State-parameter identification for accurate building energy audits

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STATE-PARAMETER IDENTIFICATION PROBLEMS FOR ACCURATE BUILDING ENERGY AUDITS

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ABSTRACT

Building performance simulation often fails to predict accurately the real energy performance, mostly due to great uncertainties in the input data. Errors in computed performance are particularly significant in the case of existing buildings, for which the amount of information about intrinsic characteristics is low. However, efficient energy retrofit operations make necessary an accurate understanding of the initial state of a building using a calibrated prediction model. Several works have investigated the use of identification techniques for model calibration. The present paper investigates the use of such techniques to derive an energy audit procedure suitable to be an efficient aid for retrofit. In particular, we study here the possibility to determine the unknown thermal conductivity of the envelope and the internal gains based on temperature measurements only. We show how the adjoint method can be used to solve efficiently the inverse problem, while providing a fast method for computing model's sensitivities.

INTRODUCTION

Building energy performance simulation often fails to predict accurately the actual energy performance, (Norford et al., 1994; Bordass et al., 2004; Demanuele et al., 2010). Errors in computed performance are particularly significant in the case of existing buildings, for which the uncertainties on intrinsic characteristics are high. Most of today's audit methods are unable to provide reliable data under cost and implementation constraints. However, efficient energy retrofit operations make necessary an accurate understanding of the models used for studying and choosing the rehabilitation procedures. This need has fostered the research for the developpement of model calibration techniques. The present paper investigates the use of inverse modelling to identify some among the most significant parameters of a model.

State-parameter identification aims at determining heat sources and intrinsic properties of a mathematical model based on partial observations of the thermal state of the building. Using optimal control theory, it can be formulated as a minimization problem, where the unknowns are sought as minimizers of a cost function evaluating the gap between the measures and the response of the model. Such inverse problems are ill-posed in the sense of Hadamard, which means that their solution, if it exists and is unique, does not depend continuously on the data, (Tikhonov and Arsenin, 1977). In order to obtain a stable numerical scheme, some regularization has to be introduced. We use here Tikhonov regularization, which consists in adding a regularization term to the cost function thus providing local convexity and transforming the initial problem into a well-posed one, (Chavent, 2009).

The minimization is done using a descent algorithm, and the gradient is computed with the adjoint state method, (Lions, 1968). This method gives a way to derive the gradient by solving a problem which has the same structure than the direct one, meaning that the same tools can be used for both computations. An additional advantage is that the minimization framework gives a complete local sensitivity analysis of the model (Griesse, 2007), as it will be illustrated in the last section of this document.

The choice of the regularization term greatly influences the shape of the reconstructed functions. The internal gains modelling the use of heating devices are discontinuous functions of time. However, the usual L^2 framework in least squares minimization with Tikhonov regularization fails to render discontinuities in time. Several works, in the field of image restoration, showed that the total variation (TV) is an efficient regularization term for the reconstruction of piecewise constant functions, (Rudin et al., 1992; Vogel and Oman, 1996; Osher et al., 2005).

We focus here on the identification of thermal conductivities and internal gains. These two parameters are reconstructed using simple ambient and wall surface temperatures. Tikhonov regularization is used in a TV framework to catch time discontinuities in internal gains. The adjoint method is employed to construct a non linear minimization algorithm. We show in the last section how the adjoint model can be used to efficiently compute model's sensitivities.

The general methodology framework is presented here on a very simple test case composed of a two-zone building for illustration purpose, see figure 1. Solar radiation is neglected.

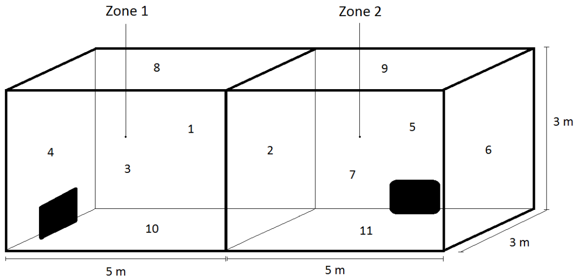

Figure 1: Case study and walls numbering

BUILDING THERMAL SIMULATION

Inverse identification aims at determining the input parameters of a mathematical model that minimize the discrepancy between the model's response and the data. We describe in this section the model used for this study.

The mathematical model describing the thermal state of the building builds upon standard multizone assumptions for the air temperatures, and partial differential equations for the heat flow in the envelope, (Boland, 1997). In this framework, the entire zone temperature and pressure are assumed to be homogeneous, and the heat transfers through the walls are assumed one-directional. The equations system is composed of eleven partial differential equations for the temperature distribution into walls, and two ordinary differential equations for the zone temperatures. All equations are coupled by radiative and convective exchange coefficients.

The evolution of the temperature T_i of the room $i \in \{1;2\}$ is governed by:

$$\begin{cases} C_i \dfrac{dT_i}{dt} = \displaystyle\sum_{p \in \mathbb{P}_i} S_p h_p^0 \left(\theta_p^s(t) - T_i\right) + W_i \\ T_i(t=0) = T_i^0 \end{cases} \quad (1)$$

where C_i is the room heat capacity in $J.K^{-1}$, S_p is the area of the wall p in m, h_p^0 is the convective exchange coefficient with the surface of the wall p whose temperature is denoted $\theta_p^s(t)$, and $W_i(t)$ the internal gains in $J.s^{-1}$. $\mathbb{P}_i$ is the index set of walls having a surface adjacent to the zone i.

The evolution of the tempeature field $\theta_p(x;t)$ inside a wall p of thickness L_p adjacent to the zone i is given by an equation of the form:

$$\begin{cases} S_p c_p \dfrac{\partial \theta_p}{\partial t} - S_p \dfrac{\partial}{\partial x}\left(k_p \dfrac{\partial \theta_p}{\partial x}\right) = 0 \\ -k_p S_p \dfrac{\partial \theta_p}{\partial x}(0;t) = S_p h_p^0 \left(T_i - \theta_p(0;t)\right) \\ \qquad + \displaystyle\sum_{m \in \mathbb{P}_i} S_p \alpha_{p;m} \left(\theta_m(0;t) - \theta_p(0;t)\right) \\ k_p S_p \dfrac{\partial \theta_p}{\partial x}(L_p;t) = S_p h_p^L \left(T_a - \theta_p(L_p;t)\right) \\ \qquad + S_p \beta_p \left(T^\infty - \theta_p(L_p;t)\right) \\ \theta_p(x,t=0) = \theta_p^0(x) \end{cases} \quad (2)$$

where c_p is the equivalent heat capacity in $J.m^{-3}.K^{-1}$, k_p is the equivalent thermal conductivity in $J.s^{-1}.m^{-1}.K^{-1}$, $\alpha_{p;m}$ is the radiative exchange coefficient with the wall m, h_p^0 is the convective exchange coefficient with the room temperature and h_p^L is the convective exchange coefficient with the outside temperature $T_a(t)$, and β_p is the radiative exchange coefficient with the sky temperature $T^\infty(t)$. Equation (2) is written for a wall with surface $(x = 0)$ adjacent to the zone i and surface $(x = L_p)$ facing outside. Of course, the various boundary conditions have to be adapted to each wall situation.

For the sake of simplicity, we will not write the heat equation inside every wall. They are all the same, only boundary conditions differ.

INVERSE PROBLEM

We present in this section the identification method based on the model described in the previous section. We suppose that we have air and surface temperature measurements obtained from sensors deployed on site for a given period of time. Let t_a be the actual time, we suppose having data starting from the initial time to the actual time : $t \in [0;t_a]$. We set $T_i^d(t)$ the inside air temperature measurements of the room $i \in \{1;2\}$, and $\theta_p^d(0;t)$ and $\theta_p^d(L_p;t)$ the temperature measurements of the surface $(x = 0)$ and $(x = L_p)$ of the wall p, respectively. More precisely, the two surface temperatures of walls 1 and 2, and two zone temperature are measured. We suppose that all the walls in contact with the outside have the same internal composition. That is, their thermal conductivity are the same, denoted k hereafter.

Based on the measurements described above, we aim at reconstructing this equivalent thermal conductivity k of the walls having a surface in contact with the outside, the equivalent thermal conductivity k_2 of the wall 2, and the internal gains in each room. The internal gains W_i heating the rooms are due to heating devices. They are typically discontinuous functions of time, and traditional identification techniques are unable to catch discontinuities in their time evolution. To deal with the discontinuous nature of the unknowns, we use here a regularization technique first developped in the field of image restoration, (Rudin et al., 1992; Vogel and Oman, 1996). It consists in looking for the unknowns in the space of bounded variation functions, thus separating, in the set of unknowns, the smoothly varying components from the rapidly varying ones. We make use of this technique here for the reconstruction of the internal gains, (Brouns et al., 2013).

Let $u = \{k; k_2; W_1, W_2\} \in \mathcal{P}$ be the vector of unknowns. It belongs to the parameters space $\mathcal{P} = (\mathbb{R}^+)^2 \times BV(0;t_a)^2$, where $BV(0;t_a)$ is the space of bounded variation functions. We set the problem as an optimization problem, consisting in the minimization of a cost function evaluating the gap between the data and the model's response. The problem reads: find

$u \in \mathcal{P}$ that minimizes

$$\begin{aligned} J(u) =& \frac{1}{2}\sum_{p=1}^{2} \|\theta_p(0;t) - \theta_p^d(0;t)\|_{\mathcal{M}}^2 \\ &+ \frac{1}{2}\sum_{p=1}^{2} \|\theta_p(L_p;t) - \theta_p^d(L_p;t)\|_{\mathcal{M}}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{2} \|T_i(t) - T_i^d(t)\|_{\mathcal{M}}^2 + \frac{\epsilon}{2}\|u\|_{\mathcal{P}}^2 \end{aligned} \tag{3}$$

with $\mathcal{M} = L^2(0;t_a)$ the measurements space, and ϵ the regularization parameter.

Levenberg-Marquardt algorithm

Since the model response is not linear with respect to the thermal conductivities, we use an iterative method based on the Levenberg-Marquardt approach (Moré, 1977), coupled with the conjugate gradient to solve the minimization problem. This technique relies on the following approximation: let $\delta u = \{\delta k; \delta k_2; \delta W_1, \delta W_2\} \in \mathcal{P}$ be a small perturbation of the unknowns ; the approximation writes:

$$\begin{aligned} &\theta_p(u+\delta u) \simeq \theta_p(u) + \delta\theta_p(\delta u), \quad p \in [\![1;11]\!] \\ &T_i(u+\delta u) \simeq T_i(u) + \delta T_i(\delta u), \quad\ \ i \in [\![1;2]\!] \end{aligned} \tag{4}$$

where δT_i and $\delta\theta_p$ are the solutions to the sensitivity model arround u, written in (5)-(7):

$$\begin{cases} C_i \dfrac{d\delta T_i}{dt} = \displaystyle\sum_{p\in\mathbb{P}_i} S_p h_p^0 \left(\delta\theta_p^s(t) - \delta T_i\right) + \delta W_i \\ \delta T_i(t=0) = 0 \end{cases} \tag{5}$$

$$\begin{cases} S_p c_p \dfrac{\partial\delta\theta_p}{\partial t} - S_p \dfrac{\partial}{\partial x}\left(k\dfrac{\partial\delta\theta_p}{\partial x}\right) = S_p \dfrac{\partial}{\partial x}\left(\delta k \dfrac{\partial\theta_p}{\partial x}\right) \\ -kS_p \dfrac{\partial\delta\theta_p}{\partial x}(0;t) = S_p h_p^0 \left(\delta T_i - \delta\theta_p(0;t)\right) \\ \qquad + \displaystyle\sum_{m\in\mathbb{P}_i} S_p \alpha_{p;m} \left(\delta\theta_m(0;t) - \delta\theta_p(0;t)\right) \\ \qquad + \delta k S_p \dfrac{\partial\theta_p}{\partial x}(0;t) \\ kS_p \dfrac{\partial\delta\theta_p}{\partial x}(L_p;t) = - S_p\left(h_p^L + \beta_p\right)\delta\theta_p(L_p;t) \\ \qquad\qquad - \delta k S_p \dfrac{\partial\theta_p}{\partial x}(L_p;t) \\ \delta\theta_p(x,t=0) \equiv 0 \end{cases} \tag{6}$$

$$\begin{cases} S_2 c_2 \dfrac{\partial\delta\theta_2}{\partial t} - S_2 \dfrac{\partial}{\partial x}\left(k_2\dfrac{\partial\delta\theta_2}{\partial x}\right) = S_2 \dfrac{\partial}{\partial x}\left(\delta k_2 \dfrac{\partial\theta_2}{\partial x}\right) \\ -k_2 S_2 \dfrac{\partial\delta\theta_2}{\partial x}(0;t) = S_2 h_2^0 \left(\delta T_1 - \delta\theta_2(0;t)\right) \\ \qquad + \displaystyle\sum_{m\in\mathbb{P}_1} S_2 \alpha_{2;m} \left(\delta\theta_m(0;t) - \delta\theta_2(0;t)\right) \\ \qquad + \delta k_2 S_2 \dfrac{\partial\theta_2}{\partial x}(0;t) \\ k_2 S_2 \dfrac{\partial\delta\theta_2}{\partial x}(L_2;t) = S_2 h_2^L \left(\delta T_2 - \delta\theta_2(L_2;t)\right) \\ \qquad + \displaystyle\sum_{m\in\mathbb{P}_2} S_2 \alpha_{2;m} \left(\delta\theta_m(0;t) - \delta\theta_2(L_2;t)\right) \\ \qquad - \delta k_2 S_2 \dfrac{\partial\theta_2}{\partial x}(L_2;t) \\ \delta\theta_2(x,t=0) \equiv 0 \end{cases} \tag{7}$$

Therefore, we introduce a new cost function:

$$\begin{aligned} \tilde{J}_u(\delta u) =& \frac{1}{2}\sum_{p=1}^{2} \|\theta_p(0;t) + \delta\theta_p(0;t) - \theta_p^d(0;t)\|_{\mathcal{M}}^2 \\ &+ \frac{1}{2}\sum_{p=1}^{2} \|\theta_p(L_p;t) + \delta\theta_p(L_p;t) - \theta_p^d(L_p;t)\|_{\mathcal{M}}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{2} \|T_i(t) + \delta T_i(t) - T_i^d(t)\|_{\mathcal{M}}^2 + \frac{\epsilon}{2}\|\delta u\|_{\mathcal{P}}^2 \end{aligned} \tag{8}$$

The new unknown is the optimal local increase δu which will be used for the next linearization step around $u_{n+1} = u_n + \delta u$.

The cost function (8) is quadratic, which ensures convexity. Its minimum corresponds to the solution of Euler's equation:

$$\delta u = \arg\min_{\delta\overline{u}\in\mathcal{P}} \tilde{J}_u(\delta\overline{u}) \iff \nabla\tilde{J}_u(\delta u) = 0 \tag{9}$$

The problem (9) is solved using the conjugate gradient method. In order to do this, the gradient of (8) is computed using the adjoint method. With this method, an exact expression of the gradient can be obtained by solving the so-called adjoint problem, which as the same structure as the initial problem (1)-(2).

Adjoint state method

Let $V_i, i \in [\![1;2]\!]$, and $A_p, p \in [\![1;11]\!]$, be the solutions to the adjoint state equations (10)-(12):

$$\begin{cases} -C_i \dfrac{dV_i}{dt} = \displaystyle\sum_{p\in\mathbb{P}_i} S_p h_p^0 \left(A_p^s(t) - V_i\right) \\ \qquad\qquad + T_i + \delta T_i - T_i^d \\ V_i(t=t_a) = 0 \end{cases} \tag{10}$$

$$
\begin{cases}
-S_p c_p \dfrac{\partial A_p}{\partial t} = S_p \dfrac{\partial}{\partial x}\left(k \dfrac{\partial A_p}{\partial x}\right) \\
\qquad + \left(\theta_p + \delta\theta_p - \theta_p^d\right)\left(\delta_0(x) + \delta_{L_p}(x)\right) \\
-k S_p \dfrac{\partial A_p}{\partial x}(0;t) = S_p h_p^0 \left(V_i - A_p(0;t)\right) \\
\qquad + \displaystyle\sum_{m \in \mathbb{P}_i} S_p \alpha_{p;m}\left(A_m(0;t) - A_p(0;t)\right) \\
k S_p \dfrac{\partial A_p}{\partial x}(L_p;t) = -S_p \left(h_p^L + \beta_p\right) A_p(L_p;t) \\
A_p(x, t = t_a) \equiv 0
\end{cases}
\tag{11}
$$

$$
\begin{cases}
-S_2 c_2 \dfrac{\partial A_2}{\partial t} = S_2 \dfrac{\partial}{\partial x}\left(k_2 \dfrac{\partial A_2}{\partial x}\right) \\
\qquad + \left(\theta_2 + \delta\theta_2 - \theta_2^d\right)\left(\delta_0(x) + \delta_{L_2}(x)\right) \\
-k_2 S_2 \dfrac{\partial A_2}{\partial x}(0;t) = S_2 h_2^0 \left(V_1 - A_2(0;t)\right) \\
\qquad + \displaystyle\sum_{m \in \mathbb{P}_1} S_2 \alpha_{2;m}\left(A_m(0;t) - A_2(0;t)\right) \\
k_2 S_2 \dfrac{\partial A_2}{\partial x}(L_2;t) = S_2 h_2^L \left(V_2 - A_2(L_2;t)\right) \\
\qquad + \displaystyle\sum_{m \in \mathbb{P}_2} S_2 \alpha_{2;m}\left(A_m(0;t) - A_2(L_2;t)\right) \\
A_2(x, t = t_a) \equiv 0
\end{cases}
\tag{12}
$$

where δ_a is the Dirac measure at a. The optimal control theory tells us that the components of $\nabla \tilde{J}_u(\delta u)$ are the following:

$$
\begin{aligned}
\left\langle \nabla \tilde{J}_u ; \delta \tilde{k} \right\rangle = & -\sum_{p \in [\![1;7]\!] \setminus \{2\}} \int_0^{t_a} \int_0^{L_p} \delta \tilde{k} S_p \frac{\partial \theta_p}{\partial x} \frac{\partial A_p}{\partial x} \\
& + \epsilon \left\langle \delta \tilde{k} ; \delta k \right\rangle
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\left\langle \nabla \tilde{J}_u ; \delta \tilde{k}_2 \right\rangle = & -\int_0^{t_a} \int_0^{L_p} \delta \tilde{k}_2 S_2 \frac{\partial \theta_2}{\partial x} \frac{\partial A_2}{\partial x} \\
& + \epsilon \left\langle \delta \tilde{k}_2 ; \delta k_2 \right\rangle
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\left\langle X_i ; \delta \tilde{W}_i \right\rangle_{L^2(0;t_a)} = & \int_0^{t_a} \delta \tilde{W}_i V_i \\
& + \epsilon \left\langle \delta \tilde{W}_i ; \delta W_i \right\rangle_{L^2(0;t_a)}
\end{aligned}
\tag{15}
$$

with $i \in \{1;2\}$.
The third and fourth components of the gradient are obtained by the projection in the space $BV(0;t_a)$ using the following computation (see (Brouns et al., 2013) for more details):

$$
\|X_i - \nabla \tilde{J}_u\|_{L^2(0;t_a)} = \min_{v \in BV(0;t_a)} \|X_i - v\|_{L^2(0;t_a)}
\tag{16}
$$

with $i \in \{1;2\}$.
Once we have the gradient, we use a descent method for solving (9). We choose the conjugate gradient method (see (Nassiopoulos and Bourquin, 2013) for more details).

IDENTIFICATION RESULTS

In order to give numerical evidence of the performance of the method, we generate simulated temperature measurements with our model. The virtual sensors are distributed on the two surfaces of the walls 1 and 2 (meanning that one of the surface temperature sensors of the wall 1 is outside of the building), and in each room. The data are sampled at $10^{-3} Hz$ (every 15 min), during 24h. We add a gausian white noise with standard deviation 10^{-2} to the data. These data are then used as inputs for the inverse algorithm.

We reconstruct simultaneously the thermal conductivities k and k_2, and the internal gains $W_i, i \in \{1;2\}$. The algorithm is initialized by 0.7 for the conductivities, and by the zero function for the internal gains. All other parameters are assumed to be known.

The figures 2 and 3 represent the reconstruction of the thermal conductivities k and k_2, respectively. The reconstruction of W_1 and W_2 are presented in the figures 4 and 5, respectively.

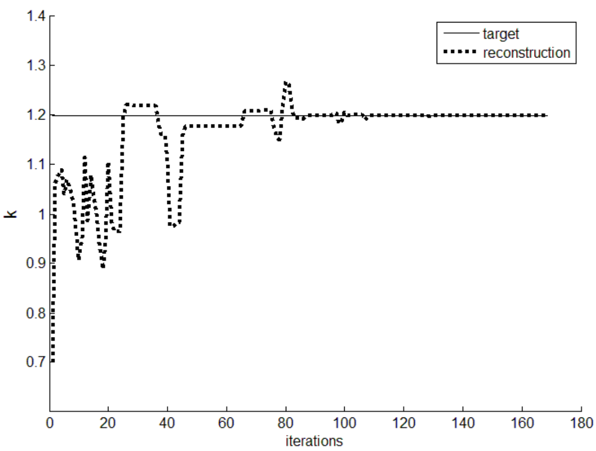

Figure 2: Reconstruction of k versus the algorithm iterations

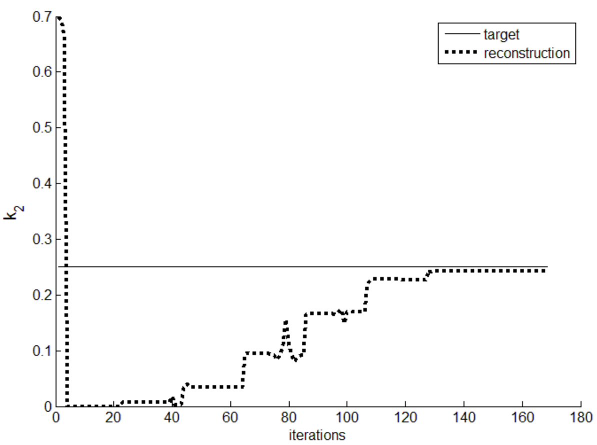

Figure 3: Reconstruction of k_2 versus the algorithm iterations

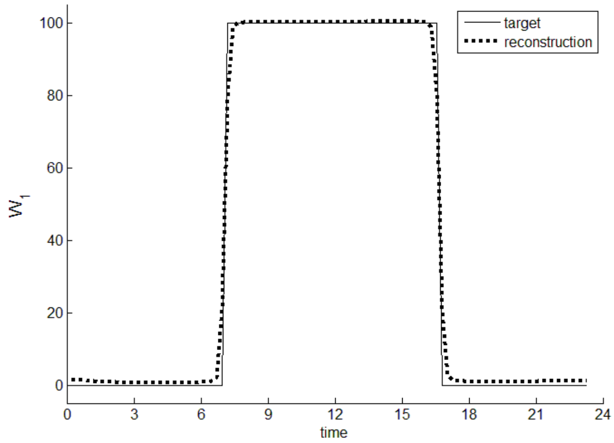

Figure 4: Reconstruction of W_1 after projection method in the BV space

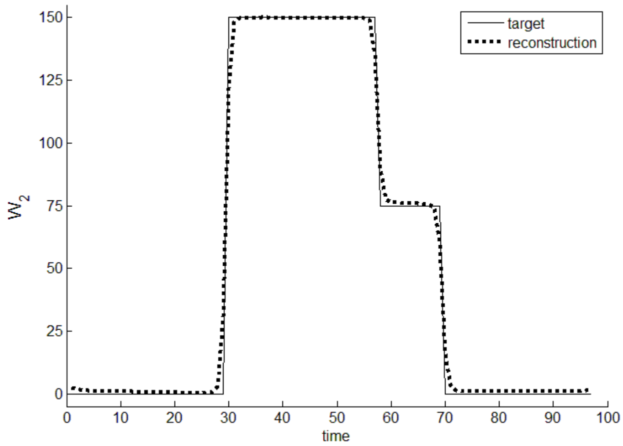

Figure 5: Reconstruction of W_2 after projection method in the BV space

COMPUTING SENSITIVITIES

When a calibrated energy performance model is available, one can accurately predict the overall performance of the building. But the next question that arises is: how robust is this prediction? In other words, how uncertainties or variations in the parameters of the model impact the overall performance? One way to analyse the behaviour of the model in that sense is to compute the sensitivities of the model's response with respect to the various parameters. We show in this section that with the adjoint method and the numerical tools presented above this kind of computation is straightforward.

A retrofit operation aims at reducing the overall energy demand of a building. This demand is computed as the overall heat needed to maintain a given setpoint temperature T_c. Neglecting the type of thermal regulation used to control the heating systems performance, one can compute the theoretical overall energy demand as a solution of an optimization problem representing a trade-off between setpoint temperature control and energy consumption. This problem can be written as: find $w^* \in L^2(0;t_a)$ such that

$$w^* = \underset{w \in L^2(0;t_a)}{\arg\min} \ \mathcal{J}(w) \tag{17}$$

where

$$\mathcal{J}(w) = \sum_{i=1}^{2} \frac{\mu}{2} \|T_i - T_c\|^2_{\mathcal{M}} + \frac{1}{2}\|w\|^2_{L^2(0;t_a)^2} \tag{18}$$

and T_i is the solution of

$$\begin{cases} C_i \dfrac{dT_i}{dt} = \displaystyle\sum_{p \in \mathbb{P}_i} S_p h_p^0 \left(\theta_p^s(t) - T_i\right) + W_i + w \\ T_i(t=0) = T_i^0 \end{cases} \tag{19}$$

In (18), μ is a parameter modulating the severity of the setpoint temperature constraint. The first term of (18) measures the gap between the model response for the rooms air temperature and the setpoint temperature, while the second measures the overall consumptions. The result of (17) represents an optimal theoretical consumption that most of the times is not reached because of the non-optimal regulation system. But one can argue that the sensitivities of w^* with respect to the parameters of the model are of the same magnitude as the sensitivities of the true consumption.

We show hereafter how the adjoint model can be used in order to compute these sensitivities in this setting. We will illustrate this method by focusing on the computation of the sensitivities with respect to two parameters: k and T^∞.

In fact, the cost function (18) depends implicitly on k and T^∞: $\mathcal{J}(w) = \mathcal{J}(w; k; T^\infty)$, so we introduce the following notation:

$$\mathcal{J}(w^*; k; T^\infty) = \min_w \mathcal{J}(w; k; T^\infty) \tag{20}$$

The optimal solution w^* also depends on k and T^∞. The sensitivity of w^* with respect to k and T^∞ is defined as the variation of w^* under perturbations δk and δT^∞ respectively. These sensitivities, denoted $\tilde{w}^*_{\delta k}$ and $\tilde{w}^*_{\delta T^\infty}$ respectively, are given by the Gâteaux derivatives:

$$\tilde{w}^*_{\delta k} = \lim_{\epsilon \to 0} \frac{w^*(k + \epsilon\delta k; T^\infty) - w^*(k; T^\infty)}{\epsilon} \tag{21}$$

$$\tilde{w}^*_{\delta T^\infty} = \lim_{\epsilon \to 0} \frac{w^*(k; T^\infty + \epsilon\delta T^\infty) - w^*(k; T^\infty)}{\epsilon} \tag{22}$$

One can show (Griesse, 2007) that the solutions of the problems (21)-(22) are in fact the solutions of the folowing problems:

$$\min_{\tilde{w} \in L^2(0;t_a)} \left\{ \sum_{i=1}^{2} \frac{\mu}{2} \|\tilde{T}_i\|^2_{\mathcal{M}} + \frac{1}{2}\|\tilde{w}\|^2_{L^2(0;t_a)^2} \right\} \tag{23}$$

with $\tilde{w} \in L^2(0; t_a)$, and $\tilde{T}_i, i \in \{1; 2\}$ the solutions of equations (24)-(25):

$$\begin{cases} C_i \dfrac{d\tilde{T}_i}{dt} = \displaystyle\sum_{p \in \mathbb{P}_i} S_p h_p^0 \left(\tilde{\theta}_p^s(t) - \tilde{T}_i \right) + \tilde{w} \\ \tilde{T}_i(t = 0) = 0 \end{cases} \tag{24}$$

$$\begin{cases} S_p c_p \dfrac{\partial \tilde{\theta}_p}{\partial t} - S_p \dfrac{\partial}{\partial x} \left(k \dfrac{\partial \tilde{\theta}_p}{\partial x} \right) = S_p \dfrac{\partial}{\partial x} \left(\delta k \dfrac{\partial \theta_p}{\partial x} \right) \\ -k S_p \dfrac{\partial \tilde{\theta}_p}{\partial x}(0; t) = S_p h_p^0 \left(\tilde{T}_i - \tilde{\theta}_p(0; t) \right) \\ \qquad + \displaystyle\sum_{m \in \mathbb{P}_i} S_p \alpha_{p;m} \left(\tilde{\theta}_m(0; t) - \tilde{\theta}_p(0; t) \right) \\ \qquad + \delta k S_p \dfrac{\partial \theta_p}{\partial x}(0; t) \\ k S_p \dfrac{\partial \tilde{\theta}_p}{\partial x}(L_p; t) = -S_p \left(h_p^L + \beta_p \right) \tilde{\theta}_p(L_p; t) \\ \qquad\qquad + S_p \beta_p \delta T^\infty - \delta k S_p \dfrac{\partial \theta_p}{\partial x}(L_p; t) \\ \tilde{\theta}_p(x, t = 0) \equiv 0 \end{cases} \tag{25}$$

where δk and δT^∞ are small perturbations of k and T^∞, respectively. The term $\theta_p(x; t)$ appearing in (25) is the solution of the initial problem (1)-(2) for the given k and T^∞.
Let $\tilde{w}^*_{\delta k}$ and $\tilde{w}^*_{\delta T^\infty}$ be the solution of the minimization (23) for the problem (21) and (22), respectively. They satisfy locally:

$$w^*|_{k+\delta k} \simeq w^*|_k + \tilde{w}^*_{\delta k} \tag{26}$$
$$w^*|_{T^\infty + \delta T^\infty} = w^*|_{T^\infty} + \tilde{w}^*_{\delta T^\infty} \tag{27}$$

Since the model's response is linear with respect to the sky temperature, the equation (27) is an equality, and is globally satisfied. The equation (26) is an approximation only satisfied for a small perturbation δk.
This sensitivity analysis could be a precious aid for retrofit. Firstly, it helps the professionals to evaluate the uncertainty of the energy performance computation with the knowledge of the uncertainties on the other parameters. Secondly, it can help the professionals to determine the best rehabilitation scenario by highlighting the most sensitive parameters.

CONCLUSION

We showed the possibility to reconstruct thermal conductivities and internal gains of a two-zone building based on six temperature sensors. We use a projection method in the space of bounded variation functions to increase the accuracy of the reconstruction of the internal gains. The procedure has been validated using sythetic data obtained from simulation, where numerical noisehas been added. We showed that the adjoint model can be used efficiently to perform a local sensitivity analysis: the problem is equivalent to a minimization problem similar to the initial inverse problem. We outlined the methodology in the case of the computation of sensitivities of the theoretical consumptions of the building with respect to its thermal conductivities and the sky temperature.

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