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SOCIAL NORMS AND INDIVIDUAL SAVINGS IN THE CONTEXT OF INFORMAL INSURANCE

ZAKI WAHHAJ

ABSTRACT. This paper develops a theory of informal insurance in the presence of an intertemporal technology. It is shown that when an insurance agreement suffers from enforcement problems, constraints on individual savings behaviour can enable the group to sustain greater cooperation. This result provides a motivation for a variety of social norms observed in traditional societies which discourage 'excessive' accumulation of wealth by individuals. The paper also shows that social norms that discourage savings are more likely to benefit poorer communities and thus, paradoxically, cause them to fall further behind even as it serves a useful purpose.

JEL Codes: D81, D91, O12, Z1

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UNIVERSITY OF OXFORD

1. INTRODUCTION

The role of culture in the process of economic development has long been a subject of interest to social scientists. In recent years, in parallel to new developments in research on formal institutions, there has also been renewed interest in the question of whether the varied economic paths of different societies through history can be traced to fundamental differences in their respective cultures, and the customs and social norms to which they give rise (see Platteau 2001 and Guiso, Sapienza and Zingales 2006 for surveys of this literature).

The discussion around social norms and economic development tends to distinguish between and concentrate on good and bad practices: on those that appear to encourage economic activity or provide an effective solution to a problem of social organisation and others that are unambiguously harmful for economic development. For example, Landes (1998) distinguishes between cultural factors such as thrift, hard work, tenacity, honesty and tolerance that promote economic development, in contrast to xenophobia and religious intolerance that stifle enterprise. Greif (1994) highlights the merits of individualist cultural beliefs as opposed to collectivist cultural beliefs for the formation of efficient agency relations in the context of medieval merchants, while Putnam (1993) attributes the greater success of modern political institutions in northern Italy to the pre-existence of a stronger civic culture.

This paper argues that a wide range of social norms may not easily fit into one or the other of these categories for a natural reason: where a cooperative agreement has to be self-enforcing, efficient norms take the form that ensures continued dependence of the individual on the service provided by the group, for it is precisely the need for this service which would induce the individual to keep his end of the bargain. In this manner, social norms appear both to constrain the economic behaviour of agents at the individual level and to promote cooperation at the level of the group. A practice that appears wasteful for an individual may be welfare-improving when the group and the surplus that it generates is taken into account.

The paper makes this argument formally in the context of self-enforcing mutual insurance agreements in an environment where agents have access to an intertemporal technology. In a self-enforcing contract, an agent is willing to assist another member of the insurance network today only to the extent he values the promise of insurance from the network

in the future. In the absence of an insurance network, own savings provide the agent an alternative means to smooth consumption in the face of adverse shocks. Therefore, increased saving on his part improves his ability to smooth consumption on his own, and thus lowers his need for the network. This means that if agents participating in an informal insurance network are discouraged from saving, greater demands can be made of them to assist a fellow member in need when such a need arises. Consequently, in the constrained efficient agreement, individuals save below the level that is individually optimal, and the group achieves a higher level of insurance than would otherwise be possible. This is the main insight of the paper.

Thus, we obtain a theoretical result which shows how social norms that constrain the accumulation of wealth within a society can be welfare improving. Interestingly, the anthropological literature provides a variety of examples of precisely such social norms. Platteau (2000: Chapter 5) provides a survey of studies on tribal societies where social beliefs that associate the accumulation of excessive wealth to manifestations of witchcraft serve as severe deterrents to wealth accumulation by individuals. In addition, the obligation of marking important events in a household – birth, marriage or death – through extravagant feasts is common to many traditional societies¹. The prevalence of such norms in traditional societies shows that it is often within the means of a social group to induce its members to engage in excessive consumption, as stipulated by the constrained efficient insurance agreement. This is not to argue that such social behaviour may have evolved specifically to ensure that households continue to depend on their social networks for insurance purposes. But more importantly, the paper provides a theoretical framework where it becomes apparent that such practices may have a positive effect on welfare, through their impact on group cooperation.

Interestingly, the type of social norms discussed here would tend to accentuate the differences between social groups that initially find it worthwhile to adopt them and those that do not. In the context of mutual insurance, we show that if there is some cost involved in forming an insurance network, it is more likely to arise in poorer communities where individuals cannot effectively smooth consumption using own assets. Richer communities fail to form insurance networks and therefore also have no reason to adopt the social constraints which facilitate informal insurance in the presence of enforcement problems. In the absence of such constraints, they can opt for a higher growth path of consumption

¹See, for example, Bloch, Rao and Desai (2004) for a discussion on excessive wedding expenditures in India and Manuh (1995) for a discussion on exorbitant funerals in West Africa.

and wealth accumulation, compared to their counterparts who start off poor and do find it worthwhile to adopt the norms that constrain savings. Thus, the initial differences in wealth which makes the relevant social norms useful among some social groups and not in others would widen over time.

Thanks to a number of significant contributions in this area in recent years, the theoretical characteristics of self-enforcing mutual insurance agreements and their implications for empirical work are well-understood today. However, for the sake of analytical simplicity, the literature has concentrated, for the most part, on an environment where no intertemporal technology is available to the agents. An important exception in this regard is Ligon, Thomas and Worrall (2000), who also consider mutual insurance in an environment where a savings technology is available. In particular, they show that in the constrained efficient agreement, the consumption stream satisfies a modified Euler equation involving an additional right-hand side term that depends on how saving affects the participation constraints. However, they do not provide any definite insights about the sign of this term and note that it is ambiguous.

The key theoretical insight of this paper is to relate this term to the premium that agents are willing to pay to participate in the insurance agreement. By doing so, we are able to show that if individual savings can be used for the purpose of self-insurance, then the sign of this term is negative and the constrained efficient contract prescribes a *lower* level of saving than the individual's preferred choice.² In other words, the constrained efficient agreement will discourage saving and encourage 'excessive' consumption. This result, in turn, has implications for social norms regarding savings behaviour in the context of informal insurance, as discussed above.

The remainder of this paper is organised as follows. Section 2 provides a more detailed survey of the theoretical and empirical literature on informal insurance. The model is presented in section 3, while sections 3.1 and 3.2 investigate self-enforcing insurance agreements and constrained efficient agreements respectively. Section 4 considers implications of the main results for communities that initially vary in the level of wealth, and section 5 concludes.

²To be precise, the 'lower level of saving' refers to a comparison between what the individual would like to save within a risk-sharing group, given the expectation of future transfers to cope with idiosyncratic shocks, and what the constrained efficient agreement would require him to do

2. RELATED LITERATURE

The idea of limited commitment as a basis of mutual insurance has been explored and developed extensively in the literature; it was first formalised in Kimball (1988) in the context of farm households in a rural community. Coate and Ravallion (1993) characterised the conditions under which the first-best insurance can be implemented under limited commitment. Kocherlakota (1996) provided a characterisation of constrained efficient agreements, and examined their long-run dynamics. Ligon, Thomas and Worrall (2002) showed that the constrained efficient agreements are characterised by a simple updating rule; specifically, that for each state of nature, there is a time-invariant interval for the ratio of marginal utilities; and in each period, this ratio adjusts by the smallest amount necessary to bring it into the current interval.

Fafchamps (1999) argued that the theoretical characteristics of informal insurance under limited commitment correspond closely to the empirical evidence on gift-giving and informal credit in rural societies. For example, Udry (1994) finds that the terms of repayment of informal credit in rural Nigeria is affected by both shocks to the creditor and the debtor; which corresponds to the characteristics of the constrained efficient insurance contract under limited commitment. Ligon, Thomas and Worrall (2002) test the limited commitment model using Indian village data and find that it can explain the consumption path of households more effectively than either the full insurance or the autarkic model.

The theoretical literature discussed thus far have generally assumed, for simplicity, that no intertemporal technology is available in the economy. An important exception to this literature is Ligon, Thomas and Worrall (2000), which investigates the properties of risk-sharing agreements in an environment where this assumption is relaxed. They show that in the constrained efficient agreement the consumption stream satisfies a modified Euler equation involving an additional right-hand side term that depends on how saving affects the participation constraints. However, they do not provide any definite insights about the sign of this term, and note that it is ambiguous. Ligon, Thomas and Worrall (2000) also provide an example where introduction of a savings technology diminishes welfare within a risk-sharing agreement.

The key theoretical insight in the current paper is to relate the additional term in the modified Euler equation to the premium that agents are willing to pay to participate in the insurance agreement. By doing so, we are able to show that if individual savings

can be used for the purpose of self-insurance, then the sign of this term is negative and the constrained efficient contract prescribes a *lower* level of saving than the individual's preferred choice. In other words, the constrained efficient agreement will discourage saving and encourage 'excessive' consumption.

Gobert and Poitevin (2006) study a related model where an agent who breaches a risk-sharing agreement also forfeits his savings. In this setting, saving functions like a collateral and always tends to slacken an agent's participation constraint. Therefore, in contrast to main result in this paper, the prescribed level of saving in a constrained efficient agreement is generally *higher* than that an individual would choose on his own.

The main result in this paper also has an interesting parallel in the literature on risk-sharing under private information. However, the mechanism at work in each case is quite different. Atkeson and Lucas (1992) study an environment where agents are subject to private idiosyncratic shocks and there is no scope for saving. They show that it is possible to induce agents to report their shocks truthfully by providing a 'front-loaded' consumption stream to those who report a bad shock in the current period, and a 'back-loaded' consumption stream to those who do not. Thus, some degree of risk-sharing is possible although shocks are unobserved. However, if agents have access to hidden storage, they would care only about the net present value of future transfers, not its timing. Then risk-sharing will break down since, under any insurance scheme, truthful reporting is no longer incentive-compatible (Cole and Kocherlakota 2001; see also Allen 1985). A social planner's ability to observe and constrain saving in this environment enables him to distinguish better between individuals with good and bad shocks for risk-sharing purposes³.

By contrast, we show in the present paper that, when no borrowing is possible, constraints on saving diminishes an agent's ability to self-insure and therefore improves his commitment to a risk-sharing agreement. In line with the recent literature, we believe that the full information, limited commitment model may be more pertinent to the study of risk-sharing in village economies where formal enforcement institutions are absent but individuals generally have good information on the assets and incomes of their neighbours.

³Yet another parallel exists for dynamic principle-agent models. Here again a distinct mechanism is at work. An agent's saving acts as a buffer against any future punishment the principle would inflict in the event of poor performance. Therefore, constraints on saving induces the agent to put in more effort in the future to avoid punishment (Rogerson 1985; Kocherlakota 2004; see also Golosov, Kocherlakota, Tsyvinski 2003 for a generalisation of the result in Rogerson 1985).

3. THE MODEL

We consider an environment with n agents, represented by the set $\mathcal{I} = \{1, \dots, n\}$. Each agent faces a stochastic income stream, denoted $\{y_t^i\}_{t=1,2,\dots,\infty}$. In each period, there are \mathcal{S} possible states of the world; and the probability of each state $s \in \mathcal{S}$ equals π_s , with $\sum_{s \in \mathcal{S}} \pi_s = 1$; i.e. the distribution of states is independantly and identically distributed over time. The income earned by person i when the realised state is s is denoted Y_s^i . Thus, if the state of the world in period t equals s , we have $y_t^i = Y_s^i$ for each $i \in \mathcal{I}$. Each agent also has access to an intertemporal technology whereby 1 unit of the good stored at the end of period t is transformed into σ units in period $t + 1$.

Individuals have the means to transfer some of their savings or income to each other in each period. The exact sequence of events within each period is as follows:

- (i) the state of nature is realised and each agent receives her output for the period, $\sigma k_{t-1}^i + y_t^i$ (where k_{t-1}^i denotes person i 's savings at the end of the preceding period);
- (ii) each agent i publicly pledges amounts $\tau_t^{ij} \geq 0$ to transfer to each agent $j \neq i$, and k_t^i to save at the end of the period;
- (iii) agents choose whether to ‘approve’ or ‘reject’ all pledges made; if pledges are universally approved and are ‘feasible’ then the transfers and savings take place as pledged. If not, no transfers take place and individuals have a second opportunity to allocate autarkic resources between consumption and saving.⁴

Given pledges, ‘feasibility’ means the following condition is satisfied for all agents:

$$(1) \quad \sigma k_{t-1}^i + y_t^i + \sum_{j \in \mathcal{I}} (\tau_t^{ji} - \tau_t^{ij}) \geq k_t^i$$

Thus, transfers and savings decisions are, in effect, collectively decided within the community. In particular, it is not possible for an agent to accept transfers from others in the community during a certain period, and then to choose a level of consumption that does not meet with their approval. This may be a reasonable approximation of reality

⁴Alternatively, we can assume that if one or more agents reject the pledges, the others implement an alternative scheme of transfers and savings that excludes the rejectors, while the rejectors revert to autarky. It will become evident in the next section that this would not affect the analysis of self-enforcing insurance agreements.

given that, in a village community, transfers and consumption would take place continuously through time, and the deviation by a community member from any consumption or saving rule, if publicly observed, can face immediate retaliation.⁵

Agent i 's preferences over different consumption streams are given by the following expression:

$$(2) \quad E \sum_{t=1}^{\infty} \beta^{t-1} u_i(c_t^i)$$

where $u_i(\cdot)$ is increasing and strictly concave. To ensure interior solutions for consumption, we also assume $\lim_{c \rightarrow 0} u_i'(c) = \infty$. Since $u_i(\cdot)$ is concave, agents prefer to smooth consumption across time and over different states of the world. They have two means of doing so; by engaging in precautionary saving using the intertemporal technology, and by participating in a mutual insurance agreement.

Before considering mutual insurance agreements, we introduce some terms and notation that will be used later in the analysis. Let $h_t = (s_1, s_2, \dots, s_t)$ denote a particular history of realised states up to and including period t . Let \mathcal{H}_t denote the set of all possible histories of states in period t . We define an *agreement* \mathcal{A} as follows:

Definition 3.1. *An agreement is a complete plan of all bilateral transfers made, as well as the level of savings chosen, by each agent, in each period, contingent on the history of states.*

According to this definition, an agreement can be described by $\mathcal{A} = \left\{ k_t^i(h_t), \{\tau_t^{ij}(h_t)\}_{j \neq i} \right\}$, $i \in \mathcal{I}, h_t \in \mathcal{H}_t, t = 1.. \infty$ ⁶. Such an agreement would specify that if the history of realised states in period t is h_t , then an agent i should make transfers $\tau_t^{ij}(h_t)$ to, and receive transfers $\tau_t^{ji}(h_t)$ from, each agent j during the period and have savings of $k_t^i(h_t)$ at the end of the period. We use the term $\tilde{\tau}_t^i(h_t)$ to denote *net* transfers by person i in period t following history h_t ; i.e.

$$(3) \quad \tilde{\tau}_t^i(h_t) = \sum_{j \neq i} (\tau_t^{ij}(h_t) - \tau_t^{ji}(h_t))$$

⁵This setup is equivalent to that adopted in Ligon, Thomas and Worrall (2000). An alternative possibility for modelling purposes would be for agents to make and accept transfers before savings decisions are made. We do not explore this route in this paper for it adds considerably to the complexity of the analytical problem.

⁶For the sake of legibility, we suppress the notation for the range of values of i, h_t, t whenever writing an agreement in this form hereafter.

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For the agreement to be feasible, we must have $\sum_{i \in \mathcal{I}} \tilde{\tau}_t^i(h_t) = 0, k_t^i(h_t) \geq 0$ for each period t and possible history h_t . An agreement implicitly defines a consumption stream if we assume that whatever assets are not saved at the end of a period will be consumed. Then the consumption stream $\{c_t^i(h_t)\}_{i \in \mathcal{I}, h_t \in \mathcal{H}_t, t=1.. \infty}$ is given by

$$(4) \quad c_t^i(h_t) = \sigma k_{t-1}^i(h_{t-1}) + Y_s^i - \tilde{\tau}_t^i(h_t) - k_t^i(h_t)$$

where s is the realised state in period t .

Note that two agreements that prescribe the same levels of savings and *net* transfers after each history also imply the same consumption streams and are identical for analytical purposes. Also note that an agreement is not equivalent to a complete strategy profile because the transfers and savings prescribed by the agreement are not contingent on the history of past actions. However, the prescriptions of an agreement are sufficiently detailed to address the question what utility levels can be sustained through mutual insurance in a subgame perfect equilibrium. Both these assertions will become apparent in the following section where we provide a characterisation of subgame perfect equilibria.

We denote by $\mathcal{A}|h_t$ the continuation agreement implied by \mathcal{A} after history h_t has been realised. Also let $\mathcal{A}_1|h_t$ denote the same continuation agreement except that all transfers in period t following history h_t equal to zero.. Define $V^i(z, \mathcal{A})$ as the utility that person i achieves in expectation with initial assets z when he participates in an agreement \mathcal{A} ; i.e.

$$(5) \quad V^i(z, \mathcal{A}) = E_0 u_i(z - \tilde{\tau}_1^i(\mathcal{A}|s) - k_1^i(\mathcal{A}|s)) \\ + E_0 \sum_{t=2}^{\infty} \beta^{t-1} u_i(\sigma k_{t-1}^i(\mathcal{A}|h_{t-1}) + y_t^i - \tilde{\tau}_t^i(\mathcal{A}|h_t) - k_t^i(\mathcal{A}|h_t))$$

where $k_t^i(\mathcal{A}|h_t)$ and $\tilde{\tau}_t^i(\mathcal{A}|h_t)$ represent the end-of-period savings of, and net transfers by, agent i as implied by agreement \mathcal{A} in period t , following history h_t .

Let $U_a^i(z)$ be the maximum expected utility that person i can achieve in autarky using initial assets z ; i.e.

$$(6) \quad U_a^i(z) = \max_{\{\tilde{k}_t^i(h_t)\}_{t=1}^{\infty}} E_0 u_i(z - \tilde{k}_1^i(s)) + E_0 \sum_{t=2}^{\infty} \beta^{t-1} u_i(\sigma \tilde{k}_t^i(h_t) + y_t^i - \tilde{k}_{t+1}^i(h_{t+1}))$$

subject to $\tilde{k}_t^i(h_t) \geq 0$ for $t = 1.. \infty, h_t \in \mathcal{H}_t$.

Let $I^i(z, \mathcal{A}_1|h_t)$ be the amount of money agent i would be willing to forego in the current period, given assets z , to participate in the continuation agreement $\mathcal{A}_1|h_t$. Then, assuming

$z > I^i(z, \mathcal{A}_1|h_t)$, we have that $I = I^i(z, \mathcal{A}_1|h_t)$ solves the following equation:

$$(7) \quad V^i(z - I, \mathcal{A}_1|h_t) = U_a^i(z)$$

The condition $z > I^i(z, \mathcal{A}_1|h_t)$ is necessary to ensure that the expression $V^i(z - I^i(\cdot), \mathcal{A}_1|h_t)$ is well-defined. The equation in (7) will frequently appear in our analysis since it will hold whenever the participation constraint is binding for person i for a particular allocation of resources. We will, in particular, be interested in the effect of additional saving by person i on a binding participation constraint, which is the subject of the following lemma.

Lemma 3.1. *Given a continuation agreement $\mathcal{A}|h_t$ if agent i has asset level $z > I^i(z, \mathcal{A}_1|h_t)$, and net transfers by agent i following history h_t equals $I^i(z, \mathcal{A}_1|h_t)$ in the agreement, then*

$$\frac{\partial V^i}{\partial z}(z, \mathcal{A}|h_t) - \frac{dU_a^i}{dz}(z) = -\frac{\partial V^i(z, \mathcal{A}|h_t)}{\partial \tilde{\tau}_t^i(\mathcal{A}|h_t)} \frac{\partial I^i(z, \mathcal{A}_1|h_t)}{\partial z}$$

Proof. We are given

$$\tilde{\tau}_t^i(\mathcal{A}|h_t) = I^i(z, \mathcal{A}_1|h_t)$$

Define $\mathcal{A}'(z)$ as an alternate agreement that corresponds to \mathcal{A} for $\tau = 1..\infty$, $h_\tau \in \mathcal{H}_\tau$ except that transfers following history h_t are a function of the parameter z : $\tau_t^i(\mathcal{A}'(z)|h_t) = I^i(z, \mathcal{A}_1|h_t)$, $\tau_t^{-i}(\mathcal{A}'(z)|h_t) = 0$. Therefore, by definition of $I^i(z, \mathcal{A}_1|h_t)$, we must have

$$V^i(z, \mathcal{A}'(z)|h_t) \equiv U_a^i(z)$$

Differentiating throughout with respect to z , we obtain

$$\begin{aligned} & \frac{dV^i}{dz}(z, \mathcal{A}'(z)|h_t) - \frac{dU_a^i}{dz}(z) \equiv 0 \\ \implies & \frac{\partial V^i}{\partial z}(z, \mathcal{A}'(z)|h_t) + \frac{\partial V^i}{\partial \tau_t^i(\cdot|h_t)}(z, \mathcal{A}'(z)|h_t) \frac{\partial I^i}{\partial z}(z, \mathcal{A}_1|h_t) - \frac{dU_a^i}{dz}(z) = 0 \end{aligned}$$

Note that we can replace $\mathcal{A}'(z)$ by \mathcal{A} since, as per the initial condition, the net transfers by person i following history h_t in the latter agreement equals $I^i(z, \mathcal{A}_1|h_t)$. Therefore, we can write

$$\frac{\partial V^i}{\partial z}(z, \mathcal{A}|h_t) - \frac{dU_a^i}{dz}(z) = -\frac{\partial V^i}{\partial \tau_t^i(\cdot|h_t)}(z, \mathcal{A}|h_t) \frac{\partial I^i}{\partial z}(z, \mathcal{A}_1|h_t)$$

□

Lemma 3.1 says that if, for a particular level of assets, an agreement provides an individual zero surplus over autarky, then the marginal effect on the surplus of an increase in assets equals the marginal change in the individual's valuation of the continuation agreement.

This change corresponds to the effect of increased saving on a binding participation constraint in a constrained efficient agreement. As will be seen, this technical result provides the crucial step of reasoning for the propositions that follow.

3.1. Self-Enforcing Agreements. In this section, we characterise agreements that can be supported in a subgame perfect equilibrium. These characterisations will be used later to analyse constrained efficient agreements. The results in this section are based on the techniques developed in Abreu (1988) and closely follows the reasoning behind a similar proposition in Kocherlakota (1996).

Proposition 3.1. *An agreement $\mathcal{A} = \{k_t^i(h_t), \{\tau_t^{ij}(h_t)\}_{j \neq i}\}$ and the associated consumption streams $\{c_t^i(h_t)\}$ can be obtained in a subgame perfect equilibrium if and only if it satisfies the following conditions:*

$$(8) \quad u_i(c_t^i(h_t)) + \beta E_t \sum_{\epsilon=t+1}^{\infty} u_i(c_\epsilon^i(h_\epsilon)) \geq U_a^i(\sigma k_{t-1}^i(h_{t-1}) + y_t^i) \\ i \in \mathcal{I}, \forall h_t \in H_t, t = 1.. \infty$$

where $c_t^i(h_t) = \sigma k_{t-1}^i(h_{t-1}) + y_t^i + \sum_{j \neq i} (\tau_t^{ji}(h_t) - \tau_t^{ij}(h_t)) - k_t^i(h_t)$.

The conditions ensure that for each person i , after each possible history, the expected utility obtained from the consumption path specified by the allocation is at least as large as the maximum expected utility that can be obtained under autarky. The reasoning behind the proposition is briefly sketched here. The proof can be found in Appendix A. If a particular allocation is obtained in a subgame perfect equilibrium, then it must satisfy the conditions above; if it does not hold for some individual after some history, she could improve her expected utility by deviating to the autarkic strategy in that subgame. Conversely, if an allocation satisfies the conditions specified above, then a strategy profile along the following lines would be subgame perfect: each individual, after each possible history, chooses transfers and savings as specified by the allocation; if any individual deviates at a point in time, then he receives no transfers thereafter. As the cost of deviation is autarky, which cannot be utility-improving by construction, this strategy profile is subgame perfect.

Note that the conditions under which an agreement is subgame perfect, and the level of welfare it provides to each agent, are fully determined by the consumption and saving levels implied by the agreement. The consumption levels, in turn, depends on the *net* transfers and savings made by each agent under each contingency, but not on the bilateral transfers among agents. Thus, in terms of constrained efficiency, two agreements that prescribe the same level of net transfers and savings, for each agent, under each contingency, are identical.

It should also be evident from Proposition 3.1 that the set of self-enforcing insurance agreements depends on the utility attained by someone who reneges on, or rejects, an agreement, but not on the utility received by other group members following such a rejection. In particular, if we assume that these other group members are able to implement an alternative risk-sharing arrangement which excludes the person who reneged on the original terms, this would not affect the set of self-enforcing agreements in any way.

3.2. Constrained Efficient Agreements. Among the allocations that can be obtained in a subgame perfect equilibrium, we concentrate, as in the related literature, on those that are constrained efficient; i.e. on allocations where agents exploit all the potential surplus that is available from mutual insurance in the face of the constraints imposed by their limited commitment.

Let $P(k^1, \dots, k^n, \underline{U}^1, \dots, \underline{U}^{n-1})$ be the maximum expected utility that agent n can obtain in a subgame perfect equilibrium, when the level of utility to be received by the other $n - 1$ agents are at least $\underline{U}^1, \dots, \underline{U}^{n-1}$, and the initial wealth levels of the n agents are given by k^1, \dots, k^n respectively. Then, using Proposition 3.1, we know that $P(k^1, \dots, k^n, \underline{U}^1, \dots, \underline{U}^{n-1})$ is given by the following programme:

$$(9) \quad P(k^1, \dots, k^n, \underline{U}^1, \dots, \underline{U}^{n-1}) = \max_{\{\tilde{\tau}_t^i(h_t), k_t^i(h_t)\}_{t=1.. \infty, i \in \mathcal{I}, h_t \in \mathcal{H}}} E_0 \sum_{t=1}^{\infty} \beta^{t-1} u_n(c_t^n(h_t))$$

$$(10) \quad \text{subject to } E_0 \sum_{t=1}^{\infty} \beta^{t-1} u_i(c_t^i(h_t)) \geq \underline{U}^i, \text{ for } i \neq n$$

$$(11) \quad \sum_{j \in \mathcal{I}} \tilde{\tau}_t^j(h_t) = 0 \text{ for } t = 1.. \infty \text{ and } \forall h_t \in \mathcal{H}_t$$

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and, for $t = 1..∞$, and $\forall i \in \mathcal{I}, \forall h_t \in \mathcal{H}_t$,

$$(12) \quad u_i(c_t^i(h_t)) + \beta E_t \sum_{\varepsilon=t+1}^{\infty} \beta^{\varepsilon-t-1} u_i(c_\varepsilon^i(h_\varepsilon)) \geq U_a^i(\sigma k_{t-1}^i(h_{t-1}) + y_t^i)$$

$$(13) \quad k_t^i(h_t) \geq 0$$

where $c_t^i(h_t) = \sigma k_{t-1}^i(h_{t-1}) + y_t^i - \tilde{\tau}_t^i(h_t) - k_t^i(h_t)$. The expression in (9) is optimised over the saving levels and net transfers only since, as mentioned in the previous section, these terms fully describe the consumption levels and outside options implied by an agreement following each history. The conditions in (10) ensure that each agent other than n receives his promised level of utility in the contract. The conditions in (12) are the participation constraints implied by Proposition (3.2) for self-enforcing agreements. The constraints in (13) says that agents cannot, at any time, hold negative assets, while (11) equates total transfers made and received within the community after each history. For each set of values $\underline{U}^1, \dots, \underline{U}^{n-1}$, the solution to this problem corresponds to a constrained efficient allocation. As noted in the previous literature, the fact that the maximisation problem includes a constraint for each possible history in each time period, and that the number of possible histories rise exponentially with each period makes it inconvenient to analyse constrained efficient allocations using such a programme.

Fortunately, there exists an alternative formulation of the maximisation problem, introduced by Thomas and Worrall (1988) and Kocherlakota (1996) that makes use of the fact that all continuation agreements of a constrained efficient agreement must also be constrained efficient⁷. Furthermore, given that all the constraints are forward-looking, and the game involves an infinite horizon, the problem of determining a constrained efficient continuation agreement is the same as the original problem whenever the asset levels and the promised utilities are the same. Therefore, the programme in (9) can also be written as a dynamic programming problem:

⁷The reasoning is provided by Ligon, Thomas & Worrall (2002). Consider a subgame perfect agreement starting at $t = 0$ that involves a continuation agreement, following some history h_t , that is not constrained efficient. Then there exists at least one other continuation agreement which is subgame perfect and Pareto superior. Replacing the original continuation agreement by the new agreement weakly relaxes the participation constraints and weakly raises expected utilities in the agreement starting at $t = 0$. Then the original agreement could not have been constrained efficient.

$$(14) \quad P(k^1, \dots, k^n, \underline{U}^1, \dots, \underline{U}^{n-1}) = \max_{\substack{\{\tilde{\tau}_s^i, k_s^i\}_{s \in \mathcal{S}, i \in \mathcal{I}} \\ \{U_s\}_{s \in \mathcal{S}, i \neq n}}} E[u_n(c_s^n) + \beta P(k_s^1, \dots, k_s^n, U_s^1, \dots, U_s^{n-1})]$$

subject to:

$$(15) \quad \lambda^i : E[u_i(c_s^i) + \beta U_s^i] \geq \underline{U}^i \text{ for } i \neq n$$

$$(16) \quad \pi_s \theta_s^i : u_i(c_s^i) + \beta U_s^i \geq U_a^i(\sigma k^i + Y_s^i) \text{ for } i \neq n$$

$$(17) \quad \pi_s \theta_s^n : u_n(c_s^n) + \beta P(k_s^1, \dots, k_s^n, U_s^1, \dots, U_s^{n-1}) \geq U_a^n(\sigma k^n + Y_s^n)$$

$$(18) \quad \pi_s \omega_s^i : k_s^i \geq 0 \text{ for } i \in \mathcal{I}$$

$$(19) \quad \zeta_s : \sum_{i \in \mathcal{I}} \tilde{\tau}_s^i = 0$$

for each $s \in \mathcal{S}$

where $c_s^i = \sigma k^i + Y_s^i - \tilde{\tau}_s^i - k_s^i$. Here, the conditions in (15) correspond to the promise keeping constraints in (10). The conditions in (16) and (17) ensure that the allocation of utilities in the first period of the agreement and the promised utilities hereafter satisfy the participation constraints for the first period. Unlike the dynamic programming problem presented in Ligon, Thomas and Worrall (2000) for a very similar problem on mutual insurance under limited commitment and a savings technology, the formulation used here involves a constraint set that is convex since the right-hand sides of the constraints are independent of the choice variables.⁸ Therefore, we can adopt the reasoning used by Ligon, Thomas and Worrall (2002) – a distinct paper where the problem involves a convex constraint set – to show that $P(\cdot)$ is decreasing and concave in each \underline{U}^i .⁹

⁸To see this, consider two feasible allocations \mathcal{A}_1 and \mathcal{A}_2 that generate consumption streams $\{c_1^i(h_t)\}$ and $\{c_2^i(h_t)\}$ and expected utilities $\{V_1^i\}$ and $\{V_2^i\}$ respectively. Any convex combination of these allocations would then produce consumption streams of the form $\{\alpha c_1^i(h_t) + (1 - \alpha) c_2^i(h_t)\}$ where $\alpha \in (0, 1)$. Since the per-period utility functions are concave, such a consumption stream would provide each agent i at least the level of utility $\alpha V_1^i + (1 - \alpha) V_2^i$. Then, since each of the original allocations satisfy the participation constraints for each possible history, so must be their convex combination. Therefore, the set of feasible allocations is convex.

⁹Consider two constrained efficient contracts \mathcal{A}_1 and \mathcal{A}_2 that award some agent i expected utilities equal to V_1^i and V_2^i respectively; while agent j receives V^j in each contract for $j = 1, 2, \dots, n-1, j \neq i$. Denote by $\{c_1^k(h_t)\}$ and $\{c_2^k(h_t)\}$, $k = 1, \dots, n$, the consumption streams corresponding to the two agreements. Any convex combination of these allocations would then produce consumption streams of the form $\{\alpha c_1^k(h_t) + (1 - \alpha) c_2^k(h_t)\}$, $k = 1, \dots, n$, where $\alpha \in (0, 1)$. Since the per-period utility functions are concave, such a consumption stream would provide agent i at least the level of utility $\alpha V_1^i + (1 - \alpha) V_2^i$, and to agent n a higher utility than the linear combination of the utilities obtained from \mathcal{A}_1 and \mathcal{A}_2 . Therefore, we have

$$P(k^1 \dots k^n, V^1 \dots \alpha V_1^i + (1 - \alpha) V_2^i \dots V^{n-1}) \geq \alpha P(k^1 \dots k^n, V^1 \dots V_1^i \dots V^{n-1}) + (1 - \alpha) P(k^1 \dots k^n, V^1 \dots V_2^i \dots V^{n-1})$$

Therefore, $P(\cdot)$ is concave in each V^i .

The basic characteristics of the constrained efficient agreement can be inferred from the first-order conditions and the Envelope equations of the problem. From the first-order condition with respect to transfers, we obtain the ratio of marginal utilities of consumption between any two agents in any state of nature:

$$(20) \quad \frac{u'_j(c_s^j)}{u'_i(c_s^i)} = \frac{(\lambda^i + \theta_s^i)}{(\lambda^j + \theta_s^j)} \text{ for } i, j = 1..n, s \in \mathcal{S}$$

where $\lambda^n = 1$. When the participation constraints of both agents are slack, $\theta_s^i = \theta_s^j = 0$, and therefore the ratio of marginal utilities is the same across all states where this condition holds. Whenever the constraint binds for one agent and not the other, the ratio shifts in favour of the first.

Using the Envelope theorem, we obtain

$$(21) \quad \frac{\partial P}{\partial U^i} = -\lambda^i, i = 1..n$$

This is the marginal rate of substitution between agents i and n . Since the function $P(\cdot)$ is concave in each U^i , we have λ^i increasing in U^i . Therefore, given the promised utilities for all agents except i and n , and the initial level of assets of each, λ^i uniquely identifies the promised utility to agent i , and a higher λ^i corresponds to a higher level of utility.

The first-order condition with respect to U_s^i yields

$$(22) \quad \lambda^i + \theta_s^i = -(1 + \theta_s^n) \frac{\partial P}{\partial U_s^i}$$

From (21), we can conclude that $\frac{\partial P}{\partial U_s^i} = -\lambda_s^i$ where λ_s^i is defined as the Lagrange multiplier on person i 's promise-keeping constraint one period into the future when the realised state is s . Substituting for $\frac{\partial P}{\partial U_s^i}$ into (22), we obtain

$$(23) \quad \lambda_s^i = \frac{\lambda^i + \theta_s^i}{1 + \theta_s^n}$$

This last equation shows how person i 's Pareto weight and therefore his promised utility evolves over time. Whenever his participation constraint binds, this tends to raise his promised utility since U_s^i is increasing in λ_s^i , and the opposite is true when the constraint is binding for some other agent. However, note that whether his utility actually rises or not also depends on the total assets available to the group in the new period.

From the first-order condition with respect to k_s^i , the Envelope condition with respect to k^i and Lemma 3.1, we obtain the following equation (the precise steps are shown in the proof of Proposition 3.2).

$$(24) \quad \frac{1}{\sigma} u'_i(c_s^i) = \beta E u'_i(c_{sr}^i) - \beta \sum_{r \in \mathcal{S}} \pi_r \frac{\theta_{sr}^i}{\lambda_s^i} \frac{\partial V^i(z, \mathcal{A}|h_t)}{\partial \tilde{\tau}_t^i(\mathcal{A}|h_t)} \frac{\partial I^i(z, \mathcal{A}_1|h_t)}{\partial z} + \frac{\omega_s^i}{\sigma \lambda_s^i (1 + \theta_s^n)}$$

where θ_{sr}^i is the Lagrange multiplier on person i 's participation constraint following the history of shocks (s, r) . Ignoring the second and third terms on the right-hand side, this equation is equivalent to the standard Euler condition which equates marginal utility in the current period to expected marginal utility in the following period after adjusting for the discount factor and the rate of return on capital. The third term on the right-hand side appears because of the non-negativity constraint on savings. As for the second-term, recall that $I^i(z, \mathcal{A}_1|h_t)$ is the payment that leaves an agent with wealth z indifferent between the continuation agreement \mathcal{A}_1 and autarky. Therefore the sign of the term $\frac{\partial I^i}{\partial z}$ depends on how an agent's valuation of the continuation agreement changes with wealth. Given that the term $\frac{\partial V^i}{\partial \tilde{\tau}_t^i}$ is always negative, if richer agents attach less value to an insurance agreement (i.e. $\frac{\partial I^i(z, \mathcal{A}_1|h_t)}{\partial z} < 0$), then the second term (including the minus sign in front of β) on the right-hand side of (24) will also be negative.

The precise interpretation of this term, based on Lemma 3.1, is that a marginal increase in saving by agent i in the first period tightens his participation constraint, to the extent that it makes him less willing to make insurance transfers in the second period. This suggests that saving is discouraged in the constrained efficient agreement compared with the standard Euler equation, which we would obtain both under first-best insurance and autarky. However, a precise statement regarding saving is complicated by the fact that the level of saving can also be affected by the extent of available insurance. Therefore, in order to make a precise statement about savings in the constrained efficient agreement, we introduce the concept of the *individually optimal level of saving* which is defined as follows.

Definition 3.2. *Given an agreement \mathcal{A} , the individually optimal level of saving for agent i in period t following history h_t is the solution to the following problem:*

$$(25) \quad \max_{x \geq 0} u_i(z_t^i(\mathcal{A}|h_t) - x) + \beta E_t V^i(\sigma x + y_{t+1}^i, \mathcal{A}|h_{t+1})$$

where $z_t^i(\mathcal{A}|h_t) = \sigma k_{t-1}^i(\mathcal{A}|h_{t-1}) - \tilde{\tau}_t^i(\mathcal{A}|h_{t-1}) + y_t^i$.

In words, the period t individually optimal level of saving for agent i is that which maximises his expected continuation utility assuming that transfers and savings decisions from period $t + 1$ onwards follow the prescriptions of the original agreement. We can think of

it as the level of saving that agent i would choose herself if she were given free reign to do so for one period following the occurrence of history h_t and provided with the guarantee that her choice of saving would have no impact on the subsequent agreement. Using this definition, we have the following proposition about savings in the constrained efficient agreement.

Proposition 3.2. *Suppose an agent's valuation of an insurance agreement is decreasing in wealth. Then, following any history, the agent's level of saving prescribed in a constrained efficient agreement is below that which is individually optimal, provided the latter is above zero and the agent's participation constraint binds in at least some state in the subsequent period. If the individually optimal level of saving following any history is equal to zero, so is the level prescribed in the agreement.*

Under what conditions is an agent's valuation of an insurance agreement decreasing in wealth? In the absence of a mutual insurance agreement, an agent can protect his own consumption against adverse income shocks by spending a part of his savings. However, a poorer agent is less disposed to do this for he runs a greater risk of running his stock completely dry through a succession of misfortunes, leaving him with no means to cope with future shocks. Therefore, in autarky, the rich would smooth consumption to a greater extent than the poor.¹⁰ Consequently, even if aversion to risk is constant in the level of wealth, an agent with a higher level of wealth places lower value on a mutual insurance agreement. This result would be further reinforced if the Bernoulli utility function exhibits decreasing absolute risk aversion. Note, however, that the condition in Proposition 3.2 involves an endogenous object: the continuation agreement implied by the constrained efficient agreement. Therefore, in Appendix B, we show formally, in the case of CARA utility, that the insurance premium for the continuation agreement is indeed decreasing in wealth.¹¹

¹⁰Note that in this statement, and in the subsequent discussion, the labels 'rich' and 'poor' refer to the wealth levels of individuals, and not to their stochastic income streams.

¹¹A sketch of the proof is provided here. In the case of CARA utility, the insurance premium is equal to the precautionary savings premium (henceforth called the EP property) for any agreement that provides contingent transfers but imposes no constraints on savings. To obtain this result, we start with the equivalent result in Kimball (1990) for the one-period case, and apply its reasoning iteratively. The insurance and precautionary savings premia for the finite-period model converge to their corresponding functions for the infinite-period model, and thus the EP property also holds for the infinite-period case. Next, consider a type of 'constrained optimal' agreement that need satisfy the participation constraints during the first m periods only, and the continuation agreement after m periods provides full insurance. Thus, if $m = 1$, then the continuation agreement from period 2 provides full insurance. The full insurance agreement prescribes the individually optimal level of saving, and therefore satisfies the EP property. It can be shown that if the EP property holds from period $t + 1$ onwards, it also holds from period t

Proposition 3.2 highlights the inherent tension between mutual insurance under lack of commitment and the ability to self-insure. Such an insurance network is most valuable for poor households that have few assets to smooth consumption on their own in the face of adverse shocks. The constrained efficient agreement would provide the household insurance against such shocks but also ensures that the household remains poor and thus dependent on the agreement so that it is, in turn, willing to provide assistance to others in the group as needed. This is done by requiring the household to save below the level that is individually optimal given its future stream of income and transfers. Thus, the agreement serves two functions: to enable poor households to smooth consumption and to ensure that they remain poor and therefore dependent on the agreement for consumption smoothing.

It is clear from the previous discussion that the level of assets of each individual or household is an important determinant of the extent of insurance that can be sustained in the community. The wealthier is a household, the better it is able to self-insure using its own assets, and therefore the less dependent it is on a social network that would provide support during hard times. This reasoning suggests that the poor are better insurance partners than the rich. We can verify, using the present theoretical framework, that this reasoning is indeed true.

Specifically, we consider the question how does replacing a poor agent in an insuring group by a richer agent affect the surplus from mutual insurance to the other $n - 1$ agents. Since there will, in general, be more than one agreement that is constrained efficient, each corresponding to a different division of the surplus, we need to make an assumption about how the surplus from insurance is divided within the group when the rich agent replaces the poor agent in the contract. To assume that the richer agent would receive the same *level* of surplus as the poor agent he replaces is not satisfactory: since the richer agent would place a lower value on any insurance agreement, providing him the same level of surplus would necessarily reduce the surplus available for the remaining agents.

onwards. Therefore, the EP property holds for $m = 1$, and consequently for all $m = 2, 3, \dots$ etc. Using the Contraction Mapping theorem, it can be shown that the sequence of such agreements ‘converge’ to the constrained efficient agreement. Therefore, the EP property is also satisfied for the constrained efficient agreement. This property is equivalent to the insurance premium being constant in wealth (Kimball, 1990). Finally, it can be shown that if the non-negativity constraint on saving is binding in period t in autarky, and the insurance premium from period t onwards is constant in wealth, then the insurance premium from period $t - 1$ onwards is decreasing in wealth.

We consider instead, as a benchmark, the case where the share of surplus for the rich agent in the new agreement is such that he is indifferent between the original agreement (that was offered to the poor agent) and the revised agreement. If this condition does not hold, it means that when a rich agent replaces a poor agent in an insurance agreement, the former must transfer a part of his extra wealth *unconditionally* to others in the insurance network. An agreement where the rich agent is required to do so may well be feasible, but, as a starting point, it is useful to consider the level of surplus from an insurance agreement to the other participants when the richer agent is not required to make such a transfer and thus may enjoy the full benefit of his extra wealth.

Using this assumption, we can show that at least one of the remaining agents participating in the insurance agreement is made worse off when the richer agent replaces the poorer household in the agreement, and receives a level of utility according to the rule above. Formally, we state this result as follows:

Proposition 3.3. *Suppose an agent's valuation of a constrained efficient insurance agreement is decreasing in wealth and his participation constraint binds in some state in the first period of the agreement. Then, increasing the initial level of assets of the agent, and raising his level of expected utility from the agreement sufficiently so that he does not prefer the old agreement to the new one, means that one or more of the remaining agents are worse off.¹²*

The intuition for this result is straightforward. Raising an agent's initial level of assets and his expected utility accordingly, tightens his participation constraint and also means that no one else can benefit from his increased wealth in the group. Then, one or more of the other agents must be worse off. If the rich agent is able to negotiate a higher level of surplus for himself than that assumed above, this will reinforce the result obtained in Proposition 3.3.

It is important to highlight here that the labels 'rich' and 'poor' refer to the wealth levels of individuals and not to their stochastic income streams. The comparative statics exercise described in Proposition 3.3 adjusts only the wealth level of an individual but keeps fixed his stochastic income stream. Nevertheless, the proposition does cover the case where

¹²Equivalently, lowering the initial level of assets of the agent, and lowering his level of expected utility from the agreement till the point where he is indifferent between the old and the new agreements, means that all of the remaining agents can be made better off.

assets are productive (this is equivalent to the condition $\sigma > 1$) and thus the ‘rich’ are able to generate more income, but this extra income is necessarily non-stochastic.

Note that the richer agent is himself receiving a smaller surplus under the assumptions made in Proposition 3.3. The reason is that although the new agreement provides him the same level of utility as he would have received from the agreement offered to the poorer individual, such an agreement is itself less valuable to him since he is richer and better able to smooth consumption in autarky. It follows that when we raise the starting level of wealth of any one agent in the community, there is no new agreement which provides every agent at least the level of surplus they received in the original agreement. This result also holds true when the first-best agreement can be implemented but it is reinforced if there are binding participation constraints after some history. Formally, we state the result in terms of a corollary to Proposition 3.3.

Corollary 3.4. *Given a constrained efficient insurance agreement, if agents’ valuation of the agreement is decreasing in wealth, then increasing the initial level of assets of any one agent means that one or more agents must receive a smaller surplus in the revised agreement.*

This result has important consequences for the formation of informal insurance groups. It means that any group of individuals (or households) would benefit more from including a poor individual in their insurance network than a rich individual, irrespective of their own wealth levels. If there are costs involved in developing an insurance network – say in gaining the trust of one’s neighbours and developing the rules and norms that would determine the obligations of the participants under different contingencies – then an individual in a poor community is more likely to find such an effort worthwhile, than an individual *with the same level of wealth* in a rich community. Reasoning thus, one may conclude that, *ceteris paribus*, insurance networks are more likely to develop in poorer communities. This idea is formalised in the next section.

4. DIVERGENCE OF COMMUNITIES WITH DIFFERING SOCIAL NORMS

In the preceding section, we learn, first, that in a constrained efficient mutual insurance agreement, an individual saves below the level that is individually optimal; and second, that the poor are better insurance partners than the rich. These two results together suggest that within a population where the means of consumption smoothing are limited

to informal insurance groups and the use of own assets, the rate of consumption growth should be higher for the rich versus the poor. With a very few extensions to the model, we can demonstrate this result formally.

Consider two communities, A and B, each consisting of n agents, situated sufficiently far apart that there is no scope of mutual insurance across the communities. At $t = 0$, the agents within each community decide whether or not to form an insurance group. Suppose that joining an insurance group involves a cost δ to an agent in both communities. From the Corollary to Proposition 3.3, it follows that if the initial level of assets in a community are sufficiently high and an agent's valuation of the constrained efficient insurance agreement is decreasing in wealth, then there is no feasible agreement which yields a surplus δ to each participating agent. Suppose that this is the case for community B, while agents in community A are poor enough that at least one such agreement is possible. Then, assuming that efficient bargaining can take place, an insurance network would develop in community A but not in B.

In community B, agents would then make consumption and savings decisions in autarky. Therefore, assuming positive savings, their consumption path would follow the standard Euler equation:

$$(26) \quad u'_j(c^j(t)) = \beta\sigma E u'_j(c^j(t+1))$$

Rearranging this equation, we obtain the expected rate of growth of consumption as measured by the ratio of marginal utilities between two successive time periods:

$$(27) \quad E \frac{u'_j(c^j(t+1))}{u'_j(c^j(t))} = \frac{1}{\beta\sigma}$$

In community A, if agents opt for an insurance agreement that is constrained efficient, then, as we have already seen, the consumption path would satisfy a modified Euler equation with an extra term on the right-hand side that is negative:

$$(28) \quad \frac{1}{\sigma} u'_i(c^i_s) = \beta E u'_i(c^i_{sr}) - \sum_{r \in \mathcal{S}} \frac{\theta_{sr}^i}{\lambda_s^i} \frac{\partial V^i(z, \mathcal{A}|h_t)}{\partial \tau_t^i(\mathcal{A}|h_t)} \frac{\partial I^i(z, \mathcal{A}_1|h_t)}{\partial z} + \frac{\omega^i}{\sigma \lambda_s^i (1 + \theta_s^n)}$$

Assume, as for community B, that the non-negativity constraint on saving is not binding. We obtain

$$(29) \quad E \frac{u'_i(c^i(t+1))}{u'_i(c^i(t))} \geq \frac{1}{\beta\sigma}$$

We have thus established the following result.

Proposition 4.1. *If participation in an insurance agreement carries a positive cost for an agent, and an agent's valuation of the constrained efficient insurance agreement is decreasing in wealth, then no mutual insurance agreement will develop in a community for sufficiently high levels of initial wealth. Whenever the non-negativity constraints on saving do not bind, the growth rate of consumption in this community, as measured by the expected period t ratio of marginal utilities of consumption between periods t and $t+1$, will be weakly greater than in a community where a constrained-efficient insurance agreement is adopted.*

Proposition 4.1 states, first, that if there are costs involved in setting up a mutual insurance agreement, then they will not appear in rich communities. The level of savings will be individually optimal, and the expected growth in consumption will be that implied by the standard Euler equation. By contrast, households in communities that are sufficiently poor will find it in their interest to put in the time and effort required to create a mutual insurance agreement. If they opt for the agreement that is constrained efficient, then households in these communities will weakly save below the level that is individually optimal, and correspondingly, they will experience a lower consumption growth path^{13,14}. The theoretical implication of this result is that the efficient 'social norm' that arises out of a need for mutual insurance reinforces the gap between communities where such a need initially exists and where it doesn't.

Applying this result to the real world should, however, be done with caution. It involves the implicit assumption that social norms that discourage saving arise specifically when and where it can lead to improved insurance within a group, a claim which is beyond the scope of this paper. Nevertheless, it is worthwhile to consider such a possibility for it carries an important implication for empirical analysis.

In a long-term comparison between two social groups, one where the social constraints are adopted and another where they are not, the negative effects of the constraints should be

¹³Note, however, that this comparison of consumption growth paths across communities is valid only if the non-negativity constraints on saving are not binding. If the individually optimal level of saving equals zero, then there are no additional disincentives to save in the constrained efficient agreement.

¹⁴Note also that the same reasoning would apply if, say, the process of group formation were such that only a subset of individuals in community B participated in the insurance agreement. That is to say, the result of slower consumption growth would still be valid for this subset of individuals.

Moreover, the result that no insurance agreement would emerge in a community that is sufficiently rich would hold as long as it is assumed that each participating agent should receive at least their autarkic utility and compensation for their cost of participation δ .

more readily observable than the positive ones. The constraints on individual behaviour imposed by the social norm would be reflected in the widening gap in consumption and wealth between the two communities. The fact that the norm also enables the poorer community to achieve greater consumption smoothing through insurance would be less apparent, for it is precisely because individuals in the richer community were able to achieve at least the same level of consumption smoothing in autarky that the same norms did not develop there. It would appear that the social norm acts, primarily, to impede economic development when, in fact, it also plays a positive role for the community.

5. CONCLUSION

A common assumption in the theoretical literature on informal insurance under lack of commitment is the absence of an intertemporal technology. This assumption may not be innocuous since, as pointed out by Besley (1995), an intertemporal technology enables an individual to engage in consumption smoothing outside of a group insurance contract. This paper attempts to fill this gap in the literature by investigating the interactions between savings behaviour and self-enforcing mutual insurance agreements when an intertemporal technology is available to agents.

The theoretical analysis shows that in the constrained efficient agreement, the accumulation of wealth by individuals will be constrained so that they remain dependent on the insurance network for consumption smoothing, and are therefore more willing to satisfy any future demands for assistance from within the group. This feature of the constrained efficient agreement have strong parallels with anthropological observations relating to social norms in traditional societies where a variety of cultural values and social beliefs serve to deter the accumulation of ‘excessive’ wealth by individuals.

This theoretical result should not be interpreted as providing a causal motive for a certain type of social norms in traditional societies. As noted by Banerjee (2002) and Greif (2006), a game-theoretic framework provides limited scope for positive analysis of social norms or institutions, given the usual presence of multiple equilibria. The exercise in this paper reveals not so much when social norms that deter savings by individuals will arise, but more significantly, that when they do, they will have a positive effect on the scope for group cooperation, an effect that will be missed in an economic analysis restricted to individual behaviour.

As mentioned in the introduction, the relation between self-enforcing mutual insurance agreements and savings behaviour investigated in this paper points to a more general argument which we reiterate here: where a cooperative agreement has to be self-enforcing, efficient norms take the form that ensures continued dependence of the individual on the service provided by the group for it is precisely the need for this service which would induce the individual to keep his end of the bargain.

This last statement suggests that the efficient social norm would tend to accentuate – or at least lead to persistence in – differences between social groups where the need for the service provided through group cooperation initially existed and where it did not. Proposition 4.1 makes this statement formally in the context of mutual insurance agreements. Herein lies the paradox of 'efficient' social norms: they enable the social group to come together to take care of an existing need among its members, and yet also encourage behaviour by individuals that would lead to the persistence of this need in the future.

6. APPENDIX A

Proof. of Proposition 3.1: First note that, following any history h_t , an individual who pledges zero transfers and the autarkic savings level in each period thereafter can guarantee himself the autarkic utility level $U_a^i(\sigma k_{t-1}^i(h_{t-1}) + y_t^i)$ in the continuation game. Therefore, if an allocation \mathcal{A} is obtained in a subgame perfect equilibrium, the conditions in (8) must be satisfied. If it is not satisfied in some period t , after history h_t for person i , then person i would obtain a higher utility by deviating to the autarkic strategy for the continuation game, which contradicts the definition of subgame perfection. Conversely, if an agreement \mathcal{A} satisfies the conditions in (8), then we can construct a strategy profile as follows. Each individual, after each possible history, pledges transfers and savings as specified in the agreement and approve of all pledges that correspond to this agreement, if all previous actions in the game follow this rule (the cooperation phase); after any deviation, each individual adopt autarkic strategies for the continuation game (the punishment phase). Then, the conditions in (8) ensure that, in the cooperation phase, a deviation in step (ii) of the stage game, when pledges are made, or in step (iii) of the game, when pledges are approved or rejected, cannot improve welfare. Likewise, in the punishment phase, a deviation cannot improve welfare because the autarkic strategies are subgame perfect. Therefore, the outlined strategy profile is also subgame perfect. \square

Proof. of Proposition 3.2: From the first-order condition with respect to k_s^i , we obtain

$$(30) \quad (1 + \theta_s^n) \beta \frac{\partial P}{\partial k_s^i} + \omega_s^i = (\lambda^i + \theta_s^i) u_i'(c_s^i)$$

Using the Envelope theorem, we obtain the following expression for $\frac{\partial P}{\partial k_s^i}$.

$$(31) \quad \frac{\partial P}{\partial k_s^i} = \sigma \sum_{r \in \mathcal{S}} \pi_r [\lambda_s^i u_i'(c_{sr}^i) + \theta_{sr}^i \{u_i'(c_{sr}^i) - U_a^{i'}(\sigma k_s^i + Y_s^i)\}]$$

where θ_{sr}^i is the Lagrange multiplier on person i 's participation constraint following the history of shocks (s, r) . Combining (30) and (31), and substituting for the term $\frac{1 + \theta_s^n}{\lambda^i + \theta_s^i}$ using (23), we obtain

$$(32) \quad \frac{1}{\sigma} u_i'(c_s^i) = \beta E u_i'(c_{sr}^i) + \beta \sum_{r \in \mathcal{S}} \pi_r \left[\frac{\theta_{sr}^i}{\lambda_s^i} \{u_i'(c_{sr}^i) - U_a^{i'}(\sigma k_s^i + Y_r^i)\} \right] + \frac{\omega_s^i}{\lambda_s^i \sigma (1 + \theta_s^n)}$$

Let \mathcal{A} be the constrained efficient agreement that corresponds to the solution to (14). Recall that the expression $\frac{\partial V^i}{\partial z}(z, \mathcal{A})$ represents the increase in utility to person i from a marginal increase in initial assets z , holding fixed all transfers and savings choices to the levels prescribed by \mathcal{A} . Using (5), we obtain

$$\frac{\partial V^i}{\partial z}(\sigma k_s^i + Y_r^i, \mathcal{A}(s, r)) = u_i'(c_2^i(\mathcal{A}(s, r))) = u_i'(c_{sr}^i)$$

Intuitively, an increase in assets following some history (s, r) must translate into an equivalent increase in consumption in that period, for savings and consumption levels in all subsequent periods are determined by the continuation agreement $\mathcal{A}(s, r)$. Therefore, we can rewrite (32) as

$$(33) \quad \frac{1}{\sigma} u_i'(c_s^i) = \beta E u_i'(c_{sr}^i) + \beta \sum_{r \in \mathcal{S}} \pi_r \left[\frac{\theta_{sr}^i}{\lambda_s^i} \left\{ \frac{\partial V^i}{\partial z}(\sigma k_s^i + Y_r^i, \mathcal{A}(s, r)) - \frac{dU_a^i}{dz}(\sigma k_s^i + Y_r^i) \right\} \right] + \frac{\omega_s^i}{\lambda_s^i \sigma (1 + \theta_s^n)}$$

Then, using Lemma 3.1, we obtain the result

$$(34) \quad \frac{\partial V^i}{\partial z}(z, \mathcal{A}(s, r)) - \frac{dU_a^i}{dz}(z) = - \frac{\partial V^i(z, \mathcal{A}(s, r))}{\partial \tilde{\tau}_t^i(\mathcal{A}(s, r))} \frac{\partial I^i(z, \mathcal{A}_1|_t(s, r))}{\partial z}$$

where $z = \sigma k_s^i + Y_r^i$. Then, substituting (34) into (33), we can write

$$(35) \quad \frac{1}{\sigma} u_i'(c_s^i) = \beta E u_i'(c_{sr}^i) - \beta \sum_{r \in \mathcal{S}} \pi_r \frac{\theta_{sr}^i}{\lambda_s^i} \frac{\partial V^i(z, \mathcal{A}(s, r))}{\partial \tilde{\tau}_t^i(\mathcal{A}(s, r))} \frac{\partial I^i(z, \mathcal{A}_1|_t(s, r))}{\partial z} + \frac{\omega_s^i}{\lambda_s^i \sigma (1 + \theta_s^n)}$$

If the value of the continuation agreement is decreasing in available assets z , then $\frac{\partial I^i(z, \mathcal{A}_1|_t(s, r))}{\partial z} < 0$. Moreover, $\frac{\partial V^i(z, \mathcal{A}(s, r))}{\partial \tilde{\tau}_t^i(\mathcal{A}(s, r))} < 0$. Therefore, if agent i 's participation constraint is binding after some history (s, r) , the second term on the right-hand side of (35) is negative.

Denote by x^* the individually optimal level of saving, as defined by the problem in (25). Let

$$\begin{aligned}\tilde{c}_s^i &= \sigma k^i + Y_s^i - \tilde{\tau}_s^i - x^* \\ \tilde{c}_{sr}^i &= \sigma x^* + Y_r^i - \tilde{\tau}_{sr}^i - k_{sr}^i\end{aligned}$$

If $x^* > 0$, we can show by contradiction that $k_s^i < x^*$. Suppose $k_s^i \geq x^*$. Then the non-negativity constraint on saving in the constrained efficient agreement is slack. Therefore, $\omega_s^i = 0$. Using the first-order condition to (25), we obtain

$$(36) \quad \frac{1}{\sigma} u_i'(c_s^i) = \beta E u_i'(\tilde{c}_{sr}^i)$$

Since $k_s^i \geq x^*$, we have $c_s^i \leq \tilde{c}_s^i$ and $c_{sr}^i \geq \tilde{c}_{sr}^i$. Therefore,

$$\frac{1}{\sigma} u_i'(c_s^i) \geq \beta E u_i'(\tilde{c}_{sr}^i)$$

Thus, assuming that agent i 's participation constraint binds following some history (s, r) , we obtain

$$\frac{1}{\sigma} u_i'(c_s^i) > \beta E u_i'(\tilde{c}_{sr}^i) - \beta \sum_{r \in \mathcal{S}} \pi_r \frac{\theta_{sr}^i}{\lambda_s^i} \frac{\partial V^i(z, \mathcal{A} | (s, r))}{\partial \tilde{\tau}_t^i(\mathcal{A} | (s, r))} \frac{\partial I^i(z, \mathcal{A}_1 | (s, r))}{\partial z} + \frac{\omega_s^i}{\lambda_s^i \sigma (1 + \theta_s^n)}$$

This last inequality contradicts the condition for constrained efficient agreements given in (35). Therefore, if $x^* > 0$, then $k_s^i < x^*$.

If $x^* = 0$, we can show by contradiction that $k_s^i = 0$. Since $x^* = 0$, we have

$$\frac{1}{\sigma} u_i'(c_s^i) \geq \beta E u_i'(\tilde{c}_{sr}^i)$$

If $k_s^i > 0$ then $c_s^i < \tilde{c}_s^i$ and $c_{sr}^i > \tilde{c}_{sr}^i$. Therefore,

$$\frac{1}{\sigma} u_i'(c_s^i) > \beta E u_i'(\tilde{c}_{sr}^i)$$

Also, if $k_s^i > 0$, then $\omega_s^i = 0$. Therefore, we have

$$\frac{1}{\sigma} u_i'(c_s^i) > \beta E u_i'(\tilde{c}_{sr}^i) - \beta \sum_{r \in \mathcal{S}} \pi_r \frac{\theta_{sr}^i}{\lambda_s^i} \frac{\partial V^i(z, \mathcal{A} | (s, r))}{\partial \tilde{\tau}_t^i(\mathcal{A} | (s, r))} \frac{\partial I^i(z, \mathcal{A}_1 | (s, r))}{\partial z} + \frac{\omega_s^i}{\lambda_s^i \sigma (1 + \theta_s^n)}$$

which, once again, contradicts the condition in (35). Therefore, if $x^* = 0$, then $k_s^i = 0$. \square

Proof. of Proposition 3.3: Consider the constrained efficient agreement that provides agents $1..(n-1)$ utility levels $\underline{U}^1, \dots, \underline{U}^{n-1}$ given initial wealth levels k^1, \dots, k^n . The utility available to agent n then equals $\underline{U}^n = P(k^1, \dots, k^n, \underline{U}^1, \dots, \underline{U}^{n-1})$. We consider the effect on the constrained efficient agreement of a small increase δ in the initial wealth level of some agent $i \neq n$ when agent i is awarded the same utility in the new agreement as he

would have obtained from the old agreement at his now higher wealth level. Denote by $\tilde{U}^i(\delta)$ the target utility level for agent i . We have

$$\tilde{U}^i(\delta) = EV^i(\sigma(k^i + \delta) + Y_s^i, \mathcal{A})$$

where \mathcal{A} is the original agreement. Then the increase in utility to agent i from the agreement following a marginal increase in wealth equals

$$\begin{aligned} \tilde{U}^i(0) &= \frac{d}{dk^i} EV^i(\sigma k^i + Y_s^i, \mathcal{A}) \\ &= \sigma E \frac{\partial V^i}{\partial z}(\sigma k^i + Y_s^i, \mathcal{A}) \\ &= \sigma E u'(c_s^i) \end{aligned}$$

where $\frac{\partial V^i}{\partial z}$ is the partial derivative of $V^i(\cdot)$ with respect to its first argument. The effect of a marginal increase in the initial wealth of agent i on the utility of agent n from the constrained efficient agreement, keeping fixed the utility of all agents other than i and n , equals

$$\frac{dP}{dk^i}(k^1, \dots, k^n, \underline{U}^1, \dots, \tilde{U}^i(0), \dots, \underline{U}^n) = \frac{\partial P}{\partial k^i} + \frac{\partial P}{\partial \underline{U}^i} \tilde{U}^{i'}(0)$$

From previous analysis, we know that

$$\begin{aligned} \frac{\partial P}{\partial k^i} &= \sigma \sum_{s \in \mathcal{S}} \pi_s \left[\lambda^i u'_i(c_s^i) + \theta_s^i \left\{ \frac{\partial V^i}{\partial z}(\sigma k^i + Y_s^i, \mathcal{A}) - \frac{dU_a^i}{dz}(\sigma k^i + Y_s^i) \right\} \right] \\ \frac{\partial P}{\partial \underline{U}^i} &= -\lambda^i \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} \frac{\partial P}{\partial k^i} + \frac{\partial P}{\partial \underline{U}^i} \tilde{U}^{i'}(0) &= \sigma \sum_{s \in \mathcal{S}} \pi_s \left[\lambda^i u'_i(c_s^i) + \theta_s^i \left\{ \frac{\partial V^i}{\partial z}(\sigma k^i + Y_s^i, \mathcal{A}) - \frac{dU_a^i}{dz}(\sigma k^i + Y_s^i) \right\} \right] \\ &\quad - \lambda^i \sigma \sum_{s \in \mathcal{S}} \pi_s u'(c_s^i) \\ &= \sigma \sum_{s \in \mathcal{S}} \pi_s \theta_s^i \left[\frac{\partial V^i}{\partial z}(\sigma k^i + Y_s^i, \mathcal{A}) - \frac{dU_a^i}{dz}(\sigma k^i + Y_s^i) \right] \end{aligned}$$

Using Lemma 3.1, we obtain the expression

$$(37) \quad -\sigma \sum_{s \in \mathcal{S}} \pi_s \theta_s^i \frac{\partial V^i(\sigma k^i + Y_s^i, \mathcal{A})}{\partial \tau_t^i(\mathcal{A})} \frac{\partial I^i(\sigma k^i + Y_s^i, \mathcal{A}_1)}{\partial z}$$

If the valuation of the insurance agreement is decreasing in wealth and agent i 's participation constraint binds in some state s , then this expression is negative. Therefore, increasing the initial wealth of agent i , providing him the compensation described above, and holding fixed utility of all agents other than i and n leads to a decrease in the level

of utility for agent n in the agreement. Therefore, at least one agent must be worse off in the agreement following an increase in the initial wealth of some agent and an increase in utility that leaves him indifferent between the old and new agreements. \square

Proof. of Corollary to Proposition 3.3: First, consider the case where some agent i 's participation constraint binds in some state in the first period of the agreement. By Proposition 3.3, raising the initial wealth of agent i and offering a revised agreement that provides him the level of utility at which he is indifferent between the revised and old agreements would make at least one of the remaining agents worse off. It follows that at least one of the remaining agents would receive a smaller surplus than in the original agreement. The effect on the surplus of agent i in adopting the new agreement equals

$$\begin{aligned} & \frac{d}{dk^i} [EV^i(\sigma k^i + Y_s^i, \mathcal{A}) - EU^i(\sigma k^i + Y_s^i)] \\ &= E \left[\frac{d}{dk^i} \{V^i(\sigma k^i + Y_s^i, \mathcal{A}) - U^i(\sigma k^i + Y_s^i)\} \right] \end{aligned}$$

Applying Lemma 3.1 to substitute for the term within the curly brackets, we obtain an expression that is negative. Therefore, agent i receives a smaller surplus in the new agreement. Since the new agreement is constrained efficient, this implies that, following the increase in wealth of agent i , there is no feasible agreement in which each agent receives the same surplus as they had received in the original agreement.

Second, consider the case where agent i 's participation constraint is slack in all states s in the first period. Then the expression in (37) is equal to zero. Therefore, following an increase in the wealth of agent i , a revised agreement which awards him the level of utility at which he is indifferent between the revised and old agreements, and holds fixed the utility levels of all agents other than i and n , also leaves unchanged the utility received by agent n . Therefore, the revised agreement also leaves unchanged the surplus above autarky received by all agents other than i . However, as shown above, if agent i 's valuation of an insurance agreement is decreasing in wealth, then he receives a smaller surplus in the revised agreement. Since this agreement is constrained efficient, it follows that there is no feasible agreement in which each agent receives the same surplus as they had received in the original agreement. \square

7. APPENDIX B

In this section, we shall show that if the Bernoulli utility function takes a CARA form, then $\frac{\partial U^i}{\partial z}(z, \mathcal{A}) < 0$, where \mathcal{A} is a constrained efficient agreement, assuming that the non-negativity constraint on saving binds in autarky following some history, and the corresponding constraint never binds under the agreement.

For this purpose, we shall make use of the Contract Mapping Theorem and, in particular, Blackwell's sufficient conditions for a contraction (Stokey, Lucas and Prescott, 1989, Chapter 3.2). The latter theorem applies to the space of bounded functions. To ensure that the frontier of possible utilities under various agreements may be represented by bounded functions, we redefine the intertemporal technology in this section as follows: Each agent has access to an intertemporal technology whereby k units of the good stored at the end of period t is transformed into $f(k)$ units in period $t + 1$. We assume that f is continuously differentiable, $f(0) = 0$, $f'(k) > 0$ for $k \in [0, \bar{k}]$, $f''(k) \leq 0$ and $f(k) \leq f(\bar{k})$ for $k > \bar{k}$. Thus, \bar{k} is effectively an upper limit on saving. Note that \bar{k} may be arbitrarily large; but as long as such a bound exists, it ensures that a function describing the frontier of constrained efficient agreements is also bounded. This will be apparent below. It is straightforward to verify that Propositions 3.1-3.3 presented in the main section of the paper will continue to hold with this modified savings technology (whether or not the upper limit \bar{k} is chosen in equilibrium). In addition, the result in Proposition 4.1 would continue to hold if the rate of return on saving are similar for the levels of saving chosen in the two communities considered.

First we show that this property holds for an agreement which prescribes the *individually optimal level of saving* (as defined in Definition 3.2) in each period. Let \mathbf{A}_{IO} be the set of all such agreements.

We define the precautionary savings premium, $\psi^i(z, \mathcal{A})$, implicitly as follows:

$$(38) \quad \frac{dU_a^i(z)}{dz} \equiv \frac{\partial V^i}{\partial z}(z - \psi^i(z, \mathcal{A}), \mathcal{A})$$

In words, the precautionary savings premium equals, corresponding to the use of the term by Kimball (1990), the deduction in initial wealth for which the individual would have the same marginal utility of money under the agreement as under autarky.

We also define the following functions to represent the utility levels attained in autarky or from an agreement $\mathcal{A} \in \mathbf{A}_{IO}$ over a finite horizon:¹⁵

$$\begin{aligned} U_\tau(z) &= \max_{(c_t, k_t)_{t=1}^\tau} E \sum_{t=1}^{\tau} \beta^{t-1} u_i(c_t) \\ \text{s.t.} \quad &: 0 \leq c_t(h_t) \leq f(k_{t-1}(h_{t-1})) + y_t^i \\ &: k_t(h_t) \geq 0 \end{aligned}$$

$$\begin{aligned} V_\tau(z, \mathcal{A}) &= \max_{(c_t, k_t)_{t=1}^\tau} E \sum_{t=1}^{\tau} \beta^{t-1} u_i(c_t) \\ \text{s.t.} \quad &: 0 \leq c_t(h_t) \leq f(k_{t-1}(h_{t-1})) + y_t^i - \tilde{\tau}_t^i(\mathcal{A}|h_t) \\ &: k_t(h_t) \geq 0 \end{aligned}$$

Corresponding to the functions $U_\tau(z)$ and $V_\tau(z, \mathcal{A})$, we define the insurance premium and the precautionary savings premium over a finite horizon as follows:

$$\begin{aligned} U_\tau(z) &\equiv V_\tau(z - I_\tau(z, \mathcal{A}), \mathcal{A}) \\ U'_\tau(z) &\equiv \frac{\partial V_\tau}{\partial z}(z - \psi_\tau(z, \mathcal{A}), \mathcal{A}) \end{aligned}$$

We begin by establishing a relationship between the insurance premium and the precautionary savings premium over a one-period horizon:

Lemma 7.1. *If $u_i(\cdot)$ has a CARA form, i.e. $u_i(x) = -\exp(-rx)$, then $I_1(z, \mathcal{A}) = \psi_1(z, \mathcal{A})$ for any agreement \mathcal{A} over a one-period horizon.*

Proof. $I_1(z, \mathcal{A})$ and $\psi_1(z, \mathcal{A})$ satisfy the following identities:

$$\begin{aligned} E u_i(z + Y_s^i - I_1(z, \mathcal{A})) &\equiv E u_i(z + Y_s^i + \tau_1(\mathcal{A}|s)) \\ E u'_i(z + Y_s^i - \psi_1(z, \mathcal{A})) &\equiv E u'_i(z + Y_s^i + \tau_1(\mathcal{A}|s)) \end{aligned}$$

Therefore, if $u_i(x) = -\exp(-rx)$, then we obtain

$$\begin{aligned} -E \exp(-r(z + Y_s^i - I_1(z, \mathcal{A}))) &\equiv -E \exp(-r(z + Y_s^i + \tau_1(\mathcal{A}|s))) \\ E \exp(-r(z + Y_s^i - \psi_1(z, \mathcal{A}))) &\equiv E \exp(-r(z + Y_s^i + \tau_1(\mathcal{A}|s))) \\ \implies E \exp(-r(z + Y_s^i - I_1(z, \mathcal{A}))) &= E \exp(-r(z + Y_s^i - \psi_1(z, \mathcal{A}))) \end{aligned}$$

¹⁵Note that although the functions $U_\tau(\cdot)$ and $V_\tau(\cdot)$ pertain to individual i , we have omitted to include the superscript in the notation for the sake of legibility in the subsequent exposition.

Therefore, $I_1(z, \mathcal{A}) = \psi_1(z, \mathcal{A})$ where \mathcal{A} can be *any* agreement. \square

Next, we can show, iteratively, that the insurance premium will be smaller than or equal to the precautionary savings premium for any agreement $\mathcal{A} \in \mathbf{A}_{IO}$ in which the non-negativity constraint on saving does not bind following any history. Let \mathcal{A}_s be the continuation agreement implied by \mathcal{A} if the realised state in the first period is s .

Lemma 7.2. *If $I_{\tau-1}(z, \mathcal{A}_s) \leq \psi_{\tau-1}(z, \mathcal{A}_s)$ for each $s \in \mathcal{S}$ and the non-negativity constraint on saving does not bind for agreement \mathcal{A} following any history, then $I_\tau(z, \mathcal{A}) \leq \psi_\tau(z, \mathcal{A})$.*

Proof. By construction,

$$(39) \quad U_\tau(z) = \max_{0 \leq c \leq z} u_i(c) + \beta EU_{\tau-1}(f(z-c) + Y_s^i)$$

$$(40) \quad V_\tau(z, \mathcal{A}) = \max_{0 \leq c \leq z} u_i(c) + \beta EV_{\tau-1}(f(z-c) + Y_s^i, \mathcal{A}_s)$$

Denote by $\hat{c}(z)$ and $c(z; \mathcal{A})$, the solutions to the maximisation problems in (39) and (40) respectively. From the first-order conditions,

$$(41) \quad u'_i(c(z; \mathcal{A})) = \beta f'(z - c(z; \mathcal{A})) EV'_{\tau-1}(f(z - c(z; \mathcal{A})) + Y_s^i, \mathcal{A}_s)$$

$$(42) \quad u'_i(\hat{c}(z)) \geq \beta f'(z - \hat{c}(z)) EU'_{\tau-1}(f(z - \hat{c}(z)) + Y_s^i)$$

We obtain (41) using the assumption that the non-negativity constraint on saving does not bind for agreement \mathcal{A} following any history. Moreover, from the definition of the insurance premium and the precautionary savings premium, we have

$$(43) \quad u_i(c(\bar{z}; \mathcal{A})) + \beta EV_{\tau-1}(f(\bar{z} - c(\bar{z}; \mathcal{A})) + Y_s^i, \mathcal{A}_s) = u_i(\hat{c}(z)) + \beta EU_{\tau-1}(f(z - \hat{c}(z)) + Y_s^i)$$

$$(44) \quad U'_{\tau-1}(f(z - \hat{c}(z)) + Y_s^i) = V'_{\tau-1}(f(z - \hat{c}(z)) - \psi_{\tau-1}(z, \mathcal{A}_s) + Y_s^i, \mathcal{A}_s)$$

where $\bar{z} = z - I_\tau(z, \mathcal{A})$. Let

$$p = f[z - \hat{c}(z)] - f[\bar{z} - c(\bar{z}; \mathcal{A})]$$

We consider the possible values of p :

- (1) Suppose $p < I_{\tau-1}(z, \mathcal{A}_s) \leq \psi_{\tau-1}(z, \mathcal{A}_s)$.

$$\text{Then } EV'_{\tau-1} < EU'_{\tau-1} \implies u'(c(\bar{z}); \mathcal{A}) < u'(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) > \hat{c}(z)$$

$$\text{But } EV_{\tau-1} > EU_{\tau-1} \implies u(c(\bar{z}); \mathcal{A}) < u(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) < \hat{c}(z) \implies$$

Contradiction.

- (2) Suppose $I_{\tau-1}(z, \mathcal{A}_s) = p < \psi_{\tau-1}(z, \mathcal{A}_s)$.
 Then $EV'_{\tau-1} < EU'_{\tau-1} \implies u'(c(\bar{z}); \mathcal{A}) < u'(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) > \hat{c}(z)$
 But $EV_{\tau-1} = EU_{\tau-1} \implies u(c(\bar{z}); \mathcal{A}) = u(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) = \hat{c}(z) \implies$
 Contradiction.
- (3) Suppose $I_{\tau-1}(z, \mathcal{A}_s) < p < \psi_{\tau-1}(z, \mathcal{A}_s)$.
 Then $EV'_{\tau-1} < EU'_{\tau-1} \implies u'(c(\bar{z}); \mathcal{A}) < u'(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) > \hat{c}(z)$
 And $EV_{\tau-1} < EU_{\tau-1} \implies u(c(\bar{z}); \mathcal{A}) > u(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) > \hat{c}(z) \implies$ No
 contradiction.
- (4) Suppose $p = I_{\tau-1}(z, \mathcal{A}_s) = \psi_{\tau-1}(z, \mathcal{A}_s)$.
 Then $EV'_{\tau-1} = EU'_{\tau-1} \implies u'(c(\bar{z}); \mathcal{A}) \leq u'(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) \geq \hat{c}(z)$
 But $EV_{\tau-1} = EU_{\tau-1} \implies u(c(\bar{z}); \mathcal{A}) = u(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) = \hat{c}(z) \implies$ No
 contradiction.
- (5) Suppose $p > \psi_{\tau-1}(z, \mathcal{A}_s) \geq I_{\tau-1}(z, \mathcal{A}_s)$
 Then $EV'_{\tau-1} > EU'_{\tau-1} \implies u(c(\bar{z}); \mathcal{A}) \geq u(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) \geq \hat{c}(z)$
 And $EV_{\tau-1} < EU_{\tau-1} \implies u(c(\bar{z}); \mathcal{A}) > u(\hat{c}(z); \mathcal{A}) \implies c(\bar{z}) > \hat{c}(z) \implies$ No
 contradiction.

The cases (1)-(5) above exhausts all possibilities. Therefore, we conclude that any one of the following conditions must hold: $I_{\tau-1}(z, \mathcal{A}_s) < p < \psi_{\tau-1}(z, \mathcal{A}_s)$ or $p > \psi_{\tau-1}(z, \mathcal{A}_s) \geq I_{\tau-1}(z, \mathcal{A}_s)$ or $p = I_{\tau-1}(z, \mathcal{A}_s) = \psi_{\tau-1}(z, \mathcal{A}_s)$. Therefore, $p \geq I_{\tau-1}(z, \mathcal{A}_s)$ for each $s \in \mathcal{S}$. Therefore,

$$\begin{aligned} EV_{\tau-1}(f(\bar{z} - c_{\tau}(\bar{z}; \mathcal{A})) + Y_s^i, \mathcal{A}_s) &\leq EU_{\tau-1}(f(z - \hat{c}_{\tau}(z)) + Y_s^i) \\ &\implies u_i(c_{\tau}(\bar{z}; \mathcal{A})) \geq u_i(\hat{c}_{\tau}(z)) \\ &\implies u'_i(c_{\tau}(\bar{z}; \mathcal{A})) \leq u'_i(\hat{c}_{\tau}(z)) \end{aligned}$$

Therefore, using the Envelope theorem,

$$\frac{\partial V_{\tau}(z, \mathcal{A})}{\partial z} \leq U'_{\tau}(z)$$

We previously established

$$\frac{\partial I_{\tau}}{\partial z}(z, \mathcal{A}) = 1 - U'_{\tau}(z) / \frac{\partial V_{\tau}(z, \mathcal{A})}{\partial z}$$

Therefore, $\frac{\partial I_{\tau}}{\partial z}(z, \mathcal{A}) \leq 0$. Therefore, $I_{\tau}(z, \mathcal{A}) \leq \psi_{\tau}(z, \mathcal{A})$. \square

Since we have established in Lemma 7.1 that $I_1(z, \mathcal{A}) = \psi_1(z, \mathcal{A})$, it follows from Lemma 7.2 that $I_{\tau}(z, \mathcal{A}) \leq \psi_{\tau}(z, \mathcal{A})$ for $\tau = 2, 3, \dots$, for any $\mathcal{A} \in \mathbf{A}_{IO}$.

Note that, by definition, $|U^i(z) - U_\tau(z)| = \beta^\tau EU^i(k_{\tau+1}(h_{\tau+1}) + Y_s^i)$. If there is a limit to wealth accumulation, i.e. $k_{\tau+1}(h_{\tau+1}) < \bar{k}$ for each τ and $h_{\tau+1}$, then we obtain $\|U^i(z) - U_\tau(z)\|_\infty \leq \beta^\tau EU^i(\bar{k} + Y_{\max}^i)$ where Y_{\max}^i is the highest income that person i can obtain in any state of the world.¹⁶ Thus, we obtain the result that $U_\tau(\cdot)$ converges uniformly to $U^i(\cdot)$. Similarly, we obtain the result that $V_\tau(\cdot, \mathcal{A})$ converges uniformly to $V^i(z, \mathcal{A})$.

Since $V_\tau(z, \mathcal{A})$ and $U_\tau(z)$ are both continuously differentiable in z (this follows from the fact that the CARA utility function is continuously differentiable), we have uniform convergence of $I_\tau(\cdot, \mathcal{A})$ to $I(\cdot, \mathcal{A})$ and uniform convergence of $\psi_\tau(\cdot, \mathcal{A})$ to $\psi(\cdot, \mathcal{A})$.

Since $I_\tau(z, \mathcal{A}) \leq \psi_\tau(z, \mathcal{A})$ for each τ , we have $I(z, \mathcal{A}) \leq \psi(z, \mathcal{A})$. Thus, we have established that $I(z, \mathcal{A}) \leq \psi(z, \mathcal{A})$ for any $\mathcal{A} \in \mathbf{A}_{IO}$.

The following lemma (which closely follows a similar result in Kimball (1990)) shows that if the insurance premium is smaller than or equal to the precautionary savings premium, then the insurance premium is also decreasing (weakly) in wealth:

Lemma 7.3. $\left(\frac{\partial I^i(z, \mathcal{A})}{\partial z} \geq 0\right) \iff (I^i(z, \mathcal{A}) \geq \psi^i(z, \mathcal{A}))$

Proof. Recall that the insurance premium, $I^i(z, \mathcal{A})$, is given, implicitly, as follows:

$$(45) \quad U_a^i(z) \equiv V^i(z - I^i(z, \mathcal{A}), \mathcal{A})$$

Differentiating throughout (45) and rearranging, we obtain

$$(46) \quad \begin{aligned} \frac{dU_a^i(z)}{dz} &= \frac{\partial V^i(z - I^i(z, \mathcal{A}), \mathcal{A})}{\partial z} \left(1 - \frac{\partial I^i}{\partial z}\right) \\ \implies \frac{\partial I^i(z, \mathcal{A})}{\partial z} &= 1 - \frac{dU_a^i(z)}{dz} / \frac{\partial V^i(z - I^i(z, \mathcal{A}), \mathcal{A})}{\partial z} \end{aligned}$$

Hence, from (38) and (46), we see that

$$\left(\frac{\partial I^i(z, \mathcal{A})}{\partial z} \geq 0\right) \iff (I^i(z, \mathcal{A}) \geq \psi^i(z, \mathcal{A}))$$

□

¹⁶If, alternatively, $\beta\sigma$ is sufficiently small, then this restriction on wealth accumulation is unnecessary to obtain the result of uniform convergence.

It follows from Lemma 7.3 that $\frac{\partial I^i(z, \mathcal{A})}{\partial z} \leq 0$ for each $\mathcal{A} \in \mathbf{A}_{IO}$. Next, we shall show that this property also holds for any $\mathcal{A} \in \mathbf{A}_{CE}$ where \mathbf{A}_{CE} is the set of constrained efficient agreements.

Let \mathbf{A}_m be the set of constrained optimal agreements which need satisfy the participation constraints during the first m periods only; i.e. the continuation agreement after the first m periods following any history provides full insurance. We shall use the Contraction Mapping Theorem to show how a sequence of agreements $\{\mathcal{A}_m(\mathbf{k}_0, \underline{\mathbf{U}})\}_{m=1}^\infty$, where $\mathcal{A}_m(\mathbf{k}_0, \underline{\mathbf{U}}) \in \mathbf{A}_m$ has been computed using initial assets $\mathbf{k}_0 = (k_0^1, k_0^2, \dots, k_0^n)$ and promised utilities $\underline{\mathbf{U}} = (\underline{U}^1, \dots, \underline{U}^{n-1})$, ‘converges’ to the corresponding constrained efficient agreement.

We define

$$\mathcal{U} = \left\{ (\mathbf{k}_0, \underline{\mathbf{U}}) : (\mathbf{k}_0, \underline{\mathbf{U}}) \in \mathbb{R}_+^{2n-1}, \exists \text{ self-enforcing agreement with } \right. \\ \left. \text{expected utility levels of at least } (\underline{U}^1, \dots, \underline{U}^{n-1}, EU_a^n(f(k_0^n) + Y_s^n)) \right\}$$

Let $C(\mathcal{U})$ be the space of bounded continuous functions defined on the set \mathcal{U} . Let $T : C(\mathcal{U}) \rightarrow C(\mathcal{U})$ be a transformation mapping $C(\mathcal{U})$ into itself, defined as follows:

$$(47) \quad T(F)(\mathbf{k}_0, \underline{\mathbf{U}}) = \max_{\substack{\{\tilde{\tau}_s^i, k_s^i\}_{s \in \mathcal{S}, i \in \mathcal{I}} \\ \{U_s^i\}_{s \in \mathcal{S}, i \neq n}}} E \left[u_n(c_s^n) + \beta F(k_s^1, \dots, k_s^n, U_s^1, \dots, U_s^{n-1}) \right]$$

subject to :

$$(48) \quad : E \left[u_i(c_s^i) + \beta U_s^i \right] \geq \underline{U}^i \text{ for } i \neq n$$

$$(49) \quad : u_i(c_s^i) + \beta U_s^i \geq U_a^i(k_0^i + Y_s^i) \text{ for } i \neq n$$

$$(50) \quad : u_n(c_s^n) + \beta F(k_s^1, \dots, k_s^n, U_s^1, \dots, U_s^{n-1}) \geq U_a^n(k_0^n + Y_s^n)$$

$$(51) \quad : 0 \leq k_s^i \text{ for } i \in \mathcal{I}$$

$$(52) \quad : \sum_{i \in \mathcal{I}} \tilde{\tau}_s^i = 0$$

for each $s \in \mathcal{S}$

where $c_s^i = f(k_0^i) + Y_s^i - \tilde{\tau}_s^i - k_s^i$, $F : \mathcal{U} \rightarrow \mathbb{R}_+$ and $F(\cdot) \in C(\mathcal{U})$.

Suppose $P_0(\cdot) : \mathcal{U} \rightarrow \mathbb{R}_+$ and $P_0(\cdot) \in C(\mathcal{U})$. Then, given the properties of $f(\cdot)$ and $P_0(\cdot)$, we can verify that $T^m P_0$ is bounded and continuous on \mathcal{U} for $m = 1, 2, \dots$. Moreover, the transformation T satisfies Blackwell’s sufficient conditions for a contraction (Stokey, Lucas and Prescott (1989), Theorem 3.3). Therefore, we can apply the Contraction Mapping

Theorem (ibid., Theorem 3.2). Hence, we obtain

$$(53) \quad \lim_{m \rightarrow \infty} \|T^m P_0(\mathbf{k}_0, \underline{\mathbf{U}}) - P(\mathbf{k}_0, \underline{\mathbf{U}})\| = 0$$

where the function $P(\mathbf{k}_0, \underline{\mathbf{U}})$ is as defined in (14)-(19). In other words, by applying the transformation T iteratively to $P_0(\mathbf{k}_0, \underline{\mathbf{U}})$, we can obtain functions arbitrarily ‘close’ to $P(\mathbf{k}_0, \underline{\mathbf{U}})$ in the sense of the sup norm.

Let $P_{FI}(\mathbf{k}_0, \underline{\mathbf{U}})$ be the maximum utility that agent n can obtain in a full-insurance agreement when the initial asset levels are described by \mathbf{k}_0 and the promised utilities of the other $n - 1$ agents given by $\underline{\mathbf{U}}$. Formally, $P_{FI}(\mathbf{k}_0, \underline{\mathbf{U}})$ is given by

$$P_{FI}(k^1, \dots, k^n, \underline{U}^1, \dots, \underline{U}^{n-1}) = \max_{\{\tilde{\tau}_t^i(h_t), k_t^i(h_t)\}_{t=1.. \infty, i \in \mathcal{I}, h_t \in \mathcal{H}}} E_0 \sum_{t=1}^{\infty} \beta^{t-1} u_n(c_t^n(h_t))$$

$$(54) \quad \text{subject to} \quad : \quad E_0 \sum_{t=1}^{\infty} \beta^{t-1} u_i(c_t^i(h_t)) \geq \underline{U}^i \text{ for } i \neq n$$

$$(55) \quad \sum_{j \in \mathcal{I}} \tilde{\tau}_t^j(h_t) = 0 \text{ for } t = 1.. \infty \text{ and } \forall h_t \in \mathcal{H}_t$$

$$(56) \quad k_t^i(h_t) \geq 0 \text{ for } t = 1.. \infty, \text{ and } \forall i \in \mathcal{I}, \forall h_t \in \mathcal{H}_t$$

where $c_t^i(h_t) = f(k_{t-1}^i(h_{t-1})) + y_t^i - \tilde{\tau}_t^i(h_t) - k_t^i(h_t)$. We can verify that P_{FI} is continuous and bounded on the set \mathcal{U} .

Then $T^m P_{FI}(\mathbf{k}_0, \underline{\mathbf{U}})$ corresponds to the utility obtained by agent m in the agreement $\mathcal{A}_m(\mathbf{k}_0, \underline{\mathbf{U}})$. Furthermore, letting $P_m = T^m P_{FI}$, we obtain the result that P_m converges uniformly to P . Next we show that the property that the insurance premium is (weakly) decreasing in wealth extends to each $\mathcal{A}_m \in \mathbf{A}_m$, for $m = 1, 2, \dots$

Lemma 7.4. *Let $\mathcal{A}_{m,s}$ be the continuation agreement implied by \mathcal{A}_{m+1} in state s , and \hat{k}_1^i the optimal level of saving in period 1 in autarky when initial assets equal z . Suppose $\frac{\partial I^i}{\partial z} \left(f(\hat{k}_1^i) + Y_s^i, \mathcal{A}_{m,s} \right) \leq 0$ for each $s \in \mathcal{S}$. Then, $\frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) \leq 0$.*

If, in addition, (i) the non-negativity constraint on saving is binding in period 1 in autarky or (ii) $\frac{\partial I^i}{\partial z} \left(f(\hat{k}_1^i) + Y_s^i, \mathcal{A}_{m,s} \right) < 0$ for some s and person i 's participation constraint is binding in state s , then $\frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) < 0$.

Proof. By construction, $\mathcal{A}_{m,s} \in \mathbf{A}_m$ for each $s \in \mathcal{S}$. Recall that $I^i(z, \mathcal{A}_{m+1})$ is the insurance premium that individual i is willing to pay to participate in agreement \mathcal{A}_{m+1} .

Hence we have

$$(57) \quad V^i(z - I^i(z, \mathcal{A}_{m+1})) \equiv U_a^i(z) \\ \implies u(z - I_{m+1}^i(z, \mathcal{A}_{m+1}) - k_1^i) + \beta EV_m^i(f(k_1^i) + Y_s^i, \mathcal{A}_{m,s}) \equiv u(z - \hat{k}_1^i) + \beta EU_a^i(f(\hat{k}_1^i) + Y_s^i)$$

where k_1^i is the level of saving at the end of period 1 prescribed by the agreement \mathcal{A}_{m+1} , and \hat{k}_1^i is the optimal level of period 1 saving under autarky. Given the assumptions $\frac{\partial I^i}{\partial z}(f(\hat{k}_1^i) + Y_s^i, \mathcal{A}_{m,s}) \leq 0$ for each s and that the non-negativity constraint on saving does not bind in the agreement, we obtain, using equation (24),

$$(58) \quad u'(z - I^i(z, \mathcal{A}_{m+1}) - k_1^i) \leq \beta f'(k_1^i) E \frac{\partial V^i}{\partial z}(f(k_1^i) + Y_s^i, \mathcal{A}_{m,s})$$

From the first-order condition of the optimisation problem for autarky, (6), we obtain

$$(59) \quad u'(z - \hat{k}_1^i) \geq \beta f'(\hat{k}_1^i) E \frac{dU_a^i}{dz}(f(\hat{k}_1^i) + Y_s^i)$$

Differentiating throughout (57) w.r.t. z and using (58) and (59), we obtain

$$(60) \quad \frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) \leq 1 - \frac{E \frac{dU_a^i}{dz}(f(\hat{k}_1^i) + Y_s^i)}{E \frac{\partial V^i}{\partial z}(f(k_1^i) + Y_s^i, \mathcal{A}_{m,s})}$$

From the definitions of $V^i(\cdot, \cdot)$ and $I^i(\cdot, \cdot)$, we obtain

$$(61) \quad V^i(z - I^i(z, \mathcal{A}_{m,s}), \mathcal{A}_{m,s}) \equiv U_a^i(z)$$

Differentiating throughout (61) w.r.t. z , we obtain

$$(62) \quad \frac{\partial V^i}{\partial z}(z - I^i(z, \mathcal{A}_{m,s}), \mathcal{A}_{m,s}) \left(1 - \frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m,s})\right) \equiv \frac{dU_a^i(z)}{dz}$$

Therefore, if $\frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m,s}) \leq 0$, then $\frac{\partial V^i}{\partial z}(z - I^i(z, \mathcal{A}_{m,s})) \leq \frac{dU_a^i(z)}{dz}$. By construction, $V^i(f(k_1^i) + Y_s^i, \mathcal{A}_{m,s}) \geq U_a^i(f(\hat{k}_1^i) + Y_s^i)$ (since the agreement \mathcal{A}_{m+1} satisfies the participation constraints in period 1). Therefore, letting $\hat{z} = f(\hat{k}_1^i) + Y_s^i$, we have $f(k_1^i) + Y_s^i \geq \hat{z} - I^i(z, \mathcal{A}_{m,s})$. Therefore

$$\frac{\partial V^i}{\partial z}(f(k_1^i) + Y_s^i, \mathcal{A}_{m,s}) \leq \frac{\partial V^i}{\partial z}(\hat{z} - I_{m,s}^i(\hat{z}), \mathcal{A}_{m,s}) < \frac{dU_a^i}{dz}(f(\hat{k}_1^i) + Y_s^i)$$

Substituting for $\frac{dU_a^i(f(\hat{k}_1^i) + Y_s^i)}{dz}$ using (62) into (60), we obtain

$$(63) \quad \frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) \leq 1 - \frac{E \frac{\partial V^i}{\partial z}(\hat{z} - I^i(\hat{z}, \mathcal{A}_{m,s})) \left(1 - \frac{\partial I^i}{\partial z}(\hat{z}, \mathcal{A}_{m,s})\right)}{E \frac{\partial V^i}{\partial z}(\sigma k_1^i + Y_s^i, \mathcal{A}_{m,s})}$$

$$(64) \quad \implies \frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) \leq 1 - E \left(1 - \frac{\partial I^i}{\partial z}(\hat{z}, \mathcal{A}_{m,s})\right)$$

The last inequality follows from the fact that $\frac{\partial V^i}{\partial z}(\hat{z} - I^i(\hat{z}, \mathcal{A}_{m,s})) > E \frac{\partial V^i}{\partial z}(f(k_1^i) + Y_s^i, \mathcal{A}_{m,s})$. From (64), we obtain $\frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) \leq E \left[\frac{\partial I^i}{\partial z}(\hat{z}, \mathcal{A}_{m,s}) \right] \leq 0$.

If the non-negativity constraint on saving is binding in period 1 in autarky, we obtain a strict inequality in (59) and therefore in (60), (63) and (64). Then, we would have $\frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) < 0$.

If $\frac{\partial I^i}{\partial z}\left(f\left(\hat{k}_1^i\right) + Y_s^i, \mathcal{A}_{m,s}\right) < 0$ for some s , we obtain a strict inequality in (64). Then, once again, we would have $\frac{\partial I^i}{\partial z}(z, \mathcal{A}_{m+1}) < 0$. \square

Note that a full-insurance agreement always prescribes the individually optimal level of saving. Thus $\mathbf{A}_{FI} \subset \mathbf{A}_{IO}$. By construction, for $\mathcal{A}_1 \in \mathbf{A}_1$, each continuation agreement $\mathcal{A}_{1,s} \in \mathbf{A}_{FI} \subset \mathbf{A}_{IO}$. We previously established that $\frac{\partial I^i(z, \mathcal{A})}{\partial z} \leq 0$ for $\mathcal{A} \in \mathbf{A}_{IO}$. Therefore, using Lemma 7.4, we obtain $\frac{\partial I^i(z, \mathcal{A}_1)}{\partial z} \leq 0$ for each $\mathcal{A}_1 \in \mathbf{A}_1$. Also, by construction, for $\mathcal{A}_m \in \mathbf{A}_m$, each continuation agreement $\mathcal{A}_{m-1,s} \in \mathbf{A}_{m-1}$ for $m \geq 2$. Therefore, applying Lemma 7.4 iteratively, we obtain the result that, if $\mathcal{A}_m \in \mathbf{A}_m$, then $\frac{\partial I^i(z, \mathcal{A}_m)}{\partial z} \leq 0$ for $m = 1, 2, \dots$. Then, using Lemma 7.3, we obtain the result $I^i(z, \mathcal{A}_m) \leq \psi^i(z, \mathcal{A}_m)$ for each m .

We have established that the sequence of functions $\{P_m\}$ converges uniformly to P . Let $\mathbf{x}_m = (\tilde{\tau}_{m,s}^i, k_{m,s}^i, U_{m,s}^i)_i$ be the solution to the optimisation problem corresponding to the transformation TP_{m-1} , and let $\mathbf{x} = (\tilde{\tau}_s^i, k_s^i, U_s^i)_i$ be the solution to the optimisation problem in (14)-(19). Then, using Theorem 3.8 in Stokey, Lucas and Prescott (1989), we can establish that $\{\mathbf{x}_m\}$ converges pointwise to \mathbf{x} .

By construction,

$$(65) \quad V^i(z, \mathcal{A}_m|s) = u_i(z - \tilde{\tau}_{m,s}^i - k_{m,s}^i) + \beta U_{m,s}^i$$

and

$$(66) \quad V^i(z, \mathcal{A}_{CE}|s) = u_i(z - \tilde{\tau}_s^i - k_s^i) + \beta U_s^i$$

where \mathcal{A}_{CE} is the agreement corresponding to the function $P(\mathbf{k}_0, \underline{\mathbf{U}}_0)$ and \mathcal{A}_m is the agreement corresponding to $P_m(\mathbf{k}_0, \underline{\mathbf{U}}_0)$. Since $(\tilde{\tau}_{m,s}^i, k_{m,s}^i, U_{m,s}^i)_i \rightarrow (\tilde{\tau}_s^i, k_s^i, U_s^i)_i$ pointwise, we obtain the result, from (65) and (66) that, for a given s , $V^i(\cdot, \mathcal{A}_m|s) \rightarrow V^i(\cdot, \mathcal{A}_{CE}|s)$ uniformly in m .

Since the Bernoulli utility function $u_i(\cdot)$ is continuous, $V^i(\cdot, \mathcal{A}_m|s)$ is continuous for each m . Then, from the definition of $I^i(\cdot)$ and $\psi^i(\cdot)$, we obtain the result that $I^i(\cdot, \mathcal{A}_m|s)$ and $\psi^i(\cdot, \mathcal{A}_m|s)$ converge uniformly to the functions $I^i(\cdot, \mathcal{A}_{CE}|s)$ and $\psi^i(\cdot, \mathcal{A}_{CE}|s)$ respectively, in m .¹⁷ We previously established that $I^i(z, \mathcal{A}_m) \leq \psi^i(z, \mathcal{A}_m)$ for any $\mathcal{A}_m \in \mathbf{A}_m$ and each m . Therefore, $I^i(z, \mathcal{A}) \leq \psi^i(z, \mathcal{A})$ for each $\mathcal{A} \in \mathbf{A}_{CE}$. Then, using Lemma 7.3, we obtain $\frac{\partial I^i(z, \mathcal{A})}{\partial z} \leq 0$ for $\mathcal{A} \in \mathbf{A}_{CE}$.

Finally, we shall show that $\frac{\partial I^i(z, \mathcal{A})}{\partial z} < 0$ for $\mathcal{A} \in \mathbf{A}_{CE}$ if the non-negativity constraint on saving binds in autarky following some history. Let $\mathcal{A}|h_t$ be the continuation agreement that is implied by \mathcal{A} from period t onwards if the realised history upto period t equals h_t . $\mathcal{A}|h_t \in \mathbf{A}_{CE}$ for each $h_t \in \mathcal{H}_t$ (otherwise, the agreement \mathcal{A} can be improved upon). Therefore, $\frac{\partial I^i(f(k_{t-1}^i(h_{t-1})) + Y_s^i, \mathcal{A}|h_t)}{\partial z} \leq 0$, where $k_{t-1}^i(h_{t-1})$ is the level of saving prescribed by the agreement \mathcal{A} following history h_{t-1} and $h_t = (h_{t-1}, s)$. Then, using Lemma 7.4, if the non-negativity constraint on saving is binding in period $t-1$ in autarky for asset level z , we obtain the result $\frac{\partial I^i(z, \mathcal{A}|h_{t-1})}{\partial z} < 0$ for each $z \geq k_{t-1}^i(h_{t-1})$. Then, applying Lemma 7.4 iteratively, we obtain the result $\frac{\partial I^i(z, \mathcal{A})}{\partial z} < 0$.

REFERENCES

- Abreu, D., 1988. On the theory of infinitely repeated games with discounting. *Econometrica* 56, 383-396.
- Allen, F., 1985. Repeated principal-agent relationships with lending and borrowing. *Economic Letters* 17, 27-31.
- Atkeson, A., Lucas, R.E., 1992. On efficient distribution with private information. *Review of Economic Studies* 59, 427-453.
- Banerjee, A. V., 2002. The uses of economic theory: Against a purely positive interpretation of theoretical results. BREAD Working Paper 7, MIT, Bureau for Research in Economic Analysis of Development, Cambridge, MA.
- Besley, T., 1995. Savings, credit and insurance. In: Behrman, J., Srinivasan, T.N. (Eds.). *Handbook of Development Economics*, Vol. 3A. North-Holland, Amsterdam, 2123-2207.
- Bloch, F., V. Rao, Desai, S., 2004. Wedding celebrations as conspicuous consumption: Signaling social status in rural India. *Journal of Human Resources* 39, 675-695.
- Coate, S., Ravallion, M., 1993. Reciprocity without commitment: Characterisation and performance of informal risk-sharing arrangements. *Journal of Development Economics* 40, 1-24.
- Cole, H. L., Kocherlakota, N., 2001. Efficient allocations with hidden income and hidden storage. *Review of Economic Studies* 68, 523-542.
- Fafchamps, M., 1999. Risk sharing and quasi-credit. *Journal of International Trade and Economic Development* 8, 257-278.
- Gobert, K., Poitevin, M., 2006. Non-commitment and savings in dynamic risk-sharing contracts. *Economic Theory* 28, 357-272.

¹⁷This follows from the result that if the sequence of functions $\{f_n\}$ uniformly converges to f and g is a continuous function, then the sequence $\{gf_n\}$ converges uniformly to gf .

- Golosov M., Kocherlakota, N., Tsyvinski, A., 2003. Optimal indirect and capital taxation. *Review of Economic Studies* 70, 569-587.
- Greif, A., 1994. Cultural beliefs and the organization of society: A historical and theoretical reflection on collectivist and individualist societies. *Journal of Political Economy* 102, 912-950.
- Greif, A., 2006. *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*. Cambridge University Press, Cambridge.
- Guiso, L., Sapienza, P., Zingales, L., 2006. Does culture affect economic outcomes?. *Journal of Economic Perspectives* 20, 23-48.
- Kimball, M., 1988. Farmers' cooperatives as behaviour toward risk. *American Economic Review* 78, 224-232.
- Kimball, M., 1990. Precautionary saving in the small and the large. *Econometrica* 58, 53-73.
- Kocherlakota, N., 1996. Implications of efficient risk sharing without commitment. *Review of Economic Studies* 63, 595-609.
- Kocherlakota, N., 2004. Figuring out the impact of hidden savings on optimal unemployment insurance. *Review of Economic Dynamics* 7, 541-554.
- Landes, D., 1998. *The Wealth and Poverty of Nations*. Norton and Company, New York.
- Ligon, E., Thomas, J.P., Worrall, T., 2000. Mutual insurance, individual savings and limited commitment. *Review of Economic Dynamics* 3, 1-47.
- Ligon, E., Thomas, J.P., Worrall, T., 2002. Informal insurance arrangements with limited commitment: Theory and evidence from village economies. *Review of Economic Studies* 69, 209-244.
- Manuh, T., 1995. Changes in marriage and funeral exchanges among the Asante. In: Guyer, J. (Eds.). *Money Matters: Instability, Values and Social Payments in the Modern History of West African Communities*. Heinemann, Portsmouth, OH.
- Platteau, J.P., 2000. *Institutions, Social Norms, and Economic Development*. Harwood Academic Publishers, Amsterdam.
- Putnam, R. D., 1993. *Making Democracy Work*. Princeton University Press, Princeton, NJ.
- Rogerson, W.P., 1985. Repeated moral hazard. *Econometrica* 53, 69-76.
- Stokey, N., Lucas, R., Prescott, E., 1989. *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge, MA.
- Thomas, J., Worrall, T., 1988. Self-enforcing wage contracts. *Review of Economic Studies* 55, 541-553.
- Udry, C., 1994. Risk and insurance in a rural credit market: An empirical investigation in northern Nigeria. *Review of Economic Studies* 61, 495-526.