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Optimal maintenance and replacement decisions under technological change with consideration of spare parts inventories

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A B S T R A C T

Classical spare parts inventory models assume that the same vintage of technology will be utilized throughout the planning horizon. However, asset replacement often occurs in the form of a new technology that renders existing spare parts inventories obsolete. This paper aims to study the impact of spare parts inventory on equipment maintenance and replacement decisions under technological change via a Markov decision process formulation. The replacement decision is complex in that one must decide with which technology available on the market to replace the current asset. Under technological change, it is shown that the do nothing and repair options have significantly more value as they allow waiting the appearance of even better technologies in the future.

Keywords:

Technology evolution
Spare parts
Equipment replacement
Maintenance
Markov decision process

1. Introduction

Maintenance is a key for ensuring the efficient use of equipment as well as an efficient production process. Repetitive breakdowns or operating in bad condition may lead to lower product quality, increased energy consumption and reduced revenue. Therefore, the objective of maintenance is not simply to overcome failures, but also to predict and prevent revenue loss at the management level. Managers must analyze all relevant information to assess the profitability of equipment, give sound investment decisions, and consider possible cost saving. In particular, high spare parts inventory is one important factor in maintenance costs. Asset downtime may increase due to waiting time if necessary spare parts are not available due to poorly managed inventory levels. Hence, the consideration of the spare parts inventory problem is an essential task of managers.

There has been intensive research to study the different aspects of spare parts inventory problems such as management issues, multi-echelon problems, age-based replacement, repairable spare parts, problems involving obsolescence, etc. A review of the spare parts inventory's problems is presented in Kennedy et al. (2002). As the authors comment, spare parts inventories totally differ from other manufacturing inventories. Their function is to assist maintenance staff in keeping equipment in operating condition. The close relation between spare parts inventories and maintenance has been discussed in several articles. Kabir and Al-Olayan (1996)

studies the joint-optimization of age-based replacement and spare parts inventory policy (s, S) . Under a block replacement strategy, Vaughan (2005) utilizes dynamic programming to solve the spare parts ordering problem while Chelbi and Ait-Kadi (2001) and Saker and Haque (2000) present management policies for a manufacturing system, aiming to optimize jointly the maintenance strategy with continuous review spare items inventory. Chien (2009) and Sheu and Chien (2004) extend the problem by also studying minimal repair for minor failures while De Smidt-Destombes et al. (2006) considers repair capacity of degraded/failed units after they were replaced by spare parts.

However, all of the above models are constructed on the assumption that the same vintage of technology will be utilized throughout the planning horizon. They do not take into account the appearance of new technology with performance improvement. This information is very important for managers who often confront the technology investment decision. They must weigh the benefits of utilizing the available equipment with their stock of spare parts and the revenues of investment in new technology. Nevertheless, there are few articles that take into account technological development in the spare parts inventory problem. They are generally based on the introduction of an economical loss when new technology appears by a cost of obsolescence. Kim et al. (1996) does not explicitly consider an obsolescence cost. They include it in the holding cost in a multi-echelon system. Cobbaert and Oudheusden (1996) develops models that can be seen as extensions of the EOQ formula for fast moving spare parts subject to sudden obsolescence risk. The authors examine the effects of obsolescence on costs under several different conditions: constant obsolescence risk and no shortages are allowed; varying obsolescence risk and no shortages are allowed and finally varying obsolescence risk with shortages. 7 parts inventory and

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maintenance strategy by considering a constant rate demand until the obsolescence time.

On the end of the spectrum, models devoted to maintenance optimization involving technological change do not consider the spare part inventory impact (Borgonovo et al., 2000; Clavareau and Labeau, 2009a, 2009b; Dogramaci and Fraiman, 2004; Hopp and Nair, 1994; Karsak and Tolga, 1998; Mercier, 2008). This gap in the literature motivates us to develop an appropriate model to meet management's requirements: optimization of maintenance cost while simultaneously updating information on technological development and considering the impact of spare parts inventory levels to make sound investment decisions. Our main objective is to examine how spare parts inventory levels will influence the replacement decision and also how much better a new technology must be in order to overcome the obsolescence of existing spare parts inventory. In order to focus on this objective, we do not consider spare parts management such as the inventory optimization problem and the potential repair of the spare parts.

We formulate a discrete-time, non-stationary Markov decision process (MDP) to determine the optimal action plan. For modeling technological evolution, we combine the uncertain appearance model (wherein technology change is characterized by the uncertain arrival time of new technology) and the geometric model. The geometric model presented by Bethuynne (2002), Borgonovo et al. (2000), Karsak and Tolga (1998), Torpong and Smith (2003), Natali and Yatsenko (2007, 2008a, 2008b) is a model where geometric functions are utilized to characterize technology change, such as the maintenance/operation cost functions in vintage equipment or in time. Unlike these articles, we present technology change by the improvement of the expected deterioration rate and the expected profit function within a period. We also consider the non-stationary likelihood of new technology's appearance over time. Thereby, we overcome the disadvantages of the geometric model proposed by Borgonovo et al. (2000). Recall Nair (1995), Nair (1997) also consider the non-stationary probability of the appearance of new technologies. However, their model is focused on the problem of capital investment decisions due to technological change rather than physical deterioration of equipment. To simplify its exposition, no salvage values are considered while we establish a reasonable salvage value function which is proportional to the current purchase price of the technology vintage, decreasing in the remaining lifetime and incurs an even greater reduction when it becomes obsolete due to new technology availability.

The remainder of this paper is structured as follows. In Section 2, we present our mathematical formulation model and its assumptions. In Section 3, the performance of our model is discussed through numerical examples. Finally, conclusions and future work are discussed in Section 4.

2. Model and assumptions

2.1. Problem statement

Consider a single system that includes multiple identical independent components. This system is accompanied by a cargo of n identical spare parts that are utilized for the maintenance of the asset. After these activities, the system will be restored to a better state. We assume that spare parts are not sold separately in the market, i.e., we cannot replenish the spare parts store when it is empty. This is a common assumption for special spare parts because it can be difficult and costly to find original and compatible spare units. In making this assumption we do not consider the optimization of the inventory policy. Our primary goal is to study the impact of the spare parts inventory level on maintenance and replacement decisions under technological change rather than

determining what should be the optimal order level/order quantity for the spare parts.

We consider the maintenance of an asset which continuously deteriorates from the as good as new state $x=0$ to the failure state denoted $x=m$. Note that the asset continues to operate in the failed state but unprofitably. Furthermore, degradation is a complex combination of a general wear process of the asset and the ageing process of its components. This complex combination is not explained here or directly modeled in the degradation process. The degradation process is assumed to be characterized by its expected degradation rate per unit of time. Degradation is not obvious and periodic inspections should be performed to determine the true state. The given inter-inspection interval τ defines the decision epochs.

We assume that only one new technology can appear in a decision interval τ . We introduce p_{i+1}^{k+1} as the non-stationary probability that technology $k+1$ appears in the interval τ given the latest available technology at decision epoch i is k . This probability is non-decreasing in time. The difference in technological generations is modeled by an improvement factor on the expected instantaneous deterioration rate and the accrued profit within a decision period.

Let (x, k, j, s) be the system state of the system at the beginning of the i th decision epoch with s spare parts in stock for the maintenance of the asset having deterioration level x . The asset in use belongs to technological generation j while the latest available technology in the market is $k, k \geq j$. In each decision epoch the possible actions are:

- 1) Do nothing: The asset continues to deteriorate until the next decision epoch and generates a profit $g_j(x)$. Note that $g_j(x)$ is the expected profit within a period when the deterioration state at the beginning of that period is x and the utilized technology is j . The spare parts inventory level is not changed, so the holding cost within this period i is sc_s where c_s is the holding cost per spare part unit in a decision period.
- 2) Imperfect maintenance restores the asset to a given deterioration level, $\max(0, x-d)$ where d models the maintenance efficiency. An imperfect maintenance cost, c_1 is incurred and θ spare parts are utilized to replace the degraded units, thus the spare part inventory level is reduced by $s-\theta$. As we assume that the maintenance time is negligible, in the next decision interval the asset deteriorates from the level $\max(0, x-d)$ and generates a profit $g(\max(0, x-d))$. Imperfect maintenance can be seen as a partial replacement of deteriorated components.
- 3) Perfect maintenance restores the asset to the initial deterioration level $x=0$ and the expected profit within next decision interval is $g(0)$. This action requires $\eta(x)$ spare parts that depends on the deterioration state of the asset. A perfect maintenance can be defined as a complete replacement of all the deteriorated items and a cost c_2 is incurred ($c_2 > c_1$). Note that as we assume the spare parts are only supplied when we buy a new asset, we can perform maintenance actions if and only if there are sufficient spare parts in stock.
- 4) Replace the asset by an available technology h in the market ($j \leq h \leq k$). The replacement time is also considered negligible. A cargo of n spare parts is supplied with the new asset. We assume the spares are only compatible with the same generation asset, hence, after replacement, the spare parts inventory level is n if we decide to replace by a newer generation asset and equals $n+s$ in the case of replacement by same-generation asset. The cost of such a replacement is given by the difference between the purchase price of the new asset $c_{i,h}$ and the salvage value $b_{i,jk}(x)$ of the existing asset. Note that the purchase price of a new technology asset can be

estimated as well as the deterioration rates. This is realistic in cases where the technical parameters and specifications of future designs may be assumed reasonably well beforehand. To capture the characteristics of technology change, we assume that the purchase price of a new asset is decreasing over time after appearance and generally increasing over technological generation while the salvage value of the exiting asset is a function of its current purchase price, its mean residual life and the difference between technological generations. After replacement, the new asset generates a profit $g_h(0)$.

Note that we could easily take into account the salvage value of the remaining spare parts in our model. However, this salvage value is not significant compared to the other values, especially in the obsolescence case with the common assumption that the spares are only compatible with the same generation asset. Hence its influence on the optimal decision can be negligible and we do not consider it.

2.2. Model formulation

In this paper, we use a non-stationary MDP formulation to find the optimal maintenance-replacement policy to maximize the expected discounted value-to-go over the finite horizon time denoted by $V^x(x, k, j, s)$.

Let $V_i(x, k, j, s)$ denote the maximum expected discounted value from the decision epoch i , $k \leq i$ to the last epoch N . Then, $V_i(x, k, j, s) = V^x(x, k, j, s)$.

We define $DN_i(x, k, j, s)$, $M_i(x, k, j, s)$, $PM_i(x, k, j, s)$, $R_i^h(x, k, j, s)$ to be the value functions of the do nothing, imperfect maintenance, perfect maintenance and replacement options at the decision epoch i , when the system state is (x, k, j, s) . $V_i(x, k, j, s)$ is the maximal value of the option set $DN_i(x, k, j, s)$, $M_i(x, k, j, s)$, $PM_i(x, k, j, s)$, $R_i^h(x, k, j, s)$. Note that the maintenance actions are realized if and only if there are sufficient spare parts in stock.

$$V_i(x, k, j, s) = \max \left\{ \begin{array}{l} DN_i(x, k, j, s), IM_i(x, k, j, s)1_{\{s > 0\}}, \\ PM_i(x, k, j, s)1_{\{s > \eta(x)\}}, R_i^h(x, k, j, s) \end{array} \right\} \quad (1)$$

where

$$DN_i(x, k, j, s) = g_j(x) - sc_s + \lambda \left[\sum_{\forall k' \in \{k, k+1\}} p_{i+1}^{k'} \sum_{\forall x' \in \{x, m\}} p_j(x'|x) V_{i+1}(x', k', j, s) \right] \quad (2)$$

$$IM_i(x, k, j, s) = -c_1 + DN_i(\max(0, x-d), k, j, s-\theta) \quad (3)$$

$$PM_i(x, k, j, s) = -c_2 + DN_i(0, k, j, s-\eta(x)) \quad (4)$$

$$R_i^h(x, k, j, s) = b_{i,j,k}(x) - c_{i,h} + DN_i(0, k, j, s+n)1_{\{h=j\}} + DN_i(0, k, j, n)1_{\{h \neq j\}} \quad (5)$$

λ is a discount factor, $\lambda \in [0, 1]$.

The value function of do nothing, $DN_i(x, k, j, s)$ is calculated from two parts. The first part is the reward action in the current period, given by the difference between the expected profit $g_j(x)$ and holding cost of spare parts c_s . The second part is the discounted revenue generated by the asset in the future with discount factor λ , the transition probability of deterioration state $p_j(x'|x)$ and the appearance probability of next technology generation, $p_{i+1}^{k'}$ (when $k' = k+1$). Note that if $k' = k$, next technology generation will not appear at next epoch with probability $p_{i+1}^k = 1 - p_{i+1}^{k+1}$.

The value function of imperfect/perfect maintenance or replacement is given by the action cost (imperfect/perfect maintenance or replacement cost) and the value acquired by system after this action. Note that the value acquired by system after maintenance or replacement action is the value function, $V_i(\cdot)$ of the system state that is affected by the respective action.

3. Numerical examples

In the previous section, we have developed a general model to find the optimal maintenance/replacement policy under technological change while examining the impact of spare parts inventories on this policy. Now we apply this model to a classical k -out-of- n system where each component lifetime is independent and exponentially distributed. The assumptions given in this section are reserved for the numerical examples to illustrate the performance of our general model and are not in any way necessary for the general model, e.g., k -out-of- n systems are only one possible application.

3.1. Input parameters

3.1.1. Probability of appearance of new technology

We define the appearance probability of new technology $k+1$ at decision epoch $i+1$, given the latest available technology at decision epoch i is k , as a time increasing function:

$$p_{i+1}^{k+1} = 1 - \delta \varepsilon^{i-k} \quad (6)$$

δ is the factor that reflects the non-appearance probability of the next generation at the next decision epoch when $k=i$. And the factor ε characterizes the increasing rate of the appearance probability of new technology over time. We have $\delta, \varepsilon \in [0, 1]$.

3.1.2. Deterioration process

We consider a k -out-of- n system, specifically 2-out-of-5, where identical independent components have time to failure T that follows an exponential distribution with parameter α_j (j is the technological generation of system). The improvement of α_j is discussed in the next paragraph.

For a new system, all components are in the good state and the deterioration state of system is $x=0$. When one component fails, the system degrades one unit, and it fails when at least 4 items fail; the system failure state is denoted by m ($m=4,5$). We assume here that imperfect maintenance IM is restricted to the replacement of one failed component, $\theta=1$, and the current system state is updated to $x-d$ with $d=1$. With perfect maintenance, we replace all failed components $\eta(x)=x$ to restore the system to its initial state. The transition probability of the deterioration state of the system is given by:

$$p_j(x'|x) = \binom{N-x}{x'-x} (1 - \exp(-\alpha_j \tau))^{x'-x} (\exp(-\alpha_j \tau))^{N-x} \quad (7)$$

$\forall x, x' \in X : \{0, 1, 2, 3, 4, 5\}$

As the components have an exponential life distribution, their conditional survivor function at age y is equal to the survivor function of a new component. If we do not consider the repair capacity of the system, the survivor function at time y , denoted by $R(x, t)$ depends only on the current deterioration state. (We have $R(m, t) = 0$).

$$R(x, t) = \sum_{z=k}^{N-x} \binom{N-x}{z} (\exp(-\alpha_j t))^z (1 - \exp(-\alpha_j t))^{N-x-z} \quad (8)$$

Therefore, the mean residual lifetime, $MRL(x)$ of system is:

$$MRL(x) = \int_0^{\infty} R(x, t) dt \quad (9)$$

3.1.3. Impact of technological evolution

We model the impact of technological evolution on the expected failure rate α_j of items with the following decreasing geometric function:

$$\alpha_j = ab^{-j-1} \quad (10)$$

Table 1
The input parameters for the Example 1.

Appearance probability & discount factor	δ	ε	λ		
	0.99	0.999	0.98		
Profit	g_0	r_1	a_1	a_2	
	415	0.05	15	1.5	
Maintenance, holding cost, failure rate	c_1	c_2	c_s	a	b
	80	120	1	7.69×10^{-2}	3.46×10^{-2}
Salvage value & purchase price	h	r	$c_{1,1}$	v	u
	0.7	0.8	50	0.998	1.2

where a, b are constant. We choose arbitrarily values for a, b in Table 1.

The profit function is also improved by technological generations. Moreover, we know that the asset will operate less efficiently when its deterioration state is greater. Therefore, the expected profit function within a decision interval τ is decreasing by deterioration state and the greater the deterioration state is, the faster the decrease of the profit function. To reflect these characteristics, we use the following function to characterize the accrued profit.

$$g_{i,j} = (g_0 - a_1 \exp(x))(a_2 - \exp[r_1(1-j)]) \quad (11)$$

where a_1, a_2, g_0, r_1 are constants, chosen arbitrarily in Table 1. In case of failure, the do nothing action is still allowed, but the profit in the next decision epoch $g(m)$ is assumed to be zero.

Additionally, under technological evolution, the purchase price of a new asset is assumed to be decreasing over time after appearance and normally increasing over technological generation as follows:

$$c_{i,k} = c_{1,1} v^{i-1} u^{k-1} \quad (12)$$

where $c_{1,1}$ is the purchase price of the first of technological generation at the first decision epoch; v and u are constants characterizing the change of the purchase price over time and over technological generation. We choose arbitrarily $c_{1,1}, v, u$ in Table 1.

We assume the salvage value is a function of the current purchase price of the technology, the difference between technological generation and the MRL. Thus, we propose the following function for the salvage value, $\forall x \in [0, 5]$; h, r are constant, chosen arbitrarily in Table 1.

$$b_{i,j,k}(x) = hr^{k-j} c_{i,j} \frac{MRL(x)}{MRL(0)} \quad (13)$$

3.2. Numerical analysis

This subsection is organized in two parts. First, a preliminary example is proposed to highlight the performance of the proposed model. Second, a restrictive sensitivity analysis is presented for the identification of some properties of the optimal policies. Due to the large number of model parameters, we have restricted the numerical analysis to the most relevant results. In this subsection, all the data are generic and with no dimension, only relative magnitudes are of interest. The inspection interval, τ is considered as a unit time. We examine the time horizon $N=200$ that is long enough to guarantee the decision optimality for the initial periods. The other parameter values are chosen (presented in Table 1) so that the estimated lifetime of system and the average appearance time of new technologies are not too long or short over the time horizon.

For numerical examples, due to the ratio between the average appearance time of new generation technologies and the time

horizon $N=200$, we consider only the possibility of five technological generations. In addition, we also assume the storage capacity for spare parts is limited to a maximal storage capacity of 20 units and the cargo of spare parts that accompanies a new asset comprises 15 units. Note that this assumption is utilized only to implement the numerical example so as not to consider an infinite possible value of the stock state and does not influence the investment decision in new technology so long as it is chosen sufficiently large as to not be a hard constraint in the optimization.

3.2.1. Impact of spare parts inventory on the optimal strategy

The optimal policy for the first ten decision epochs of Example 1 in cases where $k=1, 2, 3$ is given in Table 2. Note that with time horizon $N \geq 190$, this policy does not change.

With the current technology j , given that generation k is the latest available technology, the optimal decision is defined according to the current deterioration x and stock level s . Each column in Table 2 represents a combination of k and j given by the vector (k,j) in the header. For each vector (k,j) the different combinations of stock level and system state are given in the left and right sides of the columns, respectively. The optimal action is given for each system state at each stock level.

We find that in the cases where there is no obsolescence problem i.e., $(k,j) = (1, 1); (2, 2); (3, 3)$; the replacement decision is made only at low stock levels ($s < 5$). For example, with $(k,j) = (1, 1)$ at stock level $s=0$ when we cannot perform maintenance, thus the optimal policy prescribes replacement (R) for deterioration states greater than $x=2$. This replacement threshold is non-increasing in the used technology (j) because the profit function and the failure rate of components are improved over technological generations, and the replacement option has more value than the do nothing option.

As illustrated by Table 2, the replacement threshold is $x=2$ for $j=1$ and $x=1$ for $j=3$. When the stock level $s > 0$, imperfect/perfect maintenance (IM/PM) is performed by using spare parts to restore the asset to the previous/initial deterioration state. Consider the case $s=2$, the policy dictates the DN action when $x=0, 1$; PM when $x=2$; IM with $x=3, 4$; and finally replace with new asset at $x=5$. With a high stock level, PM demonstrates its dominance and replacement is not necessary. For example, with $(k,j) = (2, 2)$; when the stock level is greater than 5, the optimal policy prescribes DN until $x=2$, then PM .

Now we consider the influence of stock level on the optimal policy in the obsolescence cases: $(k,j) = (2, 1); (3, 1); (3, 2)$; where the available technology is greater than the one currently in use. With the chosen parameters in Table 1, the optimal decision is to replace the used asset by the latest available technology when replacement is prescribed.

We find that if the technological improvement is not significant, the firm tends to take advantage of the performance gained

Table 2
The optimal policy for Example 1.

$(k, j)=(1, 1)$		$(k, j)=(2, 1)$		$(k, j)=(2, 2)$		$(k, j)=(3, 1)$		$(k, j)=(3, 2)$		$(k, j)=(3, 3)$	
s	Action	S	Action	s	Action	s	Action	s	Action	s	Action
0	$x=0, 1: DN$ $x \geq 2: R$	0	$x=0: DN$ $x \geq 1: R$	0	$x=0: DN$ $x=1: R (i \leq 5),$ $DN (i \geq 6)$	[0,2]	$\forall x: R$	0	$x=0: DN$ $x \geq 1: R$	0	$x=0: DN$ $x \geq 1: R$
1	$x=0: DN$ $1 \leq x \leq 4: IM$ $x=5: R$	1	$x=0: DN$ $x=1:$ $R (i=2, 3)$ $IM (i \geq 4)$ $x \geq 2: R$	1	$x=0: DN$ $1 \leq x \leq 4: IM$ $x=5: R$	[3,4]	$3 \leq x \leq s: PM$ $\forall x \in X[3, s]: R$	1	$x=0: DN$ $1 \leq x \leq 3: IM$ $x=4, 5: R$	1	$x=0: DN$ $1 \leq x \leq 4: IM$ $x=5: R$
[2,4]	$x=0, 1: DN$ $2 \leq x \leq s: PM$ $s < x < 5: IM$ $x=5: R$	[2,4]	$x=0,1: DN$ $2 \leq x \leq s: PM$ $x > s: R$	[2,4]	$x=0,1 DN$ $2 \leq x \leq s: PM$ $s < x < 5: IM$ $x=5: R$	$s \geq 5$	$x \leq 2: R$ $x \geq 3: PM$	[2,4]	$x=0,1 DN$ $2 < x \leq s: PM$ $s < x < 4: IM$ $x=4, 5: R$	[2,4]	$x=0,1 DN$ $2 \leq x \leq s: PM$ $s < x < 5: IM$ $x=5: R$
$s \geq 5$	$x=0, 1: DN$ $x \geq 2: PM$	[5,10]	$x=0,1: DN$ $x \geq 2: PM$	$s \geq 5$	$x=0, 1: DN$ $x \geq 2: PM$			[5,13]	$x=0,1 DN$ $x > 2: PM$	[5, 8]	$x=0, 1: DN$ $x \geq 2: PM$
		$s=11$	$x=0: R (i \leq 4), DN(\forall i \geq 5)$ $x=1: R (i \leq 3), DN(\forall i \geq 4)$ $x \geq 2: PM$					$s=14$	$x=0: R$ $x=1: R$ $(i \leq 3),$ $DN(\forall i \geq 4)$ $x \geq 2: PM$	$s \geq 8$	$x=0: DN$ $x=1: IM$ $x \geq 2: PM$
		$s=12$	$x=0: R (i \leq 8), DN(\forall i \geq 9)$ $x=1: R (i \leq 6), DN(\forall i \geq 7)$ $x \geq 2: PM$					$s \geq 15$	$x=0, 1: R$ $x \geq 2: PM$		
		$s \geq 13$	$x=0, 1: R$ $x \geq 2: PM$								

by available equipment and their spare parts in stock before investing in technological innovation. There exists an optimal inventory level to balance the loss due to later investment in new technology: replacement should be made when the stock level is lower than the defined threshold (s_1) or greater than (s_2). For example, with $(k, j) = (2, 1)$, the profit function and the failure rate are improved, i.e., $g_2(x)/g_1(x) = 1.01$ and $\alpha_2/\alpha_1 = 0.9$. If stock levels $\in [5, 10]$, the asset operates normally until deterioration state $x = 2$, then is maintained perfectly to restore it to the initial state. At low stock levels, $s < 5$, compared with the cases $(k = j)$, the firm tends to replace earlier. For example, with $x = 2$, the replacement threshold in obsolescence case $(k, j) = (2, 1)$ is $s = 3$ while it is $s = 5$ in case $(k, j) = (1, 1)$. The greater the stock level is, the higher the holding cost is, so a high inventory level is not needed in the obsolescence case. As illustrated in Table 2, given $(k, j) = (2, 1)$ with $s \geq 13$, spare parts are utilized to perform PM when $x > 1$. This restores the asset to an initial state and a maximal profit $g(0)$ is incurred during the next period. If the deterioration state is $x = 0$ or $x = 1$, it is optimal to sell the asset in order to obtain the maximal residual values $b_{i,j,k}(0)$ or $b_{i,j,k}(1)$ and invest in new technology; else ($x > 1$) one continues to perform PM to reduce the deterioration state and stock level.

In the case where the latest available technology is a much greater improvement over the one currently in place, e.g., $(k, j) = (3, 1)$, the firm tends to invest early in new technology. It replaces the current asset by the latest available technology when the stock level is less than 2. If the stock level $s \geq 3$, with $x \geq 3$, it utilizes spare parts in stock for PM to restore the asset to as-good-as-new before selling it at the next epoch (when its deterioration state $x \leq 2$ or stock level $s \leq 2$).

3.2.2. Impact of the speed of technological evolution on the optimal strategy

In this subsection, we consider the speed of technological evolution and its effects on the optimal strategy. Note that this speed is represented by the appearance probability of new technology in next the period, given by Eq. (6) wherein the factor δ reflects the non-appearance probability of the next generation during the

Table 3
The optimal policy for the first ten decision epochs in the obsolescence case (3, 1).

$\delta=0.99$		0.8		0.6	
s	Action	s	Action	s	Action
[0, 2]	$\forall x: R$	0	$x < 2: DN$ $x \geq 2: R$	0	$x < 3: DN$ $x \geq 3: R$
		1	$x=0: DN$ $x=1, 2: IM$ $x \geq 3: R$	1	$x < 2: DN$ $x=2, 3: IM$ $x \geq 4: R$
		2	$x < 2: DN$ $x=2: PM$ $x=3: IM$ $x \geq 4: R$	2	$x < 2: DN$ $x=2: PM$ $x=3, 4: IM$ $x=5: R$
[3, 4]	$3 \leq x \leq s: PM$ $\forall x \in$ $X[3, s]: R$	[3, 4]	$x=0,1 DN$ $2 \leq x \leq s: PM$ $x > s: R$	[3, 4]	$x=0,1 DN$ $2 \leq x \leq s: PM$ $s < x < 5: IM$ $x=5: R$
$s \geq 5$	$x \leq 2: R$ $x \geq 3: PM$	$s \geq 5$	$x=0,1 DN$ $x \geq 2: PM$	$s \geq 5$	$x=0,1 DN$ $x \geq 2: PM$

next decision interval. Thus, the smaller δ is, the more rapid technology evolution is. As illustrated by the numerical examples in the case $(k, j) = (3, 1)$, when $\delta = 0.99, 0.8, 0.6\delta$, consider the first ten decision epochs (Table 3). We find that the smaller δ is, the greater the value of DN or IM/PM because the firm tends to keep the existing asset for waiting the appearance of new technology when its appearance probability during the next period is high. Consider, for example, when $\delta = 0.99$, the replacement option has significantly more value and we replace immediately the asset at the low stock levels $s \leq 2$. Contrarily, the do nothing and maintenance options demonstrate their dominance when $\delta = 0.6, 0.8$.

4. Conclusion

In this paper, we proposed a model that takes into account the spare part inventory in the maintenance/replacement problem of a stochastically deteriorating system under technological change.

It determines the maintenance/replacement strategy based on the parametric performance of the system and the technological environment. We have combined several aspects never seen before in the same model: equipment replacement, maintenance optimization, technological evolution, and spare parts inventory. However, the application of such model in practice remains difficult both for the characterization of many parameters presented and the integration of new hypotheses that would make a not solvable model.

Through our numerical examples we have shown the influence of the spare parts inventory level and technological change on the maintenance/replacement strategy. In the non-obsolescence case, it is obvious that replacement is done only at low stock levels; on the contrary, at high stock levels, the maintenance options demonstrate their dominance. In the obsolescence case, the replacement option with new technology is motivated, but the trade-off between the benefits of utilizing spare parts in store and the revenues of investment in new technology is also considered. Therefore, replacement is not done when the stock level is in the interval that is determined by the parameters of the model. The better the technological improvement is the greater value the replacement option has. However, in the case of rapid technological change, the do nothing and repair options have significantly more value as they allow the appearance of even better technologies in the future. These findings can greatly aid equipment managers in their maintenance and replacement decisions.

Some proposed assumptions can be seen as limitations of our model such as the expectation of purchase price and improvement of new technology. In fact, these can be stochastic and difficult to capture. An extension of our model could reflect the stochastic characteristics of these parameters. Furthermore, the stochastic efficiency of the imperfect maintenance action could also be included in our model or the non-stationary properties of the deterioration.

The principal objective of this paper is to consider simultaneously the influences of technological evolution and spare parts levels on the optimal maintenance/replacement strategy. Thus we have simplified the inventory problem by assuming that spare parts stock cannot be replenished without purchasing a new asset and that the quantity of spare parts supplied is determined by the manufacturer. In further research, we could take into account the ability to replenish the current spare parts and also examine the optimal inventory policy.

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