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# Insertion techniques and constraint propagation for the DARP

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**Abstract**— This paper deals with the *Dial and Ride Problem (DARP)*, while using randomized greedy insertion techniques together with constraint propagation techniques. Though it focuses here on the static version of *Dial and Ride*, it takes into account the fact that practical DARP has to be handled according to a dynamical point of view, and even, in some case, in real time contexts. So, the kind of algorithmic solution which is proposed here, aim at making easier to bridge both points of view. The model is a classical one, and considers a performance criterion which is a mix between Quality of Service (QoS) and economical cost. We first propose the general framework of the model and discuss the link with dynamical DARP, next describe the algorithm and end with numerical experiments.

## I. INTRODUCTION

Literature in the field of urban systems and geomatics hint a trend to a surge of new “on demand” flexible transportation systems (ODT): ad hoc shuttle fleets, vehicle sharing (AUTOLIB...), co-transportation (see for instance [3], [9]). This trend reflects from both environmental (climate change, overcrowded megalopolis...) and economical concerns (surge of energy prices...). It has also to be associated with technological advances: internet, mobile communication, geo-localization..., which allow efficient monitoring of complex mobility system and large sets of heterogeneous requests.

An important Operations Research model for the management of flexible reactive transportation system is the DARP, which tries to optimize the way a given fleet of vehicles meet mobility demands emanating from people, or, in some cases from some combination of people and goods. DARP is a complex problem, which admits several formulation, most of them NP-Hard. It usually does not fit well the Integer Linear Programming framework [2] and one must try do handle it through heuristic techniques: Tabu search [4], genetic algorithms [7], partial branch/bound [2], Simulated Annealing [6], VNS techniques [8], [10], Dynamic Programming [2]-[3], Insertion techniques [11]-[12]. Moreover, a basic features of DARP is that it usually derives from a dynamic context. So, algorithms for static DARP should be designed in order to take into account the fact that they will have to be adapted to dynamic and reactive context, which means synchronization mechanisms,

interactions between the users and the vehicles, and uncertainty about fore coming demands.

So, what is done inside this paper is to consider a generic DARP model with time windows and a mix QoS/Economical-Cost performance criterion, and propose algorithms for this model which are based upon randomized insertion techniques and constraint propagation, and so, which will easily adapt themselves to dynamic contexts, where demand package has to be inserted into (or eventually removed from) current vehicle schedules, in a very short time, while taking into account some probabilistic knowledge about fore coming demand packages.

The paper is organized as follows: we first introduce the problem and discuss the link between static and dynamic formulations, next describe our formal model, together with the performance criterion which we use. Then we present the general insertion mechanism together with the constraint propagation techniques which we use in order to filter insertion parameters and to select the demands to be inserted. We conclude with experimental experiments and comparison with [7] and [8].

## II. THE STANDARD DIAL A RIDE PROBLEM

### A. General Dial a Ride Problem

We can find in literature several mathematical formulations for the DARP. But, the complexity of all these linear programs doesn't allow finding an exact solution with a solver, the operation is too time consuming. In fact, it mixes a lot of booleans and plenty of fractional numbers. Refer to [4], [7] for the principal formulations.

A *Dial a Ride Problem* instance is essentially defined by:

- a *Transit* network  $G = (V, E)$ , which contains at least some specific node *Depot*, and whose arcs  $e \in E$  are endowed with riding times  $l(e) \geq 0$ , and, eventually, with other technical characteristics;
- a vehicle fleet  $\mathcal{VH}$ ;
- a *Demand* set  $\mathcal{D} = (\mathcal{D}_i, i \in I)$ , any demand  $\mathcal{D}_i$  being defined as a 6-uple  $\mathcal{D}_i = (o_i, d_i, \Delta_i, F(o_i), F(d_i), Q_i)$ , where:
  - o  $o_i \in V$  is the *origin* node of the demand  $\mathcal{D}_i$ ;

- $d_i \in V$  is the *destination* node of the demand  $\mathcal{D}_i$ ;
- $\Delta_i \geq 0$  is an upper bound (*transit bound*) on the duration of demand  $\mathcal{D}_i$ 's processing;
- $\mathcal{F}(o_i)$  is a time window related to the time  $\mathcal{D}_i$  starts being processed;
- $\mathcal{F}(d_i)$  is a time window related to the time  $\mathcal{D}_i$  ends being processed;
- $Q_i$  is a description of the load related to  $\mathcal{D}_i$ .

Dealing with such an instance means planning the handling demands of  $\mathcal{D}$ , by the fleet  $\mathcal{VH}$ , while taking into account the constraints which derive from the technical characteristics of the network  $G$ , of the vehicle fleet  $\mathcal{VH}$ , and of the 6-uples  $\mathcal{D}_i = (o_i, d_i, \Delta_i, \mathcal{F}(o_i), \mathcal{F}(d_i), Q_i)$ , and while optimizing some performance criterion which is usually a mix of an economical cost (point of view of the fleet manager) and of QoS criteria (point of view of the users).

All along this work, we are going to deal with **homogeneous fleets** and with **nominal demands**, and we shall limit ourselves to static points of view but our insertion process allows flexibility for using it in a dynamic context. Still, we shall pay special attention to cases when temporal constraints are tight.

#### B. Discussion: Dynamic versus Static DARP

DARP is essentially a problem which arise in dynamic contexts, and the trend is about reactivity delays which become smaller and smaller [5]. Basically, one should consider a system which is identified by a vehicle set  $V$ , a user community  $C$ , and a supervision system  $S$ , which, because of advances in the field of geo-localization, mobile communications and remote monitoring, permanently disposes of a full knowledge about the current state of the vehicles (position, load, roadmap...) and maintains communication with both users and vehicles. All along the time, the system (centralized or decentralized) receives user request, which, in the simplest case, are characterized by a load, an origin and a destination node, and time windows related load and unload transactions, as well as about trip duration. At some instant  $t$ , supervisor  $S$  decides to launch a scheduling process  $P$ , which consider as its input the current state  $E$  of the vehicles of  $V$ , together with the currently waiting demand set  $D$ , and which, for any demand  $d$  in  $D$ , either rejects it or insert it into the current schedule of some vehicle  $\varpi$  in  $V$ , without modifying in a significant way the way  $v$  is supposed to meet previous demands. Running  $P$  require a  $\delta$  computing time, and, at time  $t + \delta$ , propositions are transmitted to users and updated schedules are transmitted to the vehicles, which apply them until instant  $t'$ , when the whole process takes place again. Meanwhile, it may occur that some demands are dropped or that vehicles register failure (delays or user fault...) [14].

In any case, one see that, in case vehicles are moving inside a small area (a urban area) and deal with a large size set of demands, process  $P$  has to insert in a fast way a demand set  $D$  into a current schedule  $E$ , and that it has to do it in a way which keeps most features of  $E$ , and preserves the

ability of the system to efficiently deal with fore coming demands, that means with demands which are likely to be formulated after the instant  $t$  when  $P$  is launched. This point is the key one which motivates the approach which is going to be described here. We want an algorithmic framework which is going to be naturally compatible with this context: the use of insertion techniques is clearly going to fit the input  $(E, D)$  of the dynamic context, and the use of constraint propagation techniques is going to make easier uncertainty about fore coming demands handling.

Also, one should notice that, under this prospect, the virtual complete network which is going to be the key input data for the static model (see next section III.A), is, in practice, going to be a dynamic network.

### III. THE FRAMEWORK

#### A. The Considered Network

We treat here the general Dial a Ride Problem described above. It is known that we do not need to consider the whole transit network  $G = (V, E)$ , and that we may restrict ourselves to the nodes which are either the origin or the destination of some demand, while considering that any vehicle which visits two such nodes in a consecutive way does it according to a shortest path strategy. This leads us to consider the node set  $\{Depot, o_i, d_i, i \in I\}$  as made with pairwise distinct nodes, and provided with some distance function DIST, which to any pair  $x, y$  in  $\{Depot, o_i, d_i, i \in I\}$ , makes correspond the shortest path distance from  $x$  to  $y$  in the transit network  $G$ .

As a matter of fact, we also split the *Depot* node according to its arrival or departure status and to the various vehicles of the fleet  $\mathcal{VH}$ , and we consider the **input data** of a *Standard Dial a Ride Problem* instance as defined by:

- the set  $\{1..K = \text{Card}(\mathcal{VH})\}$  of the vehicles of the homogenous fleet  $\mathcal{VH}$ ;
- the common capacity  $CAP$  of a vehicle in  $\mathcal{VH}$ ;
- the node set  $X = \{DepotD(k), DepotA(k), k = 1..K\} \cup \{o_i, d_i, i \in I\}$ ;
- the distance matrix DIST, whose meaning is that, for any  $x, y$  in  $X$ ,  $\text{DIST}(x, y)$  is equal to the length, in the sense of the length function  $l$ , of a shortest path which connect  $x$  to  $y$  in the transit network  $G$ : we suppose that DIST, satisfies the *triangle inequality*.

Moreover the following characteristics, which, to any node  $x$  in  $X$ , make correspond:

- its status  $Status(x)$ : *Origin, Destination, DepotA, DepotD*; we set  $Depot = DepotD \cup DepotA$ ;
- its load  $\mathcal{CH}(x)$ :
  - if  $Status(x) \in Depot$  then  $\mathcal{CH}(x) = 0$ ;
  - if  $Status(x) = Origin$  then  $\mathcal{CH}(x) = Q_i$ ;
  - if  $Status(x) = Destination$  then  $\mathcal{CH}(x) = -Q_i$ ;
- its twin node  $\mathcal{Twin}(x)$ :
  - if  $x = DepotA(k)$  then  $\mathcal{Twin}(x) = DepotD(k)$  and conversely;
  - if  $x = o_i$  then  $\mathcal{Twin}(x) = d_i$  and conversely;

- its time window  $\mathcal{F}(x)$ : for any  $k = 1..K$ ,  $\mathcal{F}(DepotA(k)) = [0, +\infty[ = \mathcal{F}(DepotD(k))$ . Also, we suppose that any  $\mathcal{F}(x)$ ,  $x \in X$ , is an interval, which may be written  $\mathcal{F}(x) = [\mathcal{F}.min(x), \mathcal{F}.max(x)]$ ;
- its transit bound  $\Delta(x)$ : if  $x = o_i$  or  $d_i$ , then  $\Delta(x) = \Delta_i$ , and  $\Delta(x) = \Delta$  else, where  $\Delta$  is an upper bound which is imposed on the duration of any vehicle tour.

According to this construction, we understand that the system works as follows: vehicle  $k \in \{1..K\}$ , starts its journey from  $DepotD(k)$  at some time  $t(DepotD(k))$  and ends it into  $DepotA(k)$  at some time  $t(DepotA(k))$ , after having taken in charge some subset  $\mathcal{D}(k) = \{D_i, i \in I(k)\}$  of  $\mathcal{D}$ : that means that for any  $i \in I(k)$ , vehicle  $k$  arrived in  $o_i$  at time  $t(o_i) \in \mathcal{F}(o_i)$ , loaded the whole load  $Q_i$ , and kept it until it arrived in  $d_i$  at time  $t(d_i) \in \mathcal{F}(o_i)$  and unloaded  $Q_i$ , in such a way that  $t(d_i) - t(o_i) \leq \Delta_i$ . Clearly, solving the Standard Dial a Ride Problem instance related to those data ( $X, DIST, K, CAP$ ) will mean computing the subsets  $\mathcal{D}(k) = \{D_i, i \in I(k)\}$ , the routes followed by the vehicles and the time values  $t(x)$ ,  $x \in X$ , in such a way that both economical performance and quality of service be the highest possible.

### B. Discussion: Durations and Waiting Times

Many authors include what they call *service durations* in their models. That means that they suppose that loading and unloading processes related to the various nodes of  $X$  require some time amount  $\delta(x)$ , (*service time*) and, so, that they distinguish, for any node  $x \in X$ , time values  $t(x)$  (beginning of the *service*) and  $t(x) + \delta(x)$  (end of the *service*). By the same way, some authors suppose that the vehicles are always running at their maximal speed, and make a difference between the time  $t^*(x)$ ,  $x \in X$ , when some vehicle arrives in  $x$ , and the time  $t(x)$  when this vehicle starts servicing the related demand (loading or unloading process). We do not do it. Taking into account service times, which tends to augment the size of the variables of the model and to make it more complex it, has really sense only if we suppose that the service times  $\delta(x)$  depend on the current state (its current load) of the vehicle at the time the loading or unloading process has to be launched. Making explicitly appear waiting times  $t(x) - t^*(x)$  is really useful if we make appear the speed profile as a component of the performance criterion. In case none of the situation holds, the knowledge of the routes of the vehicles and of the time value  $t(x)$ ,  $x \in X$ , is enough to check the validity of a given solution and to evaluate its performance, and then it turns out that ensuring the compatibility of the model with data which involve service times and waiting times  $t(x) - t^*(x)$ ,  $x \in X$ , is only a matter of adapting the times windows  $\mathcal{F}(x)$ , the transit bounds  $\Delta(x)$ ,  $x \in X$ , and the distance matrix  $DIST$  (cf. **Fig. 1**).

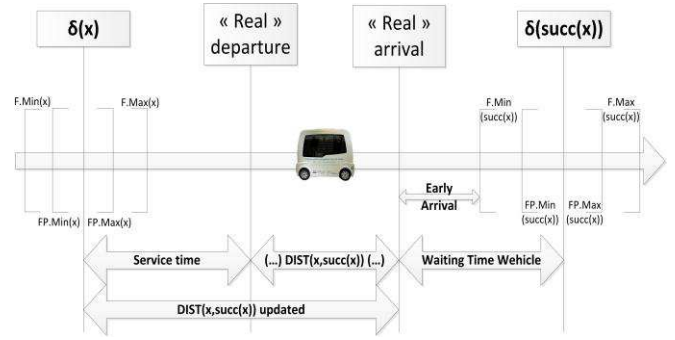


Fig. 1 Considered times between two nodes

### C. Modeling and Evaluation Techniques

The model described in this section needs some definitions, we set:

- $First(\Gamma) =$  First element of  $\Gamma$ ;  $Last(\Gamma) =$  last element of  $\Gamma$ ;
- for any  $z$  in  $\Gamma$ :
  - o  $Succ(\Gamma, z) =$  Successor of  $z$  in  $\Gamma$ ;
  - o  $Pred(\Gamma, z) =$  Predecessor of  $z$  in  $\Gamma$ ;
- for any  $z, z'$  in  $\Gamma$ :
  - o  $z \ll_{\Gamma} z'$  if  $z$  is located before  $z'$  in  $\Gamma$ ;
  - o  $z \ll_{\Gamma}^{-} z'$  if  $z \ll_{\Gamma} z'$  or  $z = z'$ .
  - o  $Segment(\Gamma, z, z') =$  the subsequence defined by all  $z''$  in  $\Gamma$  such that  $z \ll_{\Gamma}^{-} z'' \ll_{\Gamma}^{-} z'$ . If  $z = Nil$ , then  $Segment(\Gamma, Nil, z')$  denotes the subsequence defined by all  $z''$  in  $\Gamma$  such that  $z'' \ll_{\Gamma}^{-} z'$ .

In any algorithmic description, we use the symbol  $\leftarrow$  in order to denote the value assignment operator:  $x \leftarrow \alpha$ , means that the variable  $x$  receives the value  $\alpha$ . Thus, we only use symbol  $=$  as a comparator.

In order to provide an accurate description of the **output data** of our standard Dial a Ride Problem instance ( $X, DIST, K, CAP$ ), we need to talk about *tours* and related *time value sets*.

A *tour*  $\Gamma$  is a sequence of nodes of  $X$ , which is such that:

- $Status(First(\Gamma)) = DepotD$ ;  $Status(End(\Gamma)) = DepotA$ ;
- For any node  $x$  in  $\Gamma$ ,  $x \neq First(\Gamma), End(\Gamma)$ ,  $Status(x) \notin Depot$ ;
- No node  $x \in X$  appears twice in  $\Gamma$ ;
- For any node  $x = o_i$  (resp.  $d_i$ ) which appears in  $\Gamma$ , the node  $Twin(x)$  is also in  $\Gamma$ , and we have:  $x \ll_{\Gamma} Twin(x)$  (resp.  $Twin(x) \ll_{\Gamma} x$ ).

This tour  $\Gamma$  is said to be *load-valid* iff:

- for any  $x$  in  $\Gamma$ ,  $x \neq First(\Gamma)$ , we have  $\sum_{y \ll_{\Gamma} x} \mathcal{CH}(y) \leq CAP$ .

Moreover, this tour  $\Gamma$  is said to be *time-valid* iff it is possible to associate, with any node  $x$  in  $\Gamma$ , some time value  $t(x)$ , in such a way that:

(E1)

- for any  $x$  in  $\Gamma$ ,  $x \neq Last(\Gamma)$ ,  $t(Succ(\Gamma, x)) \geq t(x) + DIST(x, Succ(\Gamma, x))$ ; (*Distance Constraints*)
- for any  $x$  in  $\Gamma$ ,  $|t(Twin(x)) - t(x)| \leq \Delta(x)$ ;

- for any  $x$  in  $\Gamma$ ,  $t(x) \in \mathcal{F}(x)$ .

In case the tour  $\Gamma$  is time-valid, any time value set  $t = \{t(x), x \in X\}$ , which satisfies (E1) is said to be a *valid related time value set*. We denote by  $\mathcal{V}alid(\Gamma)$  the set of the related time value set  $t$ .

In case we need to consider  $\mathcal{F}$  as a variable, we say that  $\Gamma$  is *time-valid in relation to  $\mathcal{F}$* .

The tour  $\Gamma$  is said to be *valid* if it is both time valid and load valid.

For any pair  $(\Gamma, t)$  defined by some time-valid tour  $\Gamma$  and by some valid related time value set  $t$ , we may set:

- $\mathcal{G}lob(\Gamma, t) = t(\text{End}(\Gamma)) - t(\text{First}(\Gamma))$ : this quantity denotes the global duration of the tour  $\Gamma$ ;
- $\mathcal{R}ida(\Gamma, t) = \sum_{i \in \Gamma} (t(d_i) - t(o_i))$ ; this quantity may be viewed as a QoS criterion, and denotes the sum of the duration of the individual trips of the demanders which are taken in charge by tour  $\Gamma$ ;
- $\mathcal{W}ait(\Gamma, t) = \mathcal{G}lob(\Gamma, t) - (\sum_{x|x \neq \text{Last}(\Gamma)} \text{DIST}(x, \text{Succ}(\Gamma, x)))$ : this quantity denotes the « waiting time » of the vehicle involved in  $\Gamma$ , the waiting time related to some node  $x$  being the time the vehicle is supposed to wait before loading or unloading  $x$  in case it runs full speed on the route which connects  $\text{Pred}(\Gamma, x)$  to  $x$ .

If  $A, B, C$  are three multi-criterion coefficients, we may define the performance criterion  $\text{Cost}_{A, B, C}(\Gamma, t)$  as follows:  $\text{Cost}_{A, B, C}(\Gamma, t) = A \cdot \mathcal{G}lob(\Gamma, t) + B \cdot \mathcal{R}ida(\Gamma, t) + C \cdot \mathcal{W}ait(\Gamma, t)$ .

In section V, we use different coefficients in order to compare with other techniques found in literature. Our insertion techniques allow some flexibility for this change.

So, let us suppose that we deduced from the data  $G = (V, E)$ ,  $\mathcal{V}\mathcal{H} = (K, \mathcal{C}\mathcal{A}\mathcal{P})$ ,  $\mathcal{D} = (\mathcal{D}_i = (o_i, d_i, \Delta_i, \mathcal{F}(o_i), \mathcal{F}(d_i), Q_i), i \in I)$ , a 4-uple  $(X, \text{DIST}, K, \mathcal{C}\mathcal{A}\mathcal{P})$ , and that we are also provided with 3 multi-criterion coefficients  $A, B$  and  $C \geq 0$ . Then we see that solving the related *Standard Dial a Ride Problem* instance means computing:

- for any vehicle index  $k$  in  $1..K$ , a valid tour  $T(k)$ ;
- a time value set  $t = \{t(x), x \in X\}$ ;

in such a way that:

- the restriction of  $t$  to any  $T(k)$ ,  $k = 1..K$ , defines a valid time value set related to  $T(k)$ ;
- the tour set  $T = \{T(k), k = 1..K\}$  induces a partition of  $X$ ;
- the quantity  $\mathcal{P}erf_{A, B, C}(T, t) = \sum_{k=1..K} \text{Cost}_{A, B, C}(T(k), t)$  is the smallest possible.

#### IV. AN INSERTION ALGORITHM

##### A. Handling Constraints

Let  $\Gamma$  a tour. The algorithm which we are going to describe in this section will essentially be based upon the use of insertion techniques. Thus, we must be able to check in a fast way, whether the insertion of some demand  $\mathcal{D}_i$  inside  $\Gamma$  will maintain the validity of  $\Gamma$ , and to get an evaluation of the quality of this insertion. Since we want to pay a special attention to the case when temporal constraints are tight, we

are first going to provide ourselves with a package of constraint handling tools for testing the valid tours.

First, checking the load validity of  $\Gamma$  is easy. In order to be able to test the impact of the insertion of some demand into the tour  $\Gamma$  on the charge-validity of this tour, we associate, with any such a tour, the quantities  $\mathcal{C}(\Gamma, x)$ ,  $x \in \Gamma$ , defined by:

$$\text{for any } x \text{ in } \Gamma, \mathcal{C}(\Gamma, x) = \sum_{y|y \ll_{\Gamma} \text{ or } y = x} \mathcal{C}\mathcal{H}(y).$$

Then it comes that  $\Gamma$  is load-valid iff for any  $x$  in  $\Gamma$ ,  $\mathcal{C}(\Gamma, x) \leq \mathcal{C}\mathcal{A}\mathcal{P}$ .

Second, checking the time validity of  $\Gamma$ , according to a current time window set  $\mathcal{F}\mathcal{S} = \{\mathcal{F}\mathcal{S}(x) = [\mathcal{F}\mathcal{S}.min(x), \mathcal{F}\mathcal{S}.max(x)], x \in \Gamma\}$  may be performed through propagation of the following inference rules  $R_i$ ,  $i = 1..5$ :

**Rule  $R_1$ :**  $y = \text{Succ}(\Gamma, x)$ ;  $\mathcal{F}\mathcal{S}.min(x) + \text{DIST}(x, y) > \mathcal{F}\mathcal{S}.min(y) \models \mathcal{F}\mathcal{S}.min(y) \leftarrow \mathcal{F}\mathcal{S}.min(x) + \text{DIST}(x, y)$ ;  $\text{NFact} \leftarrow y$ ;

**Rule  $R_2$ :**  $y = \text{Succ}(\Gamma, x)$ ;  $\mathcal{F}\mathcal{S}.max(y) - \text{DIST}(x, y) < \mathcal{F}\mathcal{S}.max(x) \models \mathcal{F}\mathcal{S}.max(x) \leftarrow \mathcal{F}\mathcal{S}.max(y) - \text{DIST}(x, y)$ ;  $\text{NFact} \leftarrow x$ ;

**Rule  $R_3$ :**  $y = \text{Twin}(x)$ ;  $x \ll_{\Gamma} y$ ;  $\mathcal{F}\mathcal{S}.min(x) < \mathcal{F}\mathcal{S}.min(y) - \Delta(x, y) \models \mathcal{F}\mathcal{S}.min(x) \leftarrow \mathcal{F}\mathcal{S}.min(y) - \Delta(x, y)$ ;  $\text{NFact} \leftarrow x$ ;

**Rule  $R_4$ :**  $y = \text{Twin}(x)$ ;  $x \ll_{\Gamma} y$ ;  $\mathcal{F}\mathcal{S}.max(y) > \mathcal{F}\mathcal{S}.max(x) + \Delta(x, y) \models \mathcal{F}\mathcal{S}.max(y) \leftarrow \mathcal{F}\mathcal{S}.max(x) + \Delta(x, y)$ ;  $\text{NFact} \leftarrow y$ ;

**Rule  $R_5$ :**  $x \in \Gamma$ ;  $\mathcal{F}\mathcal{S}.min(x) > \mathcal{F}\mathcal{S}.max(x) \models \text{Fail}$ .

Propagating these rules may be performed as follows:

##### Procedure Propagate

**Input:** ( $\Gamma$ : Tour,  $L$ : List of nodes,  $\mathcal{F}\mathcal{S}$ : Time windows set related to the node set of  $\Gamma$ );

**Output:** ( $\mathcal{R}\mathcal{E}\mathcal{S}$ : Boolean,  $\mathcal{F}\mathcal{R}$ : Time windows set related to node set of  $\Gamma$ );

Not Stop;

While  $L \neq \text{Nil}$  and Not Stop do

$z \leftarrow \text{First}(L)$ ;  $L \leftarrow \text{Tail}(L)$ ;

For  $i = 1..5$  do Compute all the pairs  $(x, y)$  which make possible an application of the rule  $R_i$  and which are such that  $x = z$  or  $y = z$ ;

For any such pair  $(x, y)$  do

Apply the rule  $R_i$ ;

If  $\text{NFact}$  is not in  $L$  then Insert  $\text{NFact}$  in  $L$ ;

If  $\text{Fail}$  then Stop;

Propagate  $\leftarrow$  (Not Stop,  $\mathcal{F}\mathcal{S}$ );

##### Proposition 1

*The tour  $\Gamma$  is time-valid according to the input time window set  $\mathcal{F}\mathcal{S}$  if and only if the  $\mathcal{R}\mathcal{E}\mathcal{S}$  component of the result of a call  $\text{Propagate}(\mathcal{F}\mathcal{S}, \Gamma)$  is equal to 1. In such a case, any valid time value set  $t$  related to  $\Gamma$  and  $\mathcal{F}\mathcal{S}$  is such that: for any  $x$  in  $\Gamma$ ,  $t(x) \in \mathcal{F}\mathcal{S}(x)$ .*

### Proof

The part (only if) of the above equivalence is trivial, as well as the second part of the statement. As for the part (if), we only need to check that if we set, for any  $x$  in  $\Gamma$ :

- $\mathcal{FS}(x) = [\mathcal{FS}.min(x), \mathcal{FS}.max(x)]$ ;
- $t(x) = \mathcal{FS}.min(x)$ ;

then we get a time value set  $t = \{t(x), x \in X(\Gamma)\}$  which is compatible with  $\Gamma$  and  $\mathcal{FS}$ .

*End-Proof.*

We denote by  $\mathcal{FQ}(\Gamma)$  the time window set which result from a call  $Propagate(\Gamma, L, \mathcal{F})$ .  $\mathcal{FQ}(\Gamma)$  may be considered as the largest (in the inclusion sense) time window set which is included into  $\mathcal{F}$  and which is stable under the rules  $R_i$ ,  $i = 1..5$ , and is called the *window reduction* of  $\mathcal{F}$  through  $\Gamma$ .

### B. Evaluating a Tour

Let us consider now the tour  $\Gamma$ , provided with the window reduction set  $\mathcal{FQ}(\Gamma)$ . We want to get some fast estimation of the best possible value  $Cost_{A, B, C}(\Gamma, t) = A.Glob(\Gamma, t) + B.Ride(\Gamma, t) + C.Wait(\Gamma, t)$ ,  $t \in Valid(\Gamma)$ . We already noticed that it could be done through linear programming or through general shortest path and circuit cancelling techniques. Still, since we want to perform this evaluation process in a fast way, we design two ad hoc procedures EVAL1 and EVAL2:

- the EVAL1 procedure works in a greedy way, by first assigning to the node  $First(\Gamma)$  its largest possible time value, and by next performing a Bellman process in order to assign to every node  $x$  in  $\Gamma$  its smallest possible time value.
- the EVAL2 procedure starts from a solution produced by EVAL1, and improves it by performing a sequence of local moves, each move involving a single value  $t(x)$ ,  $x \in \Gamma$ .

**Procedure EVAL1**( $\Gamma$ : Tour): (Val: Number,  $\delta$ : value set)

For any  $x$  in  $\Gamma$ , let us set set:  $[a(x), b(x)] = \mathcal{FQ}(\Gamma)$ ;

$\delta(First(\Gamma)) \leftarrow b(First(\Gamma))$ ;  $x \leftarrow First(\Gamma)$ ;

While  $x \neq Last(\Gamma)$  do

$y < Succ(\Gamma, x)$ ;  $\delta(y) \leftarrow Sup(a(y), \delta(x) + DIST(x, y))$ ;

$x \leftarrow y$ ;  $\delta \leftarrow \{\delta(x), x \in \Gamma\}$ ;  $Val \leftarrow Cost_{A, B, C}(\Gamma, \delta)$ ;

*EVAL1*  $\leftarrow (Val, \delta)$ ;

**Procedure EVAL2**( $\Gamma$ : Tour): (Val: Number,  $\delta$ : value set)

For any  $x$  in  $\Gamma$ , let us set:  $[a(x), b(x)] = \mathcal{FQ}(\Gamma)$ ;

For any  $x$  in  $\Gamma$  do  $\delta(x) \leftarrow EVAL1(\Gamma, \mathcal{FS}).\delta$ ; Not Stop;

While Not Stop do

  Search the node  $x$  in  $\Gamma$  such that one of the two statements (E2) or (E3) below is true:

- o (E2):  $(\lambda_x < 0) \wedge (Status(x) \in \{Origin, DepotD\}) \wedge (\delta(x) \neq Inf(b(x), \delta(Succ(\Gamma, x) - DIST(x, Succ(\Gamma, x))))$ ;
- o (E3):  $(\lambda_x > 0) \wedge (Status(x) \in \{Destination, DepotA\}) \wedge (\delta(x) \neq Sup(a(x), \delta(Pred(\Gamma, x) + DIST(Succ(\Gamma, x), x)))$ ;

If *Fail*(Search) then

Stop;

*EVAL2*  $\leftarrow (\delta = \{\delta(x), x \in X(\Gamma)\}; Val = Cost_{A, B, C}(\Gamma, \delta))$ ;

Else

If (E2) then  $\delta(x) \leftarrow Inf(b(x), \delta(Succ(\Gamma, x) - DIST(x, Succ(\Gamma, x)))$ ;

Else if (E3) then  $\delta(x) \leftarrow Sup(a(x), \delta(Pred(\Gamma, x) + DIST(Pred(\Gamma, x), x))$ ;

*EVAL2*  $\leftarrow (Cost_{A, B, C}(\Gamma, \delta), \delta)$ ;

### Proposition 2

Both *EVAL1* and *EVAL2* yield a time value set  $\delta$  which is compatible with  $\Gamma$  and  $\mathcal{F}$  (with  $\Gamma$  and  $\mathcal{FQ}(\Gamma)$ ). Besides, if  $B = C = 0$ , then *EVAL1* yields an optimal value  $Val$ , that means yields the smallest possible value  $Cost_{A, B, C}(\Gamma, \delta)$ ,  $\delta \in Valid(\Gamma, \mathcal{F})$ .

### Proof

As in the description of both procedures *EVAL1* and *EVAL2*, we suppose that for any  $x$  in  $\Gamma$ , the time window  $\mathcal{FQ}(\Gamma)$  may also be written  $\mathcal{FQ}(\Gamma) = [a(x), b(x)]$ ;

The first part of the above statement is trivial. In case  $B$  and  $C = 0$ , minimizing  $Cost_{A, B, C}(\Gamma, \delta)$  means minimizing  $\delta(Last(\Gamma)) - \delta(First(\Gamma))$ . We must deal with two cases:

- First Case: there exists  $x \in \Gamma$  and  $x \neq Last(\Gamma)$  such that:
  - o  $\delta(x) = a(x)$ ;
  - o For any  $y$  such that  $x \ll_{\Gamma} y \ll_{\Gamma} Last(\Gamma)$ , we have:  $\delta(Succ(\Gamma, y)) - \delta(y) = DIST(y, Succ(\Gamma, y))$ ;

Then the stability of  $\mathcal{FQ}(\Gamma)(x)$  under the inference rule  $R_3$  allows us to deduce  $\delta(Last(\Gamma)) = a(Last(\Gamma))$ , and the result since  $\delta(First(\Gamma)) = b(First(\Gamma))$ .

- Second Case: for any  $x$  in  $X(\Gamma)$ ,  $x \neq Last(\Gamma)$ , we have  $\delta(Succ(\Gamma, x)) - \delta(x) = DIST(x, Succ(\Gamma, x))$ .

Then the result comes in an immediate way.

*End-Proof.*

$\Gamma$  being some valid tour, we denote by  $VAL1(\Gamma)$  and  $VAL2(\Gamma)$  the values respectively produced by the application of *EVAL1* and *EVAL2* to  $\Gamma$ .

### C. The Insertion Mechanism

It works in a very natural way. Let  $\Gamma$  be some valid tour, let  $\mathcal{D}_i = (o_i, d_i, \Delta_i, \mathcal{F}(o_i), \mathcal{F}(d_i), Q_i)$  be some demand whose origin and destination nodes are not in  $\Gamma$ , and let  $x, y$  be two nodes in  $\Gamma$ , such that  $x \ll_{\Gamma} y$ . Then we denote by  $INSERT(\Gamma, x, y, i)$  the tour which is obtained by:

- locating  $o_i$  between  $x$  and  $Succ(\Gamma, x)$ ;
- locating  $d_i$  between  $y$  and  $Succ(\Gamma, y)$ .

We say that the tour  $INSERT(\Gamma, x, y, i)$  results from the *insertion of demand  $D_i$  into the tour  $\Gamma$  according to the insertion nodes  $x$  and  $y$* . The tour  $INSERT(\Gamma, x, y, i)$  may not be valid. So, before anything else, we must detail the way the validity of this tour is likely to be tested.

**Testing the Load-Admissibility of INSERT( $\Gamma, x, y, i$ ).**

We only need to check, that for any  $z$  in  $\text{Segment}(\Gamma, x, y) = \{z \text{ such that } x \ll_{\Gamma} z \ll_{\Gamma} y\}$  we have,  $C(\Gamma, z) + Q_i \leq \text{CAP}$ . It comes that we may set:

**Procedure Test-Load( $\Gamma, x, y, i$ ):**

$\text{Test-Load} \leftarrow \{\text{For any } z \text{ in } \text{Segment}(\Gamma, x, y), C(\Gamma, z) + Q_i \leq \text{CAP}\};$

**Testing the Time-Admissibility of INSERT( $\Gamma, x, y, i$ ).**

It should be sufficient perform a call  $\text{Propagate}(\Gamma, \{o_i, d_i\}, \mathcal{FP}(\Gamma))$ , while using the list  $\{o_i, d_i\}$  as a starting list. Still, such a call is likely to be time consuming. So, in order to make the testing process go faster, we introduce several intermediary tests, which aim at interrupting the testing process in case non-feasibility can be easily noticed:

- the first test *Test-Node* aims at checking the feasibility of the insertion of a node  $u$ , related to some load  $Q$ , between two consecutive node  $z$  and  $z'$  of a given tour  $\Gamma$ . It only provides us with a necessary condition for the feasibility of this insertion.
- the second test *Test-Node1* aims at checking the feasibility of the insertion of an origin/destination node  $u, v$ , related to some load  $Q$ , between two consecutive node  $z$  and  $z'$  of a given tour  $\Gamma$ . Again, it only provides us with a necessary condition for the feasibility of this insertion.

**Procedure Test-Node( $\Gamma, z, z'$ : nodes in  $\Gamma, u$ : node out  $\Gamma, Q$ : load): Boolean**

Let us set, for any  $x$  in  $\Gamma, [a(x), b(x)] = \mathcal{FP}(\Gamma)(x)$ ;

Let us set:  $[\alpha, \beta] = \mathcal{F}(u)$ ;

$\text{Test node} \leftarrow (a(z) + \text{DIST}(z, u) \leq \beta) \wedge (\alpha + \text{DIST}(u, z') \leq b(z')) \wedge (a(z) + \text{DIST}(z, u) + \text{DIST}(u, z') \leq b(z')) \wedge (C(\Gamma, z) + Q \leq \text{CAP});$

**Procedure Test-Node1( $\Gamma, z, z'$ : nodes in  $\Gamma, u, v$ : nodes out  $\Gamma, Q$ : load): Boolean**

Let us set, for any  $x$  in  $\Gamma, [a(x), b(x)] = \mathcal{FP}(\Gamma)(x)$ ;

Let us set, for any  $x$  in  $\{u, v\}: [\alpha(x), \beta(x)] = \mathcal{F}(\Gamma)(u)$ ;

$\text{Test node1} \leftarrow (a(z) + \text{DIST}(z, u) \leq \beta(u) \wedge (\alpha(u) + \text{DIST}(u, v) \leq \beta(v)) \wedge (\alpha(v) + \text{DIST}(v, z') \leq b(z'))) \wedge (a(z) + \text{DIST}(z, u) + \text{DIST}(u, v) \leq \beta(v)) \wedge (a(z) + \text{DIST}(z, u) + \text{DIST}(u, v) + \text{DIST}(v, z') \leq b(z')) \wedge (\alpha(u) + \text{DIST}(u, v) + \text{DIST}(v, z') \leq b(z')) \wedge (C(\Gamma, z) + Q \leq \text{CAP});$

So, testing the admissibility of a tour  $\text{INSERT}(\Gamma, x, y, i)$  may be performed through the following procedure:

**Procedure Test-Insert( $\Gamma, x, y, i$ ): (Test: Boolean, Val: Number);**

If  $x \neq y$  then  $\text{Test} \leftarrow \text{Test-Node}(\Gamma, x, \text{Succ}(\Gamma, x), o_i, Q_i) \wedge \text{Test-Node}(\Gamma, y, \text{Succ}(\Gamma, y), d_i, Q_i)$ ;

Else  $\text{Test} \leftarrow \text{Test-Node1}(\Gamma, x, \text{Succ}(\Gamma, x), o_i, d_i, Q_i)$ ;

If  $\text{Test} = 1$  then  $\text{Test} \leftarrow \text{Test-Charge}(\Gamma, x, y, i)$ ;

If  $\text{Test} = 1$  then  $(\text{Test}, \mathcal{F}1) \leftarrow \text{Propagate}(\Gamma, \{o_i, d_i\}, \mathcal{FP}(\Gamma))$ ;

If  $\text{Test} = 1$  then  $\text{Val} \leftarrow \text{EVAL1}(\text{INSERT}(\Gamma, x, y, i), \mathcal{F}1).\text{Val}$ ;

Else  $\text{Val} \leftarrow \text{Undefined}$ ;

$\text{Test-Insert} \leftarrow (\text{Test}, \text{Val} - \text{Val1}(\Gamma))$ ;

**D. The Insertion Process**

So, this process takes as input the demand set  $\mathcal{D} = (\mathcal{D}_i = (o_i, d_i, \Delta_i, \mathcal{F}(o_i), \mathcal{F}(d_i), Q_i), i \in I)$ , the 4-uple  $(X, \text{DIST}, K, \text{CAP})$ , and 3 multi-criterion coefficients  $A, B$  and  $C \geq 0$ , and it works in a greedy way through successive insertions of the various demands  $\mathcal{D}_i = (o_i, d_i, \Delta_i, \mathcal{F}(o_i), \mathcal{F}(d_i), Q_i)$  of the demand set  $\mathcal{D}$ . The basic point is that, since we are concerned with tightly constrained time windows and transit bounds, we use, while designing the INSERTION algorithm, several constraint propagations tricks. Namely, we make in such a way that, at any time we enter the main loop of this algorithm, we are provided with:

- the set  $I_1 \subset I$  of the demands which have already been inserted into some tour  $T(k), k = 1..K$ ;
- current tours  $T(k), k = 1..K$ : for any such a tour  $T(k)$ , we know the related time windows  $\mathcal{FP}(T(k))(x), x \in T(k)$ , as well as the load values  $C(T(k), x), x \in T(k)$ , and the values  $\text{VAL1}(T(k))$  and  $\text{VAL2}(T(k))$ ;
- the knowledge, for any  $i$  in  $J = (I - I_1)$  of the set  $\text{FREE}(i)$  of all the 4-uple  $(k, x, y, v), k = 1..K, x, y \in T(k), v \in Q$ , such that a call **Test-Insert**( $T(k), x, y, i$ ) yields a result  $(1, v)$ . We denote by  $N\text{-FREE}(i)$  the cardinality of the set  $V\text{-FREE}(i) = \{k = 1..K, \text{ such that there exists a 4-uple } (k, x, y, v) \text{ in } \text{FREE}(i)\}$ :  $N\text{-FREE}(i)$  provides us with the number of vehicles which are still able to deal with demand  $\mathcal{D}_i$ .

Then, the INSERTION algorithm works according to the following scheme:

- First, it picks up some demand  $i_0$  in  $J$ , among those demands which are the most constrained, that means which are such that  $N\text{-FREE}(i_0)$  is small: more specifically, if there exists  $i$  such that  $N\text{-FREE}(i) = 1$ , then  $i_0$  is chosen in a random way among those demand indices  $i$  in  $J$  which are such that  $N\text{-FREE}(i) = 1$ ; else we select randomly in a set of demands  $j$  with  $N\text{-FREE}(j)$  inside  $\{2, N\text{-FREEMAX}\}$ .  $N\text{-FREEMAX}$  becomes a parameter of the INSERTION. (E4)
- Next, it picks up  $(k_0, x_0, y_0, v_0)$  in  $\text{FREE}(i_0)$  which simultaneously corresponds to one of the smallest values  $v$ , and to one of the smallest values  $\text{EVAL2}(\text{INSERT}(T(k), x, y, i_0)).\text{Val} - \text{VAL2}(T(k))$ : more specifically it first builds the list *L-Candidate* of the  $N_1$  (up to five) 4-uples  $(k, x, y, v)$  in  $\text{FREE}(i_0)$  with best (smallest value  $v$ ). For any such a 4-uple, it computes the value  $w = \text{EVAL2}(\text{INSERT}(T(k), x, y, i_0)).\text{Val} - \text{VAL2}(T(k))$ , and it orders *L-Candidate* according to increasing values  $w$ . Then it randomly chooses  $(k_0, x_0, y_0, v_0)$

among those  $N_2 \leq N_1$  first 4-uples in  $\mathcal{L}$ -Candidate.  $N_1$  and  $N_2$  become two parameters of the INSERTION procedure. (E5)

- Next it inserts the demand  $\mathcal{D}_{i_0}$  into  $T(k_0)$  according to the insertion nodes  $x_0, y_0$ , which means that it replaces  $T(k_0)$  by  $\text{INSERT}(T(k_0), x_0, y_0, i_0)$ ;
- Next it defines, for any  $i \in J$ , the set  $\Lambda(i)$  as being the set of all pairs  $(x, y)$  such that there exists some 4-uple  $(k_0, x', y', v)$  in  $\text{FREE}(i)$ , which satisfies:
  - o  $(x' = x)$  or  $((x' = x_0)$  and  $x' = \text{Pred}(T(k_0), x))$  or  $((x' = x_0 = y_0)$  and  $(x' = \text{Pred}(\text{Pred}(T(k_0), x))))$ ;
  - o  $(y' = y)$  or  $((y' = y_0)$  and  $y' = \text{Pred}(T(k_0), y))$  or  $((y' = x_0 = y_0)$  and  $(y' = \text{Pred}(\text{Pred}(T(k_0), y))))$ ;
- Finally, it performs, for any pair  $(x, y)$  in  $\Lambda(i)$ , a call **Test-Insert** $(T(k_0), x, y, i)$ , and it updates  $\text{FREE}(i)$  and  $\text{N-FREE}(i)$  consequently.

This can be summarized as follows:

**Procedure INSERTION**( $N_1$  and  $N_2$ : Integer): ( $T$ : tour set,  $t$ : time value set,  $\text{Perf}$ : induced  $\text{Perf}_{A, B, C}(T, t)$  value,  $\text{Reject}$ : rejected demand set);

For any  $k = 1..K$  do

$T(k) \leftarrow \{\text{DepotD}(k), \text{DepotA}(k)\}$ ;  
 $t(\text{DepotD}(k)) = t(\text{DepotA}(k)) \leftarrow 0$ ;

$I_1 \leftarrow \text{Nil}$ ;  $J \leftarrow I$ ;  $\text{Reject} \leftarrow \text{Nil}$ ;

For any  $i \in J$  do

$\text{N-FREE}(i) \leftarrow K$ ;

$\text{FREE}(i) \leftarrow$  all the possible 4-uple  $(k, x, y, v)$ ,  $k = 1..K$ ,  
 $x, y \in \{\text{DepotD}(k), \text{DepotA}(k)\}$ ,  $x \ll_{T(k)} y$ ,  $v = \text{EVAL2}(\{\text{DepotD}(k), o_i, d_i, \text{DepotA}(k)\})$ .Val;

While  $J \neq \text{Nil}$  do

Pick up some demand  $i_0$  in  $J$  as in (E4); Remove  $i_0$  from  $J$ ;

If  $\text{FREE}(i_0) = \text{Nil}$  then  $\text{Reject} \leftarrow \text{Reject} \cup \{i_0\}$

Else

Derive from  $\text{FREE}(i_0)$  the  $\mathcal{L}$ -Candidate list and Pick up  $(k_0, x_0, y_0, v_0)$  in  $\mathcal{L}$ -Candidate as in (E5);

$T(k_0) \leftarrow \text{INSERT}(T(k_0), x_0, y_0, i_0)$ ;

$\delta \leftarrow \text{EVAL2}(T(k_0)).\delta$ ; Insert  $i_0$  into  $I_1$ ;

For any  $x$  in  $T(k_0)$  do  $t(x) \leftarrow \delta(x)$ ;

For any  $i \in J$  do

$\Lambda(i) \leftarrow$  {all pairs  $(x, y)$  such that there exists some 4-uple  $(k_0, x', y', v)$  in  $\text{FREE}(i)$ , which satisfies (E6);

For any pair  $(x, y)$  in  $\Lambda(i)$  do

( $\text{Test}, \text{Val}$ )  $\leftarrow$  **Test-Insert** $(T(k_0), x, y, i)$ ;

Remove  $(k_0, x, y, v)$  from  $\text{FREE}(i)$  in case such a 4-uple exists and update  $\text{N-FREE}(i)$  consequently;

If  $\text{Test} = 1$  then insert  $(k_0, x, y, \text{Val})$  into  $\text{FREE}(i)$  and update  $\text{N-FREE}(i)$  consequently;

$\text{Perf} \leftarrow \text{Perf}_{A, B, C}(T, t)$ ;

**INSERTION**  $\leftarrow (T, t, \text{Perf}, \text{Reject})$ ;

Since the above (I1) and (I2) instruction may be written in a non deterministic way, the whole INSERTION algorithm becomes non deterministic and may be used inside some MONTE-CARLO framework:

**RANDOM-INSERTION**( $N_1, N_2, P$ : Integer) **Scheme**;

For  $p = 1..P$  do

Apply the INSERTION( $N_1, N_2$ ) procedure;

Keep the best result (the pair  $(T, t)$  such that  $|\text{Reject}|$  is the smallest possible, and which is such that, among those pairs which minimize  $|\text{Reject}|$ , it yields the best  $\text{Perf}_{A, B, C}(T, t)$  value).

## V. COMPUTATIONAL EXPERIMENTS

Our experimentations deal with the randomly generated instances of Cordeau and Laporte [4]. To analyse the behavior of our solution, we used the same objective function used in [7] and adapted in [8]. The instances have between 24 and 144 requests which have to be supported by a fleet of 3 to 13 vehicles. The maximum route duration is 480 for each vehicle and for each instance. The capacity is equal to 6 and the maximum ride time is 90.

[7] used the objective function given in equation (4), the terms penalizing the violations have been removed. Thus, we minimize travel distance ( $c$ ), excess ride time ( $r$ , cf. (1)), passenger waiting ( $l$ , cf. (2)), the total duration  $\mathcal{G}lob$  ( $g$ ) and early arrival ( $e$ , cf. Fig. 1 & (3)). We set the weight like in [7] and [8] to  $w_1=8, w_2=3, w_3=1, w_4=1, w_5=|\mathcal{D}|$ .

$$r = \sum_{k=1}^K \sum_{i \in k} (\text{Ride}(i) - \text{DIST}(o_i, d_i)) \quad (1)$$

$$l = \sum_{k=1}^K \sum_{x=\text{succ}(\text{First}(\Gamma_k))}^{\text{pred}(\text{last}(\Gamma_k))} \text{Wait}_x(\text{C}(\Gamma_k, x) - q_x) \quad (2)$$

$$e = \sum_{k=1}^K \sum_{x=\text{First}(\Gamma_k)}^{\text{pred}(\text{pred}(\text{last}(\Gamma_k)))} F.\text{Min}(\text{succ}(x)) - (\delta(x) + \text{DIST}(x, \text{succ}(x))) \quad (3)$$

$$\text{Cost} = w_1c + w_2r + w_3l + w_4g + w_5e \quad (4)$$

Table I gives the values of the  $\text{COST}$  obtained with the proposed insertion techniques using constraint propagation. We take best results over  $25.10^4$  replications with a variation in the values of  $\text{N-FREEMAX}$ ,  $N_1$  and  $N_2$  (each lower than 4). We noted only the objective function of the two works. So we compare our Insertion Techniques (IT) with the Variable Neighborhood Search (VNS) and the Genetic Algorithm (GA). Refer to [13] and [8] for the other values.

As with the VNS technique, we obtained results always better than the GA. Moreover, we often obtained better



results than the variable neighborhood search. So we found a large difference between [7] and the others works, but solutions obtained by us and by Parragh and al. [8] are close even though in R10a we obtain a large gap. In fact, time constraints of this instance are very tight and we use a simple learning algorithm without computing a precise order for introducing the demands already rejected.

Early arrivals have the largest weight in the objective function and the related column gives us numbers close to 0 (except for R10a). As a result, no vehicle arrives at a node before the beginning of a node's time window.

[8] used an Intel Pentium D computer at 3.2 GHz and the results of this paper are computed with an Intel Q8300 at 2.5 GHz (only one thread has been used). Our CPU times are close to the VNS' runs with the same number of iterations (e.g. we required only one minute for R1a and 38 minutes for R10a).

## VI. CONCLUSION

The static multi-vehicle DARP with Time Windows required approximate solutions for being able to be solved in a reasonable time. We have described an implementation of some insertion techniques using constraint propagation. This solution allows obtaining good results in little time. In addition, we formulate an objective function which optimizes quality of service. But, in order to compare with tests found in literature we prove the flexibility of our framework by changing the objective function without modification of the framework itself. Despite this change, we obtain good results.

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TABLE I.  
INSERTION TECHNIQUES (IT) COMPARED TO GA ([7]) AND VNS ([8])

Instances	Customers	Total Cost f (GA)	Total Cost f (VNS)	Total Cost f (IT)	Travel distance (IT)	Excess ride (IT)	Passenger wait. (IT)	Total duration (IT)	Early Arrival (IT)
R1a	24	4696	<b>3234.60</b>	3371.41	272.81	145.55	0.00	752.28	0.00
R2a	48	19426	14640.16	<b>8152.32</b>	495.29	711.18	46.75	1625.71	8.00
R3a	72	65306	15969.08	<b>10361.79</b>	861.78	388.59	0.00	2301.78	0.00
R5a	120	213420	23852.00	<b>14006.79</b>	1054.57	705.22	0.00	3454.57	0.00
R9a	108	333283	<b>13806.40</b>	14081.01	1056.17	805.16	0.00	3216.17	0.00
R10a	144	740890	<b>25016.46</b>	43889.79	1517.66	1568.67	85.74	4553.24	155.58
R1b	24	4762	2825.53	<b>2809.75</b>	235.80	69.16	0.00	715.81	0.00
R2b	48	13580	<b>5003.11</b>	5066.46	449.26	21.04	0.00	1409.26	0.00
R5b	120	98111	<b>12360.50</b>	12528.93	1001.21	372.68	0.00	3401.21	0.00
R6b	144	185169	16499.44	<b>16005.12</b>	1321.22	411.38	0.00	4201.22	0.00
R7b	36	9169	4601.71	<b>4480.11</b>	395.98	65.43	0.00	1115.98	0.00
R9b	108	167709	<b>13412.76</b>	13586.04	1062.19	622.11	0.00	3222.19	0.00
R10b	144	474758	<b>16420.00</b>	17546.52	1411.27	655.03	0.00	4291.27	0.00