Impact of Storage on the Efficiency and Prices in Real-Time Electricity Markets

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ABSTRACT

We study the effect of energy-storage systems in dynamic real-time electricity markets. We consider that demand and renewable generation are stochastic, that real-time production is affected by ramping constraints, and that market players seek to selfishly maximize their profit. We distinguish three scenarios, depending on the owner of the storage system: (A) the supplier, (B) the consumer, or (C) a stand-alone player. In all cases, we show the existence of a competitive equilibrium when players are price-takers (they do not affect market prices). We further establish that under the equilibrium price process, players' selfish responses coincide with the social welfare-maximizing policy computed by a (hypothetical) social planner. We show that with storage the resulting price process is smoother than without.

We determine empirically the storage parameters that maximize the players' revenue in the market. In the case of consumer-owned storage, or a stand-alone storage operator (scenarios B and C), we find that they do not match socially optimal parameters. We conclude that consumers and the stand-alone storage operator (but not suppliers) have an incentive to under-dimension their storage system. In addition, we determine the scaling laws of optimal storage parameters as a function of the volatility of demand and renewables. We show, in particular, that the optimal storage energy capacity scales as the volatility to the fourth power.

1. INTRODUCTION

The process of liberalizing electricity markets is underway worldwide. It amounts to replacing tightly regulated monopolies with lightly regulated competitive markets [29]. Electricity production is managed through scheduling decisions. In a first stage, producers commit to an energy generation schedule determined through forecasts for the demand and generation of renewable energy for the following day. In a second stage, decisions are made in real-time to compensate for forecast errors in load and production. The price volatility in real-time electricity markets[1] raises the question of the efficiency of these markets: Does the selfish behavior of the various actors lead to a socially acceptable situation?

Electricity markets are highly complex dynamical systems. They incorporate renewable energy sources, such as wind and solar, that are highly volatile; loads that are inelastic, for the most part; and generation units, that are subject to friction and real-time constraints. To avoid blackouts and due to physical constraints, real-time scheduling of energy is critical.

A model of an electricity market that takes into account these dynamical aspects is proposed in [7, 8]. The authors study the competitive equilibriums in a real-time electricity market where demand is stochastic and energy generation is subject to ramping constraints. They show that if all actors are price-takers (they do not affect market prices), then there exists a competitive equilibrium that is *efficient*: more specifically, the selfish behavior of actors leads indeed to a socially optimal scheduling of generation. However, they also show that the prices that guarantee such an equilibrium exhibit considerable volatility: they oscillate between 0 and a "choke-up" price, and do not concentrate around the marginal production cost. This model has been extended to incorporate network constraints [26, 28], or the presence of renewables [19]; see also [27] for a survey.

Our motivation in this paper is to understand the role of storage in compensating volatility in dynamic real-time electricity markets. This is highly relevant for markets with a high penetration of renewables. Exploitation of renewable energy is encouraged in many countries as a means to reduce CO_2 emissions. However, renewable energy sources, such as wind and solar photovoltaic, are not dispatchable. A side effect of their high penetration is the increase in the volatility of electricity generation and thus of prices, according to [19]. Therefore, in order to compensate for their volatility, a high penetration of renewables needs to be supported by mechanisms such as storage systems (batteries or pump-hydro) or fast-ramping generators (essentially gas-fired turbines). Storage can be operated by an energy producer, a consumer, or by a stand-alone storage operator. In the last case, the storage owner needs to generate revenue, hence energy is stored when market prices are low and is provided to the grid when prices are high. We are interested, in particular, in understanding whether the market efficiency results of [27] continue to hold; for example, whether it is socially

[1] the peak to mean ratio of prices can be as high as several thousands, one famous example being the price observed in California in 2000-2001 [17]

optimal to have stand-alone storage operators that react to real-time prices.

Contributions. We extend the wholesale real-time market model of [27] to incorporate a storage system with losses due to the charge/discharge cycles. We consider three scenarios depending on the owner of the storage: (A) the supplier, (B) the consumer, or (C) a stand-alone real-time storage operator. We show that in all three cases, the market is efficient: there exists a price process such that the selfish behaviors of the players coincide with a socially optimal use of the storage and scheduling of the generation. When the storage energy capacity is large, this price process becomes smooth. Moreover, irrespective of the considered scenario, *the same* decisions concerning bought/sold energy, real-time production, and storage system operation lead to social optimality. These decisions are enforced via *the same* incentive (pricing) scheme.

We show numerically that when the storage belongs either to the consumer or to a stand-alone real-time storage operator, the storage energy capacity that maximizes the storage owner welfare is strictly smaller than the socially optimal storage energy capacity. Consequently, even though the market is efficient when the storage parameters are fixed, consumers and stand-alone storage operators still have an incentive to under-dimension their storage systems.

Finally, we study the effect of the volatility and of the ramping capabilities of generators on the optimal storage parameters. We show that when the volatility is σ, the optimal energy capacity of the storage system scales as σ^4, and its maximum charging/discharging power scales as σ^2. When the ramping capability of the generators is ζ, the optimal storage energy capacity scales as $1/\zeta^3$, and the optimal maximum charging/discharging power scales as $1/\zeta$. We conclude that a linear increase in the ramping capability of fast-ramping generators (such as gas turbines) entails a cubic decrease in the required energy capacity. In view of the high cost of storage capacity, a paradoxical situation arises: in order to accommodate a large deployment of renewables and to compensate for the resulting generation volatility, there is an incentive to deploy conventional fast-ramping generators (that have high carbon dioxide emissions) rather than to invest in storage.

Road-Map. The rest of the paper is structured as follows. We first describe the market and storage model in Section 3. We study the control problem from a social point of view in Section 4. Section 5 contains the main theoretical results: we show the existence of competitive equilibria in all three scenarios and prove that they are socially optimal. We study the incentives for actors to install storage devices in Section 6. We investigate the relations between volatility, ramping capabilities and optimal storage energy capacity in Section 7. Finally, we conclude in Section 8.

2. RELATED WORK

A large part of the work, *e.g.*, [5, 13, 18], related to the economic aspects of storage systems investigates optimal energy storage strategies for profit maximization in the electricity market, assuming that storage owner are price-takers. For example, for a wind-farm owner the authors of [10] study optimal storage strategies in a day-ahead market. These policies often assume that prices are known. The uncertainties due to the variability of prices and their forecasting is also studied in [1, 4].

The economic viability for storage owners is also an important question. On the one hand, it is shown in [25] that there is a strong economic case for storage installations in the New York City region. A similar analysis is conducted for the PJM interconnection (US east-coast) in [21] for which the authors show a moderate storage capacity is viable. On the other hand, according to [23], "*storage is not viable from a system perspective until extremely large levels of wind power are seen on the system*" in Ireland – a country that envisioned 80% of the electrical consumption generated by wind power plants. Moreover, it is suggested in [16] that pumped-storage hydro-plant operators need to change their business model (currently electricity price arbitrage) because the potential diminishes with the increased penetration of renewables: Inexpensive energy is generated during the noon consumption peak by PV installations.

From a social planner's perspective, it is shown [22] that the undesired effects of the volatility of renewables can be mitigated via the use of energy storage, with a manageable increase in energy costs, based on a study in the UK. At a European scale, the authors of [2, 15, 24] show how to use model predictive control to update day-ahead production schedule and mitigate energy curtailments. The use of storage can also compensate for forecast uncertainties. Generation scheduling policies that minimize energy losses and the use of fast-ramping generators are developed in [3, 12].

The question of the efficiency of the control by prices of storage devices is raised in a few papers. Using traces of the real market bids data of the French day-ahead market in 2009, the authors of [14] evaluate the storage operation by a public or a private operator by simulation. They show that the public operation leads to higher social welfare and lower cost of supply than the private operation. Using traces from the Swiss market, authors of [6] also show that the optimal control for a price-taker storage owner are socially optimal. However, these two papers only consider a single-run of simulation and do not provide theoretical guarantee.

The authors of [20] go one step further and obtain theoretical guarantees on a model of a purely static setting: loads are predictable on-peak and off-peak periods, and the price depends linearly on the load. They show that for the same three cases of ownership structure as ours (storage belongs to producers, to consumers or to independent actors), the selfish behavior of price-taker agents lead to a socially optimal use of storage. However, when agents influence prices, storage tends to be underused when owned by producers or independent actors or overused if owned by consumers. The reason is that the higher the use of storage is, the smoother the price is. Our model differs greatly from [20] as it incorporates the dynamical aspects of demand and generation.

3. MARKET AND STORAGE MODEL

In this section we first present a model of a two-stage (day-ahead and real-time) electricity market consisting of two main actors, a consumer and a supplier. Both can have access to an energy storage system in the real-time stage of the market, depending on the scenario. We then describe the storage system, and we analyze three scenarios, depending on whether the storage is controlled by the supplier, by the consumer, or by a third independent actor (a real-time storage operator).

3.1 Two-stage Electricity Markets

We consider an electricity market with two stages: a day-ahead stage and a real-time stage. The two main actors or players in this market are a *supplier* who produces electricity, and a *consumer* who buys electricity and serves a fairly large number of end-users.

In the *day-ahead* market, each day, players forecast a demand profile $d^{da}(t)$ and plan generation $g^{da}(t)$ for the next day. The price of electricity is set by market mechanisms to $p^{da}(t)$, and the consumer agrees to buy an amount of electricity $g^{da}(t)$ at this price. As often in markets where demand is inelastic and unpredictable, the consumer purchases an extra amount of goods over the predicted demand as a precautionary measure. Hence we assume that $g^{da}(t) = d^{da}(t) + r^{da}$, where r^{da} is called the *fixed reserve*. In addition to the conventional energy sources, the supplier uses renewable energy sources (*e.g.*, wind, solar) that provide a volatile production. The forecast of generation of renewables constitutes a fraction of the planned generation $g^{da}(t)$, and is complemented with conventional energy.

The *real-time* market deals with situations where the actual demand cannot be met, due to the volatility of the demand and of the production of renewable energy. The actual demand at time t is $D^a(t) = d^{da}(t) + D(t)$, where $D(t)$ is referred to as the *real-time demand*, and can be either positive or negative. The renewable forecast error (the difference between the actual and the forecasted production of renewable energy) is denoted by $\Gamma(t)$. The supplier reacts in real-time to unpredictable changes in demand and renewable generation, and she produces an amount of energy $G(t)$ from additional and usually expensive sources. The total amount of electricity produced at time t is then $G^a(t) = g^{da}(t) + \Gamma(t) + G(t)$, and the corresponding real-time reserve is defined as $R(t) = G^a(t) - D^a(t)$. The energy produced in real-time is sold at a price $P(t)$ at time t. In [8], the authors characterize the competitive equilibria arising in the real-time market – they analyze the amount of real-time energy bought by the consumer, the amount of real-time energy $G(t)$ produced by the supplier, and the price $P(t)$, resulting from the strategic behavior of the market actors. In this paper, we are interested in studying the effect of the existence of storage on the real-time market. We make the same assumptions as in [8], *i.e.*:

(A1) Volatility. We model the forecast errors made in the demand and in the generation of renewable energy, $Z(t) = \Gamma(t) - D(t) + r^{da}$, as a driftless Brownian motion with volatility σ, *i.e.*, the variance of $Z(t)$ is $\sigma^2 t$.

(A2) Prices. We assume that the market actors are price takers: the actors strategically define their actions assuming that the prices are exogenous, and that they cannot influence or manipulate these prices.

(A3) Information. We suppose that the market actors share the same information: Before making a decision at time t, they have access to the exogenous data up to time t (*i.e.*, $(D(\tau), \Gamma(\tau), P(\tau), \tau \le t)$ and to all the past decisions of the other players up to time t. Mathematically, this assumption means that there exists a filtration $\{\mathcal{F}_t, t \ge 0\}$ such that all processes are $\mathcal{F}_t$-adapted.

3.2 Storage Model

We consider a storage system characterized by (i) the maximum amount of energy $B_{\max}$ it can store, *i.e.*, its energy capacity; (ii) the maximum speed $C_{\max}$ at which it can be charged, referred to as the maximum charging power; (iii) the maximum instantaneous amount of energy $D_{\max}$ we can extract from it, *i.e.*, its the maximum discharging power. We model the efficiency of a charge-discharge cycle as follows: only a fraction $\eta \in (0, 1]$ of the injected energy is stored, and the stored energy can be extracted without losses.

More precisely, let $B(t)$ denote the stored energy at time t, and let $u(t)$ be the instantaneous energy extracted from the storage. $u(t) < 0$ means that we are currently storing energy. The storage level evolves as follows:

$$\frac{\partial B}{\partial t} = -u(t)(\mathbb{1}_{\{u(t)>0\}} + \eta \mathbb{1}_{\{u(t)<0\}}). \tag{1}$$

The storage control process $u = (u(t), t \ge 0)$ satisfies the following constraints: at any time t,

$$-C_{\max} \le u(t) \le D_{\max}, \tag{2}$$

$$u(t) \ge 0 \text{ if } B(t) = B_{\max}, \text{ and } u(t) \le 0 \text{ if } B(t) = 0. \tag{3}$$

We write $u \in X_B$ if (2)-(3) are satisfied. Note that (3) is equivalent to $0 \le B(t) \le B_{\max}$ for all t.

3.3 Scenario A: Storage at the Supplier

We first focus on the case where the supplier controls the storage system. The storage system is used only in the real-time stage of the market. At this stage, deterministic processes describing day-ahead demand, generation, and prices $(d^{da}(t), g^{da}(t), p^{da}(t), t \ge 0)$ are known from the day-ahead market. The real-time market model for this scenario is represented in Figure 1. Next, we describe the strategic decisions that the actors may take, and introduce the notion of dynamic competitive equilibrium.

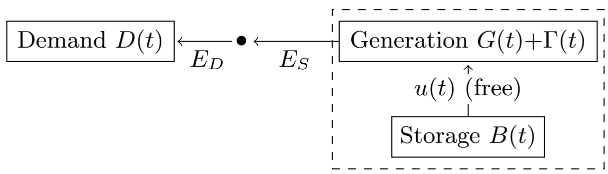


Figure 1: Real-time market model in Scenario A: storage is at the supplier.

Consumer. The strategic decisions taken over time by the consumer are represented by a process E_D, where $E_D(t)$ is the energy bought on the real-time market at time t. We denote by X_D the set of all possible processes E_D. We denote by v the consumer utility per unit of satisfied demand. When the demand exceeds the acquired energy, *i.e.*, if $E_D(t) + g^{da}(t) < D^a(t)$, the consumer bears the cost of a blackout, and suffers from a loss of utility c^{bo} per unit of unsatisfied demand. Recall that the price of electricity in the real-time market is $P(t)$ at time t, and hence the payoff $U_D(t)$ of the consumer at time t is:

$$\begin{aligned} U_D(t) &:= v \min(D^a(t), E_D(t) + g^{da}(t)) \\ &- c^{bo}(D^a(t) - E_D(t) - g^{da}(t))^+ - (P(t)E_D(t) + p^{da}(t)g^{da}(t)). \end{aligned}$$

This payoff can be decomposed as $U_D(t) = \bar{U}_D(t) + W_D(t)$, where

$$\begin{aligned} \bar{U}_D(t) &= v(d^{da}(t) + D(t)) - p^{da}(t)g^{da}(t), \\ W_D(t) &= -(v + c^{bo})(E_D(t) - D(t) + r^{da})^- - P(t)E_D(t). \end{aligned}$$

The term $\bar{U}_D(t)$ only contains quantities that cannot be controlled by the consumer in the real-time market. Thus, the quantity of interest is the second term $W_D(t)$. By abuse of notation, we refer to this second term (rather than $U_D(t)$) as the consumer's payoff.

The consumer's objective is to maximize her welfare defined as her long-run discounted expected payoff:

$$\max_{E_D \in X_D} \mathcal{W}_D := \mathbb{E} \int_0^\infty e^{-\gamma t} W_D(t)\, dt.$$

Supplier. The supplier controls three quantities: the real-time energy generation $G(t)$, the storage system via the storage control process $u(t)$, and the energy sold on the real-time market $E_S(t)$. We say that (E_S, G, u) satisfies the constraints of the supplier and we write $(E_S, G, u) \in X_S$ if

- the following ramping constraints for real-time generation are satisfied: $\forall t' > t, \quad \zeta^- \leq \frac{G(t')-G(t)}{t'-t} \leq \zeta^+$, where $\zeta^- < 0$ and $\zeta^+ > 0$ are called the ramping capabilities;
- the storage constraints are satisfied: $u \in X_B$;
- for all $t \geq 0$, the supplier sells at most $E_S(t) \leq \Gamma(t) + G(t) + u(t)$ on the real-time market.

We denote by c the marginal cost of real-time energy generation, and by c^{da} the cost of energy generation in the day-ahead market. The supplier's payoff at time t is given by:

$$U_S(t) = P(t)E_S(t) + p^{da}(t)g^{da}(t) - cG(t) - c^{da}g^{da}(t).$$

As for the consumer, we write $U_S(t){=}\bar{U}_S(t){+}W_S(t)$, where

$$\bar{U}_S(t) = p^{da}(t)g^{da}(t) - c^{da}g^{da}(t),$$
$$W_S(t) = P(t)E_S(t) - cG(t).$$

The term $\bar{U}_S(t)$ contains only quantities that cannot be controlled by the supplier in real-time. Thus, we focus on the second term, $W_S(t)$. Again, by abuse of notation we refer to $W_S(t)$ (rather than $U_S(t)$) as the supplier's payoff. Observe that the supplier's utility is increasing in E_S. Hence, setting $E_S(t) = \Gamma(t) + G(t) + u(t)$ maximizes her payoff in X_S.

The objective of the supplier is to maximize her welfare:

$$\max_{(E_S,G,u)\in X_S} \mathcal{W}_S := \mathbb{E} \int_0^\infty e^{-\gamma t} W_S(t)\, dt.$$

Dynamic Competitive Equilibrium. As in Cho and Meyn [8], we introduce the notion of dynamic competitive equilibrium. When the supplier owns and controls the storage, a dynamic competitive equilibrium is defined as follows:

Definition 1. *(Dynamic competitive equilibrium, storage at the supplier) A dynamic competitive equilibrium is a set of price and control processes $(P^e, E_D^e, E_S^e, G^e, u^e)$ satisfying*

$$E_D^e \in \arg\max_{E_D \in X_D} \mathcal{W}_D, \tag{4}$$

$$(E_S^e, G^e, u^e) \in \arg\max_{(E_S,G,u)\in X_S} \mathcal{W}_S, \tag{5}$$

$$E_D^e = E_S^e. \tag{6}$$

In the above definition, (4) means that E_D^e constitutes an optimal control from the consumer's perspective. Similarly, (5) states that (E_S^e, G^e, u^e) is optimal from the supplier's perspective. Finally, (6) is the market constraint. Note that in (4), the consumer is not subject to the supplier's constraints and vice-versa for (5). See [8] for a discussion.

3.4 Scenario B: Storage at the Consumer

When the consumer has control of the storage, her strategic decisions are represented by the process pair (E_D, u), where $u(t)$ is the storage control process, *i.e.*, the amount of power discharged at time t from the storage.

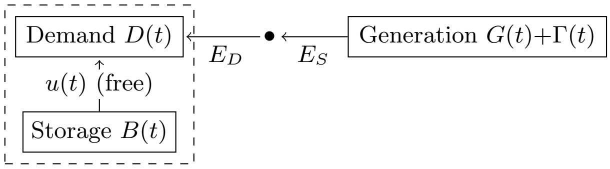


Figure 2: Market model in scenario B: storage is at the consumer.

By abuse of notation we still write the constraints of the consumer as $(E_D, u) \in X_D$. The energy bought is complemented using the storage system, and thus the consumer's payoff becomes

$$W_D(t) = -(v + c^{bo})(E_D(t){+}u(t){-}D(t){+}r^{da})^- - P(t)E_D(t).$$

The supplier controls the energy sold E_S and the real-time generation G. We say that they satisfy the supplier constraints and we write $(E_S, G){\in}X_S$ if and only if for all $t{\geq}0$, $0{\leq}E_S(t){\leq}G(t){+}\Gamma(t)$. The supplier's payoff remains the same as in the case she controls the storage: $W_S(t) = P(t)E_S(t) - cG(t)$.

In this scenario, the definition of a dynamic competitive equilibrium is similar to that presented in the case the supplier owns the storage: the only difference is that (4) and (5) are replaced respectively by $(E_D^e, u^e) \in \arg\max_{E_D, u \in X_D} \mathcal{W}_D$ and $(E_S^e, G^e) \in \arg\max_{(E_S,G)\in X_S} \mathcal{W}_S$. Note that here the expression of $\mathcal{W}_D$ is modified compared to (4).

3.5 Scenario C: Stand-Alone Storage Operator

In this scenario, the storage is owned by a third player, the *stand-alone storage operator* who seeks to maximize her profit via arbitrage on the real-time market: buying energy at low prices and selling at high prices.

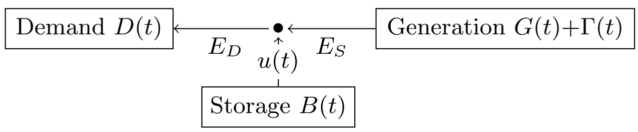


Figure 3: Market model in Scenario C: stand-alone storage operator.

The only control action of the storage operator is $u(t)$, the power discharged from the storage system at time t. Her constraints depend on the storage system parameters and are summarized writing $u \in X_B$. The payoff of the storage operator at time t is $W_B(t) = P(t)u(t)$.

The control, constraints, and payoff of the consumer are the same as in Section 3.3. In particular, her payoff is

$$W_D(t) = -(v + c^{bo})(E_D(t) - D(t) + r^{da})^- - P(t)E_D(t).$$

The control, constraints, and payoff of the supplier are the same as in Section 3.4. Again, the supplier's payoff remains equal to $W_S(t) = P(t)E_S(t) - cG(t)$.

The definition of a dynamic competitive equilibrium is again similar to the one presented in the case where the supplier owns the storage. The difference is that (5) is replaced by $(E_S^e, G^e) \in \arg\max_{(E_S,G)\in X_S} \mathcal{W}_S$, and (6) by $E_S(t) + u(t) = E_D(t)$. In addition a competitive equilibrium maximizes the welfare of the storage operator: $u^e \in \arg\max_{u\in X_B} \mathcal{W}_B$.

4. THE SOCIAL PLANNER'S PROBLEM

In this section we assume that the system is controlled by a single entity, a hypothetical social planner, who decides what is bought and sold, the generation and storage control processes, and whose objective is to maximize the system welfare over all feasible controls. After defining the social planner's optimal control problem, we characterize its solution, *i.e.*, the socially optimal controls. This analysis will be used later in the paper to state whether the market is efficient, or the strategic behavior of the various market actors has a negative impact in terms of social efficiency.

4.1 Social Welfare

A time t, the total system payoff $U(t)$ is either defined as $U(t) = U_D(t) + U_S(t)$ when the storage is controlled by the supplier (or the consumer) or as $U(t) = U_D(t)+U_S(t)+U_B(t)$ in case the storage is controlled by a stand-alone market actor. It can be easily checked that the total payoff does not depend on the market actor controlling the storage. Let us, for instance, assume that the storage is controlled by the supplier. As previously, we decompose the total payoff into a controllable part $W(t)$ and an uncontrollable part $\bar{U}(t)$. We have, for any $t \geq 0$:

$$W(t) = -(v + c^{bo})(E_D(t) - D(t) + r^{da})^- + P(t)(E_S(t) - E_D(t)) - cG(t). \quad (7)$$

When the market constraint $E_S = E_D$ is further imposed, the controllable part of the total payoff does not depend on the price $P(t)$ and is an increasing function of $E_S(t)$. Hence, the socially optimal controls satisfy: $E_S(t) = E_D(t) = \Gamma(t) + G(t) + u(t)$. These controls must also satisfy the generation and storage constraints, *i.e.*, $(\Gamma{+}G{+}u, G, u){\in}X_S$. Recall that the reserve process R is defined as the difference of the generated energy and of the demand:

$$R(t) := \Gamma(t) + G(t) - D(t) + r^{da}.$$

By abusing the notation, we write $(R, u) \in X_S$ if and only if $(\Gamma + G + u, G, u) \in X_S$. The controllable part of the total payoff can be further reduced and written as

$$W(t) = -(v + c^{bo})(R(t) + u(t))^- - cR(t). \quad (8)$$

A social planner want to maximize the social welfare $\mathcal{W}$:

$$\mathcal{W} := \mathbb{E}\int_0^\infty e^{-\gamma t}[-(v + c^{bo})(R(t) + u(t))^- - cR(t)]dt.$$

over all couple of processes $(R, u) \in X_S$. The solution of this problem is denoted by $(R^\star, u^\star)$, and the corresponding optimal controls at the various market actors are $E_D^\star = E_S^\star = R^\star + u^\star + D - r^{da}$, $G^\star = R^\star + D - \Gamma - r^{da}$, and $u^\star$.

4.2 Socially Optimal Controls

The following theorem characterizes the socially optimal controls. These controls define, for any given reserve r and storage state b, the way real-time generation evolves, *i.e.*, $g = \frac{\partial G}{\partial t}$, and the storage control process u. All proofs are presented in Appendix.

THEOREM 1. *The socially optimal controls satisfy: (Generation) For each b, there exists a threshold $\phi(b)$ such that the socially optimal generation control $g^\star$ is ζ^+ if $r < \phi(b)$ and ζ^- if $r > \phi(b)$. ϕ is a nonincreasing function. (Storage) The optimal storage control $u^\star$ satisfies:*

$$u^\star = \begin{cases} \max(-r, -C_{\max}) & \text{if } r \geq 0 \text{ and } b < B_{\max}, \\ \min(-r, D_{\max}) & \text{if } r < 0 \text{ and } b > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

The value function of the social planner problem is:

$$V(r, b) = \sup_{(R,u)\in X_S} \mathbb{E}_{(r,b)} \int_0^\infty e^{-\gamma t} \times \Big[-(v + c^{bo})(R(t) + u(t))^- - cR(t)\Big]\, dt, \quad (10)$$

where (r, b) is the initial condition: $R(0) = r$ and $B(0) = b$, and $\mathbb{E}_{(r,b)}[\cdot] := \mathbb{E}[\cdot|R(0) = r, B(0) = b]$.

In the proof of the above theorem, we establish that $\frac{\partial V}{\partial r}$ is well defined (almost everywhere), and that the threshold function ϕ characterizing the optimal generation control is $\phi(b) = \sup\{r : \frac{\partial V}{\partial r}(r, b) \geq 0\}$. The optimal storage control $u^\star$ has a simple interpretation: When the reserve is positive, the supplier first serves the demand and puts the remaining energy in the storage while ensuring that storage constraints are satisfied (e.g. the charging power cannot exceed $C_{\max}$). If the reserve is negative, the supplier can serve only part of the demand using the generated energy and has to extract energy from the storage to serve the remaining demand (if this is at all possible).

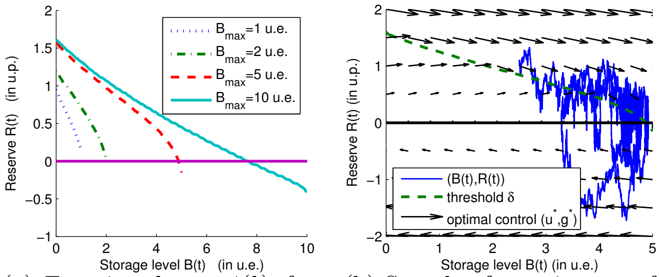
Under the socially optimal controls, we denote by $B^\star$ the storage level process. The system dynamics are characterized by $(R^\star, B^\star)$ and admit a steady-state whose stationary distribution is denoted by π. Under the assumption described in Section 3, $R^\star$ is a Brownian motion with variance σ and whose average drift varies over time when the generation control $g^\star$ switches values (e.g. from ζ^+ to ζ^-).

4.3 Numerical Example and Energy Units

To illustrate the socially optimal controls defined in Theorem 1, we plot the threshold function $b \mapsto \phi(b)$ in Figure 4(a) for different values of the storage energy capacity $B_{\max}$. We have the storage level b on the x-axis and the reserve r on the y-axis. Using the same representation, a sample-path of (R^*, B^*) is plotted in Figure 4(b) (solid blue line). The vector field corresponds to the optimal controls (u^*, g^*) and the dashed line to the function $b \mapsto \phi(b)$.

In all figures, the storage energy capacity $B_{\max}$ is expressed in *units of energy* (u.e.) and the maximum charging/discharging powers $C_{\max}$ and $D_{\max}$ are expressed in *units of power* (u.p.). For a variability σ and a ramping capability ζ, we choose that one unit of energy is equal to σ^4/ζ^3 and one unit of power corresponds to σ^2/ζ. We refer to Section 7 for the reasons for these scalings.

For example, let us consider the scenario envisioned for the UK in 2020 [3, 12] where wind power is used to cover 20% of the total electricity consumption – this corresponds to 26GW of peak power. At the scale of the country, the variability of the demand is small compared to the variability of the wind generation. This variability is due to uncertain forecast. Using the same data as [12], the square of the



(a) Function $b \mapsto \phi(b)$ for various values of the storage energy capacity $B_{\max}$.

(b) Sample of a trajectory of the optimal reserve and storage processes. $B_{\max} = 5\,$u.e.

Figure 4: Illustration of the optimal control law for the social planer's problem.

volatility of the wind generation is $\sigma^2 = 0.6\,(GW)^2/h$. This means that if the ramping capability $\zeta = 1$GW/h, one unit of energy corresponds to 360MWh and one unit of power to 600MW. For a ramping capability of $\zeta = 2$GW/h, one unit of energy corresponds to 45MWh and one unit of power to 300MW.

Throughout the paper, all numerical values are obtained by discretizing in time and space the original model to obtain a discrete Markov decision process. This control problem is then solved using brute-force dynamic programming to obtain the function $b \mapsto \phi(b)$. The simulation of a sample-path (Figures 4(b) and 5) is obtained using ϕ in a discrete-time continuous-space model and the steady-state performance indicator (Figures 6 to 11(b)) are computed by computing the stationary measure of the discrete-time discrete-space model. Unless otherwise specified, the parameters used for the simulations are σ=1, ζ^+=1, ζ^-=3, $B_{\max}$=1 u.e., $C_{\max}$=$D_{\max}$=3 u.p., η=1, γ=0.01, c=1 and $(v + c^{bo})$=5.

5. DYNAMIC COMPETITIVE EQUILIBRIA AND MARKET EFFICIENCY

Under a dynamic competitive equilibrium, the price process is such that the decisions taken over time by the various market actors maximize their respective welfares. In this section, we first prove that for all scenarios, the market is efficient in the sense that any dynamic competitive equilibrium maximizes the social welfare. We then prove the existence of such an equilibrium: we provide explicit expressions for the equilibrium price process and for the strategic controls used by the various market actors in equilibrium. We conclude the section by showing numerically the effect on prices of the presence of storage: in absence of storage, prices are volatile and can only take two values, 0 and the "choke-up" price $v + c^{bo}$ [8], whereas with storage, prices are smoother and oscillate around the marginal production cost c as the storage energy capacity grows large.

5.1 Market Efficiency

We first assume that dynamic competitive equilibria exist, and we prove that these equilibria are efficient in the sense that the corresponding controls maximize the system social welfare. This result is often referred to as the *social welfare theorem* in economics. A similar result has been established in [27] without storage. We show that the market remains efficient even in presence of storage.

THEOREM 2. *(Social Welfare Theorem) Assume that a dynamic competitive equilibrium exists. Then:*
(i) any competitive equilibrium maximizes the social welfare;
(ii) conversely, for any control processes (E^e_D, E^e_S, G^e, u^e) maximizing the social welfare, there exists a price process P^e such that $(P^e, E^e_D, E^e_S, G^e, u^e)$ is a competitive equilibrium.

5.2 Equilibria: Existence and Properties

If dynamic competitive equilibria exist, we know that they are socially efficient (even in presence of storage). We show that competitive equilibria indeed exist and characterize the corresponding price and control processes. More precisely, we identify a price process $P^\star$ that can lead to an equilibrium (in fact, as it turns out, $P^\star$ is the only price process leading to an equilibrium), and show that if $(E^\star_D, E^\star_S, G^\star, u^\star)$ are socially optimal controls, $(P^\star, E^\star_D, E^\star_S, G^\star, u^\star)$ is a competitive equilibrium

Let $(R^\star, u^\star)$ be the reserve-storage control process maximizing the social welfare (starting at $R^\star(0) = r$ and $B^\star(0) = b$). Denote by $B^\star$ the corresponding storage level process, and define the price process $P^\star$ as:

$$P^\star(t) = \begin{cases} 0 & \text{if } R^\star(t) + u^\star(t) > 0, \\ \eta \frac{\partial V}{\partial b}(R^\star(t), B^\star(t)), & \text{if } R^\star(t) + u^\star(t) = 0, R^\star(t) > 0, \\ \frac{\partial V}{\partial b}(R^\star(t), B^\star(t)), & \text{if } R^\star(t) + u^\star(t) = 0, R^\star(t) \le 0, \\ v + c^{bo}, & \text{if } R^\star(t) + u^\star(t) < 0. \end{cases} \tag{11}$$

THEOREM 3. *Let $P^\star, G^\star$ and $u^\star$ be defined as above. Then*

(i) $(P^\star, E^\star_D{=}\Gamma{+}G^\star{+}u^\star, E^\star_S{=}\Gamma{+}G^\star{+}u^\star, G^\star, u^\star)$ *is a competitive equilibrium when the storage is at the supplier.*

(ii) $(P^\star, E^\star_D = \Gamma + G^\star, E^\star_S = \Gamma + G^\star, G^\star, u^\star)$ *is a competitive equilibrium when the storage is at the consumer.*

(iii) $(P^\star, E^\star_D = \Gamma + G^\star + u^\star, E^\star_S = \Gamma + G^\star, G^\star, u^\star)$ *is a competitive equilibrium when there is a stand-alone storage owner.*

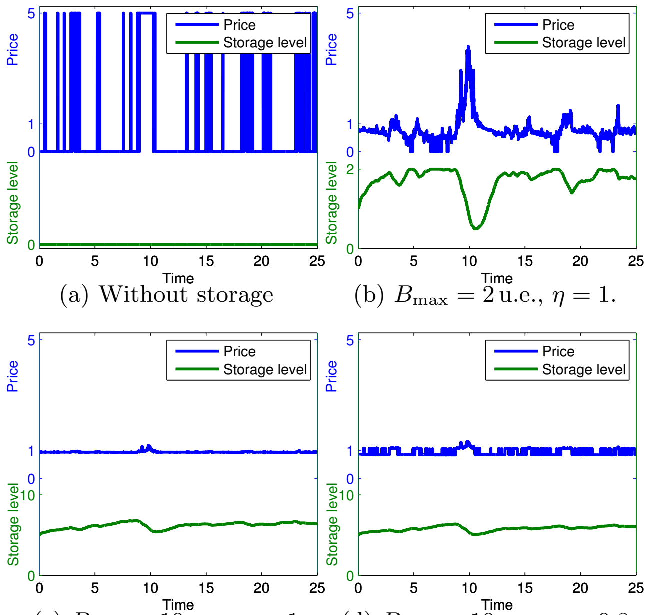
In particular, this theorem implies that at the equilibrium, the price, generation and storage control processes are the same in all three scenarios. Moreover these controls maximize the social welfare.

5.3 Equilibrium Price Distribution: the Impact of Storage

At the equilibrium, the price process (11) is a function of the optimal reserve and storage control processes. We plot in Figure 5 the evolution over time of the prices $P^*(t)$ and of the storage level $B^*(t)$. We fix the maximum charging and discharging powers to $C_{\max}$=$D_{\max}$=3 u.p. and we compare the results for four values of energy capacity of storage: $B_{\max}$=0 u.e. (*i.e.* no storage), 1 u.e., 3 u.e. and 10 u.e., where 1 u.p. = σ^2/ζ and 1 u.e.=σ^4/ζ^3 (see §4.3). We use the same random seed so that $D(t){-}\Gamma(t)$ is the same in all four cases.

As shown in [27], when there is no storage the prices oscillate between 0 and the "choke-up" price $(v + c^{bo})$ (Figure 5(a)). When $B_{\max} > 0$, the prices always remain between 0 and $(v{+}c^{bo})$, but they are smoother. When $B_{\max}$ is large and $\eta = 1$ (Figure 5(c)), the prices are almost constant and close to the marginal cost of production $c = 1$. When $B_{\max}$ is large and $\eta = 0.8$ (Figure 5(d)), the prices oscillate around $c = 1$, in this case, between 0.88 and $1.1 = 0.88/\eta$.

The optimal reserve and storage level process (R^*, B^*) is stationary. We compute numerically the distribution of

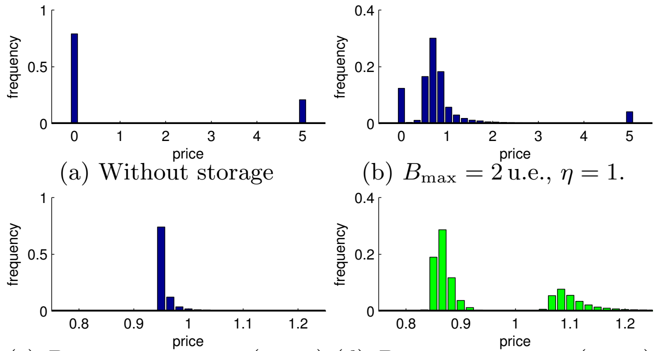


(a) Without storage (b) $B_{\max} = 2\,\text{u.e.}$, $\eta = 1$.

(c) $B_{\max} = 10\,\text{u.e.}$, $\eta = 1$ (d) $B_{\max} = 10\,\text{u.e.}$, $\eta = 0.8$

Figure 5: Evolution of prices and storage level over time for various storage energy capacities $B_{\max}$. For $B_{\max} = 10\,\text{u.e.}$, we compare $\eta = 0.8$ and $\eta = 1$.

prices in the steady-state regime of the process (R^*, B^*). The results are reported in Figure 6. Again, we observe that when there is no storage, the price takes only the two values 0 and $(v + c^{bo})$. When $\eta = 1$, the price tends to concentrate around the marginal production cost $c = 1$ as $B_{\max}$ increases. For $\eta = 0.8$, the price signals whether the storage owner should charge (price≈0.88) or discharge (price≈1.1).



(a) Without storage (b) $B_{\max} = 2\,\text{u.e.}$, $\eta = 1$.

(c) $B_{\max}$=10 u.e., η=1 (zoom) (d) $B_{\max}$=10ue, η=0.8(zoom)

Figure 6: Steady-state distribution of prices for various storage energy capacities $B_{\max}$. For $B_{\max} = 10\,\text{u.e.}$, we zoom on c=1 to compare $\eta = 0.8$ and $\eta = 1$.

6. STRATEGIC INVESTMENT IN STORAGE

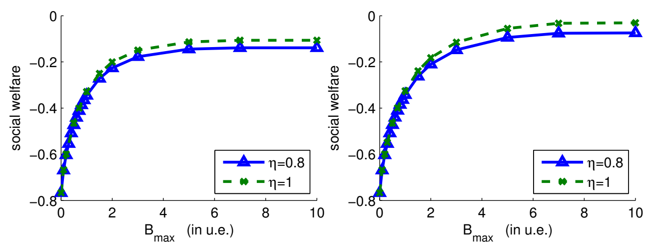
In this section, we study numerically the welfare of the different actors as a function of the energy capacity of storage and of the maximum charging/discharging powers. We first define and compute the socially optimal energy capacity (§6.1). We then show that when the supplier owns the storage, her optimal energy capacity is the socially optimal one. However, when the storage controlled by the consumer or by an independent actor, their optimal energy capacity is strictly lower. Finally, we show in §6.3 that the same results hold for the maximum charging/discharging powers.

6.1 Socially Optimal Energy Capacity

The total payoff, given by (8), is composed of two terms: the benefit for the consumer minus the cost of blackout, equal to $-(v + c^{bo})(R(t) + u(t))^-$, and minus the cost of producing the energy $-cG(t)$. Thus, if the initial reserve and storage process (r, b) is distributed as the stationary distribution π of the optimal reserve and storage processes, the expected social welfare is

$$- \mathbb{E}_{(r,b)\sim\pi} \int_0^\infty e^{-\gamma t} \left[(v + c^{bo})(R^* + u^*)^- + cR^* \right] dt$$
$$= \frac{1}{\gamma} \mathbb{E}_{(r,b)\sim\pi} \left[-(v + c^{bo})(R^* + u^*)^- - cR^* \right]. \quad (12)$$

The social welfare increases as the energy capacity ($B_{\max}$) or maximum charging/discharging power ($C_{\max}$ and $D_{\max}$) of the storage system increases. Thus, if we neglect the cost of installing additional energy capacity, the greater the storage system is, the greater the social welfare is.



(a) $C_{\max} = 1\,\text{u.p.}$ (b) $C_{\max} = 3\,\text{u.p.}$

Figure 7: Expected social welfare in the stationary regime as a function of the energy capacity $B_{\max}$.

In practice, however, storage capacity is expensive. Thus, installing additional storage capacity is worthwhile only as long as the resulting welfare gain is important. In Figure 7, we plot the average social welfare in a stationary regime, given by (12), as function of the storage capacity $B_{\max}$ for two values of the maximum charging/discharging power, $C_{\max} = D_{\max} = 1\,\text{u.p.}$ and $C_{\max} = D_{\max} = 3\,\text{u.p.}$, and for two values of the storage efficiency, $\eta = 0.8$ and $\eta = 1$. We observe that in all cases, the gain in welfare is important for low storage capacities and saturates rapidly. For example, when $C_{\max} = 1\,\text{u.p.}$, Figure 7(a), the saturation occurs for $B_{\max} \approx 4\,\text{u.e.}$. For $C_{\max} = 3\,\text{u.p.}$, the saturation occurs for $B_{\max} \approx 7\,\text{u.e.}$. We call these values the socially optimal storage capacities.

6.2 Storage Operator's Revenue

6.2.1 Storage at the Supplier

Let us first assume that the storage belongs to the supplier. As the market is efficient, the reserve process R and storage control u are equal to the socially optimal reserve processes R^* and u^*. Thus, at time t, the supplier sells $R^*(t) + u^*(t)$ at prices $P^*(t)$ and has a cost of production of $-cR^*(t)$. Her instantaneous payoff is $P^*(t)(R^*(t) + u^*(t)) - cR^*(t)$. The equilibrium price $P^*(t)$ is equal to

0 when $R^*(t) + u^*(t) > 0$ and to $(v + c^{bo})$ when $R^*(t) + u^*(t) < 0$. This shows that the supplier's payoff equals $-(v + c^{bo})(R^*(t) + u^*(t))^- - cR^*(t)$, which is exactly the total payoff. This shows that if the storage belongs to the supplier, her welfare increases as the energy capacity increases in the same proportion as the social welfare. Hence, the optimal energy capacity from a supplier-owned storage perspective is the same as the socially optimal capacity.

6.2.2 *Storage at the Consumer and at a Stand-Alone Storage Operator*

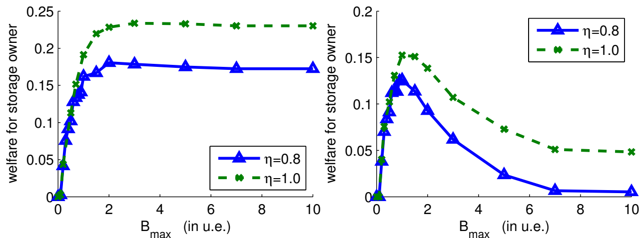
When the the consumer owns the storage, her payoff at time t is equal to $-(v + c^{bo})(R^*(t) + u^*(t))^- - P^*(t)R^*(t)$. The price $P^*(t)$ is equal to 0 when $R^*(t) + u^*(t) > 0$ and to $v + c^{bo}$ when $R^*(t) + u^*(t) < 0$. Thus we can write

$$\begin{aligned} P^*(t)R^*(t) &= P^*(t)(R^*(t) + u^*(t)) - P^*(t)u^*(t) \\ &= -(v + c^{bo})(R^*(t) + u^*(t))^- - P^*(t)u^*(t). \end{aligned}$$

This shows that the consumer's payoff is equal to $P^*(t)u^*(t)$. If the storage is owned by a stand-alone storage operator, her instantaneous payoff will be exactly the same: at time t, a quantity $u^*(t)$ is sold at price $P^*(t)$. Thus, the average welfare of the consumer is equal to that of a stand-alone storage operator:

$$\mathbb{E}_{(r,b)\sim\pi} \int_0^\infty dte^{-\gamma t} P^*(t)u^*(t) = \frac{1}{\gamma} \mathbb{E}_{(r,b)\sim\pi}[P^\star \cdot u^\star]. \quad (13)$$

In Figure 8, we plot the average welfare of a stand-alone storage operator as a function of the energy capacity $B_{\max}$. The system parameters are the same as those used to compute the social welfare in Figure 7. When $C_{\max} = D_{\max} = 1$, the expected welfare saturates for $B_{\max} = 2$ u.e. and diminishes slightly afterward. When $C_{\max} = D_{\max} = 3$ u.p., the expected welfare is maximal for $B_{\max} \approx 1.5$ u.e. and decreases sharply afterward. It diminishes almost to zero for $\eta = 0.8$ when $B_{\max}$ goes to infinity. In both cases, the expected welfare is maximal for a finite energy capacity and this capacity is much lower than the socially optimal capacity. This means that the consumers and the stand-alone storage operators have an incentive to undersize their storage.



(a) $C_{\max} = D_{\max} = 1$ u.p. (b) $C_{\max} = D_{\max} = 3$ u.p.

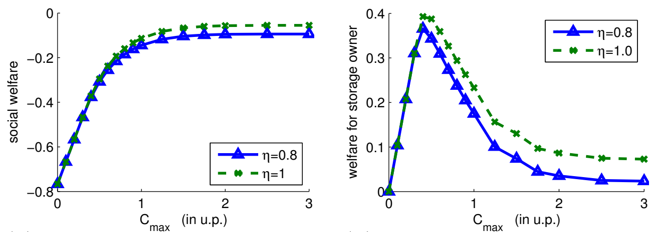
Figure 8: Expected welfare of the consumer when she owns the storage or of the stand-alone storage operator as a function of the energy capacity $B_{\max}$.

This result seems paradoxical, as it implies that even if we neglect the cost of storage, a storage owner would make less money with a larger energy capacity. The explanation comes from the price-taking assumption (A2). When the energy capacity grows, the price variations diminish. As a stand-alone storage owner gains only from buying at low price and selling at high price, her gain diminishes as the prices variability decreases. This situation is radically different when the supplier owns the storage. In this case, a higher storage leads to lower losses and therefore diminishes the production cost. This results in higher gain for the supplier even with a large storage.

6.3 Optimal Maximum Charging/Discharging Powers

As we just observed, when the storage belongs to the consumer or to a third party, the storage owner has an incentive to undersize her energy capacity, compared to a social planner. Figure 9 shows that we have a similar phenomenon regarding the maximum charging/discharging power.

In Figure 9(a), we plot the expected social welfare as a function of the maximum charging/discharging power $C_{\max} = D_{\max}$. This curve is similar to Figure 7 and the optimal charging/discharging powers are $C_{\max} = D_{\max} \approx 1$ u.p.. In Figure 9(b), we plot the expected welfare of a stand-alone storage operator. As for the energy capacity, it has a maximum for a relatively small value $C_{\max} = D_{\max} \approx 1$ u.p. and then decreases quickly. We plot these curves for $B_{\max} = 5$ u.e. but their shapes are similar for other values of $B_{\max}$, even small values like $B_{\max} = 0.5$ u.e..



(a) Social welfare or welfare of the supplier when she owns the storage (scenario A). (b) Welfare for the customer in scenario A or for a stand-alone storage operator.

Figure 9: Welfare of the players as a function of maximum charging/discharging power $C_{\max} = D_{\max}$.

7. VOLATILITY AND STORAGE SIZING

We now study the effect of σ (the volatility of the demand and renewable generation process) on the way the storage should be dimensioned. We show that if the ramping capability of the generators remains unchanged, then the storage energy capacity needed to mitigate this volatility scales as σ^4, and that the required charging/discharging powers scale as σ^2. If the volatility is fixed and the ramping capability ζ varies, then the storage capacity needed to mitigate the volatility scales as $1/\zeta^3$, and the maximum charging/discharging powers should scale as $1/\zeta$. In particular, this implies that to maintain a fixed optimal energy capacity, ζ should scale as $\sigma^{4/3}$.

This supports the idea that increasing the amount of power coming from renewables could lead to an increase in carbon dioxide emissions. Doubling the amount of renewables would result in multiplying the volatility by 2. In this case, it is probably easier and less expensive to install generators with $2^{4/3} \approx 2.5$ times larger ramping capability rather than to multiply the energy capacity by 16. However, these fast-ramping generators (*e.g.* gas turbines) tend to emit more carbon dioxide than conventional generators.

7.1 Scaling Laws for the Storage Capacities

Let us recall that σ is the volatility[2] of the difference of demand and of the renewable generation process and that ζ^- and ζ^+ are the ramping capabilities of the real-time generators. The next theorem shows that if the storage capacity $B_{\max}$ is scaled as σ^4/ζ^3 and the maximum charging/discharging powers $C_{\max}$ and $D_{\max}$ are scaled as σ^2/ζ, then the social welfare scales as σ^2/ζ.

In Section 6, we defined the socially optimal storage parameters. They correspond to the knee of the curve of Figures 7 and 9(a) beyond which installing new storage capacity leads to a negligible increase of the social welfare. Therefore, this theorem implies that the optimal storage energy capacity needed to accommodate the volatility scales as σ^4/ζ^3, and the maximum charging/discharging powers scale as σ^2/ζ:

$$B_{\max} = \Theta(\sigma^4/\zeta^3) \quad \text{and} \quad C_{\max} = \Theta(\sigma^2/\zeta).$$

THEOREM 4. *Let R, B be the socially optimal reserve and storage level processes when the system parameters are*

$$(\sigma, \zeta^+, \zeta^-, B_{\max}, C_{\max}, D_{\max}, \gamma).$$

Then, when the system parameters become

$$(x\sigma, y\zeta^+, y\zeta^-, \frac{x^4}{y^3}B_{\max}, \frac{x^2}{y}C_{\max}, \frac{x^2}{y}D_{\max}, \frac{x^2}{y^2}\gamma),$$

the corresponding socially optimal processes $R^{x,y}, B^{x,y}$ are

$$R^{x,y}(t) = \frac{x^2}{y}R\left(\frac{y^2}{x^2}t\right) \quad \text{and} \quad B^{x,y}(t) = \frac{x^4}{y^3}B\left(\frac{y^2}{x^2}t\right).$$

Moreover, let $\mathcal{W}$ be the social welfare for the initial parameters. The social welfare for the rescaled parameters is $(x^2/y)\mathcal{W}$.

The proof of this theorem consists in verifying that the proposed scaling works. It is detailed in [11].

7.2 Practical Implications

Theorem 4 implies that if ζ and σ are multiplied by the same factor x, then the optimal storage parameters increase linearly in x. Figure 10 illustrates this fact: we plot the optimal social welfare as a function of the available $B_{\max}$ for four values of x. Each time the remaining parameters are the same, and we choose $C_{\max} = D_{\max} = 1000$ to release the charge/discharge constraints of the storage. Figure 10 shows that for $x = 1$ (respectively $2, 4, 6$), the optimal energy capacity is approximately 8 (respectively 15, 30 and 50), that is to say linear in x (as expected).

For a fixed ramping constraint ζ, Theorem 4 states that the optimal energy capacity increases as the scaling factor of σ to the fourth power. In Figure 11(a) we scale ζ by the a fixed constant $y = 3$, we scale σ by a various factors $x = 1, 2, 3, 4$, and we plot the resulting optimal social welfare. For the four values of x, the respective optimal values of the energy capacity are approximately $7, 100, 400, 1000$ u.e., which correspond to roughly $\Theta(x^4)$, as expected.

[2]Note that the volatility corresponds to the standard deviation and not to the variance: as $D-\Gamma$ is a Brownian motion, this means that $D(t)-\Gamma(t)-D(0)+\Gamma(0)$ has a variance $\sigma^2 t$.

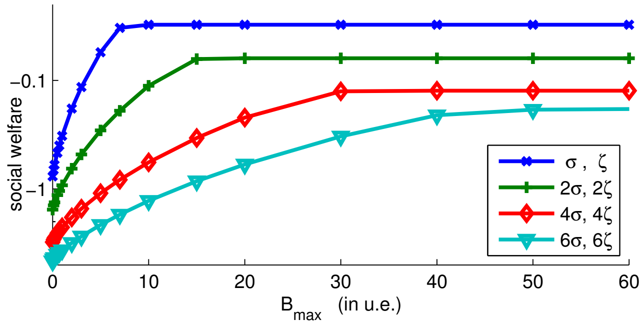
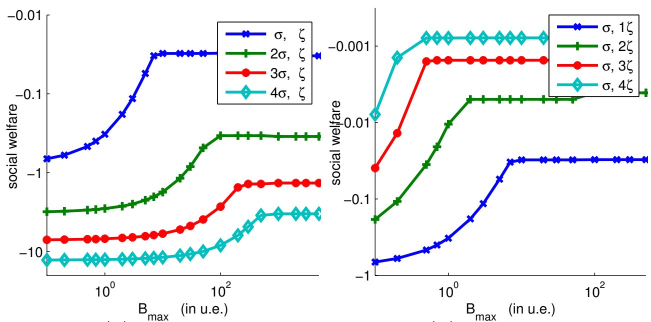


Figure 10: Social welfare as a function of $B_{\max}$ when σ and ζ are scaled by the same factor. $B_{\max}$ is expressed in unit of energy with $1\,\text{u.e.} = \sigma^4/\zeta^3$. The y-axis is in log-scale.

For a fixed scaling of the volatility σ, by Theorem 4 the optimal storage size decreases as the cube of the scaling factor of the ramping constraint ζ. This is illustrated in Figure 11(b), where we consider a fixed σ, and we scale ζ by factors $y = 1, 2, 3, 4$. We plot the optimal social welfare, and we observe the decreasing values of optimal energy capacity $10, 1.5, 0.5, 0.3$ ($\approx \Theta(y^{-3})$).



(a) Fixed ζ (b) Fixed σ

Figure 11: Social welfare as a function of the energy capacity $B_{\max}$ when one of σ or ζ is fixed. The plot is log-log scale. $B_{\max}$ is expressed in unit of energy with $1\,\text{u.e.} = \sigma^4/\zeta^3$.

8. CONCLUSION

We have shown that under the price-taking assumption, electricity markets remain efficient when we introduce storage capabilities in the system: they lead to an efficient allocation of generation and storage control and smooth the prices. However, we have shown that there is no incentive for consumers or third-party actors to install large storage devices, despite the fact that the required storage capacity needed to accomodate real-time flucuations explodes with high variability.

There are still many open questions. A first question concerns the case of oligopolies: Does the market remain efficient if a small number of players can influence prices? Another question is whether the market efficiency results also hold in a physical network as described in [27]. More specifically, how are these results affected by the placement of storage devices in such a network? Finally, the lack of incentives for actors to install large storage capacity raises the question of designing political incentives to encourage the development of storage systems.

APPENDIX

A. PROOF OF THEOREM 1

To derive socially optimal controls, we use the following structural properties for the value function V:

LEMMA 1. *(i) $(r,b) \mapsto V(r,b)$ is concave.*
(ii) V is sub-additive: for all $r^- \le r^+$ and $b^- \le b^+$,

$$V(r^-,b^-) + V(r^+,b^+) \le V(r^-,b^+) + V(r^+,b^-).$$

(ii) For all $b^- \le b^+$, and r:

$$0 \le V(r,b^+) - V(r,b^-) \le (b^+ - b^-)(v + c^{bo}).$$

The proof of the above lemma is postponed at the end of this section. Let us first prove the optimality of $g^\star$. We start from a reserve $R(0) = r$ and storage $B(0) = b$. The storage control is fixed and optimal. Note that due to the convexity of V, $\frac{\partial V}{\partial r}$ is well defined almost everywhere. Let $\delta > 0$. We denote by $d(\delta) = \int_0^\delta Z(t)dt$, and by $b' = B(\delta)$. Consider two production controls g_1 and g_2 different over $[0,\delta]$, but equal to the optimal control after δ. Denote by V_1 and V_2 the expected social welfare obtained using g_1 and g_2, respectively. Define $\xi_1 = \frac{1}{\delta}\int_0^\delta g_1(t)dt$ and $\xi_2 = \frac{1}{\delta}\int_0^\delta g_2(t)dt$. We can easily show that:

$$\begin{aligned} V_1 - V_2 &= e^{-\gamma\delta}\mathbb{E}[V(r + \xi_1\delta - d(\delta), b') \\ &\quad - V(r + \xi_2\delta - d(\delta), b')] + O(\delta^2) \\ &= \delta(\xi_1 - \xi_2)\frac{\partial V}{\partial r}(r,b) + O(\delta^2). \end{aligned}$$

We deduce that the optimal production control $g^\star$ is such that $g^\star = \zeta^+$ if $\frac{\partial V}{\partial r} > 0$ and $g^\star = \zeta^-$ if $\frac{\partial V}{\partial r} < 0$. Now since V is concave, the threshold function $\phi(b) = \sup\{r : \frac{\partial V}{\partial r} \ge 0\}$ is well defined, and for $r < \phi(b)$ (resp. $r > \phi(b)$), the optimal production control is $g^\star = \zeta^+$ (resp. ζ^-). The fact that ϕ is decreasing is directly deduced from (ii) in Lemma 1.

Next we prove the optimality of $u^\star$. We establish that $u^\star$ is optimal when starting from a reserve $r = R(0)$ over a small time interval $[0,\delta]$. The production control is fixed, and we denote by $r' = R(\delta)$. Let $b = B(0)$.

Case 1: $r > C_{\max}$ and $\delta > 0$ is such that $R(s) \ge C_{\max}$ for all $s \in [0,\delta]$ (a.s.). We have $u^\star(s) = -C_{\max}$ for all $s \in [0,\delta]$. If this control is used, the expected social welfare is denoted by V_1. Now assume that instead we use the control $u \in [-C_{\max}, D_{\max}]$, in which case, the expected welfare is V_2. At time δ, under control $u^\star$, the state of the storage is $b + c_1$, and under u, it is $b + c_2$. We have:

$$c_1 = \eta\delta C_{\max} \ge -\int_0^\delta (\eta u(s)1_{u(s)<0} + u(s)1_{u(s)\ge 0})ds = c_2.$$

Now, observe that there is no blackout in $[0,\delta]$ under both controls, and hence:

$$V_1 - V_2 = e^{-\gamma\delta}\mathbb{E}[V(r', b + c_1) - V(r', b + c_2)] \ge 0.$$

Case 2. $r \in (0, C_{\max})$ and $\delta > 0$ is such that $C_{\max} \ge R(s) \ge 0$ for all $s \in [0,\delta]$ (a.s.). We have $u^\star(s) = -R(s)$ for all $s \in [0,\delta]$. If this control is used, the expected social welfare is denoted by V_1. Now assume that instead we use the control $u \in [-C_{\max}, D_{\max}]$, in which case, the expected welfare is V_2. At time δ, under control $u^\star$, the state of the storage is $b + c_1$, and under u, it is $b + c_2$. We have: $c_1 = \int_0^\delta \eta R(s)ds$, and

$$\begin{aligned} c_2 &= -\int_0^\delta (\eta u(s)1_{u(s)<0} + u(s)1_{u(s)\ge 0})ds \\ &\le -\int_0^\delta \eta(u(s)1_{u(s)\le -R(s)} + u(s)1_{-R(s)<u(s)<0})ds \\ &\le -\int_0^\delta \eta(u(s)1_{u(s)\le -R(s)} - R(s)1_{-R(s)<u(s)<0})ds. \end{aligned}$$

We deduce that: $c_2 - c_1 \le \eta\int_0^\delta (R(s) + u(s))^- ds$. Now observe that under control u, if $R(s) + u(s) < 0$ we have a blackout, and hence:

$$\begin{aligned} V_1 - V_2 =& (v + c^{bo})\int_0^\delta ds e^{-\gamma s}(R(s) + u(s))^- \\ &+ e^{-\gamma\delta}\mathbb{E}[V(r', b + c_1) - V(r', b + c_2)]. \end{aligned}$$

From (iii) in Lemma 1,

$$V_1 - V_2 \ge (1 - \eta)(v + c^{bo})\int_0^\delta ds(R(s) + u(s))^- \ge 0.$$

Case 3: $r = C_{\max}$. This case can be handled by combining the arguments involved in the two first cases, and decomposing the interval $[0,\delta]$ into the two sets where $R(s) \ge C_{\max}$ and $0 \le R(s) < C_{\max}$.

The cases where $r \le 0$ can be treated in a similar way.

A.1 Proof of Lemma 1

For (i), we define by $(R_1^\star, u_1^\star)$ (resp. $(R_2^\star, u_2^\star)$) the optimal reserve and storage control processes when starting at (r_1, b_1) (resp. (r_2, b_2)). For any $\theta \in (0,1)$, the processes $(\theta R_1^\star + (1-\theta)R_2^\star, \theta u_1^\star + (1-\theta)u_2^\star)$ correspond to feasible controls when the system starts in state $(r,b) = (\theta r_1 + (1-\theta)r_2, \theta b_1 + (1-\theta)b_2)$. Hence:

$$\begin{aligned} V(r,b) &\ge \mathbb{E}\int_0^\infty e^{-\gamma t}W(R(t), u(t))dt \\ &\ge \theta V(r_1, b_1) + (1-\theta)V(r_2, b_2), \end{aligned}$$

where the last inequality is obtained by concavity of $W(r,u)$. Thus V is concave.

(ii) can be proved using the same method as for (i). Regarding (iii), let $b^- \le b^+$ and u_- and u_+ be optimal controls starting from b^- and b^+. Let B_+ be the storage level process starting at b^+ under control u_+. We introduce the control u (and the corresponding storage level process B that starts from b^-):

$$u(t) = \begin{cases} u_+(t) & \text{if } B(t) > 0 \text{ or } u_+(t) < 0 \\ 0 & \text{otherwise} \end{cases}$$

By abuse of notation, we denote $V(u)$ the value function of the control u. As u_- is optimal starting from b^- and u is a valid control starting from b^-, we have:

$$\begin{aligned} &V(b^+) - V(b^-) = V(u_+) - V(u) \\ &= \mathbb{E}\left[\int_0^\infty e^{-\gamma t}(v + c^{bo})[(R(t) + u(t))^- - (R(t) + u_+(t))^-]dt\right] \\ &\le \mathbb{E}\left[\int_0^\infty (v + c^{bo})(u_+(t) - u(t))dt\right] \le (v + c^{bo})(b^+ - b^-). \end{aligned}$$

The first inequality is obtained remarking that $x \mapsto -(R+x)^-$ is 1-lipschitz. The last inequality is obtained by combining the facts that $b^+ - B_+(T) = \int_0^T f(u_+(t))dt$, $b^- - B(T) = \int_0^T f(u(t))dt$, where $f(x) = x(1_{x>0} + \eta 1_{x\leq 0})$, and that by construction $B(T) \leq B_+(T)$ for any T – the inequality is deduced by letting $T \to \infty$.

B. PROOF OF THEOREM 2

To simplify notations, we define $E = (E_D, E_S, G, u)$ and we write $E \in X_C$ if $E_D \in X_D$ and $(E_S, G, u) \in X_S$. To emphasize the dependence of the social welfare on the process E, we use the notations $\mathcal{W}(E)$. Similarly we denote by $\mathcal{W}_D(E_D, P)$ and $\mathcal{W}_S(E_S, G, u, P)$ the welfares of the consumer and the supplier. Moreover, we define the inner product of two stochastic processes F_1 and F_2 as:

$$\langle F_1, F_2\rangle := \mathbb{E}\int_0^\infty e^{-\gamma t}F_1(t)F_2(t)dt.$$

Using this notation, the consumer's welfare becomes $\mathcal{W}_D := \langle W_D, \mathbf{1}\rangle$, where $\mathbf{1}$ is the constant process, always equal to 1. The proof is detailed for scenario A[3].

To prove the result, we proceed as in [27] and we interpret the social welfare optimization problem as the problem of maximizing $\mathcal{W}_D(E_D, P) + \mathcal{W}_S(E_S, G, u)$ subject to the constraints $E \in X_C$ and $E_D = E_S$. The last constraint is relaxed, and the price process P serves as corresponding Lagrange multipliers. The Lagrangian is:

$$\begin{aligned}\mathcal{L}(E, P) &= -\mathcal{W}(E) + \langle P, E_D - E_S\rangle \\ &= -\mathcal{W}_D(E_D, P) - \mathcal{W}_S(E_D, G, u, P).\end{aligned}$$

The dual function h is defined as: $h(P) = \inf_{E\in X_C}\mathcal{L}(E, P)$. Let $E \in X_C$. If $E_D = E_S$, weak duality holds:

$$h(P) \leq \mathcal{L}(E, P) = -\mathcal{W}(E).$$

We further establish that for $E^e \in X_C$ such that $E^e_D = E^e_S$, (E^e, P) is a competitive equilibrium if and only if $h(P) = -\mathcal{W}(E)$.

1. Assume that (E^e, P) is a competitive equilibrium. Then:

$$\begin{aligned}h(P) &= \inf_{E\in X_C}\mathcal{L}(E, P) \\ &= \inf_{E\in X_C}(-\mathcal{W}_D(E_D, P) - \mathcal{W}_S(E_S, G, u, P)) \\ &= -\sup_{E_D\in X_D}\mathcal{W}_D(E_D, P) \\ &\qquad - \sup_{(E_S,G,u)\in X_S}\mathcal{W}_S(E_S, G, u, P) \\ &= -\mathcal{W}_D(E^e_D, P) - \mathcal{W}_S(E^e_S, G^e, u^e, P) \\ &= \mathcal{L}(E^e, P) = -\mathcal{W}(E).\end{aligned}$$

2. Conversely, assume that under (E^e, P), $h(P) = -\mathcal{W}(E)$. Since $(E^e_D, E^e_S) \in X_T$, we deduce that: $h(P) = \mathcal{L}(E^e, P)$, which in turn implies that:

$$\begin{aligned}-\sup_{E_D\in X_D}\mathcal{W}_D(E_D, P) - \sup_{(E_S,G,u)\in X_S}\mathcal{W}_S(E_S, G, u, P) \\ = -\mathcal{W}_D(E^e_D, P) - \mathcal{W}_S(E^e_S, G^e, u^e, P).\end{aligned}$$

We conclude that E^e_D maximizes the welfare from the consumer's perspective, and that (E^e_S, G^e, u^e) maximizes supplier's welfare under price process P.

From the above analysis, we deduce that any competitive equilibrium maximizes the social welfare. Now let (E^e, P^e) be a competitive equilibrium, and let $E \in X_C$ be socially optimal. Since both E and E^e maximize the social welfare, we have: $-\mathcal{W}(E) = -\mathcal{W}(E^e) = h(P^e)$. This implies that (E, P^e) is a competitive equilibrium.

C. PROOF OF THEOREM 3

The proof of the theorem consists in showing that under the price process $P^\star$, the controls maximizing the social welfare are also optimal from the consumer's and supplier's perspectives.

The technical difficulties of the proof are concentrated on the following result (Lemma 2) that relates the price process $P^\star$ and the value function of the social planner problem V. It is the key results to show that the social and selfish problems coincide.

LEMMA 2. *Let $P^\star$ be the price process defined by Equation (11) and V the value function for the social planner's problem, defined by Equation (10). Then, we have:*

$$\frac{\partial V}{\partial r}(r, b) = \mathbb{E}_{r,b}\int_0^\infty e^{-\gamma t}(P^\star(t) - c)dt.$$

The proof of this lemma is technical and is detailed at the end of the section (§C.4).

We now detail how to use this results when the storage is at the supplier (§C.1), the consumer (§C.2) or is an standalone player (§C.3).

C.1 Storage is at the Supplier

We now prove that, under price $P^\star$, the choice $E^\star_S = E^\star_D = \Gamma + G^\star + u^\star$ optimizes both the consumer's welfare and the supplier's welfare.

Consumer – The consumer's welfare is

$$\mathbb{E}\int_0^\infty e^{-\gamma t} - (v + c^{bo})(E_D(t) - D(t) + r^{da})^- - P^\star(t)E_D(t)dt.$$

As there is no ramping constraints in the consumer's optimization (*i.e.*, $E_D \in X_D$), an optimal strategy is myopic in the sense that for any $t \geq 0$, an optimal strategy $E^\star_D(t)$ satisfies $E^\star_D(t) \in \arg\max_e\{-(v + c^{bo})(e - D(t) + r^{da})^- - P^\star(t)e\}$.

Recall that $R^\star = G^\star + \Gamma^\star - D + r^{da}$. We consider three cases:

1. When $P(t) = 0$ (*i.e.* $R^\star(t) + u^\star(t) > 0$) any $e \geq D(t) - r^{da}$ maximizes the consumer's payoff. Thus $E^\star_D(t) = \Gamma(t) + G^\star(t) + u^*(t) > D(t) - r^{da}$ is optimal for the consumer.
2. When $0 < P^\star(t) < (v + c^{bo})$ (*i.e.* when $R^\star(t) + u^\star(t) = 0$), her payoff is maximized for $E^\star_D(t) = D(t) - r^{da} = \Gamma(t) + G^\star(t) + u^*(t)$.
3. When $P(t) = v + c^{bo}$, her payoff is maximized for any $E^\star_D(t) \leq D(t) - r^{da}$. Thus, $E^\star_D(t) = \Gamma(t) + G^\star(t) + u^*(t) < D(t) - r^{da}$ is optimal for the consumer in that case.

Supplier. It remains to show that $(G^\star, u^\star)$ maximizes the welfare of the supplier. By assumption (A1-A3), the real

[3] The only difference for the two other scenarios concerns the notations *i.e.*, replacing $\mathcal{W}_D(E_D, P)$ and $\mathcal{W}_S(E_S, G, u, P)$ by $\mathcal{W}_D(E_D, u, P)$ and $\mathcal{W}_S(E_S, G, P)$ for scenario B and by $\mathcal{W}_D(E_D, P)$, $\mathcal{W}_S(E_S, G, P)$ and $\mathcal{W}_D(u, P)$ for scenario C

time price $P(t)$ and and the generation of renewable energy $\Gamma(t)$ are uncontrollable by the players. Moreover, as $E_S(t) \leq \Gamma(t)+u(t)+G(t)$ and the payoffs of the supplier is increasing in E_S, a supplier will chose $E_S(t) = \Gamma(t)+u(t)+G(t)$. Thus, a direct computation shows that the welfare of the supplier can be solely expressed as a function of the reserve process R and the control process u. Her payoff can be written (up to uncontrollable parts) as $W_S(t) = P^\star(t)(R(t)+u(t)) - cR(t)$.

We define by X_R the set of possible production controls: $(R, u) \in X_S$ iff $R \in X_R$ and $u \in X_B$. This decomposition is possible because the constraints on production and storage are not coupled. Observe that the optimal control problem that the supplier solves is equivalent to:

$$\max_{(R,u)\in X_S} \mathbb{E} \int_0^\infty e^{-\gamma t}(P^\star(t)(R(t) + u(t)) - cR(t))dt.$$

The corresponding value function $V_S(r', b)$ is:

$$V_S(r', b) = \sup_{(R,u)\in X_S} \mathbb{E}_{(r',b)} \int_0^\infty e^{-\gamma t} [P^\star(t)(R(t) + u(t)) - cR(t)]dt.$$

We have $V_S(r', b) = V_R(r') + V_B(b)$ where

$$V_R(r') = \sup_{R\in X_R} \mathbb{E}_{r'} \int_0^\infty e^{-\gamma t}(P^\star(t) - c)R(t)dt \qquad (14)$$

$$V_B(b) = \sup_{u\in X_B} \mathbb{E}_b \int_0^\infty e^{-\gamma t}P^\star(t)u(t)dt. \qquad (15)$$

Optimality of $R^\star$: From the above expression, we simply deduce that:

$$\frac{\partial V_R}{\partial r'} = \mathbb{E}_{r'} \int_0^\infty e^{-\gamma t}(P^\star(t) - c)dt.$$

From Lemma 2, we conclude that $\frac{\partial V_S}{\partial r} = \frac{\partial V}{\partial r}$, and hence $R^\star$ also maximizes the supplier's welfare.

Optimality of $u^\star$: As $P^*(t) \geq 0$, it should be clear that $\partial V_B/\partial b \geq 0$. Next, we show that $\partial V_B/\partial b \leq (v + c^{bo})$. Now let $b \leq b'$. Using the same argument as in the proof of (iii) of Lemma 1, we obtain:

$$V_B(b') - V_B(b) \leq (v + c^{bo})(b' - b).$$

This shows that $\partial V_B/\partial b \leq (v + c^{bo})$.

We show next that when $0 < P^*(t) < v + c^{bo}$, $\partial V_B/\partial b = P^*(t)$. To see that, we consider a discharge control $d_\epsilon(t) = \sqrt{\epsilon}\mathbf{1}_{t\leq\sqrt{\epsilon}}$, which is such that $\int_t d_\epsilon(t)dt = \epsilon$. Let u_b and $u_{b+\epsilon}$ be the optimal control starting from b and $b + \epsilon$. We have:

$$V(b+\epsilon) - V(b) \geq V(u_b + d_\epsilon) - V(u_b) = P^*(t)\epsilon + o(\epsilon)$$
$$V(b+\epsilon) - V(b) \leq V(u_{b+\epsilon}) - V(u_{b+\epsilon} - d_\epsilon) = P^*(t)\epsilon + o(\epsilon)$$

Therefore, $\partial V_B/\partial b = P^*(t)$.

As $0 \leq \partial V_B/\partial b \leq (v + c^{bo})$, it should be clear that when $P^*(t) = 0$ (or $P^*(t) = v+c^{bo}$), the control u that maximizes the welfare of the storage owner satisfies $u(t) = -C_{\max}$ (or $P^*(t) = D_{\max}$). Furthermore, when $0 < P^*(t) < v + c^{bo}$, $\partial V_B/\partial b = \partial V/\partial b$, and hence $u^\star$ is also optimal in this case.

C.2 Storage is at the Consumer

Consumer. Recall from §3.4 that the consumer's payoff is

$$-(v + c^{bo})(E_D(t) + u(t) - D(t) + r^{da})^- - P^\star(t)E_D(t).$$

Let us denote $E_u(t) = E_D(t) + u(t)$. The consumer's payoff can be rewritten as

$$-(v + c^{bo})(E_u(t) - D(t) + r^{da})^- - P^\star(t)E_u(t) + P^\star(t)u(t)).$$

As $E_D(t)$ is unconstrainted, $E_u(t)$ is also unconstrainted. Therefore, the process E_u^* that maximizes the welfare associated with $-(v + c^{bo})(E_u(t) - D(t) + r^{da})^- - P^\star(t)E_u(t)$ is the same as the process of E_D that maximizes the consumer's welfare in §C.1: $E_u^*(t) = \Gamma(t) + G^*(t) + u^*(t)$.

Moreover, as E_D is unconstrainted, the choice of u is not constrained by the choice of E_u. Thus, maximizing the welfare associated with $P^\star(t)u(t)$ is the same as maximizing (15) in §C.1. This shows that $u^*(t)$ is optimal from a consumer's perspective.

Combining the two results, we get that $E_D^*(t) = \Gamma(t) + G^*(t)$ and $u^*(t)$ are optimal for the consumer.

Supplier. As in §C.1, a direct computation shows that the supplier seeks to maximize (14) and has the same constraints as in §C.1. Thus, its optimal generation process is also G^* and $E_S = \Gamma + G^*$.

C.3 Stand-Alone Storage Operator

The proof for this scenario comes from the proof for the scenario when the storage is at the supplier. When there is a stand-alone storage operator, we have three payers:

- A consumer, that has the same welfare and constraints of the consumer of C.1.
- A supplier that optimizes Equation (14).
- A storage operator that optimizes Equation (15).

We have shown in C.1 that that the optimization problem of a supplier that controls both generation and storage can be decomposed in two independent optimization problems: one for the generation (Eq (14)) and one for the use of the storage (Eq. (15)). Hence, even if Equations (14) and (15) are controlled by two independent players, they will lead to the same socially optimal controls.

C.4 Smoothness of Prices

Before proving Lemma 2, we first start by a results that provide a direct formula for the derivative of the value function.

LEMMA 3. *Let V be the value function associated to the social planner's problem (Equation (10)). Then:*

(i) Let $R^\star, B^\star$ be the reserve and storage process starting from (r, b) and let T and τ be the first time that $B^\star(t)$ hits 0 and $B_{\max}$ for the first time, i.e.,

$$T = \inf\{t \mid B^\star(t) = 0\} \textit{ and } \tau = \inf\{t \mid B^\star(t) = B_{\max}\}.$$

$$\frac{\partial V}{\partial b} = (v + c^{bo})\mathbb{E}\left[e^{-\gamma T}\mathbf{1}_{\{T<\tau\}}\right]. \qquad (16)$$

(ii) $\frac{\partial V}{\partial b}(r, b)$ is continuous in (r, b) for all $(r, b) \in \mathbb{R}^* \times (0; B_{\max})$.

(iii) If $r < 0$, then $\lim_{b\to 0} \partial V/\partial b(r, b) = (v + c^{bo})$.

(iv) If $r > 0$, then $\lim_{b\to B_{\max}} \partial V/\partial b(r, b) = 0$.

PROOF. Let $(r, b) \in \mathbb{R}^* \times (0; B_{\max})$ and let $\varepsilon > 0$. Let (R, B) be an optimal reserve and storage process starting from (r, b) and let u be the associated storage control. We define a storage process B_ε and its storage control u_ε by:

$$B_\varepsilon(0) = b + \varepsilon$$
$$u_\varepsilon(t) = \begin{cases} \max(-R(t), -C_{\max}) & \text{if } R(t){\geq}0 \wedge B_\varepsilon(t){<}B_{\max}, \\ \min(-R(t), D_{\max}) & \text{if } R(t){<}0 \wedge B_\varepsilon(t){>}0, \\ 0 & \text{otherwise.} \end{cases}$$

We also define the two associated stopping times T_ε and τ_ε:

$$T_\varepsilon = \inf\{t \mid B_\varepsilon(t) = 0\};$$
$$\tau_\varepsilon = \inf\{t \mid B_\varepsilon(t) = B_{\max}\}.$$

By definition of B and B_ε, for all t, we have

$$B(t) \leq B_\varepsilon(t) \leq B(t) + \varepsilon.$$

In particular, this implies that $B_{-\varepsilon}$ hits 0 before $B(t)$. Hence: $T \leq T_\varepsilon$ and $\tau_\varepsilon \leq \tau$.

Let us now compare the welfare of the reserve and storage process R, B_ε and R, B_ε. We have:

$$W(R, B_\varepsilon) - W(R, B) = \mathbb{E} \int_0^\infty e^{-\gamma t}(v + c^{bo})\Big[(R(t) + u(t))^- - (R(t) + u_\varepsilon(t))^-\Big]dt.$$

By definition of u and u_ε, we have $u(t) \leq u_\varepsilon(t)$ for all t. Hence, for all t, we have, $(R(t) + u(t))^- \geq (R(t) + u_\varepsilon(t))^-$.

Assume that for a given sample path, we have $T_\varepsilon < \tau_\varepsilon$. Then for all $t < T_\varepsilon$, either $u(t) = u_\varepsilon(t)$ or $B(t) = 0$, $R(t) < 0$ and $B_\varepsilon(t) > 0$. In particular, this implies that

- $(R(t) + u(t))^- \geq (R(t) + u_\varepsilon(t))^- = u_\varepsilon(t) - u(t)$ for all $t < T_\varepsilon$;
- $u(t) = u_\varepsilon(t)$ for all $t < T$; and
- $B(T_\varepsilon) \leq B_\varepsilon(T_\varepsilon) = 0$.
- For all $t > T_\varepsilon$, we have $B(t) \leq B_\varepsilon(t)$ and $u(t) = u_\varepsilon(t)$.

Hence:

$$\begin{aligned} \int_0^\infty e^{-\gamma t}\Big[(R(t) + u(t))^- - (R(t) + u_\varepsilon(t))^-\Big]dt \\ = \int_T^{T_\varepsilon} e^{-\gamma t}(u_\varepsilon(t) - u(t))dt \\ \geq e^{-\gamma T_\varepsilon} \int_T^{T_\varepsilon} (u_\varepsilon(t) - u(t))dt \\ = \varepsilon e^{-\gamma T_\varepsilon}. \end{aligned}$$

Therefore,

$$W(R, B_\varepsilon) - W(R, B) \geq (v + c^{bo})\mathbb{E}\left[\mathbf{1}_{\{T_\varepsilon < \tau_\varepsilon\}} e^{-\gamma T_\varepsilon}\right].$$

Recall that T is the time at which the storage hits 0. Since the derivative of B is of the same sign as R, we have $R(T) \leq 0$. Moreover, it should be clear that $R(T) \neq 0$ almost surely.

Assume that $R(T) < 0$. As the function $t \mapsto R(t)$ is continuous, then there exists $\delta, \xi > 0$ such that if $t \leq T + \xi$, then $R(t) \leq -\delta$. This implies that if $\varepsilon \leq \delta\xi$, then $T_\varepsilon \leq T + \xi$. Hence, T_ε converges to T almost surely as ε goes to 0. Similarly, we have also $\tau_\varepsilon \to \tau$. This shows that the right derivative of V with respect to b is greater than (16).

Let us now consider a sequence of initial condition (r_b, b_n) that goes to (r, b) as n goes to infinity. Because of the form of the optimal control (Theorem 1), it should be clear that the corresponding sequence of optimal reserve and storage processes $R_n^\star, B_n^\star$ converges to $R^\star, B^\star$ almost surely. Moreover, if $R(T) \neq 0$, the corresponding stopping time T_n and τ_n converge almost surely to T and τ. This implies that the right derivative of V w.r.t b is continuous in (r, b). As V is concave, this implies that V is differentiable everywhere and satisfies (16). This concludes the proof of (i) and (ii).

(iii) and (iv) comes directly from Equation (16):

- When $r < 0$ and $b \to 0$, we have $T < \tau$ almost surely and $\lim_{b\to 0} T = 0$. This implies (iii).
- When $r > 0$ and $b \to B_{\max}$, we have $T > \tau$ almost surely. This implies (iv).

□

C.4.1 *Proof of Lemma 2*

Let ε (typically small) and let R, B be the optimal reserve and storage process starting from (r, b). Let u_ε be a storage control process on $[0; \varepsilon]$ that is equal to the optimal storage control when the reserve is $R{+}\varepsilon$ and the initial storage level is $B(0)$. u_ε is defined as in (9):

$$u_\varepsilon(t) = \begin{cases} \max(-R(t), -C_{\max}) & \text{if } R(t) \geq 0 \text{ and } B_\varepsilon < B_{\max}, \\ \min(-R(t), D_{\max}) & \text{if } R(t) < 0 \text{ and } B_\varepsilon > 0, \\ 0 & \text{otherwise.} \end{cases}$$

with $B_\varepsilon(t) = B(0) + \int_0^t -u_\varepsilon(s)(\mathbb{1}_{\{u_\varepsilon(s)>0\}} + \eta\mathbb{1}_{\{u_\varepsilon(s)<0\}})ds$.

The process (R, B) is optimal when starting from (r, b). Thus:

$$V(r, b) = \mathbb{E}\Big[\int_0^\varepsilon e^{-\gamma t}W(R(s), B(s))ds{+}e^{-\gamma\varepsilon}V(R(\varepsilon), B(\varepsilon))\Big].$$

The process $(R + \varepsilon, B_\varepsilon)$ is suboptimal when starting from $(r + \varepsilon, b)$. Hence, we have:

$$\begin{aligned} V(r{+}\varepsilon.b) &\geq \mathbb{E}\Big[\int_0^\varepsilon e^{-\gamma t}W(R(s){+}\varepsilon, B_\varepsilon(s))ds{+}e^{-\gamma\varepsilon}V(R(\varepsilon){+}\varepsilon, B_\varepsilon)\Big] \\ &= \mathbb{E}\Big[\int_0^\varepsilon e^{-\gamma t}W(R(s) + \varepsilon, B_\varepsilon(s))ds \\ &\quad + e^{-\gamma\varepsilon}[V(R(\varepsilon) + \varepsilon, B_\varepsilon(\varepsilon)) - V(R(\varepsilon) + \varepsilon, B(\varepsilon))] \\ &\quad + e^{-\gamma\varepsilon}V(R(\varepsilon) + \varepsilon, B(\varepsilon))\Big] \end{aligned}$$

This shows that $V(r, b) - V(r + \varepsilon, b)$ is greater than:

$$\begin{aligned} \mathbb{E}\Big[\int_0^\varepsilon e^{-\gamma t}(W(R, B) - W(R + \varepsilon, B_\varepsilon))ds \\ + e^{-\gamma\varepsilon}[V(R(\varepsilon) + \varepsilon, B_\varepsilon(\varepsilon)) - V(R(\varepsilon) + \varepsilon, B(\varepsilon))] \\ + e^{-\gamma\varepsilon}V(R(\varepsilon, B(\varepsilon)) - V(R(\varepsilon) + \varepsilon, B(\varepsilon))\Big] \end{aligned} \quad (17)$$

Let us now define a process B_ε on all $t \geq 0$. It is left-continuous on all interval of the form $(k\varepsilon; (k+1)\varepsilon]$, and corresponds to the storage control u_ε on these interval. It is reseted to $B(k\varepsilon)$ at $(k\varepsilon)^-$. This means that for all $t \in (k\varepsilon; (k+1)\varepsilon]$:

$$B(t) = B(k\varepsilon) + \int_0^s -u_\varepsilon(s)(\mathbb{1}_{\{u_\varepsilon(s)>0\}} + \eta\mathbb{1}_{\{u_\varepsilon(s)<0\}})ds.$$

Although B_ε is not a valid storage process on $[0; T]$ (it can be discontinuous on the points $k\varepsilon$), it is valid on all interval

$[k\varepsilon;(k+1)\varepsilon]$. Thus, iterating the inequality (17), $V(r,b) - V(r+\varepsilon,b)$ is greater than or equal to

$$\mathbb{E}\Big[\sum_{k=1}^{T/\varepsilon}\int_{(k-1)\varepsilon}^{k\varepsilon} e^{-\gamma t}(W(R,B)-W(R+\varepsilon,B_\varepsilon)ds \tag{18}$$

$$+e^{-k\gamma\varepsilon}V(R(k\varepsilon)+\varepsilon,B(k\varepsilon))-V(R(k\varepsilon)+\varepsilon,B_\varepsilon(k\varepsilon))\Big] \tag{19}$$

$$+e^{-\gamma T}\mathbb{E}[V(R(T),B(T))-V(R(T)+\varepsilon,B(T))]. \tag{20}$$

For all $k\in\{1\dots,T/\varepsilon\}$, we define G_k and H_k by:

$$G_k := \int_{(k-1)\varepsilon}^{k\varepsilon} e^{-\gamma t}(W(R,B)-W(R+\varepsilon,B_\varepsilon)ds$$
$$H_k := V(R(k\varepsilon)+\varepsilon,B(k\varepsilon)) - V(R(k\varepsilon)+\varepsilon,B_\varepsilon(k\varepsilon))$$

We distinguish 5 cases:

1. For all $t\in I_k$: $R^\star(t)\in(C_{\max};+\infty)$. In that case, for all $t\in I_k$, we have $u(t)=u_\varepsilon(t)=-C_{\max}$ if $B(t)=B_\varepsilon(t)<B_{\max}$ and $u(t)=u_\varepsilon(t)=0$ otherwise. Thus: $B(k\varepsilon)=B_\varepsilon(k\varepsilon)$ and we have:

$$G_k = \int_{(k-1)\varepsilon}^{k\varepsilon} e^{-\gamma t}c\varepsilon dt = \varepsilon\int_{(k-1)\varepsilon}^{k\varepsilon}(-P^\star(t)+c)dt.$$
$$H_k = 0.$$

2. For all $t\in I_k$: $R^\star(t)\in(-\infty;-D_{\max}-\varepsilon)$. In that case, for all $t\in I_k$, we have $u(t)=u_\varepsilon(t)=D_{\max}$ if $B(t)=B_\varepsilon(t)>0$ and $u(t)=u_\varepsilon(t)=0$ otherwise. Thus, we again have $B(k\varepsilon)=B_\varepsilon(k\varepsilon)$ and:

$$G_k = \int_{(k-1)\varepsilon}^{k\varepsilon} e^{-\gamma t}(c-(v+c^{bo}))\varepsilon dt$$
$$= \varepsilon\int_{(k-1)\varepsilon}^{k\varepsilon}(-P^\star(t)+c)dt.$$
$$H_k = 0.$$

3. For all $t\in I_k$: $R^\star(t)\in(0;C_{\max}-\varepsilon)$. In that case, if $B_\varepsilon(t)<B_{\max}$ for all $t\in I_k$, then we have $u(t)=-R(t)$ and $u_\varepsilon(t)=-R(t)-\varepsilon$. This leads to $B_\varepsilon(k\varepsilon)=B(k\varepsilon)+\eta\varepsilon^2$. Hence:

$$G_k = \varepsilon\int_{(k-1)\varepsilon}^{k\varepsilon} e^{-\gamma t}cdt.$$
$$H_k = V(R(k\varepsilon)+\varepsilon,B(k\varepsilon)) - V(R(k\varepsilon)+\varepsilon,B(k\varepsilon)+\eta\varepsilon^2)$$
$$= -\eta\varepsilon^2\frac{\partial V}{\partial b}(R(k\varepsilon)+\varepsilon,B(k\varepsilon))+o(\varepsilon^2)$$
$$= -\varepsilon^2P^\star(k\varepsilon)+o(\varepsilon^2),$$
$$= \varepsilon\int_{(k-1)\varepsilon}^{k\varepsilon} e^{\gamma(k\varepsilon-t)}(-P^\star(t))dt+o(\varepsilon^2).$$

The last two equalities contains terms in $o(\varepsilon^2)$ due to the fact that derivative $\partial V/\partial b$ is continuous in (r,b) and that $P^\star(k\varepsilon)=(\partial V/\partial b)(R(k\varepsilon),B(k\varepsilon))$. Note that because R is continuous on $[0;T]$, it is uniformly continuous. Hence, the hidden constants in the $o(\varepsilon)$ are uniform and do not depend on k.

Note that if there exists t such that $B_\varepsilon(t)=B_{\max}$, then the same equality holds using Lemma 3-(iv).

4. For all $t\in I_k$: $R^\star(t)\in(-D_{\max};-\varepsilon)$. If $B(t)>0$ for all $t\in I_k$, then we have $u(t)=-R(t)$ and $u_\varepsilon(t)=-R(t)-\varepsilon$. This leads to $B_\varepsilon(k\varepsilon)=B(k\varepsilon)+\varepsilon^2$. Thus, we have:

$$G_k = \varepsilon\int_{(k-1)\varepsilon}^{k\varepsilon} e^{-\gamma t}cdt$$
$$H_k = V(R(k\varepsilon)+\varepsilon,B(k\varepsilon)) - V(R(k\varepsilon)+\varepsilon,B(k\varepsilon)+\varepsilon^2)$$
$$= -\varepsilon^2\frac{\partial V}{\partial b}(R(k\varepsilon)+\varepsilon,B(k\varepsilon))+o(\varepsilon^2)$$
$$= -\varepsilon^2P^\star(k\varepsilon)+o(\varepsilon^2)$$
$$= \varepsilon\int_{(k-1)\varepsilon}^{k\varepsilon} e^{\gamma(k\varepsilon-t)}(-P^\star(t))dt+o(\varepsilon^2).$$

As before, if there exists t such that $B(t)=0$, then the same inequality holds using Lemma 3-(iv).

5. If none of the above property is satisfied, then there exists $t\in I_k$ such that $R^\star(t)\notin\{-D_{\max},0,C_{\max}\}\pm\varepsilon$. By definition of the total payoff Eq.(8), we have:

$$|W(R(t),B(t))-W(R(t)+\varepsilon,B_\varepsilon(t))|\le(v+c^{bo})\varepsilon.$$

Moreover, by definition of u_ε, we have $|B_\varepsilon(k\varepsilon)-B(k\varepsilon)|\le\varepsilon^2$. As $|\partial V/\partial b|\le(v+c^{bo})$, this implies that

$$|V(R(k\varepsilon),B(k\varepsilon))-V(R(k\varepsilon)+\varepsilon,B_\varepsilon(k\varepsilon))|\le(v+c^{bo})\varepsilon^2.$$

In particular, this implies $G_k=O(\varepsilon^2)$ and $H_k=O(\varepsilon^2)$.

By lemma 4 (that we prove in the next section), there are at most $o(1/\varepsilon)$ intervals of the form $[(k-1)\varepsilon;k\varepsilon]$ that corresponds to the fifth case of the previous analysis.

$$\sum_{k=1}^{T/\varepsilon}G_k+e^{-k\gamma\varepsilon}H_k = \varepsilon\int_0^T e^{-\gamma t}(c-P^\star(t))dt+o(\varepsilon)$$

This shows that, almost surely,

$$\lim_{\varepsilon\to0}\frac{1}{\varepsilon}\left(\sum_{k=1}^{T/\varepsilon}G_k+e^{-k\gamma\varepsilon}H_k\right) = \int_0^T e^{-\gamma t}(P^\star(t)-c)dt.$$

Moreover, it should be clear that $\left|\sum_{k=1}^{T/\varepsilon}G_k+e^{-k\gamma\varepsilon}H_k\right|/\varepsilon\le\int_0^T e^{-\gamma t}(v+c^{bo}+c)dt\le(v+c^{bo}+c)/\gamma$ almost surely. Hence, applying the dominated convergence, we can exchange the expectation and the limit as ε goes to 0 to show that as ε goes to 0, Eq.(18-19)$/\varepsilon$ is equal to

$$\lim_{\varepsilon\to0}\frac{1}{\varepsilon}\mathbb{E}\left(\sum_{k=1}^{T/\varepsilon}G_k+e^{-k\gamma\varepsilon}H_k\right) = \mathbb{E}\lim_{\varepsilon\to0}\frac{1}{\varepsilon}\left(\sum_{k=1}^{T/\varepsilon}G_k+e^{-k\gamma\varepsilon}H_k\right)$$
$$= \int_0^T e^{-\gamma t}(P^\star(t)-c)dt.$$

Moreover, Equation (20) is less than $e^{-\gamma T}(v+c^{bo}+c)\varepsilon/\gamma$ and is therefore negligible for high T. Hence, by letting T goes to infinity, we have

$$\frac{\partial V}{\partial r}\le\mathbb{E}\int_0^\infty e^{-\gamma t}(P^\star(t)-c)dt.$$

The converse inequality can be obtained using similar arguments. □

C.4.2 Lemma 4

LEMMA 4. *Let $R^\star$ be the optimal reserve process. For all $T < \infty$, with probability one, the number of intervals $[k\varepsilon, (k+1)\varepsilon]$ such that $k \leq T/\varepsilon$ and $R^\star(t)$ is at distance ε of 0, C_{max} or $D_{\max}$ in this interval grows slowlier that $1/\varepsilon$. More precisely, if we denote $S_\varepsilon = [-D_{\max} - \varepsilon, D_{\max+\varepsilon}] \cup [-\varepsilon, \varepsilon] \cup [C_{\max} - \varepsilon, C_{\max} + \varepsilon]$, then:*

$$\lim_{\varepsilon \to 0} \varepsilon \cdot (\#\{k \mid \exists t \in [k\varepsilon; (k+1)\varepsilon] : R^\star(t) \in S_\varepsilon\}) = 0 \ a.s.$$

PROOF. For all $\varepsilon > 0$ and $k < T/n$, we denote $I_{\varepsilon,k} := [k\varepsilon; (k+1)\varepsilon]$. We define the set A_ε by

$$A_\varepsilon = \bigcup \Big\{ I_{\varepsilon,k} \mid \exists t \in I_{n,\varepsilon} \text{ s.t. } R^\star(t) \in S_\varepsilon \Big\}.$$

We denote by A the limit of the A_ε: $A = \bigcap_{\varepsilon > 0} A_\varepsilon$. The limit set A is the closure of the set B, defined by

$$B := \{t \mid R^\star(t) \in \{-D_{\max}, 0, C_{\max}\}\}.$$

As $R^\star(t)$ is continuous, B is closed and therefore $A = B$.

The reserve process $R^\star$ is the sum of a Brownian motion $Z = \Gamma - D$ of volatility σ and a function G that is constraints by the ramping constraints ζ_+ and ζ_-. This shows that

$$\begin{aligned} P(R^\star(t) = 0) &= P(Z(t) - G^\star(t) = 0) \\ &= \lim_{\delta \to 0} P(Z(t) - Z(t-\delta) = G^\star(t) - Z(t-\delta)) \\ &\leq \lim_{\delta \to 0} P(Z(t) - Z(t-\delta) + G^\star(t-\delta) \in [-\zeta^-\delta; \zeta^+\delta]) \\ &\leq \lim_{\delta \to 0} (\zeta^- + \zeta^+)\delta \frac{1}{\sigma\sqrt{\delta}\sqrt{2\pi}} = 0. \end{aligned}$$

The last inequality comes from the fact that $Z(t) - Z(t-\delta) + G^\star(t-\delta)$ has a normal distribution of variance $\delta\sigma^2$ and the one before from the ramping constraints of $G^\star$.

Therefore, for all t, $R^\star(t) \neq 0$ almost surely. Hence, by Fubini, the expectation of the Lebesgue measure of A is zero. This shows that with probability one, the Lebsgue measure of A_ε goes to 0 as ε goes to 0 and concludes the proof. $\square$

D. PROOF OF THEOREM 4

Let (R, B) be a reserve and storage level process when the parameters of the system are $(\sigma, \zeta^+, \zeta^-, B_{\max}, C_{\max}, D_{\max}, \gamma)$. Let $G^{x,y}$ be a generation process defined by

$$G^{x,y}(t) = \frac{x^2}{y} G\left(\frac{y^2}{x^2} t\right)$$

The generation process G satisfies the ramping constraints ζ^- and ζ^+, *i.e.* for all $t > t'$, we have $\zeta^-(t-t') \leq G(t) - G(t') \leq \zeta^+(t-t')$. This implies that for all $t > t'$, we have $y\zeta^- \leq G^{x,y}(t) - G^{x,y}(t') \leq y\zeta^+(t-t')$. Thus, the generation process $G^{x,y}$ satisfies the constraints $y\zeta^-$ and $y\zeta^+$.

Let $Z^{x,y}$ be a demand minus renewable generation process defined by

$$Z^{x,y}(t) = \frac{x^2}{y} Z\left(\frac{y^2}{x^2} t\right)$$

Recall that if Y is a Brownian motion with volatility σ, then $t \mapsto \alpha Y(\beta t)$ is a Brownian motion with volatility $\alpha\sqrt{\beta}$. As the process Z is a Brownian motion with volatility σ, the process $Z^{x,y}$ is a Brownian motion with volatility

$$\frac{x^2}{y}\sqrt{\frac{y^2}{x^2}}\sigma = x\sigma.$$

Finally, we define the storage level process $B^{x,y}$ and its corresponding control $u^{x,y}$ as

$$B^{x,y}(t) = \frac{x^4}{y^3} B\left(\frac{y^2}{x^2} t\right) \quad \text{and} \quad u^{x,y}(t) = \frac{x^2}{y} u\left(\frac{y^2}{x^2} t\right)$$

By Equation (1), the storage B evolves as

$$\frac{\partial B}{\partial t} = -u(t)(\mathbb{1}_{\{u(t)>0\}} + \eta \mathbb{1}_{\{u(t)<0\}}).$$

The derivatice of $B^{x,y}$ with respect to time is x^2/y times the derivative of B with respect to time. Thus, $B^{x,y}$ evolves as

$$\begin{aligned} \frac{\partial B^{x,y}}{\partial t} &= -\frac{x^2}{y} u(t)(\mathbb{1}_{\{u(t)>0\}} + \eta \mathbb{1}_{\{u(t)<0\}}) \\ &= -u^{x,y}(t)(\mathbb{1}_{\{u(t)>0\}} + \eta \mathbb{1}_{\{u(t)<0\}}). \end{aligned}$$

This shows that $u^{x,y}$ corresponds indeed to the control of $B^{x,y}$. Moreover, since $C_{\max}$ and $D_{\max}$ have been scaled by x^2/y, $u^{x,y}$ is a valid control. Therefore, the storage level process $B^{x,y}$ is a valid storage process for the scaled model.

Finally, as the reserve process R and the storage control u are both multiplied by x^2/y, the social payoff, defined by Equation (8) is multiplied by x^2/y.

This shows that there is a one to one correspondence between the reserve and storage level processes for the parameters $(\sigma, \zeta^+, \zeta^-, B_{\max}, C_{\max}, D_{\max}, \gamma)$ and for the parameters $(x\sigma, y\zeta^+, y\zeta^-, \frac{x^4}{y^3} B_{\max}, \frac{x^2}{y} C_{\max}, \frac{x^2}{y} D_{\max}, \frac{x^2}{y^2}\gamma)$. Hence, if the process (R, B) is optimal for the first set of parameters, then the process $(R^{x,y}, B^{x,y})$ is optimal for the scaled parameters.

E. REFERENCES

[1] H. Abgottspon, M. Bucher, and G. Andersson. Stochastic dynamic programming for unified shortand medium-term planning of hydro power considering market products. In *International Conference on Probabilistic Methods Applied to Power Systems(PMAPS), Istanbul, Turkey*, 2012.

[2] M. Arnold and G. Andersson. Model predictive control of energy storage including uncertain forecasts. In *Power Systems Computation Conference (PSCC), Stockholm, Sweden*, 2011.

[3] A. Bejan, R. Gibbens, and F. Kelly. Statistical aspects of storage systems modelling in energy networks. *46th Annual Conference on Information Sciences and Systems*, 2012.

[4] E. Castronuovo and J. Lopes. On the optimization of the daily operation of a wind-hydro power plant. *Power Systems, IEEE Transactions on*, 19(3):1599–1606, 2004.

[5] E. D. Castronuovo and J. A. P. cas Lopes. Optimal operation and hydro storage sizing of a wind-hydro power plant. *International Journal of Electrical Power & Energy Systems*, 26(10):771 – 778, 2004.

[6] S. Chatzivasileiadis, M. Bucher, M. Arnold, T. Krause, and G. Andersson. Incentives for optimal integration of fluctuating power generation. 2011.

[7] M. Chen, I. Cho, and S. Meyn. Reliability by design in distributed power transmission networks. *Automatica*, 42(8):1267–1281, 2006.

[8] I. Cho and S. Meyn. Efficiency and marginal cost pricing in dynamic competitive markets with friction. *Theoretical Economics*, 5(2):215–239, 2010.

[9] R. Durrett. *Probability: Theory and examples.* Wadsworth & Brooks/Cole Advanced Books & Software, 1996.

[10] J. Garcia-Gonzalez, R. de la Muela, L. Santos, and A. González. Stochastic joint optimization of wind generation and pumped-storage units in an electricity market. *IEEE Transactions on Power Systems*, 23(2):460–468, 2008.

[11] N. Gast, J. Le Boudec, A. Proutière, and D. Tomozei. Impact of storage on the efficiency and prices in real-time electricity markets. Technical report, EPFL. http://infoscience.epfl.ch/record/183149, 2013.

[12] N. Gast, D. Tomozei, and J. Le Boudec. Optimal storage policies with wind forecast uncertainties. *Greenmetrics*, 2012.

[13] F. Graves, T. Jenkin, and D. Murphy. Opportunities for electricity storage in deregulating markets. *The Electricity Journal*, 12(8):46–56, 1999.

[14] X. HE, W. Delarue, E. abd D'Haeseleer, and J.-M. Glachant. Coupling electricity storage with electricity markets: a welfare analysis in the french market. *TME working paper - Energy and Environment*, 2012.

[15] K. Heussen, S. Koch, A. Ulbig, and G. Andersson. Unified system-level modeling of intermittent renewable energy sources and energy storage for power system operation. *Systems Journal, IEEE*, 6(1):140 –151, march 2012.

[16] M. Hildmann, A. Ulbig, and G. Andersson. Electricity grid in-feed from renewable sources: A risk for pumped-storage hydro plants? In *8th International Conference on the European Energy Market (EEM)*, pages 185 –190, 2011.

[17] P. Joskow and E. Kahn. A quantitative analysis of pricing behavior in california's wholesale electricity market during summer 2000. Technical report, National Bureau of Economic Research, 2001.

[18] M. Korpaas, A. T. Holen, and R. Hildrum. Operation and sizing of energy storage for wind power plants in a market system. *International Journal of Electrical Power & Energy Systems*, 25(8):599 – 606, 2003.

[19] S. Meyn, M. Negrete-Pincetic, G. Wang, A. Kowli, and E. Shafieepoorfard. The value of volatile resources in electricity markets. In *CDC*, pages 1029–1036. IEEE, 2010.

[20] R. Sioshansi. Welfare impacts of electricity storage and the implications of ownership structure. *Energy Journal*, 31(2):173, 2010.

[21] R. Sioshansi, P. Denholm, T. Jenkin, and J. Weiss. Estimating the value of electricity storage in pjm: Arbitrage and some welfare effects. *Energy Economics*, 31(2):269–277, 2009.

[22] G. Strbac, A. Shakoor, M. Black, D. Pudjianto, and T. Bopp. Impact of wind generation on the operation and development of the UK electricity systems. *Electric Power Systems Research*, 77(9):1214 – 1227, 2007.

[23] A. Tuohy and M. O'Malley. Impact of pumped storage on power systems with increasing wind penetration. In *Power & Energy Society General Meeting.*, pages 1–8. IEEE, 2009.

[24] A. Ulbig and G. Andersson. On operational flexibility in power systems. In *IEEE PES General Meeting, San Diego, USA*, 2012.

[25] R. Walawalkar, J. Apt, and R. Mancini. Economics of electric energy storage for energy arbitrage and regulation in new york. *Energy Policy*, 35(4):2558–2568, 2007.

[26] G. Wang, A. Kowli, M. Negrete-Pincetic, E. Shafieepoorfard, and S. Meyn. A control theorist's perspective on dynamic competitive equilibria in electricity markets. In *Proc. 18th World Congress of the International Federation of Automatic Control (IFAC)*, 2011.

[27] G. Wang, M. Negrete-Pincetic, A. Kowli, E. Shafieepoorfard, S. Meyn, and U. Shanbhag. Dynamic competitive equilibria in electricity markets. In *Control and Optimization Theory for Electric Smart Grids*. Springer, 2011.

[28] G. Wang, M. Negrete-Pincetic, A. Kowli, E. Shafieepoorfard, S. Meyn, and U. Shanbhag. Real-time prices in an entropic grid. In *Innovative Smart Grid Technologies (ISGT), IEEE PES*, pages 1–8. IEEE, 2012.

[29] R. Wilson. Architecture of power markets. *Econometrica*, 70(4):1299–1340, 2003.