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# A Two-grid Finite Element / Reduced Basis method for fluid dynamics problems

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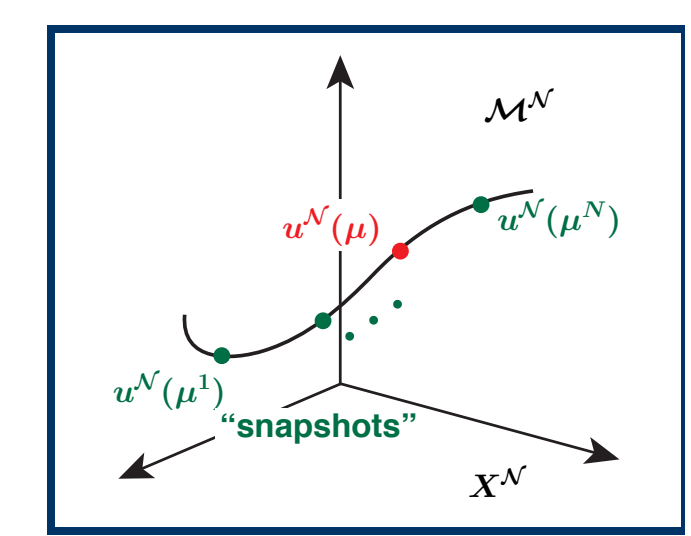
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## Introduction:

Given  $\sigma$  a set of parameters (boundaries conditions, physicals parameters,...) computing the solution  $u^{\mathcal{N}}(\sigma)$  using classical discretization techniques such as finite element methods are time consuming. The reduced basis (R.B.) methods exploit the parametric structure of the governing PDEs to construct rapidly, convergent and computationally efficient approximations. These methods rely on the fact that the set of solutions  $u^{\mathcal{N}}(\sigma)$ , depending on parameters  $\sigma$ , is a manifold  $\mathcal{M}^{\mathcal{N}}$  with a simple structure. We can select a set of parameters  $(\sigma_1, \dots, \sigma_N)$  in such way that  $\mathcal{M}^{\mathcal{N}}$  can be approximated by  $W_N^{\mathcal{N}} = \text{span}(u^{\mathcal{N}}(\sigma_1), \dots, u^{\mathcal{N}}(\sigma_N))$ .



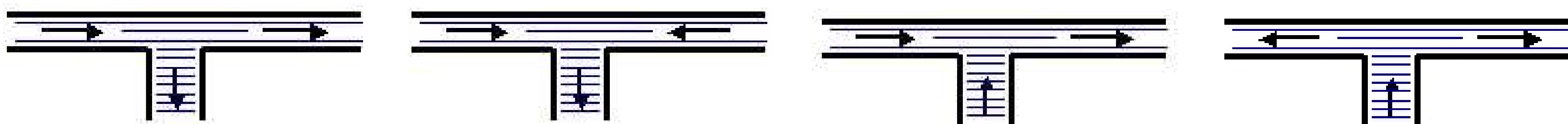
$$\mathcal{M}^{\mathcal{N}} = \{ u^{\mathcal{N}}(\sigma), \sigma \in \mathcal{D} \}$$

How to actually compute the reduced solution  $u_N(\sigma)$  for a given  $\sigma$ ? The reduced basis method is a Galerkin approach within  $W_N^{\mathcal{N}}$ . This technique is promising if  $N$  is small!!  $N \ll \mathcal{N}$  ( $\mathcal{N}$  begin the dimension of the classical discretization space  $X^{\mathcal{N}}$  and  $\mathcal{N}$  the kolmogorov width of  $\mathcal{M}^{\mathcal{N}}$ ).

→ One of the keys of the reduced basis method is the decomposition of the computational work into an OFFLINE and ONLINE stage.

## Motivation:

In an industrial framework, for optimization processes for instance the reduced basis methods have a great potential. One of the keys of this technique is the decomposition of the computational work into an off-line and on-line stage. However in some situation, it's not possible to perform all the off-line computations required with an efficient performance of the reduced method. For example when the simulation code is used as a black box, one won't be able to perform a very fast and cheap online stage. For this reason, we proposed an alternative method. The aim of this work is to provide tests to validate and generalize our method to fluid dynamics problems governed by the incompressible steady state Navier-Stokes equations. Let us consider a physical domain  $\Omega$  with its boundary  $\partial\Omega = \Gamma_{in} \cup \Gamma_{wall} \cup \Gamma_{out}$ . On  $\Gamma_{in}$ , a velocity  $\vec{v}_{in}$  is prescribed, which has a parabolic profile:  $\vec{v}_{in} = V_{in} f(\vec{x}) \vec{n}|_{\Gamma_{in}}$ . On  $\Gamma_{wall}$  we impose homogeneous Dirichlet boundary condition and on  $\Gamma_{out}$  we impose homogeneous Neumann boundary condition. Here, we are interested in studying the two-grid Finite Element / Reduced Basis method applied to a fluid dynamics problem for T-Junction of pipes parametrized by the velocity magnitude  $V_{in}$  of the inlet flow.



Examples of T-junctions of pipelines

## How :

For a stable implementation of the reduced basis method, it is required to build a better basis than the one composed with the  $u^{\mathcal{N}}(\sigma_i)$ , usually by a Gramm-Schmidt method. Let denotes by  $\{\phi_i^{\mathcal{N}}\}_{i=1, \dots, N}$  these orthonormalized basis of  $W_N^{\mathcal{N}}$ .

The standard reduced basis method aims at evaluating the coefficients  $\alpha_i^h(\sigma)$  intervening in the decomposition of  $u_N(\sigma_i)$  in the basis of the  $\phi_i^{\mathcal{N}}$ , those can appears as a substitute to the optimal coefficients

$$\gamma_i^h(\sigma) = (u^{\mathcal{N}}(\sigma), \phi_i^{\mathcal{N}})_{L^2}$$

intervening in the decomposition of the  $L^2$ -projection of  $u^{\mathcal{N}}(\sigma)$  on  $W_N^{\mathcal{N}}$ . Our alternative method consists in proposing an other surrogate to the  $\gamma_i^h(\sigma)$  defined by  $\gamma_i^H(\sigma) = (u^{\mathcal{N}_H}(\sigma), \phi_i^{\mathcal{N}})_{L^2}$ .

Since the computation of  $u^{\mathcal{N}_H}(\sigma)$ , for  $\mathcal{N}_H \gg \mathcal{N}$  is less expensive than the one of  $u^{\mathcal{N}}(\sigma)$ , using the industrial code with the mesh size  $H$  (chosen adequately) to construct the  $\gamma_i^H(\sigma)$  is still cheap enough. Besides, to improve even further the accuracy of this technique we propose to do a simple rectification of the results.

This treatment will insure that for each value of the parameters  $\sigma_i$ ,  $i = 1, \dots, N$ , used in the construction of the reduced basis, the method return exactly the  $L^2$ -projection of  $u_N(\sigma_i)$  on the  $W_N^{\mathcal{N}}$ . We denote by  $T^N$  the matrix associated to this rectification such that:

$$T^N \begin{pmatrix} \gamma_1^H(\sigma_1) & \dots & \gamma_1^H(\sigma_N) \\ \vdots & & \vdots \\ \gamma_N^H(\sigma_1) & \dots & \gamma_N^H(\sigma_N) \end{pmatrix} = \begin{pmatrix} \gamma_1^h(\sigma_1) & \dots & \gamma_1^h(\sigma_N) \\ \vdots & & \vdots \\ \gamma_N^h(\sigma_1) & \dots & \gamma_N^h(\sigma_N) \end{pmatrix}.$$

For each new value of  $\sigma$ , we will replace the  $\gamma_i^H(\sigma)$  coefficients by

$$\tilde{\gamma}_i^H(\sigma) = \sum_{k=1}^N T_{ik}^N \gamma_k^H(\sigma).$$

## OFFLINE stage

- Construction of a reduced approximation's space.
  - Computation of a sample of solutions (black box software)
  - Selection of  $N$  solutions to build the reduced basis (F.E. Library).
- Orthonormalisation of the reduced basis functions (F.E. Library).
- Preparation for the post-processing.
  - Computation of the  $N$  coarse solutions  $u^{\mathcal{N}_H}(\sigma_i)$  (black box software)
  - Construction of matrix  $T^N$  (F.E. Library).

## ONLINE stage

- Computation of the coarse solution  $u^{\mathcal{N}_H}(\sigma)$ . (black box software)
- Compute the coefficient  $\gamma_i^{\mathcal{N}_H}(\sigma)$ . (F.E. Library)
- Apply the post-processing on the  $\gamma_i^{\mathcal{N}_H}(\mu)$ . (F.E. Library)
- Build the output  $s(u_N^H(\sigma))$ . (F.E. Library)

## Numerical experimentation :

In this application we are interested in the evaluation of the flux velocity vector  $\vec{u}(\sigma) = (u_x, u_y)$  for any set of parameter  $\sigma = V_{in} \in [0.01; 0.5] \text{ cm.s}^{-1}$ .

