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# Simplicial Homology for Future Cellular Networks

Anaïs Vergne, Laurent Decreusefond, Philippe Martins, *Senior Member, IEEE*

**Abstract**—Simplicial homology is a tool that provides a mathematical way to compute the connectivity and the coverage of a cellular network without any node location information. In this article, we use simplicial homology in order to not only compute the topology of a cellular network, but also to discover the clusters of nodes still with no location information. We propose three algorithms for the management of future cellular networks. The first one is a frequency auto-planning algorithm for the self-configuration of future cellular networks. It aims at minimizing the number of planned frequencies while maximizing the usage of each one. Then, our energy conservation algorithm falls into the self-optimization feature of future cellular networks. It optimizes the energy consumption of the cellular network during off-peak hours while taking into account both coverage and user traffic. Finally, we present and discuss the performance of a disaster recovery algorithm using determinantal point processes to patch coverage holes.

**Index Terms**—Future cellular networks, Self-Organizing Networks, simplicial homology.

## I. INTRODUCTION

**L**ONG Term Evolution (LTE) is the 3GPP standard specified in Releases 8 and 9. Its main goal is to increase both capacity and speed in cellular networks. Indeed, cellular network usage has changed over the years and bandwidth hungry applications, as video calls, are now common. Achieving this goal for both capacity and speed costs a lot of money to the network operator. A solution to limit operation expenditures is the introduction of Self-Organizing Networks (SON). 3GPP standards have indeed identified self-organization as a necessity for future cellular networks [1]. Self-organization is the ability for a cellular network to automatically configure itself and adapt its behavior without any manual intervention. Therefore, SON features can be divided into self-configuration, self-optimization, and self-healing functions. We will define and describe the features we are interested in, for a further reading a full description of SON in LTE can for instance be found in [2].

First, self-configuration functions aim at the plug-and-play paradigm: new transmitting nodes should be automatically configured and integrated to the existing network. Upon arrival of a new node, the neighboring nodes update their dynamic neighbor tables thanks to the Automatic Neighbor Relation (ANR) feature. Among self-configuration functions, we can find the dynamic frequency auto-planning problem. It is a known problem from spectrum-sensing cognitive radio where equipments are designed to use the best wireless channels in order to limit interference [3]. The different nodes of the

secondary cognitive network have to choose the best frequency to use in order to maximize the coverage and minimize the interference with the base stations of the primary network. However there is no hierarchy between the nodes of future cellular networks, all nodes pertain to the primary network. Therefore the cognitive network solutions can not be used here. Moreover, while in earlier releases, static frequency planning was preferred, it has become a critical point to allow dynamic configuration since the network has a dynamic behavior with arrivals and departures of base stations, and does not always follow a regular pattern with the introduction of Femtocells and Heterogeneous Networks (HetNet).

The second main SON feature is the category of the self-optimization functions, which defines the ability of the network to adapt its behavior to different traffic scenarios. Indeed, in LTE cellular networks, eNode-Bs (eNB) have multiple configurable parameters. An example is output power, so cells sizes can be configured when capacity is the limitation rather than coverage. Moreover, fast and reliable X2 communication interfaces connect eNBs. So the whole network has the capability to adapt to different traffic situations. Then, users traffic can be observed via eNBs and User Equipments (UEs) measurements. Therefore, the self-optimization functions aim at using these traffic observations to adapt the whole network, and not only each cell independently, to the traffic situation. One case where self-optimization is often needed is the adaptation to off-peak hours. Typically a cellular network is deployed to match daily peak hours traffic requirements. Therefore during off-peak hours, the network is daily under-used. This leads to a huge unneeded amount of energy consumption. An idea is thus to switch-off some of the eNBs during off-peak hours, while other eNBs adjust their configuration parameters to keep the entire area covered. In case of a growth in traffic, the switched-off eNBs could be woken up to satisfy the users demand.

The third and last of the SON main functions is self-healing. In future cellular networks, nodes would be able to appear and disappear at any time. Since the cellular network is not only constituted of operated base stations anymore, the operator does not control the arrivals or departures of nodes. But the disappearances of nodes can be more generalized: for example in case of a natural disaster (floods, earthquakes or tsunamis...), several nodes do disappear at once. The self-healing functions aim at reducing the impacts from the failures of nodes must it be in isolated cases, like the turning off of a Femtocells, or more serious cases where the whole network is damaged. We are interested in this latter case, where some of the nodes are completely destroyed. However cellular networks are not necessarily built with redundancy and then can be sensitive to such damages. Coverage holes can appear resulting in no signal for communication at all in a whole area. Paradoxically,

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reliable and efficient communication is especially needed in such situations. Therefore, solutions for damage recovery for the coverage of cellular networks are much needed.

In this article, we use simplicial homology to comply with the self-organization requirements of future cellular networks. Simplicial homology provides a way to represent any wireless network without any location information, and compute its topology. A cellular network is then represented by a combinatorial object called abstract simplicial complex, and its topology is characterized in two dimensions by the so-called first two Betti numbers: the number of connected components and the number of coverage holes. But the simplicial complex representation does not only allow the topology computation, but it also gives geographical information, such as which nodes are in some clusters, or which ones are more homogeneously distributed. We use this simplicial complex representation in three algorithms that answer three specific aspects of SON in future cellular networks. First, we propose a frequency auto-planning algorithm which, for any given cellular network, provides a frequency planning minimizing the number of frequencies needed for a given accepted threshold of interference. The algorithm calls several instances of a reduction algorithm, introduced in [4], for the allocation of each frequency. Using simplicial complex representation combined to the reduction algorithm allows us to obtain a homogeneous coverage between frequencies. In a second part, we enhance the reduction algorithm to satisfy any user traffic. The reduction algorithm, as it is presented in [4], only satisfies perfect connectivity and coverage. However, in cellular networks, especially in urban areas, coverage is not the limiting factor, capacity is. So the optimal solution is not optimal coverage anymore but depends on the required traffic. We present an enhanced reduction algorithm to reach an optimally used network. Finally, we present an algorithm for disaster recovery of wireless networks first introduced in [5]. Given a damaged cellular network, the algorithm first adds too many nodes then runs the reduction algorithm of [4] to reach an optimal result. For the addition of new nodes we propose the use of a determinantal point process which has the inherent ability to locate areas with low density of nodes: namely coverage holes. We thoroughly evaluate the performance of our algorithm compared to the classic recovery algorithm: the greedy algorithm for the set cover problem, comparing their complexity, number of added nodes, and robustness through a slight move of the added nodes.

The remainder of this article is organized as follows. After a section on related work on self-configuration, self-optimization and recovery of future cellular networks in Section II, we introduce some simplicial homology definitions as well as the reduction algorithm for simplicial complexes used all along the article in Section III. Then in Section IV, we introduce our frequency auto-planning algorithm. The energy conservation algorithm is presented in Section V. Finally, we provide the disaster recovery description and performance evaluation in Section VI.

## II. RELATED WORK

### A. Self-configuration in future cellular networks

Configuration of the different nodes (eNBs, relays, Femto-cells) of a cellular network has to be done during the deployment of the network, but also upon the arrival and departure of any node. The classic manual configuration done for previous generations of cellular networks can not be operated in future cellular networks: changes in the network occur too often. Moreover, the dissemination of private Femtocells leads to the presence in the network of nodes with no access for manual support. So the future cellular networks are heterogeneous networks with no regular pattern for their nodes. They need to be able to self-configure themselves. The initial parameter that a node needs to configure are its IP address, its neighbor list and its radio access parameters. IP addresses are out of the scope of this work, but we will discuss the two other parameters.

The selection of the nodes to put on one's neighbor list can be based on the geographical coordinates of the nodes and take into account the antenna pattern and transmission power [6]. However, this approach does not consider changing radio environment, and requires exact location information which can be easy to obtain for eNBs, but not for Femtocells. The authors of [7] propose a better criterion for the configuration of the neighbor list: each node scans in real time the Signal to Interference plus Noise Ratio (SINR) from other nodes, then the nodes which SINR are higher than a given threshold are included in the neighbor list. The neighbor list of a node is then equivalent to connectivity information between nodes. This is the only information needed in order to build the simplicial complex representing a given cellular network.

Among radio access parameters, we can find frequency but also propagation parameters since the apparition of beam-forming techniques via MIMO. Let us focus on the former which is the subject of Section IV. The frequency planning problem was first introduced for GSM networks. However the constraints were not the same: the frequency planning was static with periodic manual optimizations, and in simulations, base stations were regularly deployed along an hexagonal pattern. With the deployments of Femtocells, outdoor relays, and Picocells, future cellular networks vary from GSM network in two major points. First, cells do not follow a regular pattern anymore, then they can appear and disappear at any time. Therefore the frequency planning problem has to be rethought in an automatic way. A naive idea for frequency auto-planning would simply be applying the greedy coloring algorithm to the sparse interference graph [8]. However, even if the provided solution may be optimal for the number of needed frequencies, the utilization of each frequency can be disparate: one can be planned for a large number of nodes compared to another planned for only few of them. Then if the level of interference increase (more users, or more powered antennas), this could lead to communication problems for the over-used frequency, and a whole new planning is needed. On the contrary, a more homogeneous resource utilization can be more robust if interference increase, since there are less nodes using the same frequency on average. We provide here a frequency auto-planning algorithm which aims at a

more homogeneous utilization of the resources. Moreover, the planning of frequency channels for new nodes that do not interfere with existing nodes while still providing enough bandwidth is still an open problem. It has been addressed in the cognitive radio field, but these algorithms usually enable opportunistic spectrum access [9]. However, it is not possible to extend this type of algorithm to the frequency allocation of new nodes in cellular networks. Indeed, the new nodes would be part of the primary network, with a quality of service to achieve, so their frequency allocation needs to be guaranteed and not opportunistic. The algorithm we propose aims at allocating frequency channels to future cellular networks with non-regular deployment, so upon the arrival of a new node, the whole network is re-configured.

### B. Self-optimization in future cellular networks

In order to ensure that future cellular networks are still efficient in terms of both Quality of Service (QoS) and costs, the self-configuration is not sufficient. Indeed, future cellular networks have the ability to adjust their parameters to match different traffic situation. Periodic optimization based on log reports, and operated centrally is not an effective solution in terms of speed and costs. That is why we need self-optimization. Self-optimization can be classified in three types depending on its goal.

First we can consider load balancing optimization. There is multiple ways to adapt a cellular network to different loads: it is for example possible to adapt the resources available in different nodes. These schemes were mainly introduced for GSM [10], and then CDMA [11], but the universal frequency reuse of LTE and LTE-Advanced diminishes their applicability. Then one can adapt the traffic strategy with admission controls on given cells and forced handovers [12]. However, as the previous solution, it is not very suitable for OFDMA networks which require hard handover. Finally it is possible to modify the coverage of a node by changing either its antennas radiation pattern [13] or the output power [14]. We use this latter approach to reach an optimal result for our algorithm: we adapt the coverage radius of each node to be the minimum required to cover a given area.

The second type self-optimization is the capacity and coverage adaptation via the use of relay nodes [15], while the third is interference optimization. Our energy conservation algorithm presented in Section V could lie in this third category as the simplest approach towards interference control is switching off idle nodes. It is done based on cell traffic for Femtocells in [16]: after a given period of time in idle mode, the node puts itself on stand-by. However, if one wants to take into account the whole network, it has to consider the coverage of the network before disconnecting, which is not the case of Femtocells, which are by definition redundant to the base stations network. Without considerations of traffic, we proposed in [4] an algorithm that reduce power consumption in wireless networks by putting on stand-by some of the nodes without impacting the coverage. We can also cite [17] that proposes a game-theoretic approach in which nodes are put on stand-by according to a coverage function, but unmodified

coverage is not guaranteed. In both these works, only coverage is taken into account. This approach could eventually fit the requirements of cellular network in non-urban cells, if their deployment has coverage redundancy. But it is not valid for urban cells, where it is not coverage but capacity that delimits cells. Our present algorithm goes a little bit further by adapting the switching-off of the nodes to the whole network situation, considering both traffic and coverage.

### C. Recovery in future cellular networks

The first step of recovery in cellular networks is the detection of failures. The detection of the failure of a cell occurs when its performance is considerably and abnormally reduced. In [18], the authors distinguish three stages of cell outage: degraded, crippled and catatonic. This last stage matches with the event of a disaster when there is complete outage of the damaged cells. After detection, compensation from other nodes can occur through relay assisted handover for ongoing calls, adjustments of neighboring cell sizes via power compensation or antenna tilt. In [19], the authors not only propose a cell outage management description but also describe compensation schemes. These steps of monitoring and detection, then compensation of nodes failures are comprised under the self-healing functions of future cellular networks described in [20].

In Section VI, we are interested in what happens when self-healing is not sufficient. In case of serious disasters, the compensation from remaining nodes and traffic rerouting might not be sufficient to provide service everywhere. In this case, the cellular network needs a manual intervention: the adding of new nodes to compensate the failures of former nodes. However a traditional restoration with brick-and-mortar base stations could take a long time, when efficient communication is particularly needed. In these cases, a recovery trailer fleet of base stations can be deployed by operators [21], it has been for example used by AT&T after 9/11 events. But a question remains: where to place the trailers carrying the recovery base stations. An ideal location would be adjacent to the failed node. However, these locations are not always available because of the disaster, plus the recovery base stations may not have the same coverage radii than the former ones. Therefore a new deployment for the recovery base stations has to be decided, in which one of the main goal is complete coverage of damaged area. This can be viewed as a mathematical set cover problem, where we define the universe as the area to be covered and the subsets as the balls of radii the coverage radii. Then the question is to find the optimal set of subsets that cover the universe, considering there are already balls centered on the existing nodes. It can be solved by a greedy algorithm [22],  $\epsilon$ -nets [23], or furthest point sampling [24], [25]. But these mathematical solutions provide an optimal mathematical result that do not consider any flexibility at all in the choosing of the new nodes positions, and that can be really sensitive to imprecisions in the nodes positions.

For a further reading, a complete survey on SON for future cellular networks is given in [26].



### III. PRELIMINARIES

#### A. Simplicial homology

First we need to remind some definitions from simplicial homology for a better understanding of the simplicial complex representation of cellular networks.

When representing a cellular network, one's first idea will be a geometric graph, where nodes are represented by vertices, and an edge is drawn whenever two nodes can communicate with each other. However, the graph representation has some limitations; first of all there is no notion of coverage. Graphs can be generalized to more generic combinatorial objects known as simplicial complexes. While graphs model binary relations, simplicial complexes represent higher order relations. A simplicial complex is a combinatorial object made up of vertices, edges, triangles, tetrahedra, and their  $n$ -dimensional counterparts. Given a set of vertices  $V$  and an integer  $k$ , a  $k$ -simplex is an unordered subset of  $k+1$  vertices  $[v_0, v_1, \dots, v_k]$  where  $v_i \in V$  and  $v_i \neq v_j$  for all  $i \neq j$ . Thus, a 0-simplex is a vertex, a 1-simplex an edge, a 2-simplex a triangle, a 3-simplex a tetrahedron, etc.

Any subset of vertices included in the set of the  $k+1$  vertices of a  $k$ -simplex is a face of this  $k$ -simplex. Thus, a  $k$ -simplex has exactly  $k+1$   $(k-1)$ -faces, which are  $(k-1)$ -simplices. For example, a tetrahedron has four 3-faces which are triangles. A simplicial complex is a collection of simplices which is closed with respect to the inclusion of faces, i.e. all faces of a simplex are in the set of simplices, and whenever two simplices intersect, they do so on a common face. An abstract simplicial complex is a purely combinatorial description of the geometric simplicial complex and therefore does not need the property of intersection of faces. For details about algebraic topology, we refer to [27].

We consider the nodes of a cellular network, and their coverage radii. The Čech abstract simplicial complex then provides the exact representation of the network's coverage. Its construction for a fixed coverage radius  $r$  for all the network's nodes is given:

*Definition 1 (Čech complex):* Given  $(X, d)$  a metric space,  $\omega$  a finite set of points in  $X$ , and  $r$  a real positive number. The Čech complex of  $\omega$ , denoted  $C_r(\omega)$ , is the abstract simplicial complex whose  $k$ -simplices correspond to  $(k+1)$ -tuples of vertices in  $\omega$  for which the intersection of the  $k+1$  balls of radii  $r$  centered at the  $k+1$  vertices is non-empty.

However, the Čech complex can be hard to compute, and requires some geographical information that is not always available. For instance Femtocells are not GPS-enabled. There exists an approximation to the coverage Čech complex that is only based on the connectivity information: the so-called neighbor list of each nodes of a SON-capable cellular network. The Vietoris-Rips abstract simplicial complex is defined as follows:

*Definition 2 (Vietoris-Rips complex):* Given  $(X, d)$  a metric space,  $\omega$  a finite set of points in  $X$ , and  $r$  a real positive number. The Vietoris-Rips complex of parameter  $2r$  of  $\omega$ , denoted  $R_{2r}(\omega)$ , is the abstract simplicial complex whose  $k$ -simplices correspond to unordered  $(k+1)$ -tuples of vertices in  $\omega$  which are pairwise within distance less than  $2r$  of each

other.

It is possible to build the Čech or the Vietoris-Rips complex of a cellular network with different coverage radii using different parameters associated with each one of the nodes.

Given an abstract simplicial complex, its topology can be computed via linear algebra computations. The so-called Betti numbers are defined to be the dimensions of the homology groups and are easily obtained by the rank-nullity theorem, but they also have a geometrical meaning. Indeed, the  $k$ -th Betti number of an abstract simplicial complex  $X$  is the number of  $k$ -th dimensional holes in  $X$ . In two dimensions we are only interested in the first two Betti numbers:  $\beta_0$  counts the number of 0-dimensional holes, that is the number of connected components, and  $\beta_1$  counts the number of holes in the plane, i.e. coverage holes. Therefore computing the Betti numbers of an abstract simplicial complex representing a cellular network gives the topology of the initial network. It is the exact topology when using the Čech complex representation. But with the Vietoris-Rips complex representation, it is possible to have so-called triangular holes in the network that do not appear in the complex. The probability of that happening in computed in [28], and in our case where it is upper-bounded by about 0.03% for a cellular network simulated with a Poisson point process.

#### B. Reduction algorithm

In this section, we recall the steps of the reduction algorithm for abstract simplicial complexes presented in [4] that we will use all along this article. The algorithm takes as input an abstract simplicial complex: here it is the complex representing the cellular network, and a list of boundary vertices that can be given by the network operator. This list of boundary vertices allow us to know the area of the network that we can not obtain by location information.

Then the goal of the reduction algorithm is to remove vertices from the abstract simplicial complex without modifying its Betti numbers. That translates to a network by turning off nodes from the network without modifying nor its connectivity neither its coverage.

To cover an area, only 2-simplices are needed. So the first step of the reduction algorithm is to characterize the superfluosness of the 2-simplices of the complex for its coverage. To do that, we define a degree for every 2-simplices: a degree of a 2-simplex is the size of the largest simplex it is the face of. Next, in order to remove vertices, and not 2-simplices, we need to transmit the superfluosness information of its 2-cofaces (2-simplices it is a face of) to a vertex via what is called an index. An index of a vertex is defined to be the minimum of the degrees of the 2-simplices it is a face of. Indeed, a vertex is as sensitive for the coverage as its most sensitive 2-simplex. The boundary vertices are given a negative index to mark them as unremovable by the algorithm: we do not want the covered area to be shrunk.

Finally, the indices give an optimal order for the removal of the vertices: the greater the index of a vertex, the bigger the cluster it is part of, and the more likely it is superfluous for the coverage of its abstract simplicial complex. Therefore, the

vertices with the greatest index are candidates for removal: one is chosen randomly. If its removal does not change the homology, i.e. if it does not modify its Betti numbers  $\beta_0$  and  $\beta_1$ , then it is effectively removed. Otherwise it is flagged as unremovable the same way the boundary vertices are. The algorithm goes on until every remaining vertex is unremovable, thus achieving optimal result.

For more information on the reduction algorithm we refer to [4].

#### IV. SELF-CONFIGURATION FREQUENCY AUTO-PLANNING ALGORITHM

##### A. Problem formulation

In the frequency planning problem, the topology of the network is not relevant since it is not modified. So we use the simplicial complex representation only for the characterization of clusters without location information.

First we need to build the abstract simplicial complex representing the cellular network. In future cellular networks, every transmitting nodes (eNBs, Femtocells, relays...) have a neighbor list created and updated with the ANR feature. In simulations and figures, we choose to consider a communication radius for each node. Then, any other nodes within the communication radius of a node is added to its neighbor list. With this neighbor list information, we can build the Vietoris-Rips complex representing the network, each node is represented by a 0-simplex and each neighbor, either 1-way or 2-way, relationship is represented by a 1-simplex. The other simplices are then created with only the 1-simplices information.

The goal of a frequency planning algorithm is to assign frequencies to every network's nodes so that the interference between them is minimum using the smallest number of frequencies possible. In this article, we only consider the one frequency per node case, and co-channel interference: interference between two nodes using the same frequency. However, the main idea of the algorithm can be extended to several frequencies per node, and interference between different frequencies by considering group of frequencies. In order to represent the interference, we introduce a so-called rejection radius. The rejection radius defines around every node a rejection disk. Then if one node is within the rejection disk of another node, we consider that they shall not share the same frequency or the interference level will be too high for reliable communication within at least one of the two cells. This rejection radius defines the interference threshold that is acceptable in the cellular network. It is the interference criterion we will use for our auto-planning algorithm.

##### B. Algorithm description

We consider a cellular network as we can see an example in the following figures. In Fig. 1, we can see every node with their communication radius, and the Vietoris-Rips complex representation of the network. In Fig. 2, we can observe the rejection radii of the nodes. Here they are set to be half the communication radii of each node. The interference between nodes is then represented in an interference graph: an edge is

added if at least one of its extremity is in the rejection disk of the other one. We can easily see that the configuration of Fig. 2 will need 4 frequencies because of the tetrahedron on the left.

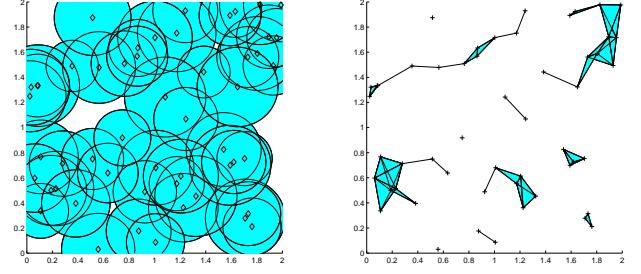


Fig. 1. A cellular network and its communication complex representation.

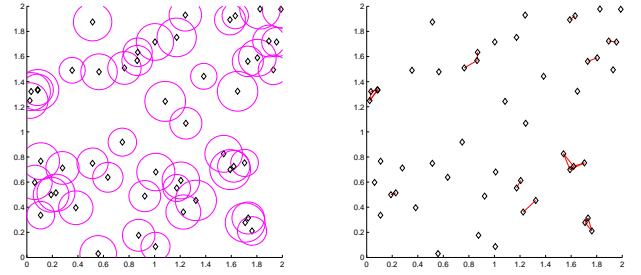


Fig. 2. Interference graph.

The algorithm begins by computing the degrees of 2-simplices and the indices of vertices, i.e. the nodes, as defined for the reduction algorithm presented in Section III. Then we apply a modified version of the reduction algorithm: the order in which the nodes are removed is still decided by the indices but the stopping condition is not the same anymore. Indeed we are not interested into achieving optimal coverage anymore. Therefore, instead of stopping when the maximum index among every remaining nodes is below a given number, the algorithm stops when there is no more nodes in any rejection disk of any other node. The nodes of the resulting simplicial complex are assigned the first frequency.

Then all the removed nodes are collected, and the corresponding abstract simplicial complex recovered. This complex is a subset of the initial complex so there is no need to build another one from scratch. The next step is then to reapply the modified coverage reduction algorithm to this recovered complex to obtain a second set of nodes to which we assign the second frequency. The algorithm goes on until every node has an assigned frequency.

At the end, we have a frequency assigned to every node. We ensured that no two nodes sharing the same frequency will be too close to each other: interference will be under a given threshold. Moreover, the use of our coverage reduction algorithm with the optimized order for nodes removal allows us to obtain a homogeneous usage of every frequency.

The frequency planning scheme obtained by our algorithm for the configuration of Fig. 1 is represented in Fig. 3. A

different color represent a different frequency. We can see that our algorithm has planned four frequencies (black, red, green and blue) of which we can see the communication area for each one in Fig. 4.

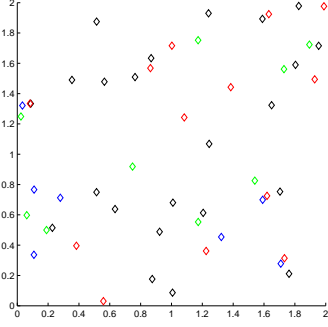


Fig. 3. Frequency planning scheme.

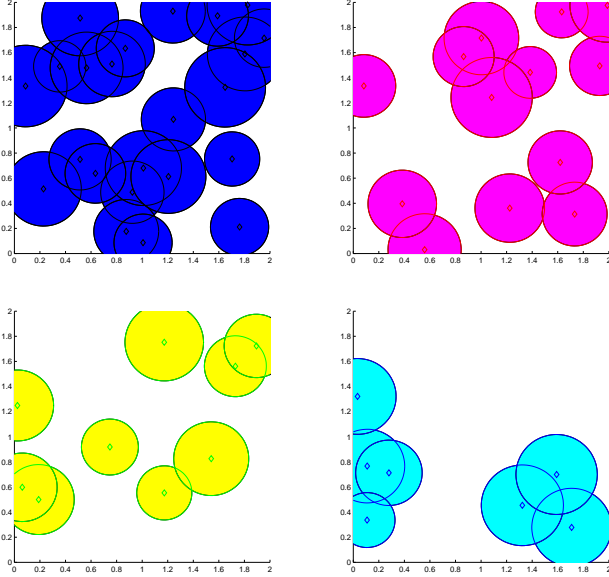


Fig. 4. Coverage for each frequency.

We give in Algorithm 1 the full frequency auto-planning algorithm: it requires the set of nodes  $\omega$ , and their communication and rejection radii, then returns the list of assigned frequencies for every node of  $\omega$ . We introduce three parameters in the algorithm description that we define here for a better understanding. First of them is the number of planned nodes: it represents the number of nodes to which we assigned a frequency already. Then, the ‘Interference’ parameter is a binary number to represent if there are at least two nodes with potentially the same frequency within the rejection disk of each other. Finally the ‘Frequency’ parameter, for every node, is where the frequency assigned to it is stored.

### C. Performance comparison

In this section we compare the performance of our frequency auto-planning algorithm to the greedy coloring al-

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#### Algorithm 1 Frequency auto-planning algorithm

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**Require:** Set  $\omega$  of  $N$  vertices, for each vertex  $v$  its communication radius  $r_v$ , and its rejection radius  $R_v$ .  
 Computation of the Vietoris-Rips complex  $X = \mathcal{R}_r(\omega)$   
 Computation of  $D_1(X), \dots, D_{s_2}(X)$   
 Computation of  $I[v_1(X)], \dots, I[v_{s_0}(X)]$   
 $I_{\max} = \max\{I[v_1(X)], \dots, I[v_{s_0}(X)]\}$   
 $N_{\text{planned}} = 0$   
 Interference = 1  
 $X' = X$   
 $i = 0$   
**while**  $N_{\text{planned}} < N$  **do**  
   **while** Interference == 1 **do**  
 Draw  $w$  a vertex of index  $I_{\max}$   
 $X' = X' \setminus \{w\}$   
 Computation of  $D_1(X'), \dots, D_{s_2}(X')$   
**for**  $i = 1 \rightarrow s'_0$  **do**  
   **if**  $I[v_i(X')] == I_{\max}$  **then**  
     Recomputation of  $I[v_i(X')]$   
   **end if**  
**end for**  
 $I_{\max} = \max\{I[v_1(X')], \dots, I[v_{s'_0}(X')]\}$   
 Interference = 0  
**for all**  $u, v \in X'$  **do**  
   **if**  $\|v - u\| < \max(R_v, R_u)$  **then**  
     Interference = 1  
   **end if**  
**end for**  
**end while**  
**for all**  $v \in X'$  **do**  
   Frequency( $v$ ) =  $i$   
    $N_{\text{planned}} = N_{\text{planned}} + 1$   
**end for**  
 $X' = X \setminus X'$   
 $i = i + 1$   
**end while**  
**return** List of assigned frequencies Frequency( $v$ ),  $\forall v \in \omega$ .

---

gorithm. Indeed, the frequency planning can be viewed as a graph coloring problem. If one considers the interference graph, then the optimal number of frequencies to assign is the chromatic number of the interference graph. The greedy coloring algorithm provides a coloring assigning the first new color available for each node. Therefore, the greedy coloring algorithm is a frequency planning algorithm. And the number of frequencies planned is at most the maximum node degree of the interference graph plus one. The greedy coloring gives especially good results for sparse graphs as the interference graph is. So the first parameter that we will use to compare the performance of both algorithm is the number of planned frequencies.

We simulate the set of nodes with a Poisson point process of intensity  $\lambda = 12$  on a square of side  $a = 2$ . The communication radii are sampled uniformly between  $a/10$  and  $2/\sqrt{\pi\lambda}$  for each node. Then the rejection radii are equal to half their corresponding communication radii. The results are

obtained in mean over  $10^4$  configurations. For each realization of the Poisson process, we compute the number of frequencies planned by the greedy coloring algorithm that we denote  $N_g$ . Then on all realizations with a given  $N_g$ , we compute the mean number of frequencies, denoted  $N_f$ , planned by our algorithm, so we can see the difference between the two algorithms. We also indicate which percentage of the  $10^4$  simulations these scenarios are, the occurrence is added for statistical information.

$N_g$	2	3	4	5	6
$\mathbb{E}[N_f N_g]$	4.00	5.04	5.83	6.43	7.13
Occurrence	8.8%	55.6%	29.4%	5.3%	0.7%

TABLE I  
MEAN NUMBER OF PLANNED FREQUENCIES.

In Table I, we can see the mean number of frequencies planned by our algorithm given the number of frequencies planned by the greedy coloring algorithm. We can see that there is a difference between the two solutions: it is not negligible in the beginning, but it decreases with the number of frequencies. Thus, our algorithm reaches its optimal performance when the number of frequencies grows for the same mean number of nodes, that is to say when there are clusters of nodes.

However, even if the greedy coloring algorithm fares well in the number of planned frequency, it leads to a disparate utilization of frequencies. Indeed, if there is only one clique of maximum size, one frequency will be only used for one node of this clique, and for no other node in the whole configuration. Therefore, the greedy coloring algorithm give good results for a homogeneous network, but not for a cluster network for example. For an optimal utilization of frequencies, each frequency should cover the whole area, but it is not always achievable if there are not enough nodes to cover several times the whole area. Our algorithm aims at a more homogeneous utilization of each ressource. In order to show that, we compare the percentage of area covered by each frequency on the total covered area for our frequency auto-planning algorithm and for the greedy coloring algorithm. We consider an area to be covered by a frequency if it is inside the communication disk of a node using this frequency. The percentages are given in mean over  $10^4$  simulations using the same setting as in the first comparison.

We can see in Table II the percentage of area covered by each frequency planned by our algorithm and the greedy coloring algorithm. The results are presented depending on the number of planned frequencies, we also indicate the number of simulations these results concern for statistical relevance. For our algorithm, even if the percentage decreases with the order in which the frequencies are planned, which is logical since the first frequency is planned in first and so on, we can see that a rather homogeneous coverage is provided. Doing that, our algorithm maximizes the usage of each resource. We can see that for the greedy coloring algorithm, the frequencies are not used equally: the first two frequencies are always a lot more planned than the other ones, the latter are thus under-used.

Greedy coloring algorithm

$N_g$	3	4	5	6
$f_1$	97.3%	97.0%	96.7%	96.3%
$f_2$	47.6%	50.1%	49.9%	51.1%
$f_3$	12.9%	17.7%	18.8%	20.2%
$f_4$		7.2%	8.8%	8.5%
$f_5$			6.2%	6.7%
$f_6$				6.2%
Occurrence	55.6%	29.4%	5.3%	0.7%

Frequency auto-planning algorithm

$N_f$	3	4	5	6
$f_1$	71.3%	62.1%	54.2%	49.2%
$f_2$	62.3%	56.4%	51.7%	47.4%
$f_3$	35.6%	46.4%	45.7%	42.9%
$f_4$		24.3%	35.6%	37.8%
$f_5$			18.8%	28.1%
$f_6$				15.3%
Occurrence	7.0%	23.0%	29.4%	22.6%

TABLE II  
MEAN PERCENTAGE OF COVERED AREA FOR EACH FREQUENCY

## V. SELF-OPTIMIZATION ENERGY CONSERVATION ALGORITHM

### A. Problem formulation

We are now interested in the self-optimization of a cellular network previously configured. Indeed, during off-peak hours a cellular network is under-used, we propose an algorithm that aims at reducing the energy consumption when user traffic is reduced.

First we want to represent the considered cellular network and its topology with an abstract simplicial complex. The network is constituted of transmitting nodes and their associated coverage disks. In future cellular networks, these coverage disks can vary in size by modifying the configuration parameters of the base stations. We choose to consider here the maximum size of these disks in order to maximize the coverage of each node so that a maximum number of nodes can be switched-off. We can build the Čech complex representing exactly the topology of the cellular network, or its approximation the Vietoris-Rips complex. Then we have to consider boundary nodes differently from the other ones. Indeed, these nodes allow us to delimit the area to cover.

Finally we have to define how we represent user traffic in the cellular network. In our simulations, we choose to create groups of nodes, then for each group, the user demand is represented by a given size of subset of nodes from this group to be kept on. All the sizes of the requested subsets for every group represent the required quality of service (QoS) of the cellular network. This QoS metric is quite artificial, but our algorithm can take into account any QoS as long as it is defined in terms of required number of ressources from a given pool available to a given group of nodes. We choose to implement this simplistic metric because it is the easiest one to implement.

To create the groups of nodes in simulations, we have to take some care. Indeed, these groups have to make sense geographically, but we do not have location information. So in order to consider clusters of nodes, every group is defined by a maximum simplex, a simplex which is not the face of



any other simplex. The groups are created so that every node pertains to exactly one group. Then, for every group of  $k$  nodes, an integer is uniformly drawn between 1 and  $k$ , this integer is the required number of nodes to keep on.

### B. Algorithm description

We consider a cellular network with transmitting nodes and their maximal coverage radii that we represent with an exact or approximated coverage simplicial complex. We can see an example of network and its Vietoris-Rips complex associated in Fig. 5. The boundary nodes are defined by the convex hull and are in red in the figure.

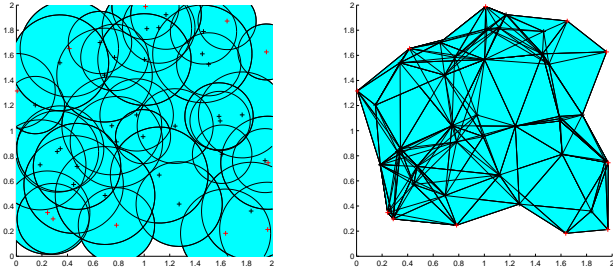


Fig. 5. A cellular network and its coverage representation.

Then for the configuration of Fig. 5, we represent the QoS groups of nodes in different colors, and give a table with their corresponding size and required QoS in Fig. 6.

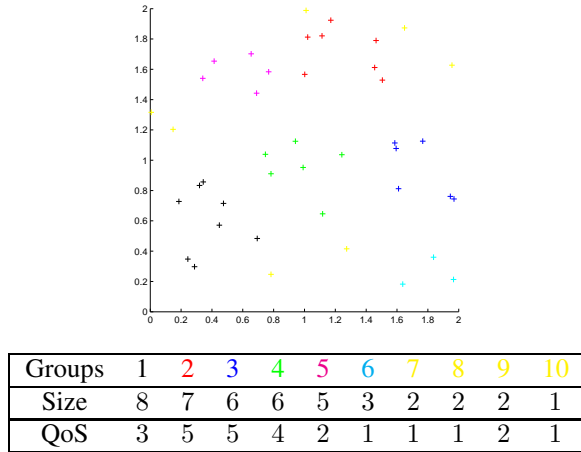


Fig. 6. QoS groups and required QoS.

The algorithm then begins by the computation of the Betti numbers. They characterize the topology of the network that we do not want to modify. Then the degrees of the 2-simplices and the indices of the vertices are computed a first time as defined in Section III. The principle of the reduction algorithm is afterwards applied. However while the order of removal for the nodes is the same, the breaking point is different. Instead of stopping when the area is covered by a minimum number of nodes, the algorithm stops when each group of nodes has been reduced to the size of its required QoS. We can see the obtained result for the configuration of Fig. 5 and Fig. 6 in Figure 7. The kept nodes are circled.

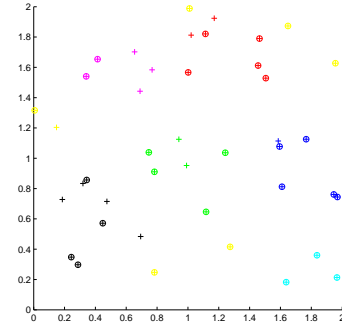


Fig. 7. Kept nodes.

The cellular network and its simplicial complex representation is represented in Fig. 8.

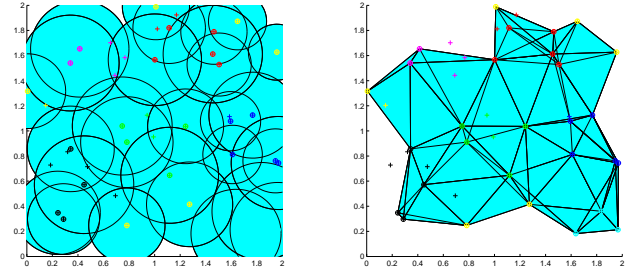


Fig. 8. The network after the first step.

Finally, given this configuration of switched-on nodes, the algorithm tries to reduce as much as possible the coverage radii of each node without creating a coverage hole. The order in which the coverage radii are examined is random. In Fig. 9, we can see the final configuration of the cellular network with the optimized coverage radii and its coverage abstract simplicial complex for the configuration of Fig. 5.

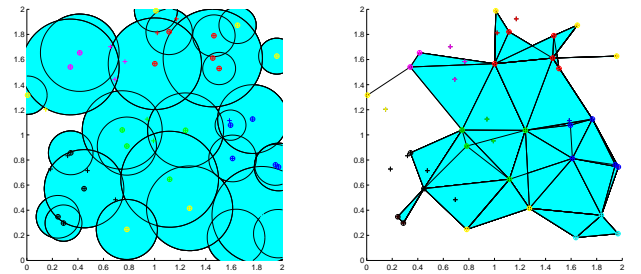


Fig. 9. Final configuration.

In the end, we have a configuration of nodes to keep switched-on that is optimal. From the first part of the algorithm, we ensure that enough nodes are kept on to satisfy the QoS representing user traffic. Then, in the second part of the algorithm, we ensure that no energy is spend uselessly by optimizing the size of the serving cells.

We give in Algorithm 2 the full energy conservation algorithm. It requires the set of nodes  $\omega$  and their maximum

coverage radii, then returns the list of kept nodes with their new coverage radii. For our simulations, we choose to give advantage to the larger simplices for the constitution of the groups. Thus, the first group will consist of the largest simplex, or one randomly chosen among the largest ones, then simplices of smaller size will become groups until every node is part of a group. It is possible to consider other rules for the constitution of the groups, but it has to follow one condition: every node must pertain to exactly one group. The groups are represented by the variable ‘Group’, that for each node gives its group number. Then ‘QoS’ represents the minimum number of nodes required for a given group, while ‘Size’ represents its number of nodes. We can see that the breaking point of the ‘while’ loop of the algorithm takes the ‘QoS’ parameter into account. Then a node can be removed if and only if it does not modify the number of connected component and the number of coverage holes, and it is not needed for the QoS requirements. Coverage radii reduction for each node is done in the last loop.

### C. Performance

In this section, we compare the performance of our algorithm to an optimal, not always achievable solution. Indeed we do not know of a energy conservation algorithm that can switch-off nodes during off-peak hours while maintaining coverage. Thus, we compare the number of switched-on nodes after the execution of our energy conservation algorithm, to the number of nodes needed for the QoS, given by the ‘QoS’ parameter. It is important to note that this optimal solution is not always achievable because it does not take into account that the area is to stay covered. Some nodes have to be kept for traffic reasons, while other are kept to maintain connectivity and/or coverage.

Our simulation results are computed on  $10^4$  configurations of Poisson point processes of intensity  $\lambda = 6$  on a square of size  $a = 2$  with a fixed boundary of nodes on the square boundary. The coverage radii are sampled uniformly between  $a/10$  and  $2/\sqrt{\pi\lambda}$ , except for the boundary nodes for which it was set to  $a/3$ . For each group the ‘QoS’ number is a sampled integer between zero and the size of the group. We denote by  $N_o$  the optimal number of nodes, and by  $N_k$  the number of kept nodes with our energy conservation algorithm. First we compute the percentage of simulations for which we have a given difference between the obtained number and the optimal number of kept nodes.

$N_k - N_o$	0	1	2	3	4	
Occurrence	1.3%	4.2%	8.5%	12.5%	15.4%	
	5	6	7	8	9	$\geq 10$
	15.4%	14.3%	11.0%	7.5%	4.9%	4.7%

TABLE III

OCCURRENCES OF GIVEN DIFFERENCES BETWEEN  $N_k$  AND  $N_o$ .

We can see in Table III the percentage of simulations in which the number of kept nodes is different from the optimal number of nodes. For 1.3% of the simulations the optimal number is reached. In 82.8% of our simulations the difference between the optimal and the effective number of kept nodes

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### Algorithm 2 Energy conservation algorithm

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**Require:** Set  $\omega$  of  $N$  vertices, for each vertex  $v$  its coverage radius  $r_v$ .  
 Computation of the Čech complex  $X = \mathcal{C}_r(\omega)$  or the Vietoris-Rips complex  $X = \mathcal{R}_{2r}(\omega)$   
 Creation of the list of boundary vertices  $L_C$   
 Computation of  $\beta_0(X)$  and  $\beta_1(X)$   
 Computation of  $D_1(X), \dots, D_{s_2}(X)$   
 Computation of  $I[v_1(X)], \dots, I[v_{s_0}(X)]$   
**for all**  $v \in L_C$  **do**  
    $I[v] = -1$   
**end for**  
 $I_{\max} = \max\{I[v_1(X)], \dots, I[v_{s_0}(X)]\}$   
 $N_{\text{group}} = 0$   
**for all** Simplex  $S_k \in X$  from largest to smallest **do**  
   **if**  $\forall v \in S_k, \text{Group}(v) == 0$  **then**  
      $N_{\text{group}} = N_{\text{group}} + 1$   
      $\forall v \in S_k \text{Group}(v) = N_{\text{group}}$   
      $\text{Size}(N_{\text{group}}) = k + 1$   
     Draw  $\text{QoS}(N_{\text{group}})$  among  $\{0, \dots, k + 1\}$   
   **end if**  
**end for**  
**while**  $I_{\max} > 2$  **and**  $\text{Size} \geq \text{QoS}$  **do**  
   Draw  $w$  a vertex of index  $I_{\max}$   
    $X' = X \setminus \{w\}$   
   Computation of  $\beta_0(X'), \beta_1(X')$   
   **if**  $\beta_0(X') \neq \beta_0(X)$  **and**  $\beta_1(X') \neq \beta_1(X)$  **and**  
    $\text{Size}(\text{Group}(w)) \leq \text{QoS}(\text{Group}(w))$  **then**  
      $I[w] = -1$   
   **else**  
      $\text{Size}(\text{Group}(w)) = \text{Size}(\text{Group}(w)) - 1$   
     Computation of  $D_1(X'), \dots, D_{s'_{k_0}}(X')$   
     **for**  $i = 1 \rightarrow s'_0$  **do**  
       **if**  $I[v_i(X')] == I_{\max}$  **then**  
         Recomputation of  $I[v_i(X')]$   
       **end if**  
     **end for**  
      $I_{\max} = \max\{I[v_1(X')], \dots, I[v_{s'_0}(X')]\}$   
      $X = X'$   
   **end if**  
**end while**  
**for all**  $v \in X$  taken in random order **do**  
    $X' = X$   
   **while**  $\beta_0(X') == \beta_0(X)$  **and**  $\beta_1(X') == \beta_1(X)$  **do**  
     Reduce  $r_v$   
   **end while**  
    $X = X'$   
**end for**  
**return** List of kept vertices  $v$  and their coverage radii  $r_v$ .

---

is smaller than 7, and it never exceeds 18. The number of boundary nodes is 16, these nodes are not removable (in order to never shrink the covered area) even if they are not needed for the traffic. So an average of 5.16 nodes to cover the area on top of those needed for user traffic seems plausible.

To have more advanced comparison, for the  $10^4$  configu-

rations, we compute the optimal number of nodes. Then for each optimal number of nodes  $N_o$  that occurred the most, we compute the mean number of kept nodes  $\mathbf{E}[N_k|N_o]$  over the simulations which have  $N_o$  for optimal number. The results are given in Table IV. For comparison, we also compute the difference between  $N_o$  and  $N_k$  in percent. We finally indicate which percent of our  $10^4$  simulations these cases occur to show the relevance of these statistical results.

$N_o$	22	23	24	25	26
$\mathbf{E}[N_k N_o]$	28.95	29.52	30.04	30.69	31.30
Difference	31.6%	28.4%	25.1%	22.8%	20.4%
Occurrence	5.0%	6.2%	7.0%	7.4%	7.9%

	27	28	29	30	31	32
$\mathbf{E}[N_k N_o]$	31.85	32.68	33.25	33.95	34.64	35.38
Difference	18.0%	16.7%	14.7%	13.1%	11.7%	10.6%
Occurrence	7.4%	7.5%	7.2%	6.3%	5.9%	4.8%

TABLE IV  
MEAN NUMBER OF KEPT NODES  $\mathbf{E}[N_k|N_o]$  FOR A GIVEN OPTIMAL NUMBER  $N_o$ .

We can see in Table IV that the more nodes are needed, the less difference there is between our result and the optimal one. Indeed, if more nodes are needed, there is a great chance that these nodes can cover the whole area.

## VI. DISASTER RECOVERY ALGORITHM

In this section we present a disaster recovery algorithm introduced in [5] of which we remind the main idea and investigate more thoroughly the performance.

### A. Main idea

The disaster recovery algorithm aims at restoring a damaged cellular network. Thus, we consider a cellular network presenting coverage holes and possibly many disconnected components. The Čech complex or its approximation the Vietoris-Rips complex is build based on the set of nodes and their coverage radii. We also need a list of boundary nodes, which can be fictional, in order to know the whole area to be covered. To patch the damaged network provides the locations where to put new nodes, so in this section locations of nodes are known. We also restrict ourselves to a fixed common coverage radius for every nodes, even if the main idea can be extended. We can see an example of damaged cellular network with a square boundary of fictional nodes and its representation by the Vietoris-Rips complex in Fig. 10.

The algorithm then adds new nodes in addition to the set of existing nodes. For the addition of nodes, several methods, deterministic or random, are possible. But we choose here to focus here on the best fitted random method: determinantal point processes. However, the number of new added nodes is not computed as in determinantal point processes, but via an incrementation. It is first set to the minimum number of nodes to cover the whole area minus the number of existing nodes. When the nodes are added, the two first Betti numbers are computed. Then if there are more than one connected component, or any coverage hole, other new nodes are added. Their number is incremented with a random variable following

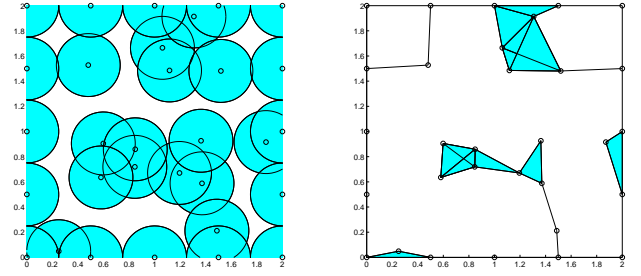


Fig. 10. A damaged cellular network with a fixed boundary.

an exponential growth: first set to 1, it is doubled every time new nodes are added and the network is still not patched.

Using determinantal point processes for the addition of nodes allows us to not only take into account the number of existing nodes via the computation of the number of added nodes, but also their locations. Indeed, the existing nodes are taken as the first drawn points of the point process, then every new point is drawn following a density law that is valued at zero at every node (existing or newly added) location and grows to be maximum the furthest from every node. The principle behind this method is explained in [5], and how to simulate determinantal point processes is detailed in [29]. The addition of new nodes stops as soon as the network is repaired: one connected component and no coverage hole.

We can see the first step of the disaster recovery algorithm illustrated in Fig. 11 for the cellular network of Fig. 10. Existing nodes are black circles while added nodes are red plusses.

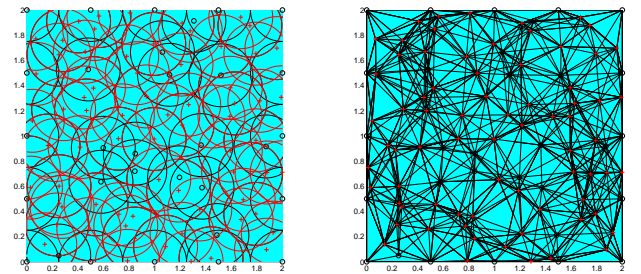


Fig. 11. With the determinantal addition method with a Ginibre basis.

Finally, the next step of our approach is to run the reduction algorithm presented in Section III which maintains the topology of the repaired cellular network. At this step, we remove some of the new nodes we just virtually added in order to achieve an optimal result with a minimum number of really added nodes. We can see in Fig. 12 an execution of the reduction algorithm on the intermediate configuration of Figure 11 which constitutes of the second and final step of the disaster recovery algorithm. Removed nodes are represented by blue diamonds.

We give in Algorithm 3 the outline of the algorithm. The algorithm requires the set of  $N_i$  initial nodes  $\omega_i$ , the fixed coverage radius  $r$ , as well as the list of boundary nodes  $L_b$ .

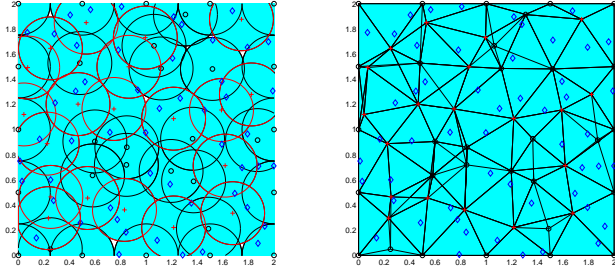


Fig. 12. The coverage reduction algorithm run on the determinantal method example.

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**Algorithm 3** Disaster recovery algorithm

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**Require:** Set of vertices  $\omega_i$ , radius  $r$ , boundary vertices  $L_b$   
 Computation of the Čech complex  $X = \mathcal{C}_r(\omega)$  or the Vietoris-Rips complex  $X = \mathcal{R}_{2r}(\omega)$   
 $N_a = \lceil \frac{a^2}{\pi r^2} \rceil - N_i$   
 Addition of  $N_a$  vertices to  $X$  following a determinantal point process  
 Computation of  $\beta_0(X)$  and  $\beta_1(X)$   
 $u = 1$   
**while**  $\beta_0 \neq 1$  or  $\beta_1 \neq 0$  **do**  
    $N_a = N_a + u$   
    $u = 2 * u$   
   Addition of  $N_a$  vertices to  $X$  following chosen method  
  
   Computation of  $\beta_0(X)$  and  $\beta_1(X)$   
**end while**  
 Reduction algorithm on  $X$   
**return** List  $L_a$  of kept added vertices.

---

### B. Performance comparison

We now compare the performance results of our disaster recovery algorithm to the best known coverage recovery algorithm: the greedy algorithm for the set cover problem along two parameters first the mean number of added nodes, and the smoothed robustness that we define. The greedy algorithm lays a grid of potential new nodes, and adds first the node the furthest from every other existing node, and so on until the furthest node is in the coverage of an existing or added node. Therefore, one can note that our homology algorithm with a grid addition method gives the exact same result as the greedy algorithm: the number et locations of new nodes are the same.

We compared the mean number of added nodes following four different scenarios for the initial state. Indeed, the performance of a recovery algorithm can be really affected depending on how damaged is the cellular network to repair. So the different scenarios are defined by the mean percentage of covered area before running a recovery algorithm. In Table V, we present the mean final number of added nodes for the greedy algorithm and our homology algorithm over  $10^4$  configurations for each scenario. Every configuration is simulated on a square of side  $a = 1$ , and nodes with identical coverage radius of  $r = 0.25$  for the construction of the Vietoris-Rips complex.

% of area initially covered	20%	40%	60%	80%
Greedy algorithm	3.65	3.36	2.82	1.84
Homology algorithm	4.47	3.85	2.98	1.77

TABLE V  
MEAN FINAL NUMBER OF ADDED NODES  $\mathbf{E}[N_f]$

The numbers of nodes added in the final state both with our recovery algorithm and the greedy algorithm are roughly the same. Nonetheless, we can see that our algorithm performs a little bit worse than the greedy algorithm in the less covered area scenarios. Indeed the greedy algorithm takes advantage of its grid layout and perfect spacing between added nodes. However, our homology algorithm gives better result in the more covered scenarios, thanks to the inherent ability of determinantal point processes to locate the coverage holes.

In order to show the advantages of our disaster recovery algorithm we choose to evaluate the compared robustness of the two algorithms when the added nodes positions are slightly moved, i.e. when the practical positioning does not strictly follow the theoretical positioning. In order to do this, we apply a Gaussian perturbation to each the added nodes position. The covariance matrix of the perturbation is given by  $\Sigma = \sigma^2 \text{Id}$  with  $\sigma^2 = 0.01$ , which means that the standard deviation for each node is of  $\sigma = 0.1$ . Other simulations parameters are unchanged, results in Table VI and VII are given in mean over  $10^4$  simulations. First, we compute the average number of holes  $\beta_1$  created by the Gaussian perturbation in Table VI. Then in Table VII, we count the percentage of simulations in which the number of holes is still zero after the Gaussian perturbation on the new nodes positions.

% of area initially covered	20%	40%	60%	80%
Greedy algorithm	0.68	0.67	0.48	0.35
Homology algorithm	0.58	0.52	0.36	0.26

TABLE VI  
MEAN NUMBER OF HOLES  $\mathbf{E}[\beta_1]$  AFTER THE GAUSSIAN PERTURBATION

% of area initially covered	20%	40%	60%	80%
Greedy algorithm	40.7%	45.2%	58.8%	68.9%
Homology algorithm	54.0%	58.0%	68.8%	76.1%

TABLE VII  
PROBABILITY THAT THERE IS NO HOLE  $\mathbf{P}(\beta_1 = 0)$  AFTER THE GAUSSIAN PERTURBATION

We can see that the perturbation on the number of holes decreases with the percentage of area initially covered, since the initial nodes are not perturbed. Our homology algorithm clearly performs better, even in the least covered scenarios, there are less than half of the simulations that create coverage holes, which is not the case for the greedy algorithm. The greedy algorithm also always create more coverage holes in mean than our disaster recovery algorithm for the same nodes positions perturbation. Therefore our algorithm seems more fitted to the disaster recovery case when a recovery network is deployed in emergency both indoor, via Femtocells, and outdoor, via a trailer fleet, where exact GPS locations are



not always available, and exact theoretical positioning is not always followed.

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