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# An Observer with Measurement-triggered Jumps for Linear Systems with Known Input \*

F. Ferrante, F. Gouaisbaut, R. G. Sanfelice, S. Tarbouriech

## Abstract

This paper deals with the estimation of the state of linear time invariant systems for which measurements of the output are available sporadically. An observer with jumps triggered by the arrival of such measurements is proposed and studied in a hybrid systems framework. The resulting system is written in estimation error coordinates and augmented with a timer variable that triggers the event of new measurements arriving. The design of the observer is performed to achieve uniform global asymptotic stability (UGAS) of a closed set including the points for which the state of the plant and its estimate coincide. Furthermore, a computationally tractable design procedure for the proposed observer is presented and illustrated in an example.

## I. INTRODUCTION

State observer design is undoubtedly a difficult problem, with high relevance in applications. Indeed, observers can be employed to obtain an estimation of certain state variables, which are not directly accessible or also to reduce the number of the sensors used in control systems. Many of the most interesting recent applications pertain to controlled systems linked together through data networks. The nature of such networks may often introduce time delays, asynchronism, packages drop-out, and communication channel limitations; see, for example, [13]. Moreover, in modern distributed systems, the communication mechanisms across the network are governed by logic statements, which aim at reducing the required bandwidth over the communication channel; see, for example, [20], [21]. Such mechanisms lead to an intermittent availability of the measured variable. In this setting, the classical paradigm of continuously measured variables needs to be reconsidered to face the new challenges induced by data network constraints. Indeed, an observer can exploit the measured output only at discrete time instants, which are *a priori* unknown. In fact, the estimation algorithm can be actually governed by an event-triggered mechanism (see [1] for further details).

In this paper, we focus on the estimation problem for linear systems where the input injected into the plant is known and the measured output is gathered in an intermittent fashion. Building from the idea in [15], we propose an open-loop observer along with a suitable event-triggered updating of the estimated state. Since the evolution of the considered observer exhibits both continuous-time behavior and instantaneous updating, we provide a hybrid model of the observer including the triggering logic. Then, using a Lyapunov function, we propose a condition which guarantees global uniform asymptotic stability of the estimation error as well as robustness with respect to bounded perturbations, in an input-to-state stability sense (see [18]). Finally the obtained condition is turned into a design algorithm for the proposed observer based on the solution of a set of linear matrix inequalities. The proposed hybrid model allows us to effectively exploit the properties of the time domain of the solutions to the resulting hybrid system, as for example, the persistence in jumping. This not only provides a more tight comprehension of the system behavior but also enables us to construct a more general Lyapunov function, so as to overcome the convexity issues induced by non-uniformity in sampling time, which are pointed out also in [15] and moreover characterizing the effect of a measurement noise by the mean of an input-to-state stability bound on the estimation error.

The paper is organized as follows. Section II presents the system under consideration, the problem we intend to solve, and the hybrid modeling of the proposed observer. Section III is dedicated to the main results, which provide a solution to the stated estimation problem. Section IV is devoted to numerical issues and it ends providing a convex design algorithm for the proposed observer. In Section V, the effectiveness of the approach is illustrated through a numerical example.

**Notation:** The set  $\mathbb{N}_0$  is the set of the positive integers including zero and  $\mathbb{R}_{\geq 0}$  represents the set of the nonnegative real scalars. For every complex number  $\omega$ ,  $\Re(\omega)$  and  $\Im(\omega)$  stand respectively for the real and the imaginary part of  $\omega$ .  $\mathbf{I}$  denotes the identity matrix whereas  $\mathbf{0}$  denotes the null matrix (equivalently the null vector) of appropriate dimensions. For a matrix  $A \in \mathbb{R}^{n \times m}$ ,  $A'$  denotes the transpose of  $A$  and  $\|A\|$  denotes the Euclidean induced norm.  $\text{He}(A) = A + A'$ . For two symmetric matrices,  $A$  and  $B$ ,  $A > B$  means that  $A - B$  is positive definite. In partitioned symmetric matrices, the symbol  $\star$  stands for symmetric blocks. The matrix  $\text{diag}\{A_1; \dots; A_n\}$  is the block-diagonal matrix having  $A_1, \dots, A_n$  as diagonal blocks. For a vector  $x \in \mathbb{R}^n$ ,  $x'$  denotes the transpose of  $x$ , whereas  $\|x\|$  denotes the Euclidean norm. For a function  $s \in [0, +\infty) \rightarrow \mathbb{R}^n$ ,  $\|s\|_t = \sup_{\tau \in [0, t]} \|s(\tau)\|$ . Let  $X$  be a given set,  $\text{Co}\{X\}$  represents the convex hull of  $X$ .  $\delta\mathbb{B}$  is the closed ball with radius  $\delta$  of appropriate dimension in the Euclidean norm. A function  $\alpha: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to belong to the class  $\mathcal{K}$  if it is continuous, zero at zero, and strictly increasing. A function  $\alpha: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to belong to class  $\mathcal{K}_\infty$  if it belongs to the class  $\mathcal{K}$  and is unbounded. A function  $\beta: \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to belong to class  $\mathcal{KL}$  if it is nondecreasing in its first argument, nonincreasing in its second argument, and  $\lim_{s \rightarrow 0^+} \beta(s, t) = \lim_{t \rightarrow +\infty} \beta(s, t) = 0$ . A function  $\beta: \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to belong to class  $\mathcal{KLL}$  if, for each  $r \in \mathbb{R}_{\geq 0}$ , the functions  $\beta(\cdot, \cdot, r)$  and  $\beta(\cdot, r, \cdot)$  belong to class  $\mathcal{KL}$ .

## II. PROBLEM STATEMENT

### A. System description

Consider the following continuous-time linear system:

$$\begin{aligned} \dot{z} &= Az + Bu \\ y &= Mz \end{aligned} \tag{1}$$

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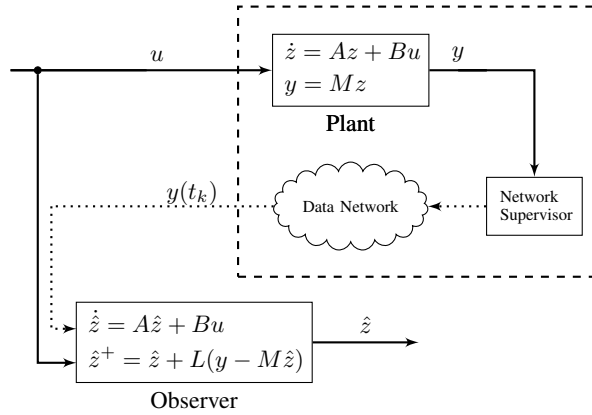


Fig. 1: State estimation for a linear system with output gathered through a data network

where  $z \in \mathbb{R}^n$ ,  $y \in \mathbb{R}^q$  and  $u \in \mathbb{R}^p$  are, respectively, the state, the measured output, and the input of the system, while  $A$ ,  $B$  and  $M$  are constant matrices of appropriate dimensions. Assume also that the input  $u$  belongs to the class of the measurable and locally bounded functions  $u: [0, \infty) \rightarrow \mathbb{R}^p$ . Suppose that the input  $u$  is measurable. We want to design an observer providing an estimation  $\hat{z}$  of the state  $z$  when the output  $y$  is available only at some times  $t_k$ , for  $k \in \mathbb{N}_0$ , not known *a priori*. Figure 1 illustrates such a setting in the context of network control. Suppose that  $\{t_k\}_0^{+\infty}$  is a strictly increasing unbounded real sequence of times. Furthermore (see [14], [3] and the references therein) we assume also that there exist two positive real scalars  $T_1, T_2$  with  $T_1 < T_2$  such that

$$T_1 \leq t_{k+1} - t_k \leq T_2. \quad (2)$$

Notice that the condition (2) prevents the existence of accumulation points in the sequence  $\{t_k\}_0^{+\infty}$  (see [10]) and, hence, it avoids the existence of Zeno behavior, which is typically undesired in practice. Hence, since the information on the output  $y$  is available in an impulsive fashion, motivated by the work of [15], to solve the considered estimation problem, we want to design an observer with jumps in its state following the law:

$$\dot{\hat{z}} = A\hat{z} + Bu \quad \text{when } t \notin \{t_k\}_0^{+\infty} \quad (3a)$$

$$\hat{z}(t_k^+) = \hat{z}(t_k) + L(y(t_k) - M\hat{z}(t_k)) \quad \text{when } t \in \{t_k\}_0^{+\infty} \quad (3b)$$

where  $L$  is a real matrix of appropriate dimensions to be designed. It is worthwhile to point out that in [19] the same observer is adopted to state estimation in presence of quantized measurement.

Following the lines of [17], the state estimation problem can be formulated as a set stabilization problem. Namely, defining the set

$$\mathcal{A}_s = \{(z, \hat{z}) \in \mathbb{R}^{2n} : z = \hat{z}\} \quad (4)$$

our goal is to design the matrix  $L$  such that  $\mathcal{A}_s$  is globally asymptotically stable. At this stage, as usual in estimation problems, one considers the estimation error defined as  $\varepsilon := z - \hat{z}$ . Through such a change of variable, the set  $\mathcal{A}_s$  defined in (4) is replaced as

$$\mathcal{A}_\varepsilon = \{\varepsilon \in \mathbb{R}^n : \varepsilon = 0\} \quad (5)$$

while the error dynamics are given by the following dynamical system with jumps:

$$\dot{\varepsilon} = A\varepsilon \quad \text{when } t \notin \{t_k\}_0^{+\infty} \quad (6a)$$

$$\varepsilon(t_k^+) = (\mathbf{I} - LM)\varepsilon(t_k) \quad \text{when } t \in \{t_k\}_0^{+\infty}. \quad (6b)$$

Due to the linearity of the system (1), the estimation error dynamics and the dynamics of  $z$  are decoupled. Then, for the purpose of stabilization of the set  $\mathcal{A}_\varepsilon$ , one can effectively neglect the plant state and just consider system (6).

*Remark 1:* Notice that assuming the knowledge of the input is not so restrictive. Indeed in many real situations all the devices exploited to control and supervise the plant are located in the same location. For example, in an observer based decentralized controller the input  $u$  is often directly accessible by the observer. This situation is depicted in Figure 1, where the dotted edges refer to impulsive data streams, while the solid edges to continuous data streams.

## B. Hybrid modeling

The fact that the observer experiments jumps when a new measurement is available suggests that such a innovation processes is governed by a hybrid system. Thus due to this, we represent the whole system composed by the observer (3) and the logic triggering jumps as a hybrid system (see [12] where similar techniques are adopted to model a finite time convergent observer).

However, in order to do that, one needs to model the hidden time-driven mechanism triggering the observer jumps. To this end, in this work, we augment the system state with an auxiliary timer variable  $\tau$ , which tracks the time flow and triggers a jump whenever a certain condition is verified. This allows to describe the time-driven jump triggering mechanism as a state-driven jump triggering mechanism, which leads to a model that can be efficiently represented by relying on the

framework for hybrid systems proposed in [8].

Namely, we make  $\tau$  decreasing as the ordinary time increases and, whenever it reaches zero, trigger a jump. Moreover, after a jump occurs  $\tau$ , is re-initialized to some value belonging to the interval  $[T_1, T_2]$  and, after the reset it flows again. Therefore, the whole system composed by the state  $\varepsilon$  and the timer variable  $\tau$  can be represented by the following hybrid system:

$$\mathcal{H}_\varepsilon \begin{cases} \left. \begin{array}{l} \dot{\varepsilon} = A\varepsilon \\ \dot{\tau} = -1 \end{array} \right\} (\varepsilon, \tau) \in C \\ \left. \begin{array}{l} \varepsilon^+ = (\mathbf{I} - LM)\varepsilon \\ \tau^+ \in [T_1, T_2] \end{array} \right\} (\varepsilon, \tau) \in D \end{cases} \quad (7a)$$

with the flow set and the jump set defined as

$$\begin{aligned} C &= \{(\varepsilon, \tau) \in \mathbb{R}^{n+1} : \tau \in [0, T_2]\} \\ D &= \{(\varepsilon, \tau) \in \mathbb{R}^{n+1} : \tau = 0\}. \end{aligned} \quad (7b)$$

For this system, we denote by  $\tilde{x} = [\varepsilon' \tau']'$  the state and by  $f$  and  $G$ , respectively, the flow map and the jump map, *i.e.*,

$$f(\tilde{x}) = \begin{bmatrix} A\varepsilon \\ -1 \end{bmatrix} \quad (8a)$$

$$G(\tilde{x}) = \begin{bmatrix} (\mathbf{I} - LM)\varepsilon \\ [T_1, T_2] \end{bmatrix}. \quad (8b)$$

Notice that, in order to make the hybrid system (7) an accurate description of the real time-triggered phenomenon, which governs the feedback innovation process, the variable  $\tau$  needs to definitely belong to the interval  $[0, T_2]$ . Then the set  $\mathcal{A}_\varepsilon$  defined in (5) can be slightly modified to take into account this further requirement. This leads to the following set

$$\mathcal{A} = \{(\varepsilon, \tau) \in \mathbb{R}^{n+1} : \varepsilon = 0, \tau \in [0, T_2]\}. \quad (9)$$

Since  $\mathcal{A}$  is closed, let  $x$  be a given vector, the distance from  $x \in \mathbb{R}^{n+1}$  to  $\mathcal{A}$  is defined as follows:

$$|x|_{\mathcal{A}} = \inf_{y \in \mathcal{A}} \|x - y\|.$$

It turns out that for every  $\tilde{x} \in C \cup D \cup G(D)$ ,  $|\tilde{x}|_{\mathcal{A}} = \|\varepsilon\|$ . Finally, the problem we intend to solve can be formulated as follows:

*Problem 1:* Given the matrices  $A$ ,  $B$ , and  $M$  of appropriate dimensions and two positive scalars  $T_1$  and  $T_2$  such that  $T_1 < T_2$ , compute a matrix  $L \in \mathbb{R}^{n \times q}$  such that the set  $\mathcal{A}$  defined in (9) is Uniform Global Asymptotic Stability (UGAS) for the hybrid system (7).

We consider the following notion of UGAS for general hybrid system  $\mathcal{H}$ .

*Definition 1:* ([8]) Let  $\mathcal{A}$  be closed. The set  $\mathcal{A}$  is

- uniformly globally stable for hybrid system  $\mathcal{H}$  if there exists a class- $\mathcal{K}_\infty$  function  $\alpha$  such that any solution  $\phi$  to  $\mathcal{H}$  satisfies  $|\phi(t, j)|_{\mathcal{A}} \leq \alpha(|\phi(0, 0)|_{\mathcal{A}})$  for all  $(t, j) \in \text{dom } \phi$ ;
- uniformly globally attractive for  $\mathcal{H}$  if for each  $\omega > 0$  and  $r > 0$  there exists  $T > 0$  such that, for any solution  $\phi$  to  $\mathcal{H}$  with  $|\phi(0, 0)|_{\mathcal{A}} \leq r$  every solution is complete and

$$\lim_{\substack{(t, j) \in \text{dom } \phi \\ t+j \rightarrow +\infty}} |\phi(t, j)|_{\mathcal{A}} = 0$$

- uniformly globally asymptotically stable for  $\mathcal{H}$  if it is both uniformly globally stable and uniformly globally attractive.

Concerning the existence of solutions to system (7), relying on the concept of solution proposed in [8, Definition 2.6], it is straightforward to check that for every initial condition  $\tilde{x}(0, 0) \in C \cup D$  every solution to  $\mathcal{H}$  is complete. In addition, we can characterize the domain of these solutions. Indeed, the variable  $\tau$ , acting as a timer, guarantees that for every initial condition  $\tilde{x}(0, 0) \in C \cup D$ , at least for  $j \geq 1$ ,  $t_{j+1} - t_j \in [T_1, T_2]$ . Therefore, the domain of the solutions to  $\mathcal{H}$  can be written as follows:

$$\text{dom } \phi = ([t_0, t_1] \times \{0\}) \cup \left( \bigcup_{j \in \mathbb{N} \setminus \{0\}} ([t_j, t_{j+1}] \times \{j\}) \right) \quad (10)$$

$$\begin{aligned} T_1 &\leq t_{j+1} - t_j \leq T_2 \quad \forall j \in \mathbb{N} \setminus \{0\} \\ 0 &\leq t_1 - t_0 \leq T_2 \end{aligned}$$

It should be noticed that the structure of the foregoing hybrid time domain implies that  $\forall (t, j) \in \text{dom } \phi$

$$t \leq T_2(j + 1) \quad (11)$$

### III. MAIN RESULTS

#### A. Conditions for Uniform Global Asymptotic Stability

The following result provides conditions for the UGAS of the set  $\mathcal{A}$  defined in (9) for system (7).

*Theorem 1:* Given two positive scalars  $T_1$  and  $T_2$  such that  $T_1 < T_2$ . If there exist a symmetric positive definite matrix  $P \in \mathbb{R}^{n \times n}$  and a matrix  $L \in \mathbb{R}^{q \times n}$  such that

$$(\mathbf{I} - LM)' e^{A'v} P e^{Av} (\mathbf{I} - LM) - P < \mathbf{0}, \quad \forall v \in [T_1, T_2], \quad (12)$$

then the set  $\mathcal{A}$  defined in (9) is UGAS for the hybrid system (7).

*Proof:* Consider the following Lyapunov function candidate for the hybrid system (7) defined for every  $\tilde{x} \in \mathbb{R}^{n+1}$ :

$$V(\tilde{x}) = \varepsilon' e^{A'\tau} P e^{A\tau} \varepsilon \quad (13)$$

To prove the claim, we rely on the stability result provided in [8, Proposition 3.24]. To this end, notice that there exist two positive scalars  $\alpha_1, \alpha_2$  such that

$$\alpha_1 |\tilde{x}|_{\mathcal{A}}^2 \leq V(\tilde{x}) \leq \alpha_2 |\tilde{x}|_{\mathcal{A}}^2 \quad \forall \tilde{x} \in C \cup D \cup G(D) \quad (14)$$

Namely, due to the positive definiteness of  $P$  and the non-singularity of the matrix  $e^{A\tau}$  for every  $\tau$ , by continuity arguments, one can set

$$\alpha_1 = \min_{\tau \in [T_1, T_2]} \lambda_{\min} \left( e^{A'\tau} P e^{A\tau} \right) \quad (15)$$

$$\alpha_2 = \max_{\tau \in [T_1, T_2]} \lambda_{\max} \left( e^{A'\tau} P e^{A\tau} \right) \quad (16)$$

where  $\lambda_{\min}(\cdot)$  and  $\lambda_{\max}(\cdot)$  denote, respectively, the smallest and the largest eigenvalue of the their matrix argument. By straightforward calculations one gets

$$\nabla V(\tilde{x})' = [2\varepsilon' e^{A'\tau} P e^{A\tau} \quad \varepsilon' e^{A'\tau} (A'P + PA) e^{A\tau} \varepsilon].$$

Exploiting the fact that the matrices  $e^{A\tau}$  and  $A$  commute, one has

$$\langle \nabla V(\tilde{x}), f(\tilde{x}) \rangle = 0 \quad \forall \tilde{x} \in C. \quad (17)$$

Notice that, for every  $g \in G(\tilde{x})$ , there exists a real scalar  $v$  belonging to the interval  $[T_1, T_2]$  such that

$$g = \begin{bmatrix} (\mathbf{I} - LM)\varepsilon \\ v \end{bmatrix}$$

Then, for every  $g \in G(\tilde{x})$ , one has

$$\begin{aligned} V(g) - V(\tilde{x}) &= \varepsilon' (\mathbf{I} - LM)' e^{A'v} P e^{Av} (\mathbf{I} - LM) \varepsilon \\ &\quad - \varepsilon' e^{A'\tau} P e^{A\tau} \varepsilon. \end{aligned}$$

Whenever  $\tilde{x} \in D$ , from (7b), we have that  $\tau=0$ . Then, we have

$$V(g) - V(\tilde{x}) = \varepsilon' \left( (\mathbf{I} - LM)' e^{A'v} P e^{Av} (\mathbf{I} - LM) - P \right) \varepsilon.$$

Hence, by virtue of relation (12), it follows that there exists a positive small enough scalar  $\beta$  such that, for every  $v \in [T_1, T_2]$ ,

$$V(g) - V(\tilde{x}) \leq -\beta \varepsilon' \varepsilon = -\beta |\tilde{x}|_{\mathcal{A}}^2, \quad \forall \tilde{x} \in D, \forall g \in G(\tilde{x}). \quad (18)$$

Now, let  $\phi$  be a solution to (7). As shown in (11),  $(t, j) \in \text{dom } \phi$  implies  $t \leq T_2(j+1)$ . Hence, for all  $T > 0$  such that  $t+j \geq T$ , one gets

$$j \geq \frac{T - T_2}{T_2 + 1}.$$

Therefore, applying [8, Proposition 3.24], for which, in this case,  $N_r = \frac{T_2}{T_2+1}$  and  $\gamma_r(T) = \frac{T}{T_2+1}$ , thanks to relations (17) and (18), the set  $\mathcal{A}$  defined in (9) is UGAS for system (7).  $\blacksquare$

*Remark 2:* Notice that assuming relation (12) to hold implies that the eigenvalues of  $e^{Av} (\mathbf{I} - LM)$  are strictly contained in the unit circle for every  $v$  belonging to  $[T_1, T_2]$ .

### B. Effect of measurement noise

Until now, the measured output was assumed to be perfectly known at sampling times  $t_k$ . However, in real situations, the measured output can be affected by a measurement noise. Hence, having some insight on the robustness of hybrid system (7) with respect to a bounded measurement noise is undoubtedly an interesting issue.

To this end, denoting the measurement noise as  $\eta : [0, +\infty) \rightarrow \delta\mathbb{B}$ , the measured output is defined by

$$y = Mx + \eta.$$

Then, the hybrid system (7) is rewritten as follows:

$$\mathcal{H}_\eta \left\{ \begin{array}{l} \dot{\varepsilon} = A\varepsilon \\ \dot{\tau} = -1 \end{array} \right\} (\varepsilon, \tau) \in C \quad (19a)$$

$$\mathcal{H}_\eta \left\{ \begin{array}{l} \varepsilon^+ = (\mathbf{I} - LM)\varepsilon - L\eta \\ \tau^+ \in [T_1, T_2] \end{array} \right\} (\varepsilon, \tau) \in D$$

with

$$C = \{(\varepsilon, \tau) \in \mathbb{R}^{n+1} : \tau \in [0, T_2]\}$$

$$D = \{(\varepsilon, \tau) \in \mathbb{R}^{n+1} : \tau = 0\}. \quad (19b)$$

Thus, the flow map remains defined as in (8a) while the jump map results in

$$\tilde{G}(\tilde{x}, \eta) = \begin{bmatrix} (\mathbf{I} - LM)\varepsilon - L\eta \\ [T_1, T_2] \end{bmatrix} \quad (20)$$

It seems natural in our setting to rest on the input-to-state-stability (ISS) concept introduced in [18] for continuous-time nonlinear systems and recently extended to hybrid systems in [4], [5]. Notice that, this extension of ISS to hybrid systems deals with hybrid signals as external perturbations, and for such class of signals, a suitable supremum norm is provided. However, in our case, the perturbation  $t \mapsto \eta(t)$  is a purely continuous-time signal, so it needs to be transformed to a hybrid signal to fit in the framework proposed by Cai and Teel. To this end, as shown in [16], given a solution  $\phi$  to  $\mathcal{H}_\eta$ , the signal  $t \mapsto \eta(t)$  can be represented as a hybrid signal  $\eta_{\mathcal{H}}$  defined as

$$\eta_{\mathcal{H}}(t, j) = \eta(t), \quad \forall (t, j) \in \text{dom } \phi. \quad (21)$$

Now, if for the hybrid signal  $\eta_{\mathcal{H}}$  we consider the hybrid supremum norm  $\|\eta_{\mathcal{H}}\|_{(t,j)}$  in [5], due to (21), it turns out that for such signal one has  $\|\eta_{\mathcal{H}}\|_{(t,j)} = \|\eta_{\mathcal{H}}\|_t$  for every  $(t, j) \in \text{dom } \phi$ . Then, the standard continuous-time supremum norm can be employed for the signal  $\eta_{\mathcal{H}}$ .

Now, the ISS approach leads to the following result, which extends Theorem 1 showing that condition (12) actually suffices to guarantee the ISS property for the hybrid system (19).

*Theorem 2:* Given two positive scalars  $T_1, T_2$  such that  $T_1 < T_2$ . If there exist a symmetric positive definite matrix  $P \in \mathbb{R}^{n \times n}$  and a matrix  $L \in \mathbb{R}^{q \times n}$  satisfying condition (12), then the set  $\mathcal{A}$  defined in (9) is ISS with respect to  $\eta$  for the hybrid system (19).

*Proof:* Consider the Lyapunov function defined in (13). Since the measurement noise  $\eta$  does not act on the flow map, as in the proof of Theorem 1, one gets

$$\langle \nabla V(\tilde{x}), f(\tilde{x}) \rangle = 0 \quad \forall \tilde{x} \in C. \quad (22)$$

For each  $g \in \tilde{G}(\tilde{x}, \eta)$  one gets

$$V(g) - V(\tilde{x}) = \varepsilon' \left( (\mathbf{I} - LM)' e^{A'v} P e^{Av} (\mathbf{I} - LM) + \right. \\ \left. - e^{A'\tau} P e^{A\tau} \right) \varepsilon - 2\eta' L' e^{A'v} P e^{Av} (\mathbf{I} - LM) \varepsilon + \\ + \eta' L' e^{A'v} P e^{Av} L \eta$$

where  $v$  is a real scalar belonging to the interval  $[T_1, T_2]$ . Whenever  $\tilde{x} \in D$ , from (19b), we have  $\tau = 0$ . Then, we get

$$V(g) - V(\tilde{x}) = \varepsilon' \left( (\mathbf{I} - LM)' e^{A'v} P e^{Av} (\mathbf{I} - LM) - P \right) \varepsilon \\ - 2\eta' L' e^{A'v} P e^{Av} (\mathbf{I} - LM) \varepsilon + \\ + \eta' L' e^{A'v} P e^{Av} L \eta \quad \forall g \in G(\tilde{x}, \eta), \forall \tilde{x} \in D. \quad (23)$$

Now, from (12), there exists a small enough positive real scalar  $\beta$  such that, for every  $v \in [T_1, T_2]$  and every  $\varepsilon$

$$\varepsilon' \left( (\mathbf{I} - LM)' e^{A'v} P e^{Av} (\mathbf{I} - LM) - P \right) \varepsilon \leq -\beta \varepsilon' \varepsilon.$$

By completing squares, one gets

$$V(g) - V(\tilde{x}) \leq -\frac{1}{2}\beta\varepsilon'\varepsilon + \frac{2}{\beta}\eta'\eta \left\| L'e^{A'v}P \left( \mathbf{I} + e^{Av}(\mathbf{I} - LM)(\mathbf{I} - LM)'e^{A'v}P \right) e^{Av}L \right\|. \quad (24)$$

Thanks to (12), as pointed out in Remark 2, one has  $\|e^{A'v}(\mathbf{I} - LM)\| < 1$  and then  $V(g) - V(\tilde{x}) \leq -\frac{1}{2}\beta\varepsilon'\varepsilon + \rho\|L\|^2\eta'\eta$ , where

$$\rho = \frac{2}{\beta}\|P\|(1 + \|P\|) \max_{v \in [T_1, T_2]} \left( \|e^{A'v}\|^2 \right).$$

The above relationship, together with (14), yields

$$V(g) \leq e^\theta V(\tilde{x}) + \|L\|^2 \rho \eta' \eta \quad \forall \tilde{x} \in D, \forall g \in G(\tilde{x}) \quad (25)$$

where  $\theta = \ln \left( 1 - \frac{\beta}{2\alpha_2} \right)$  and  $\alpha_2$  is defined in (16).

Then, from (25) and (22), and considering the definition of  $\eta_{\mathcal{H}}$  provided in (21), it turns out that given a solution  $\phi$  to hybrid system (19)

$$V(\phi(t, 0)) = V(\phi(0, 0)), \quad \forall t \in [0, t_1] \quad (26a)$$

$$\begin{aligned} V(\phi(t, j)) &\leq e^{\theta j} V(\phi(0, 0)) + \\ &\rho \|L\|^2 \sum_{i=0}^{j-1} e^{\theta i} \eta'_{\mathcal{H}}(t_i, i) \eta_{\mathcal{H}}(t_i, i), \quad \forall j \geq 1 \end{aligned} \quad (26b)$$

$$\forall (t, j) \in \text{dom } \phi.$$

Now since by definition  $\theta$  is negative  $\forall (t, j) \in \text{dom } \phi$  such that  $j \geq 1$  we have

$$V(\phi(t, j)) \leq e^{\theta j} V(\phi(0, 0)) + \frac{\rho \|L\|^2}{1 - e^\theta} \|\eta\|_t^2 \quad (27)$$

Moreover, being the input dependent term in the right-hand side of (27) non-negative, thanks to (26a), we have that (27) holds for every  $(t, j) \in \text{dom } \phi$  as well. By using (14), for every  $(t, j) \in \text{dom } \phi$  one gets

$$|\phi(t, j)|_{\mathcal{A}} \leq \frac{\alpha_2}{\alpha_1} e^{\theta j} |\phi(0, 0)|_{\mathcal{A}} + \frac{\rho \|L\|^2}{(1 - e^\theta)\alpha_1} \|\eta\|_t^2. \quad (28)$$

Thanks to relation (11) there exist two positive real scalars  $\gamma$  and  $R$  such that

$$\theta j \leq R - \gamma(t + j), \quad \forall (t, j) \in \text{dom } \phi \quad (29)$$

One has

$$\begin{aligned} |\phi(t, j)|_{\mathcal{A}} &\leq e^{-\gamma(t+j)} e^{R\frac{\alpha_2}{\alpha_1}} |\phi(0, 0)|_{\mathcal{A}} + \frac{\rho \|L\|^2}{(1 - e^\theta)\alpha_1} \|\eta\|_t^2 \\ &\forall (t, j) \in \text{dom } \phi \end{aligned} \quad (30)$$

then from (30) we have

$$\begin{aligned} |\phi(t, j)|_{\mathcal{A}} &\leq \max \left\{ 2e^{-\gamma(t+j)} e^{R\frac{\alpha_2}{\alpha_1}} |\phi(0, 0)|_{\mathcal{A}}, \right. \\ &\left. \frac{2\rho \|L\|^2}{(1 - e^\theta)\alpha_1} \|\eta\|_t^2 \right\}, \quad \forall (t, j) \in \text{dom } \phi. \end{aligned} \quad (31)$$

According to [5, Definition 2.3] the set  $\mathcal{A}$  is uniformly input-to-state stable with respect to  $\eta$  for the hybrid system (19). ■

*Remark 3:* The above analysis actually deals only with perturbations on the jump map. On the other hand, due to unmodeled dynamics, perturbations may affect also the flow map. Thus, analyzing the behavior of the hybrid system  $\mathcal{H}_\varepsilon$  in presence of a wider class of perturbation is a relevant matter. At this stage, one should notice that the way we adopted to model the hybrid system (7) leads to a hybrid system which is structurally robust with respect to bounded perturbations on the data. Namely, the hybrid system (7) is well-posed in the sense defined in [8, Definition 6.2]. Thus, the UGAS property of the set  $\mathcal{A}$  defined in (9) for the nominal system  $\mathcal{H}_\varepsilon$  holds (practically) the perturbed system as well. More specifically, provided that the set (9) is UGAS for the hybrid system  $\mathcal{H}_\varepsilon$ , then for each compact set  $\mathcal{M}$  of the state space and each  $\omega > 0$ , there exists a function  $\kappa \in \mathcal{KLL}$ , and a scalar  $\delta^* > 0$  such that for each  $\delta \in [0, \delta^*]$ , every solution  $\phi_p$  to the perturbed system  $\mathcal{H}_\varepsilon^p$  from  $\mathcal{M}$  satisfies, for all  $(t, j) \in \text{dom } \phi_p$ ,  $|\phi_p(t, j)|_{\mathcal{A}} \leq \kappa(\phi_p(t, j), t, j) + \omega$ . It is worthwhile to remark that getting a hybrid system exhibiting the above mentioned well-posedness property may not be trivial and it actually derives from suitable choices done throughout the modeling stage.

#### IV. NUMERICAL DESIGN PROCEDURE

In the previous section a condition to establish the UGAS and ISS properties, respectively, for systems (7) and (19) was provided. However, due to its form, such a condition is not computationally tractable to obtain a solution to Problem 1. Indeed, from a numerical standpoint, condition (12) has two drawbacks: it is not convex in  $P$  and  $L$ , and it needs to be verified for infinitely many values of  $v$ . The relevance of the second drawback is evident at a first sight, while the lack of convexity is a severe constraint, since non-convex problems often leads to NP-hard problems: see for example, [2]. Thus, in order to make the problem numerically tractable, some manipulations are needed. To this end, the following result provides a first step toward a convex design procedure for the proposed observer.

*Proposition 1:* Let  $T_1$  and  $T_2$  be two given positive scalars such that  $T_1 < T_2$ . If there exist a symmetric positive definite matrix  $P \in \mathbb{R}^{n \times n}$ , a matrix  $J \in \mathbb{R}^{q \times n}$ , and a matrix  $F \in \mathbb{R}^{n \times n}$  such that for every  $v \in [T_1, T_2]$

$$\begin{bmatrix} -\text{He}(F) & F - JM & e^{A'v}P \\ \star & -P & \mathbf{0} \\ \star & \star & -P \end{bmatrix} < \mathbf{0} \quad (32)$$

then the matrices  $P$  and  $L = F^{-1}J$  satisfy condition (12).

*Proof:* By defining  $\xi = \begin{bmatrix} \varepsilon^+ \\ \varepsilon \end{bmatrix}$ ,  $\bar{B} = [-\mathbf{I} \quad \mathbf{I} - LM]$  and  $Q = \begin{bmatrix} e^{A'v}Pe^{Av} & \mathbf{0} \\ \star & -P \end{bmatrix}$  the satisfaction of relation (12) is equivalent to

$$\xi'Q\xi < 0, \quad \forall \xi: \bar{B}\xi = \mathbf{0}, \quad \forall v \in [T_1, T_2]. \quad (33)$$

Then according to Finsler lemma (see [6]) (33) holds if and only if there exists a matrix  $\mathcal{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$  such that

$$Q + \mathcal{F}\bar{B} + \bar{B}'\mathcal{F} < \mathbf{0}, \quad \forall v \in [T_1, T_2]. \quad (34)$$

By setting  $F_1 = F$ ,  $F_2 = \mathbf{0}$  and labeling  $FL = J$ , one has

$$\begin{bmatrix} e^{A'v}Pe^{Av} - \text{He}(F) & F - JM \\ \star & -P \end{bmatrix} < \mathbf{0}, \quad \forall v \in [T_1, T_2]. \quad (35)$$

Finally, by Schur complement, one gets (32) and this concludes the proof. ■

*Remark 4:* Notice that condition (32) is convex with respect to the unknown matrices  $F$ ,  $L$ , and  $P$ .

To design effectively the observer, one needs to avoid finding a solution to (32) for infinitely many values of  $v$ . To overcome this problem, we provide a technique to embed the term  $e^{Av}$  whenever  $v$  belongs to the interval  $[T_1, T_2]$  in a convex set obtaining in this way a convex design procedure composed by a finite number of conditions. This technique consists in finding some matrices  $X_1, X_2, \dots, X_\nu \in \mathbb{R}^{n \times n}$ , such that  $e^{Av} \in \text{Co}\{X_1, X_2, \dots, X_\nu\}$  whenever  $v \in [T_1, T_2]$ . To this end, consider the following well known expression

$$\begin{aligned} e^{Av} &= \sum_{i=1}^{\sigma_r} \sum_{j=1}^{m_i^r} R_{ij} e^{\lambda_i v} \frac{v^{j-1}}{(j-1)!} + \\ &+ \sum_{i=1}^{\sigma_c} \sum_{j=1}^{m_i^c} 2e^{\Re(\lambda_i)v} \Re(e^{\Im(\lambda_i)v} R_{ij}) \frac{v^{j-1}}{(j-1)!} \end{aligned} \quad (36)$$

where  $\sigma_r$  is the number of distinct eigenvalues,  $\sigma_c$  the number of distinct complex-conjugates eigenvalues pairs.  $m_i$  and  $m_c$  are respectively the multiplicity of the real eigenvalue  $\lambda_i$  and of the complex-conjugates eigenvalues pair  $\lambda_i, \lambda_i^*$  in the minimal polynomial of the matrix  $A$ .  $R_{ij}$  are opportune real  $n \times n$  matrices. At this stage, note that the matrices  $R_{ij}$  are the residual associated to the partial fraction expansion of  $(sI - A)^{-1}$ . Notice that several methods can be adopted to compute such matrices. In this work, we rely on the procedure proposed in [11]. Once the value of the residual are known, to build a polytopic embedding of  $e^{Av}$  one can proceed in a similar manner of [9]. Namely,

$$\begin{aligned} \{X_1, \dots, X_\nu\} &= \left\{ \sum_{i=1}^{\sigma_r} \sum_{j=1}^{m_i^r} R_{ij} \beta_{ij} + \sum_{i=1}^{\sigma_c} \sum_{j=1}^{m_i^c} \gamma_{ij} + \gamma_{ij}^* : \right. \\ &\left. \beta_{ij} \in \{\overline{\beta_{ij}}, \underline{\beta_{ij}}\}, \gamma_{ij} \in \{\overline{\gamma_{ij}}, \underline{\gamma_{ij}}\}, \gamma_{ij}^* \in \{\overline{\gamma_{ij}^*}, \underline{\gamma_{ij}^*}\} \right\}, \end{aligned} \quad (37)$$



where

$$\begin{aligned}
\overline{\beta}_{ij} &= \max_{v \in [T_1, T_2]} e^{\lambda_i v} \frac{v^{j-1}}{(j-1)!} \\
\underline{\beta}_{ij} &= \min_{v \in [T_1, T_2]} e^{\lambda_i v} \frac{v^{j-1}}{(j-1)!} \\
\overline{\gamma}_{ij} &= \max_{v \in [T_1, T_2]} 2e^{\Re(\lambda_i)v} \cos(\Im(\lambda_i)v) \frac{v^{j-1}}{(j-1)!} \\
\underline{\gamma}_{ij} &= \min_{v \in [T_1, T_2]} 2e^{\Re(\lambda_i)v} \cos(\Im(\lambda_i)v) \frac{v^{j-1}}{(j-1)!} \\
\overline{\gamma}_{ij}^* &= \max_{v \in [T_1, T_2]} -2e^{\Re(\lambda_i)v} \sin(\Im(\lambda_i)v) \frac{v^{j-1}}{(j-1)!} \\
\underline{\gamma}_{ij}^* &= \min_{v \in [T_1, T_2]} -2e^{\Re(\lambda_i)v} \sin(\Im(\lambda_i)v) \frac{v^{j-1}}{(j-1)!}
\end{aligned} \tag{38}$$

The proposed technique leads to the following result.

*Corollary 1:* Let  $T_1$  and  $T_2$  be two given positive scalars such that  $T_1 < T_2$ . Let  $\{X_1, \dots, X_\nu\}$  be the matrices obtained by (37). If there exist a symmetric positive definite matrix  $P \in \mathbb{R}^{n \times n}$ , a matrix  $J \in \mathbb{R}^{q \times n}$ , and a matrix  $F \in \mathbb{R}^{n \times n}$  such that, for every  $i = 1, \dots, \nu$ ,

$$\begin{bmatrix} -\text{He}(F) & F - JM & X_i P \\ \star & -P & \mathbf{0} \\ \star & \star & -P \end{bmatrix} < \mathbf{0} \tag{39}$$

then the matrices  $P$  and  $L = F^{-1}J$  satisfy condition (12).

*Proof:* Since  $e^{Av} \in \text{Co}\{X_1, X_2, \dots, X_\nu\}$  whenever  $v \in [T_1, T_2]$ , then there exist  $\xi_1, \dots, \xi_\nu$  positive scalars dependent on  $v$ , such that

$$e^{Av} = \sum_{i=1}^{\nu} \xi_i(v) X_i, \quad \sum_{i=1}^{\nu} \xi_i = 1. \tag{40}$$

Then replacing in (32) the term  $e^{Av}$  with the expression given in (40) leads to

$$\begin{bmatrix} -\text{He}(F) & F - JM & \sum_{i=1}^{\nu} \xi_i(v) X_i' P \\ \star & -P & \mathbf{0} \\ \star & \star & -P \end{bmatrix} < \mathbf{0} \tag{41}$$

which, by the mean of the constraint on the  $\xi_i$  in (40), is equivalent to

$$\sum_{i=1}^{\nu} \xi_i(v) \begin{bmatrix} -\text{He}(F) & F - JM & X_i' P \\ \star & -P & \mathbf{0} \\ \star & \star & -P \end{bmatrix} < \mathbf{0}. \tag{42}$$

Hence by the virtue of (39) and Proposition 1 the matrices  $P$  and  $L = F^{-1}J$  satisfy condition (12) and this concludes the proof.  $\blacksquare$

The foregoing result represents an effective solution to Problem 1, which finally can be solved by the Algorithm 1 given below.

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#### Algorithm 1 Observer design

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- 1: Find the residual matrices  $R_{ij}$  in (36)
  - 2: Compute the scalars  $\overline{\beta}_{ij}, \underline{\beta}_{ij}, \overline{\gamma}_{ij}, \underline{\gamma}_{ij}, \overline{\gamma}_{ij}^*, \underline{\gamma}_{ij}^*$  as in (38)
  - 3: Compute the matrices  $\{X_1, \dots, X_\nu\}$  as in (37)
  - 4: Solve (39) with respect to  $J, P$  and  $H$
  - 5:  $L \leftarrow H^{-1}J$
  - 6: **return**  $L$
- 

#### V. ILLUSTRATIVE EXAMPLE

Consider the mass-spring system proposed by [7], which is defined by the following data:

$$\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 0 \\ 2 & -2 & 0 & -2 \end{bmatrix}, \quad M = [1 \ 0 \ 0 \ 0] \\
B' &= [0 \ 0 \ 1 \ 0]
\end{aligned} \tag{43}$$

consider also  $u(t) = \sin(t)$ . By fixing  $T_1 = 0.2$  and  $T_2 = 3$  Algorithm 1 yields to:

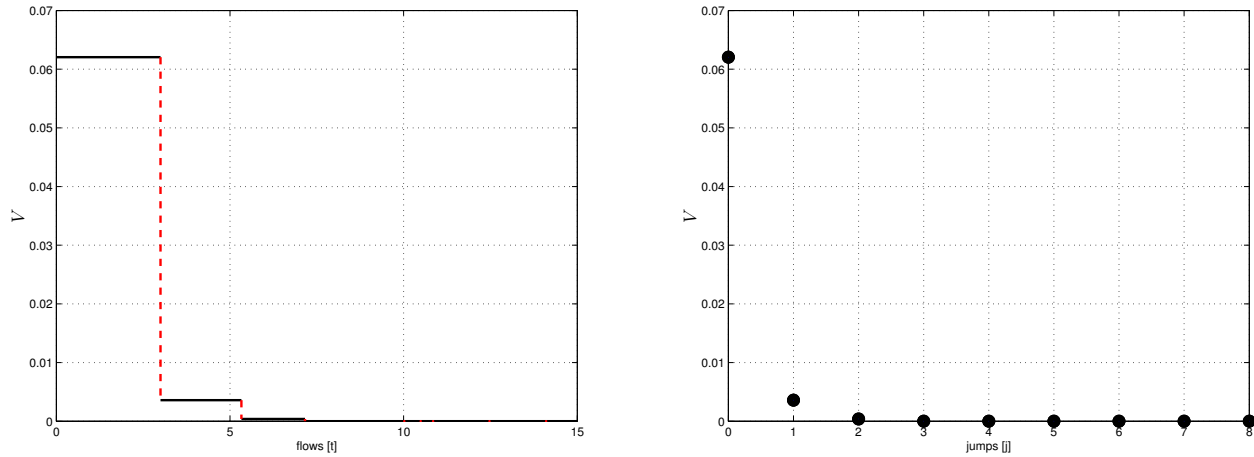
$$P = \begin{bmatrix} 0.1180 & 0.2460 & 0.1889 & 0.1491 \\ 0.2460 & 1.1788 & 1.0392 & 0.9646 \\ 0.1889 & 1.0392 & 0.9407 & 0.8778 \\ 0.1491 & 0.9646 & 0.8778 & 0.8328 \end{bmatrix}, \quad L = \begin{bmatrix} 1.0000 \\ -0.9433 \\ -0.6773 \\ 1.6274 \end{bmatrix}. \quad (44)$$

Figure 2 depicts the projection onto ordinary time  $t$  and onto jump time  $j$  of the Lyapunov function  $V(\phi(t, j))$ . Similarly, Figure 3 depicts the projection onto ordinary time  $t$  of the states  $z(t, j)$  and  $\hat{z}$ , in this simulation the sampling instants are selected randomly in the interval  $[T_1, T_2]$  according to a standard Gaussian distribution. Simulations show that the estimation appears to have a quick convergence toward the plant state  $z$ , namely the estimation and the state  $z$  are nearly overlapped after three jumps.

## VI. CONCLUSION

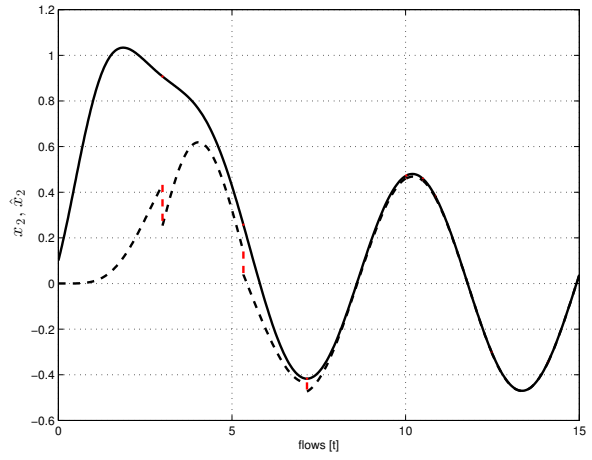
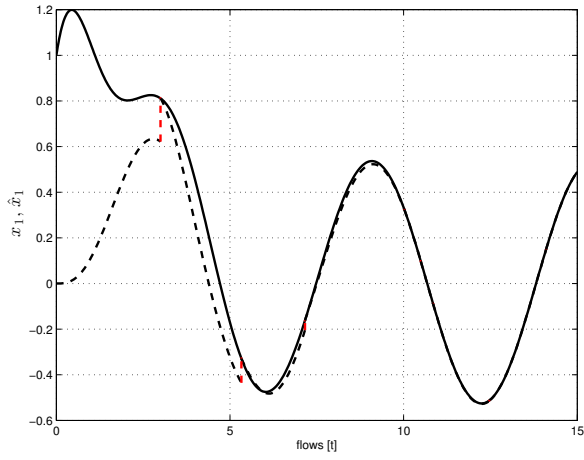
This paper proposed a methodology to model and design, through a convex setup, an event-triggered observer to estimate the state of a linear plant, whenever the output is measured in an impulsive fashion. Moreover, the proposed observer is shown to be ISS with respect to measurement noise and having a degree of robustness with respect to small enough bounded perturbations.

The results in this paper suggest several directions of research on event-triggered observers. For example, the setting allows to consider a design problem for the updating logic of  $\tau$ , in order to somehow schedule the sampling instants. Moreover, also the design of an observer-based controller in presence of impulsive output measurement represents certainly an interesting outlook.

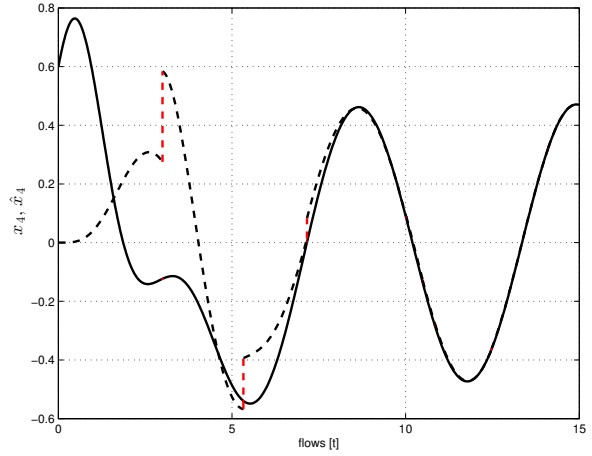
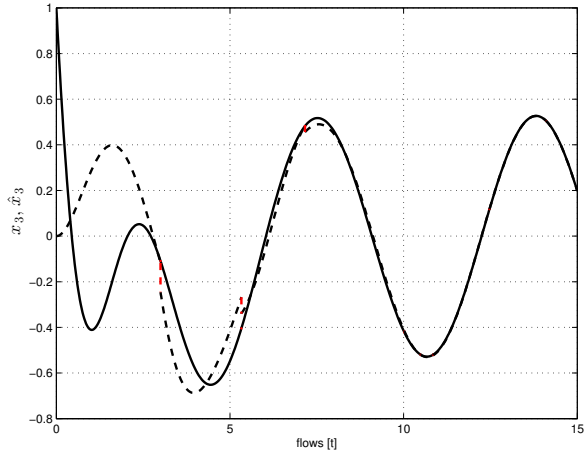


(a) Projection onto ordinary time  $t$  of the Lyapunov function  $V(\phi(t, j))$       (b) Projection onto jump time  $j$  of the Lyapunov function  $V(\phi(t, j))$

Fig. 2: The evolution of the Lyapunov function  $V(\phi(t, j))$ .



(a) Projection onto ordinary time  $t$  of  $z_1(t, j)$  (solid) and  $\hat{z}_1(t, j)$  (dashed). (b) Projection onto ordinary time  $t$  of  $z_2(t, j)$  (solid) and  $\hat{z}_2(t, j)$  (dashed).



(c) Projection onto ordinary time  $t$  of  $z_3(t, j)$  (solid) and  $\hat{z}_3(t, j)$  (dashed). (d) Projection onto ordinary time  $t$  of  $z_4(t, j)$  (solid) and  $\hat{z}_4(t, j)$  (dashed).

Fig. 3: The evolution of the states  $z$  and  $\hat{z}$  projected onto ordinary time  $t$ .

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