Rescaled Weibull tail-distributions

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Abstract

We exhibit a new stability property of Weibull tail-distributions. It is shown that a rescaled random variable with a Weibull tail-distribution still has a Weibull tail-distribution and with the same Weibull tail-coefficient.

Let us define recall that a Weibull tail-distribution is defined through its distribution function by:

$$F(x) = 1 - \exp\left\{-x^{1/\theta}\ell(x)\right\}, \quad x > 0 \tag{1}$$

where ℓ is a slowly-varying function. Our main result is the follwing. It permits to link negative Weibull tail-distributions with classical Weibull tail-distributions, see Table 1 for examples, [22] for mixture properties and [20] for a review. A sufficient condition for subexponentiality has been established in [19], Theorem 1. A new diagnostic tool has been proposed in [21].

Theorem. *Let X be a random variable from a Weibull-tail model (1). Let $Y = \lambda X$ with $\lambda > 0$. Then, Y follows a Weibull tail-distribution with associated Weibull tail-coefficient θ.*

Proof. Let us consider

$$P(Y \leq y) = P(\lambda X \leq y) = P(X \leq y/\lambda) = F(y/\lambda) = 1 - \exp\left\{-y^{1/\theta}\tilde{\ell}(y)\right\},$$

with $\tilde{\ell}(y) = \lambda^{-1/\theta}\ell(y/\lambda)$. The conclusion follows.

On the basis of this result, the estimation of θ can be achieved by applying classical Weibull-tail index estimators to X or λX. Berred [5] uses record values while most of estimators are based on the largest observations of the sample: Asimit *et al.* [1], Beirlant *et al.* [2, 3, 4], Broniatowski [6], Diebolt *et al.* [7, 8], Dierckx *et al.* [9], Gardes *et al.* [10, 11, 12, 13, 18], Girard [14], Goegebeur *et al.* [15, 16] and Mercadier *et al.* [17].

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Distribution	θ
Gaussian $\mathcal{N}(\mu, \sigma^2)$	$1/2$
Gamma $\Gamma(\alpha, \lambda)$	1
Weibull $\mathcal{W}(\alpha, \lambda)$	$1/\alpha$

Table 1: Weibull tail-distributions