions, then CO/NO<sub>X</sub> emissions, are minimized with the strategies presented above, which are compared.

Fuel consumption minimization strategy is the reference and the corresponding fuel consumption and pollutant emissions are obtained with instantaneous minimization of the Hamiltonian (21). The constant value of  $\lambda$  is found by a binary search while respecting the final SOC constraint (19).

4.1.  $NO_X$  emissions/fuel consumption compromise Tradeoffs between  $NO_X$  emissions and fuel consumption are chosen and minimized with different strategies.

The first tradeoffs between *engine* NO<sub>X</sub> emissions and fuel consumption

$$\dot{m}_{eng}(u) = (1 - \alpha_{NOx}) \, \dot{m}_{fuel}(u) + \alpha_{NOx} \, \dot{m}_{eng \, NOx}(u)$$
(34)

are minimized with strategy A, for different values of  $\alpha_{NOx}$ , from  $\alpha_{NOx} = 0$ , which is the reference, to the maximal value allowed by the battery size. Note that the maximal battery demand translates into the maximum SOC deviation obtained during an optimal simulation.

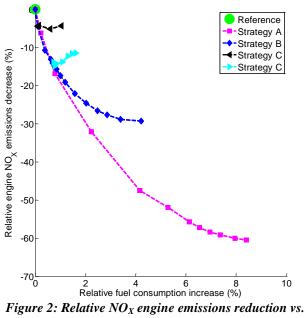
Next, tradeoffs between *vehicle*  $NO_X$  emissions and fuel consumption

$$\dot{m}_{veh}(u) = (1 - \alpha_{NOx}) \, \dot{m}_{fuel}(u) + \alpha_{NOx} \, \dot{m}_{veh \, NOx}(u)$$
(35)

are minimized with strategies B and C.

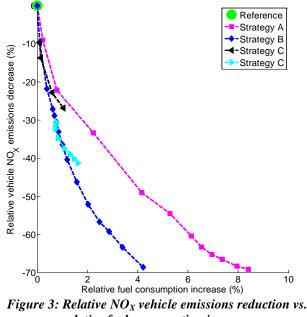
For strategy B, the results are obtained by minimizing a Hamiltonian such as (30) for different  $\alpha_{NOx}$  values in (35) and a choice of  $\lambda_{20} = 0$  as in (33). For strategy C, the results are obtained for two fixed  $\alpha_{NOx}$  values in (35) and different values of  $\lambda_{20}$  in (32).

The results presented in [*Figure 2*] show that strategy A yields the largest reductions in engine pollution for the lowest increases in fuel consumption. As expected, strategy A is the best one to minimize the *engine*  $NO_X$  emissions.



relative fuel consumption increase

[*Figure 3*] shows that strategy B and strategy C are more efficient in minimizing the *vehicle* NO<sub>X</sub> emissions. At a fixed tradeoff between NO<sub>X</sub> vehicle pollution and fuel consumption  $m_{veh}(u)$  as defined in (35) a judicious choice of  $\lambda_{20}$  in strategy C can reduce the NO<sub>X</sub> vehicle emissions [11][12][13]. However, strategy B can lead to better results provided that a good choice of  $\alpha_{NOx}$  has been made in  $m_{veh}(u)$  and a zero 3WCC temperature co-state has been chosen.



relative fuel consumption increase

Next, [*Figure 4*] reveals that strategy C implies a stronger use of the battery than strategy B, which is not desirable. The battery demands are represented by the maximum SOC deviation obtained during a driving cycle with the optimal control.

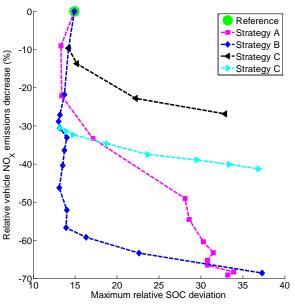


Figure 4: Relative NO<sub>x</sub> vehicle emissions reduction vs. maximum relative SOC deviation (zero co-state)

[*Figure 5*] shows the trajectories of *SOC*, relative 3WCC temperature  $\theta_{cata}$ , and relative cumulative normalized NO<sub>X</sub> engine emissions  $\dot{m}_{eng NOX}$  and vehicle emissions  $\dot{m}_{veh NOX}$  for the strategies A and B. The parameters have been chosen to ensure the same fuel consumption decrease. The 3WCC conversion consideration by the strategy B ensures a better NO<sub>X</sub> conversion than the strategy C with the same level of battery solicitations.

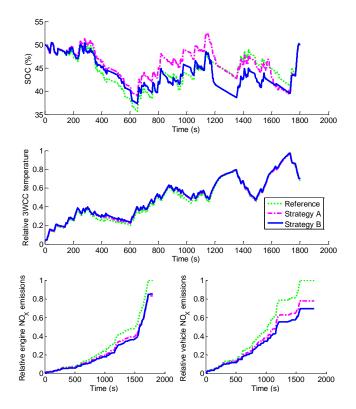


Figure 5: SOC, 3WCC temperature, NO<sub>X</sub> engine and vehicle emissions trajectories

# 4.2. CO/NO<sub>X</sub> emissions / fuel consumption compromise

Similarly as (34) and (35), tradeoffs between CO and NO<sub>X</sub> engine emissions and fuel consumption, with  $\alpha_{CO} = \alpha_{NOX}$ ,  $\dot{m}_{eng}(u(t), t) =$ 

$$(1 - \alpha_{CO} - \alpha_{NOx})\dot{m}_{fuel}(u) + \alpha_{CO}\dot{m}_{eng\ CO}(u)$$
(36)  
+  $\alpha_{NOx}\dot{m}_{eng\ NOx}(u)$ 

are minimized with strategy A, and tradeoffs between CO and NO<sub>X</sub> vehicle emissions and fuel consumption, with  $\alpha_{CO} = \alpha_{NOX}$ 

(37)

$$\begin{aligned} \dot{n}_{veh}(u) &= \\ (1 - \alpha_{NOx} - \alpha_{CO}) \dot{m}_{fuel}(u) + \alpha_{CO} \dot{m}_{veh CO}(u) \\ &+ \alpha_{NOx} \dot{m}_{veh NOX}(u) \end{aligned}$$

are minimized with strategies B and C.

For the three strategies, the vehicle emissions with respect to fuel consumption are shown on [*Figure 6*], for HC, [*Figure 7*], for NO<sub>X</sub>, and [*Figure 8*], for CO.

Again, strategies B and C are better than strategy A to minimize vehicle emissions, and, compared to strategy C, strategy B leads to better reduction of vehicle pollutant emissions, including HC.

Note that other compromises can be easily built with different objectives concerning CO,  $NO_X$  and/or HC pollutants.

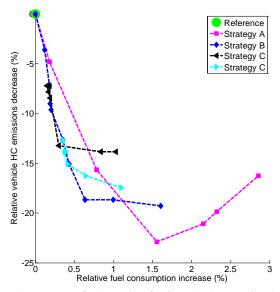


Figure 6: Relative HC vehicle emissions reduction vs. relative fuel consumption increase

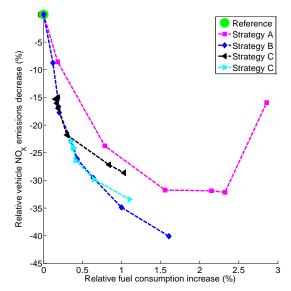


Figure 7: Relative NO<sub>x</sub> vehicle emissions reduction vs. relative fuel consumption increase

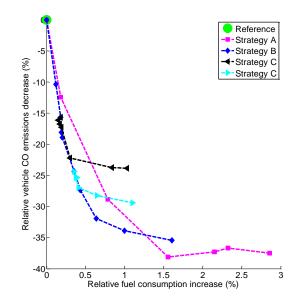


Figure 8: Relative CO vehicle emissions reduction vs. relative fuel consumption increase

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### 5. Conclusion

Optimal strategies have been proposed to minimize fuel consumption while taking pollutant emissions into consideration. A simple tradeoff between *engine* pollution and fuel consumption can be minimized with the PMP, ensuring good results.

These results can be improved if the strategy takes the 3WCC behavior into account. Two ways are proposed: a tradeoff between *vehicle* pollution and fuel consumption minimized including the 3WCC temperature dynamics in the Hamiltonian (strategy C), or not (strategy B). Introducing a second dynamics improves the results, but better results are found with lower battery demands with a zero second co-state, simply by changing the compromise between vehicle pollution and fuel consumption.

To conclude, the fuel consumption minimization with pollution constraint doesn't require considering directly the 3WCC temperature in the minimization method, as in strategy C. The simplicity and better results of strategy B are preferable for a future on-line adaptation. This is reinforced by the frequent difficulties in deducing the 3WCC temperature and its associated co-state in a real environment.

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