

Temperature and strain rate influence on AA5086 Forming Limit Curves: experimental results and discussion on the validity of the M-K model

Xingrong Chu, Lionel Leotoing, Dominique Guines, Eric Ragneau

*Université Européenne de Bretagne, France, INSA-LGCGM-EA 3913
20 Av. des Buttes de Coësmes, CS70839, 35708 Rennes Cedex 7, France*

Email: lionel.leotoing@insa-rennes.fr

Abstract

Due to the high-strength to weight ratio, corrosion resistance, good workability and weldability characteristics, aluminium alloys are increasingly used in many sectors. Researches on formability of aluminium alloy sheets have always been a hot topic these last years while very few works taking into both temperature and strain rate effects on formability limits can be found in the literature. In this study, the formability of sheet metal AA5086 is investigated at different temperatures (20, 150 and 200°C) and strain rates (0.02, 0.2 and 2 s⁻¹) through a Marciniak test setup. Experimental results show that the formability of AA5086 increases with temperature and decreases with forming speed. Based on the analytical M-K theory, a Finite Element (FE) M-K model is proposed to predict the Forming Limit Curves (FLCs). A modified Ludwick hardening law with temperature and strain rate functions is proposed to describe the thermo-elasto-viscoplastic behavior of the material. The influence of the initial imperfection (f_0) sensitivity in the FE M-K model is discussed and a strategy to calibrate f_0 is proposed. The agreement

between experimental and numerical FLCs indicates that the FE M-K model can be an effective model for predicting sheet metal formability under different operating conditions if the initial imperfection value is calibrated for each forming condition.

Keywords: Forming Limit Curves (FLCs); Marciniak test; M-K model; Aluminium alloys

1. Introduction

Sheet metal forming is widely used for producing various structural components, especially in automotive and aeronautic industries. With an ongoing anxiety about fuel consumption and environment protection (reducing CO_2 emission), mass reduction has become necessary. Application of lightweight materials, such as aluminium alloys, has been considered as an interesting alternative, especially in the car or aircraft body design field due to their high-strength to weight ratio, corrosion resistance, good workability and weldability characteristics. A major drawback for aluminium alloys is their low formability at ambient temperature compared to the traditional mild steels [1][2], which slows down their applications. With innovative warm forming methods, the formability of aluminium alloys can be greatly improved. Moreover, under warm forming conditions, the strain rate begins to play a predominant role in determining the sheet metal formability. Hence, characterizing the aluminium alloy formability at elevated temperatures and for a wide range of strain rates is crucial for controlling the success of the designed part forming process.

One efficient tool to assess the sheet metal formability in the literature is the Forming Limit Diagram (FLD) developed by Keeler and Backofen in the

1960s [3], which has been extensively adopted in experimental and numerical investigations. In the FLD, the Forming Limit Curves (FLCs) are capable of predicting the strain levels that can lead to material failure under different strain paths. Experimentally, the Nakazima and Marciniak tests have been proposed in international standard ISO 12004-2 to determine the FLCs for sheet metal at ambient temperature and for quasi-static loadings. These two tests can be extended to elevated temperatures and high strain rates. Temperature and strain rate effects on the FLCs of AA5083-O have been studied by Naka et al. [4] with a Marciniak stretch-forming test on a range of forming speeds (0.2 - 200 mm/min) and temperatures (20 - 300°C). Experimental results showed that the FLC was not sensitive to forming speed at ambient temperature but the level increased drastically with decreasing forming speed at temperatures ranging from 150 to 300°C. Li and Ghosh [5] investigated the formability of three automotive aluminium alloy sheets AA5754, AA5182 and AA6111-T4 by forming rectangular designed parts at a strain rate of 1 s^{-1} from 200 to 350°C. It was shown that temperature had a significant positive effect on the sheet drawing formability and the intensity of this effect varied for the three materials. Recently, Mahabunphachai and Koç [6] investigated the formability of AA5052 and AA6061 alloy sheets at different temperatures (ambient temperature to 300°C) and strain rates (0.0013 and 0.013 s^{-1}) through bulge tests. Classically, the formability was found to increase with temperature and decrease with strain rate. Palumbo and Tricarico [7] investigated the formability (evaluated by the Limit Drawing Ratio - LDR) of AA5754-O through a designed warm deep drawing equipment. A remarkable LDR rise of about 44% compared to ambient temperature was

obtained at punch speed of 1 mm/min and temperature of 110°C in the blank center. Wang et al. [8] studied the formability of AA2024 with the cup punch test, the results also showed that both temperature and punch velocity had a strong influence on the formability.

As well known, the experimental characterization of formability is a complicated and time consuming procedure. The difficulty is emphasized by temperature and strain rate conditions which require the development of dedicated devices. To facilitate the formability evaluation, many analytical and numerical models have been proposed to analyze the necking process and then predict the formability of sheet metal. Among these models, although the Marciniak-Kuczinsky (M-K) theory has been proposed for a long time, it is still widely used due to its simplicity. Lots of works about the determination of FLCs with M-K model at ambient temperature and without strain rate consideration can be found in the current literature, very few studies are concerned with temperature and strain rate effects. The analytical M-K theory assumes an initial thickness imperfection which leads to the onset of localized necking. The main disadvantage of the M-K model is that the results are greatly dependent on this initial imperfection value (f_0), the level of FLCs increases with the value of f_0 [9]. The yield function also affects the right hand side of analytical FLCs [10]. The influence of initial groove orientation ψ_0 in the analytical M-K model has been extensively discussed [10][11][12]. It is well known that for the right hand side of FLCs, the critical minimum strains are obtained with a constant value of ψ_0 ($\psi_0 = 0$), while for the left hand side, the value of the angle must be evaluated in order to minimize the limit strains. The FLC prediction for AA3003-O with M-K

model was carried out by Ahmadi et al. [13] with different yield functions and work-hardening models, compared to experimental data, good numerical result was obtained by implementing BBC2003 and Voce hardening law. The prediction of FLCs for aluminium alloys AA6016-T4 and AA5182-O was studied by Aretz [14] with the M-K model, good results were found with experimental data. All the related works about the M-K model were mainly carried out at ambient temperature. Little work about the M-K model was presented at high temperature, much rare for the coupling of temperature and strain rate. With the analytical M-K model, Khan and Baig [15] recently determined the FLCs of AA5082-O under different temperatures (23, 100 and 200°C) and strain rates (10^{-4} , 10^{-2} and $10^0 s^{-1}$). A positive strain rate effect on the FLCs at 23 and 100°C was observed while a negative effect at 200°C was found. But the initial imperfection value was not mentioned in this work. The FLCs of AA5182-O from 25 to 260°C were determined by Abedrabbo et al. [16] with the analytical M-K model and a constant imperfection value (0.996). The predicted FLCs showed an improvement of the formability with temperature. Unfortunately, these two studies were not validated by experimental results.

M-K model has been proved to be an effective tool for predicting sheet metal formability at ambient temperature and the value of f_0 can be defined according to the best fit between theoretical and experimental results. Up to now, for the use of M-K model at elevated temperatures and different strain rates, no guideline for the choice of f_0 can be found in the literature. Due to the sensitive character of the imperfection value f_0 , a proper value should be predefined to make the M-K model reliable. In this work, the experimental

formability of AA5086 at different temperatures (20, 150 and 200°C) and strain rates (0.02, 0.2 and 2 s^{-1}) is firstly evaluated with a Marciniak test setup. Secondly, uniaxial tensile tests for the same range of temperatures and strain rates are proposed to determine the flow stresses of AA5086. Then, a modified Ludwick hardening law incorporating temperature and strain rate functions is adopted to correlate the sheet metal flow stresses. Finally, the predicted FLCs are obtained from a dedicated FE M-K model at the studied conditions. The comparison between numerical and experimental results is given and the initial imperfection f_0 calibration strategy at different temperatures and strain rates is discussed.

2. Experimental procedure and results

2.1. Marciniak test setup

To carry out the formability tests at different temperatures, a Marciniak test setup is chosen (Figure 1) and two independent dedicated heating systems have been designed. The specimen is heated by heat conduction thanks to eight heaters plugged into the up and bottom blankholders. To ensure constant temperature in the specimen during the test, an additional heater is inserted into the punch. A mica sheet is inserted between the blankholder and the die to improve heat efficiency. The validity of the heating system has been confirmed by temperature measurements with external sensors stuck on the specimen and the punch. The image acquisition system includes a high speed and resolution camera, an optical mirror and an external illumination source. The schematic view of the system is shown in Figure 2. The distance between the mirror and the specimen remains constant through the test.

Different from international standard ISO 12004-2, a dedicated specimen with a non-uniform thickness is adopted in this work (Figure 3). A smaller

Figure 2: Schematic view of the image acquisition system

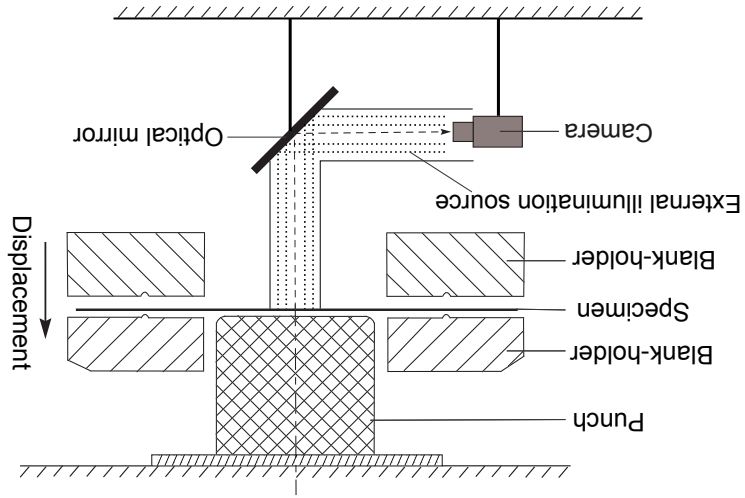
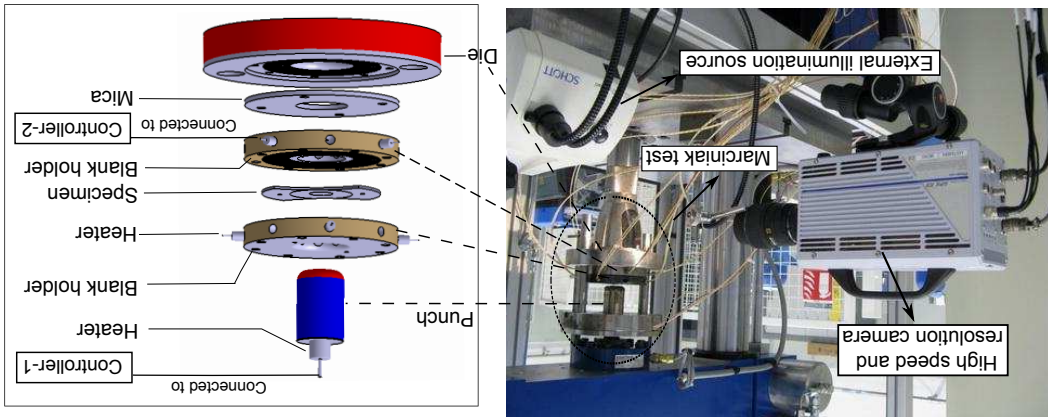


Figure 1: Marciniak test setup equipped with a heating system



The commercial digital image correlation program CORRELA 2006 permits to follow the specimen deformation during the test.

thickness of 0.8mm is set in the central part of the specimen and this thickness ensures a strain localization in this zone. For this material, the influence of the machining on the sheet metal formability is very low. By changing the specimen width (W), different strain paths are followed and the whole FLC from uniaxial stretching ($W = 10\text{mm}$) over plane strain condition ($W \approx 50\text{mm}$) to biaxial stretching ($W = 100\text{mm}$) is built as shown in Figure 4. Three temperatures (20, 150 and 200°C) and punch speeds (0.1, 1 and 10 mm/s) are tested, corresponding to an average strain rate of 0.02, 0.2 and 2 s^{-1} .

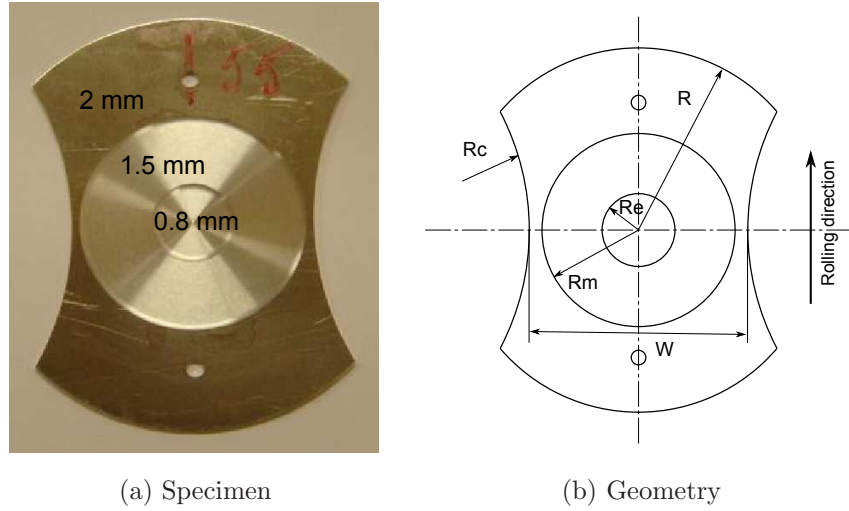


Figure 3: Dedicated specimen

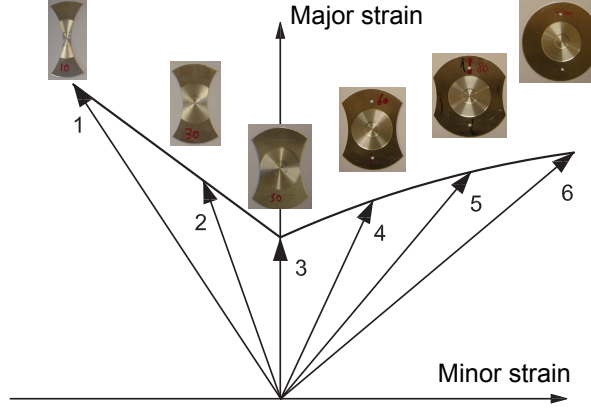


Figure 4: Specimen width and strain paths on the FLC

2.2. Experimental results

A modified 'position-dependent' standard criterion inspired by international standard ISO 12004-2 is adopted in this work to determine the FLCs. As explained in the standard, on both sides of a necked but not cracked specimen, a second order inverse polynomial function is fitted on the major strain values (ε_{11}) to determine the limit strain (ε_{11}^{limit}) at the onset of necking (Figure 5). Different from the standard, the limit strain value of ε_{22} is directly calculated from the measured strain path β_{exp} through the expression $\varepsilon_{22}^{limit} = \beta_{exp} \varepsilon_{11}^{limit}$. This method limits data scatter on the FLC especially near the plane strain condition (low minor strain values).

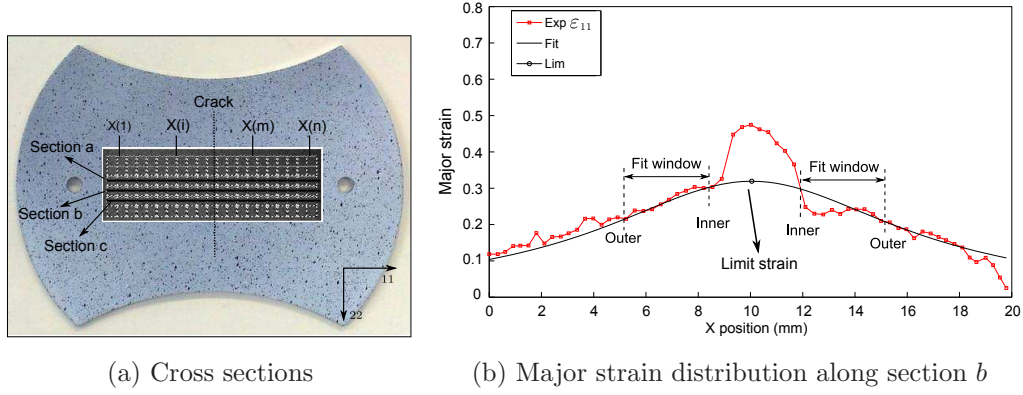


Figure 5: Position-dependent method to evaluate limit strains

The experimental FLCs for the different conditions are shown in Figure 6. At first sight, both temperature and strain rate significantly affect the sheet metal formability. In the literature, it is widely accepted that there is no strain rate effect on the formability of AA5XXX alloys [4] at ambient temperature. Hence, the limit strains at 20°C under 10 mm/s are taken as references for all the forming speeds at ambient temperature. The values of FLC_0 (value of the major strain under plane strain condition) at the different temperatures and strain rates are shown in Figure 7. The positive effect of the temperature and the negative effect of the strain rate on the formability are clearly observed. For the highest forming speed (10mm/s), the whole FLCs at 20 and 150°C are very close whereas a marked increase of formability is observed at 200°C (an increment of 80% is observed between the FLC_0 at 200°C and the one at 20°C). For 1 mm/s, the limit strains are much more sensitive to the temperature, when the temperature grows up from 20°C to 150°C and 200°C, the FLC_0 increments are respectively 24% and 181%. In

the case of the lowest forming speed (0.1mm/s), the difference between the FLCs at 20 and 150°C is very significant. For a given temperature, the FLC_0 increases with low forming speeds. At 150°C , when the forming speed reduces from 10mm/s to 1mm/s and 0.1mm/s , the order of the FLC_0 increment is 35% and 92%, respectively.

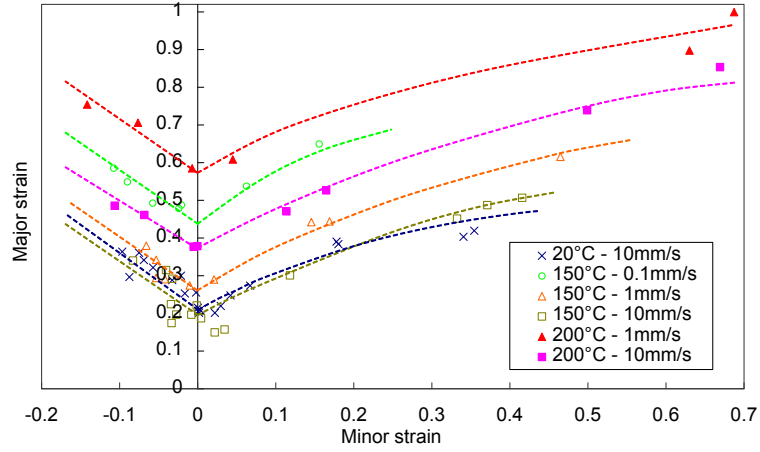


Figure 6: FLCs of AA5086

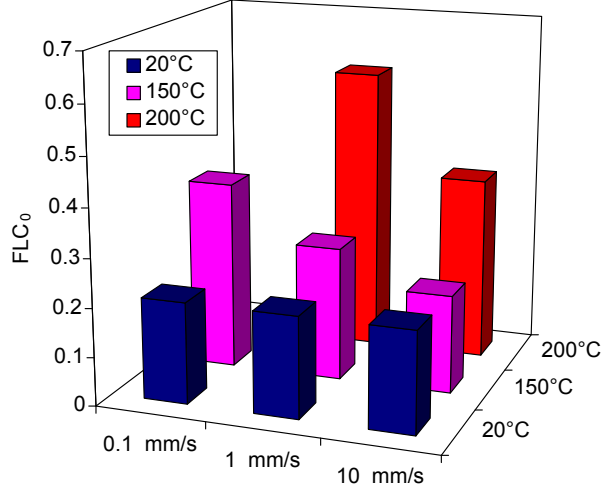


Figure 7: FLC_0 at the different tested temperatures and strain rates

In spite of a strong influence of temperature and strain rate on the level of the FLCs, the global shape of the FLCs is slightly modified for all the strain paths (Figure 6). It is worthwhile to notice that the positive effect of temperature can be compensated by the increase of forming speed. As an example, the FLC_0 at 150°C and 0.1mm/s is a little higher than the one at 200°C and 10mm/s (Figure 7).

3. M-K predictive model

3.1. Introduction of FE M-K model

In analytical studies based on M-K theory, the implemented hardening models are usually simplistic and not always representative of the actual behavior of the material. The difficulty in implementing dedicated hardening models, especially with temperature and strain rate functions, limits its application. Based on the M-K theory, a finite element (FE) M-K model was

proposed by Zhang et al. [17] to determine FLCs. The FE M-K model is shown in Figure 8. Similar to the analytical M-K model, an initial imperfection value is introduced by defining two different thicknesses in zone a (t_a) and zone b (t_b). In current model, t_a is set to 1 mm, different initial imperfection values of $f_0 = t_b/t_a$ can be obtained by changing t_b values.

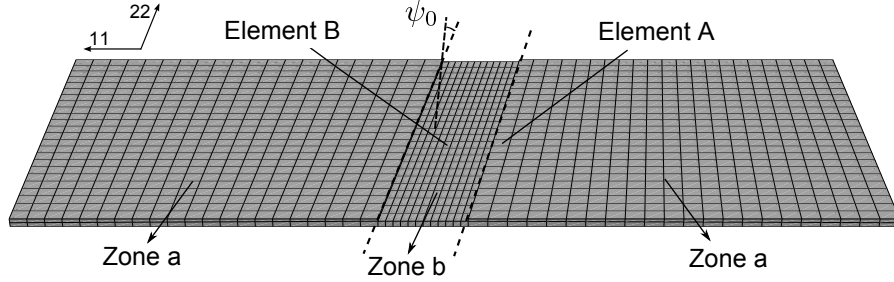


Figure 8: FE M-K model

The model is meshed by hexahedral elements. Due to the initial thickness imperfection, different equivalent plastic strain evolutions can be found in zone a and zone b (Figure 9). When the equivalent plastic strain increment ratio ($\Delta\bar{\epsilon}_p^B/\Delta\bar{\epsilon}_p^A$) of element B and A exceeds 7 [17], localized necking is assumed to occur and the corresponding major and minor strain of element A at this moment are noted as one point on the FLC.

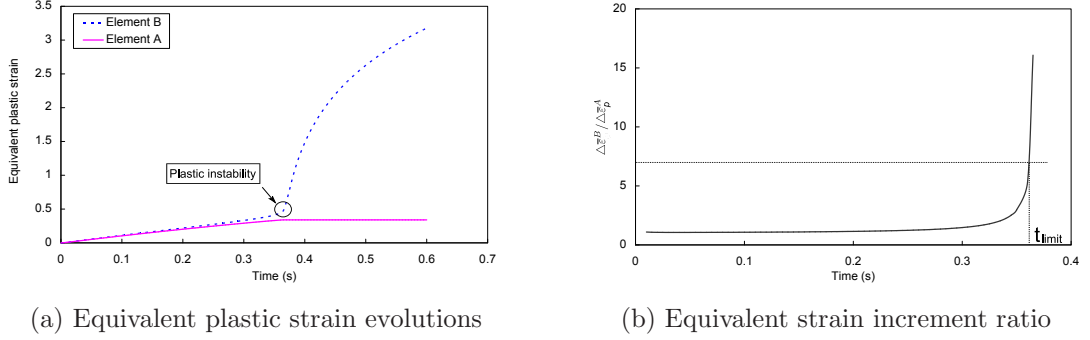


Figure 9: Temporal criterion to determine forming limit values in FE M-K model

By imposing different displacement ratios in the in-plane directions, the limit strains for all the strain paths of the FLC can be determined. Through ABAQUS user-defined subroutine UHARD, different hardening laws can be implemented into the FE M-K model to describe the material flow stress.

3.2. Hardening identification

To identify the hardening behaviour of the AA5086, uniaxial tensile tests are carried out at different temperatures (20, 150 and 200°C) and tensile speeds (1, 10 and 100 mm/s) on a servo-hydraulic testing machine equipped with a heating furnace. These tensile speeds permit to reach the strain rate values measured during Marciniak tests. Figure 10 shows the geometry and dimensions of the standard tensile specimen. All the specimens are machined along the rolling direction.

The hardening model which describes the sheet metal flow stress can affect significantly the accuracy of the simulation results. A classical Ludwick's law incorporating temperature and strain rate functions is first proposed to

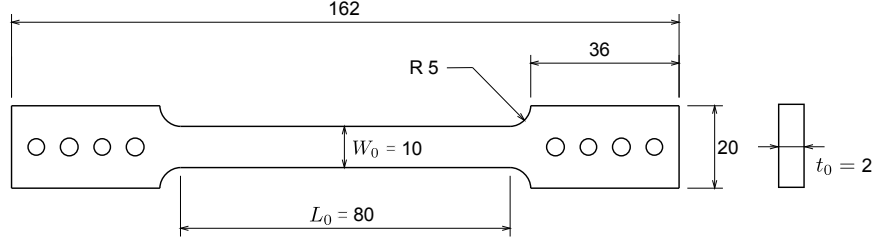


Figure 10: Geometry and dimensions of tensile test specimen (in mm)

describe the thermo-elasto-viscoplastic behavior of AA5086 as shown in Eq (1) :

$$\bar{\sigma} = \sigma_0(T) + (K_0 - K_1 T) \bar{\varepsilon}_p^{(n_0 - n_1 T)} \dot{\bar{\varepsilon}}_p^{m_0 \exp(m_1 T)} \quad (1)$$

where $\bar{\sigma}$ is the equivalent stress, $\bar{\varepsilon}_p$ and $\dot{\bar{\varepsilon}}_p$ are respectively the equivalent plastic strain and the equivalent plastic strain rate. A linear expression with temperature is chosen for both the strain hardening coefficient $K = (K_0 - K_1 T)$ and the strain hardening index $n = (n_0 - n_1 T)$. The strain rate sensitivity index $m = m_0 \exp(m_1 T)$ evolves exponentially with temperature. The evolution of the initial stress $\sigma_0(T)$ with temperature is given by :

$$\sigma_0(T) = \sigma_0 \left(1 - \frac{T}{T_m} \exp \left(Q \left(1 - \frac{T_m}{T} \right) \right) \right) \quad (2)$$

where $T_m=627^\circ\text{C}$ is the melting temperature, $\sigma_0=134.6\text{MPa}$ is the initial yield stress at ambient temperature and $Q=0.556$.

The identified parameters of the proposed hardening model are shown in Table 1. At ambient temperature, $m = 0.00017$, this value is very small which

confirms that the material is strain rate insensitive for low temperatures. For the highest temperature (200°C), m value grows up to 0.052 and leads to a positive strain rate effect on the flow stresses.

Table 1: Identified parameters for the proposed Ludwick's hardening model

K_0 (MPa)	K_1 (MPa/°C)	n_0	n_1 (1/°C)	m_0	m_1 (1/°C)
537.41	0.9753	0.5667	0.00072	0.000088	0.0319

The comparisons between predicted flow stresses and experimental curves are shown in Figure 11, Figure 12 and Figure 13. These results confirm that the proposed Ludwick's model can give a good flow stress description of AA5086 under all tested conditions.

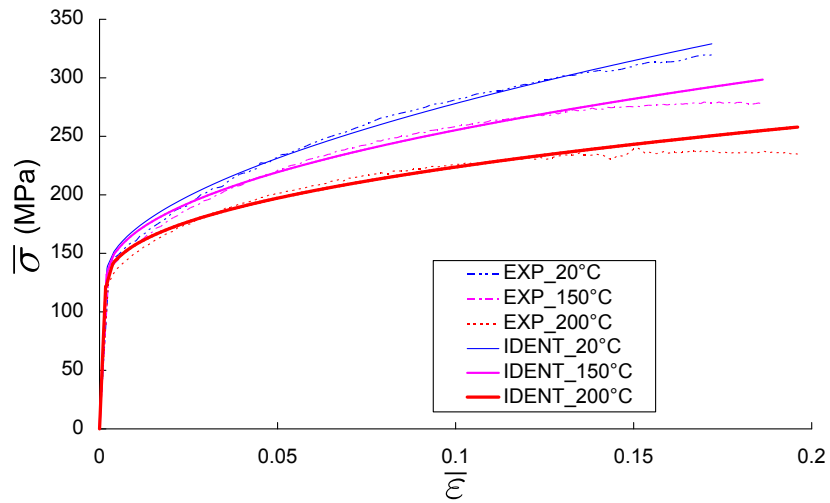


Figure 11: True stress-strain curves of AA5086 at 1 mm/s and correlation with Ludwick's model predictions

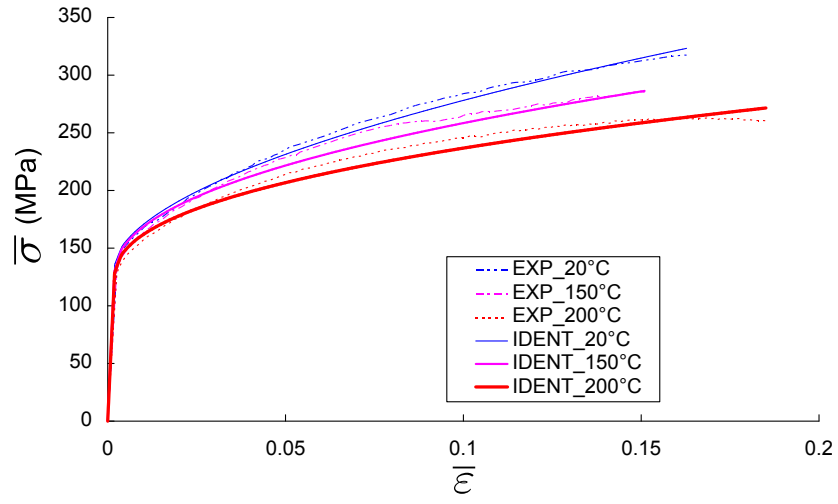


Figure 12: True stress-strain curves of AA5086 at 10 mm/s and correlation with Ludwick's model predictions

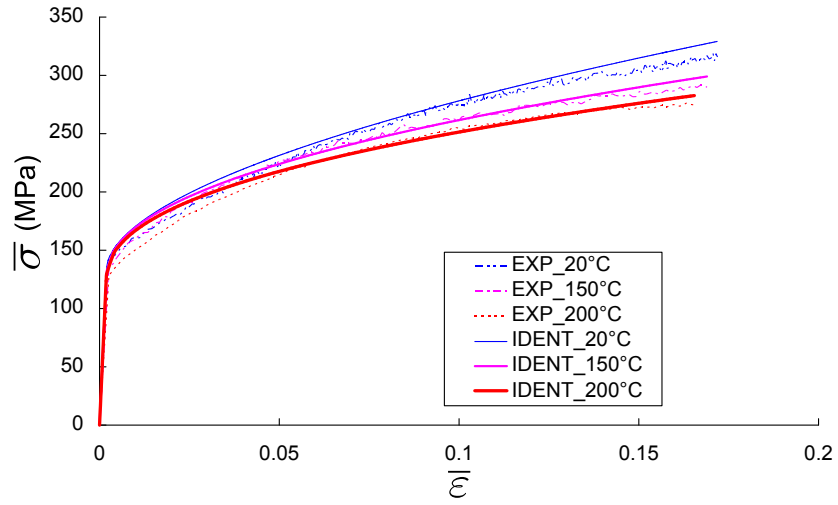


Figure 13: True stress-strain curves of AA5086 at 100 mm/s and correlation with Ludwick's model predictions

3.3. Yield function sensitivity

The yield function has been proved to be important for determining the FLCs in analytical M-K model. All the results in literature [18] show that the left hand side of the FLCs and the FLC_0 do not depend on the yield criterion while the different yield functions can lead to differences in the prediction of the right hand side of forming limit diagrams. To illustrate this purpose, the isotropic Mises's criterion is compared with the classical anisotropic Hill48 yield criterion. The anisotropy of this alloy is relatively low in the plane of the sheet and Hill48 yield criterion can give an acceptable description of this anisotropy. For Hill48 yield criterion, the equivalent stress $\bar{\sigma}$ is expressed by a quadratic function of the following type :

$$2\bar{\sigma}^2 = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 \quad (3)$$

where F , G , H , L , M and N are material constants (table 2). The direction x corresponds to the rolling direction, y the transverse direction and z the normal direction.

F	G	H	L	M	N
0.7	0.637	0.363	1.5	1.5	1.494

Table 2: Hill48 yield parameters

By implementing these two yield functions, the FLCs are determined with the proposed Ludwick's model and for a given geometrical imperfection f_0 of 0.98. As for analytical M-K model, the same conclusions can be drawn (Figure 14), yield function has no influence on the left hand side of the

FLCs and on the FLC_0 . For the following predictions with temperature and strain rate, the discussion on the validity of the M-K model is led with the conservative isotropic criterion of von Mises since the critical point FLC_0 is not impacted by the choice of the yield criterion.

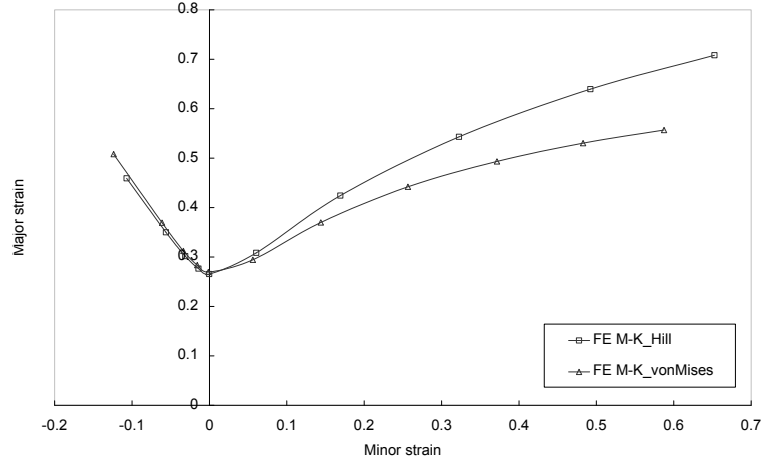


Figure 14: Yield function influence on the FLCs in FE M-K model

The imperfection orientation ψ_0 (Figure 8) must be chosen in order to get the minimum limit strains for each strain path in the left hand side of the forming limit diagram. As demonstrated above, yield function has no influence in this part of the FLD. Numerically, for this material, the minimum limit strains are always obtained with $\psi_0 = 0$ (groove perpendicular to the rolling direction) for the whole FLC. This result is in accordance with the necking band orientations observed in all the Marciniak tests and especially for small width specimens (Figure 15).

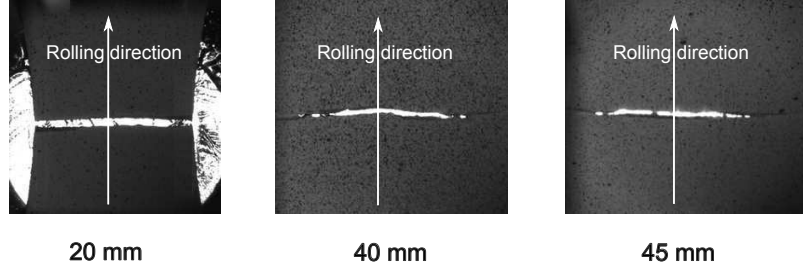


Figure 15: Necking and crack orientations for different values of specimen width (W)

3.4. Imperfection factor sensitivity

With the proposed Ludwick's model, the influence of initial imperfection value f_0 is shown in Figure 16. Similar to the analytical M-K model, the predictive forming limit curves from FE M-K model are also quite sensitive to f_0 . Then a calibration step is essential to fix the value of f_0 . The aim of the following part is to discuss the calibration strategy of the FE M-K model and to verify if the imperfection value must be determined for each temperature and strain rate.

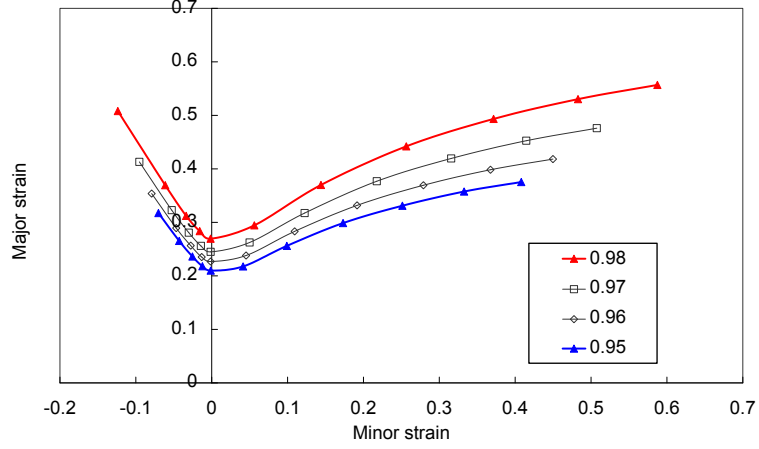


Figure 16: Influence of f_0 on the FLCs in the FE M-K model

4. Numerical FLCs results and discussion

4.1. Calibration strategy for the M-K model

The calibration of the geometrical imperfection f_0 can be formulated as an inverse analysis problem. By changing the boundary displacements (BD) of the FE M-K model, the simulated limit strains with different strain paths are available. The values of three typical points (uniaxial tension (UT), plane strain tension (PT), biaxial tension (BT)) on the experimental FLCs can be used as input experimental data. Comparing the input experimental values and the simulated ones by means of a minimum cost function (Figure 17), the best fit value of f_0 can be determined. In order to minimize the number of experimental tests, this method is applied with only one typical point for each calibration step. Then, with the three typical points, three different FLCs can be determined, they are noted as $FLC_{uniaxial}$, FLC_{plane} and $FLC_{biaxial}$ for convenience.

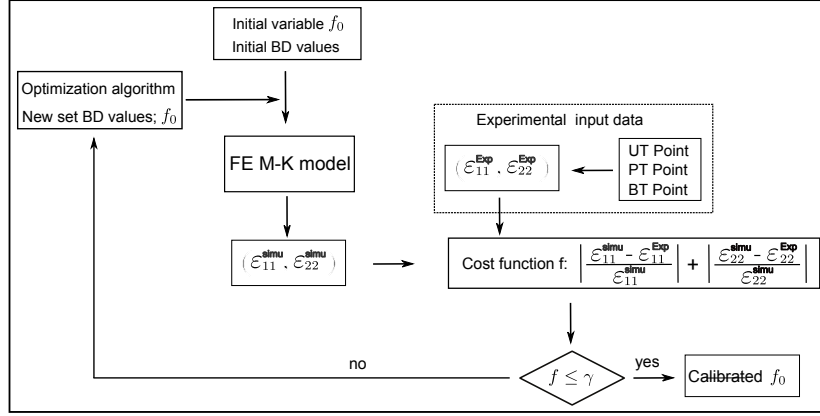


Figure 17: f_0 calibration methodology

As shown in Figure 18, the choice of the experimental point for the calibration can lead to different experimental curves. At 20°C, UT and PT points give the same results whereas BT point overestimates the global level of the FLC. At 200°C, the discrepancy between the predictive FLC curves is higher, BT points still overestimates the formability and UT point gives very conservative results. For these two temperatures, the BT point seems not appropriate, this can be explained by the high sensitivity of the right hand side of the FLCs to the yield criterion **which makes this point not really stable**. Finally, the calibration method based on the PT point is preferred, it constitutes the best compromise for the two temperatures and overall it permits the prediction of accurate forming limits near the plane strain region **without any influence of the modeled yield criterion**. This region is frequently the critical one for the forming of industrial parts. Then, considering all the factors above, the calibration method from PT point is adopted in the following work.

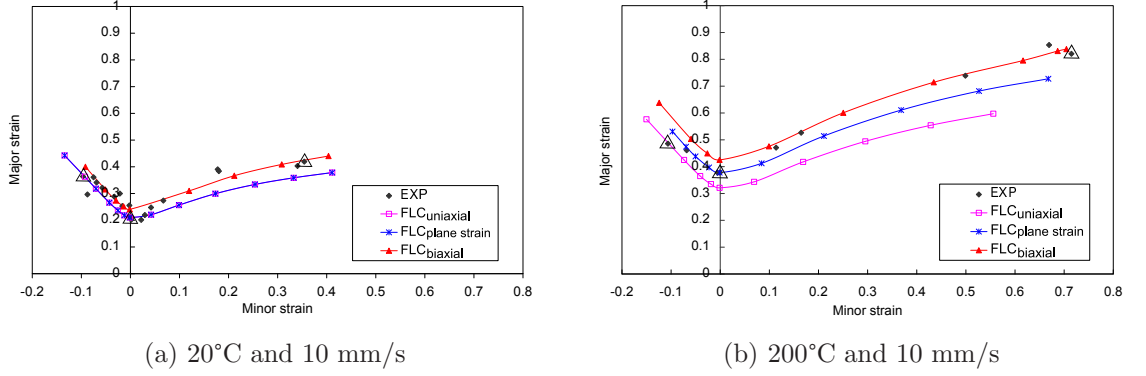


Figure 18: Comparison of f_0 calibration methods

4.2. Calibration for each condition

With the proposed calibration method, the calibrated f_0 values from the identified Ludwick's hardening model under each forming condition are shown in Table 3. It is found that the value of the calibrated f_0 varies with temperature and strain rate.

Table 3: Calibrated f_0 for the different forming conditions

Temperature (°C)	Forming speed (mm/s)	Calibrated f_0
20	10	0.9507
150	10	0.97
200	10	0.9927
150	1	0.99
200	1	0.99985
150	0.1	0.99985

The calibrated f_0 values from Table 3 are used to predict FLCs for the

tested temperatures and strain rates, the results are shown in Figures 19 to 21. Good formability predictions are observed over the tested temperature and strain rate ranges, especially for the left hand side of the FLCs. As already mentioned, the little conservative prediction in the right hand side of the FLCs is certainly caused by the isotropic yield criterion. Finally, the FE M-K model could be an efficient tool on condition that the geometrical imperfection was calibrated for each forming condition. Nevertheless, only one test in plane strain condition is sufficient to calibrate the model and to plot the whole FLC.

Due to the geometrical definition of the imperfection in the M-K model, the imperfection value should not be influenced by temperature or forming speed. This is a limitation of the M-K model, the definition of the imperfection value is very simplistic in this model and does not take into account complex phenomena at the scale of the microstructure for example, like dislocation movements or recrystallization mechanisms which are affected by the forming temperature or strain rates.

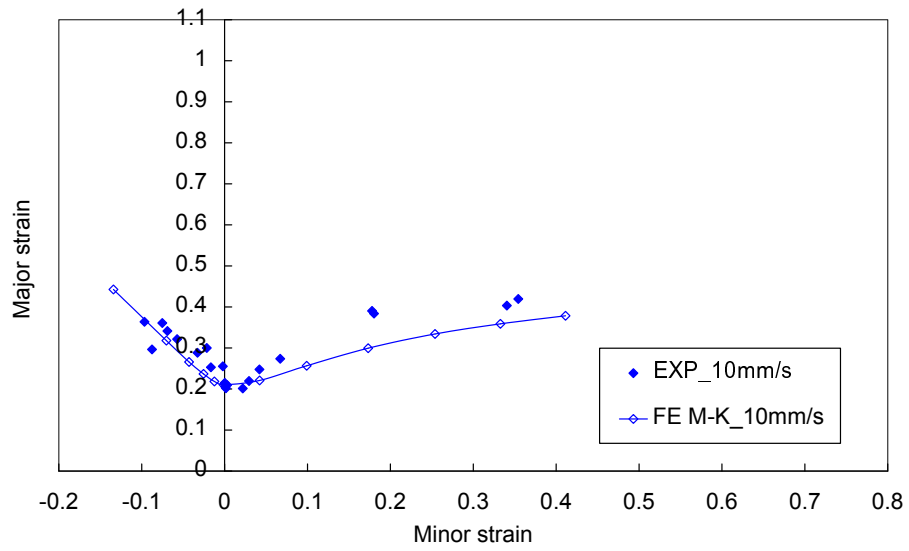


Figure 19: Predicted FLCs at 20°C with Ludwick model

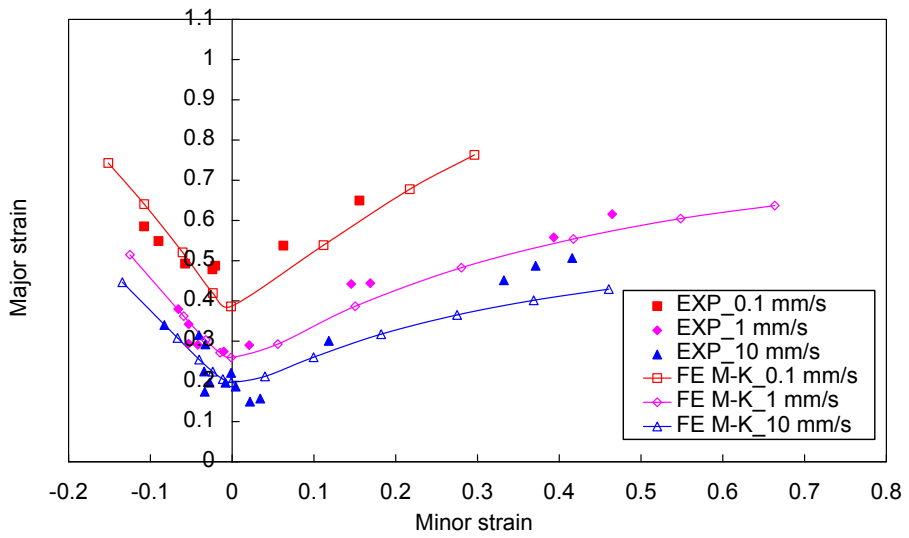


Figure 20: Predicted FLCs at 150°C with Ludwick model

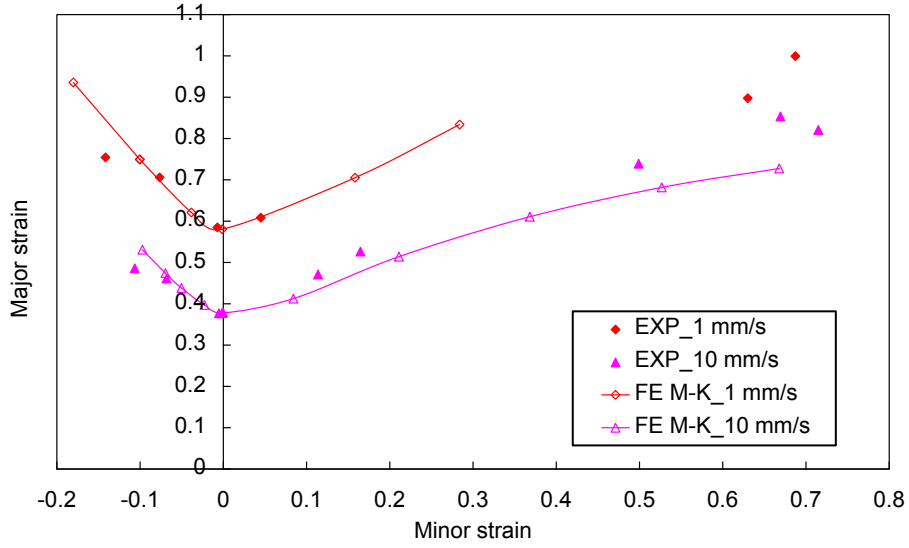


Figure 21: Predicted FLCs at 200°C with Ludwick model

5. Conclusion

In this work, the AA5086 formability at different temperatures (20, 150 and 200°C) and strain rates (0.02, 0.2 and 2 s^{-1}) has been experimentally investigated. The predictions of a FE M-K model have been evaluated by using a Ludwick's hardening model identified in the same range of temperature and strain rate. The experimental and predictive FLCs are compared and the following conclusions can be drawn :

- Both temperature and strain rate play a predominant role in evaluating the formability of AA5086 sheet metal. The formability is improved with increasing temperature and decreasing strain rate. The strain rate effect is emphasized at high temperatures.
- The determination of FLCs from the predictive M-K model is very

sensitive to the value of the geometrical imperfection. A calibration step is then essential to make reliable the predictions of this model. The experimental point from a plane strain path (zero minor strain) permits a good calibration of the FE M-K model for all the forming conditions.

- The calibrated values of the geometrical imperfection vary with the forming conditions which limits the use of the predictive M-K model without any experimental data. Nevertheless, only one test in plane strain condition for each forming condition can be sufficient to calibrate the model and to give an accurate estimation of the whole FLC.

- [1] A. Heinz, A. Haszler, C. Keidel, S. Moldenhauer, R. Benedictus, and W.S. Miller. Recent development in aluminium alloys for aerospace applications. *Materials Science and Engineering A*, 280:102 – 107, 2000.
- [2] W.S. Miller, L. Zhuang, J. Bottema, A.J. Wittebrood, P. De Smet, A. Haszler, and A. Vieregge. Recent development in aluminium alloys for the automotive industry. *Materials Science and Engineering A*, 280:37 – 49, 2000.
- [3] S.P. Keeler and W.A. Backofen. Plastic instability and fracture in sheets stretched over rigid punches. *Transactions of the ASM*, 56:25 – 48, 1963.
- [4] T. Naka, G. Torikai, R. Hino, and F. Yoshida. The effects of temperature and forming speed on the forming limit diagram for type 5083 aluminum-magnesium alloy sheet. *Journal of Materials Processing Technology*, 113(1-3):648 – 653, 2001.

- [5] D. Li and A.K. Ghosh. Biaxial warm forming behavior of aluminum sheet alloys. *Journal of Materials Processing Technology*, 145(3):281 – 293, 2004.
- [6] S. Mahabunphachai and M. Koç. Investigations on forming of aluminum 5052 and 6061 sheet alloys at warm temperatures. *Materials and Design*, 31(5):2422 – 2434, 2010.
- [7] G. Palumbo and L. Tricarico. Numerical and experimental investigations on the warm deep drawing process of circular aluminum alloy specimens. *Journal of Materials Processing Technology*, 184(1-3):115 – 123, 2007.
- [8] L. Wang, M. Strangwood, D. Balint, J. Lin, and T.A. Dean. Formability and failure mechanisms of AA2024 under hot forming conditions. *Materials Science and Engineering A*, 528(6):2648 – 2656, 2011.
- [9] C. Zhang, L. Leotoing, D. Guines, and E. Ragneau. Theoretical and numerical study of strain rate influence on AA5083 formability. *Journal of Materials Processing Technology*, 209(8):3849 – 3858, 2009.
- [10] M. Ganjiani and A. Assempour. An improved analytical approach for determination of forming limit diagrams considering the effects of yield functions. *Journal of Materials Processing Technology*, 182(1-3):598 – 607, 2007.
- [11] J.W. Signorelli, M.A. Bertinetti, and P.A. Turner. Predictions of forming limit diagrams using a rate-dependent polycrystal self-consistent plasticity model. *International Journal of Plasticity*, 25(1):1 – 25, 2009.

- [12] A. Zadpoor, J. Sinke, and R. Benedictus. Optimization of computational procedures for full-model strain path dependent analysis of forming limits. In *International Deep Drawing Research Group*, 2008.
- [13] S. Ahmadi, A.R. Eivani, and A. Akbarzadeh. An experimental and theoretical study on the prediction of forming limit diagrams using new BBC yield criteria and M-K analysis. *Computational Materials Science*, 44(4):1272 – 1280, 2009.
- [14] H. Aretz. An extension of hill’s localized necking model. *International Journal of Engineering Science*, 48:312 – 331, 2010.
- [15] A.S. Khan and M. Baig. Anisotropic responses, constitutive modeling and the effects of strain-rate and temperature on the formability of an aluminum alloy. *International Journal of Plasticity*, 27:522 – 538, 2011.
- [16] N. Abedrabbo, F. Pourboghrat, and J. Carsley. Forming of AA5182-O and AA5754-O at elevated temperatures using coupled thermo-mechanical finite element models. *International Journal of Plasticity*, 23(5):841 – 875, 2007.
- [17] C. Zhang, L. Leotoing, G. Zhao, D. Guines, and E. Ragneau. A methodology for evaluating sheet formability combining the tensile test with the M - K model. *Materials Science and Engineering A*, 528(1):480 – 485, 2010.
- [18] H. Vegter and A. Van Den Boogaard. A plane stress yield function for anisotropic sheet material by interpolation of biaxial stress states. *International Journal of Plasticity*, 22:557–580, 2006.