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# Rational Investor Sentiment in a Repeated Stochastic Game with Imperfect Monitoring

March 22, 2010

## Abstract

We consider a repeated stochastic coordination game with imperfect public monitoring. In the game any pattern of coordinated play is a perfect Bayesian Nash equilibrium. Moreover, standard equilibrium selection arguments either have no bite or they select an equilibrium that is not observed in actual plays of the game. We give experimental evidence for a unique equilibrium selection and explain this very robust finding by equilibrium selection based on behavioral arguments, in particular focal point analysis, probability matching and overconfidence. Our results have interesting applications in finance because the observed equilibrium exhibits momentum, reversal and excess volatility. Moreover, the results may help to explain why technical analysis is a commonly observed investment style.

*Keywords:* Coordination games, behavioral equilibrium selection, experimental asset markets, behavioral finance, investor sentiment, technical analysis.

*JEL-Classification:* C72, C91, C92, D83, G12.

# 1 Introduction

*Ninety percent of what we do is based on perception. It doesn't matter if that perception is right or wrong or real. It only matters that other people in the market believe it. I may know it's crazy, I may think it's wrong. But I lose my shirt by ignoring it.*

“Making Book on the Buck”  
Wall Street Journal, Sept. 23, 1988, p. 17

Real life abounds with situations where people do best, individually and collectively, by coordinating their actions: communication is only possible if we speak the same language, living together in families, cities or countries is hindered if we do not obey the same rules of conduct, software would be extremely costly to develop if everyone were using a different computer platform, and trading is facilitated if it takes place at common market places or exchanges. A case in which coordination is of particular importance is a financial market. Assuming that investors do not hold assets till maturity, there is a natural incentive to buy those assets that other investors would also buy so that one can sell them with capital gains. In all these cases there are several ways in which people could potentially coordinate (rules, norms, technology standards etc.), so the coordination problem is non-trivial even though there is no conflict of interest. Such problems can be modelled as coordination games, where each player has the same number of strategies and there are multiple, typically Pareto-ranked equilibria. Equilibrium behavior in a coordination game requires that each player correctly anticipates the equilibrium the other players are aiming at, which seems difficult to achieve in a one-shot interaction. By contrast, in repeated interaction it may be possible to learn to coordinate on one equilibrium over time if each player observes the past actions of others. However, in reality an individual often only observes a noisy signal about what others have done in the past, and this may seriously impede the learning process. An investor, for example, can profit in the short-run if she invests according to the average expectation in the market. However, if she only observes asset prices which do not reveal individual trades she may get a very diffuse view of the average expectations of traders in the market.

For these reasons we consider a game where coordination is impeded by the fact that players observe only their own payoffs and these payoffs merely provide a noisy signal about the actions chosen by others. Hence, players can never verify whether coordination has been achieved in the past and therefore cannot simply repeat the successful action in the future. Our game is a simple majority game with a finite number of strategic players who can choose between two actions,  $u$  and  $d$ . A player receives a positive payoff if her choice matches the majority and a zero payoff otherwise. The majority choice, however, is determined not only by the actions of the strategic players but also by noise, which randomly adds weight to  $u$  or  $d$  and which can overrule the strategic players with a small probability. Each player observes only the choice of the majority, while the individual actions of the other strategic players and the noise remain hidden. Hence, the game is an example of a stochastic coordination game with imperfect public monitoring (see Abreu et al., 1990). Any pattern of coordinated play is a perfect Bayesian Nash equilibrium of the repeated game, so the problem of equilibrium selection is severe.

Standard dynamic models of learning, adaptive play or evolution<sup>1</sup> have no bite here since all pure strategy equilibria of the stage game are strict and payoff equivalent. To our knowledge the tracing procedure of Harsanyi and Selten (1988) is the only equilibrium selection theory that gives a reasonably narrow (and even unique) prediction for our game. Their procedure selects the inefficient equilibrium with random behavior (in each period all players mix between  $u$  and  $d$  with equal probability). This equilibrium, however, is not observed in actual plays of the game. Indeed we are able to give robust experimental evidence for a different unique equilibrium selection. In this equilibrium the strategic players simply choose the previous period's outcome as their action. We call this equilibrium the "switch" equilibrium because actual play is coordinated on  $u$  or  $d$  until the noise overrules the strategic players and everybody switches to the other action. We explain this very robust finding by equilibrium selection based on behavioral arguments, in particular focal point analysis, probability matching and overconfidence.

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<sup>1</sup>For an overview see the books by Weibull (1995), Vega-Redondo (1996), Samuelson (1997), Young (1993) and Fudenberg and Levine (1998).

While a contribution to behavioral equilibrium selection may itself be of interest for game theory, we are interested in this simple game also for its application to financial markets, which we used for the framing of the laboratory experiment. Imagine that, when a player chooses  $u$  ( $d$ ), he initiates an order to buy (sell) one unit of an asset. Then the strategic players may be interpreted as managers of mutual funds, pension funds or hedge funds, while the noise is representing actions exogenous to the coordination game like the dividends paid by firms or the buying and selling of traders that is not related to rational investing. Indeed the payoff function of our game matches the reward function of those managers. In every period they are assessed in terms of the gains/losses resulting from the actions they have taken in that period. If the manager decides to buy (sell) and prices go up (down) in this period, she will get a positive reward. Otherwise she will get a lower reward. The actual price movement reflected in the majority rule then is a simple version of the law of demand and supply.

Keeping up the analogy to a coordination game, our paper shows that some of the main mechanisms underlying excess volatility, short-term momentum and long-term reversal<sup>2</sup> can be explained as the outcome of a repeated beauty contest with noise. Indeed the “switch” equilibrium has all these features. The volatility (measured in terms of variance) of prices is higher than that of the exogenous noise (given by the die), there are phases in which prices continue in the same direction, and every now and then prices change their direction, reverting to the long-term average. Note that our model explains these upwards and downwards trends of asset markets, commonly called “investor sentiment”, as a perfect Bayesian Nash equilibrium. That is to say, we give a rational explanation of investor sentiment. In particular, in contrast to Barberis et al. (1998), the traders in our model have no misperceptions about the statistical distribution of the exogenous random process.

The results of our simple game may help to explain why technical analysis

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<sup>2</sup> Empirical evidence has shown that stock prices often deviate substantially from their fundamental values and are more volatile than the dividends (Shiller, 1981). Moreover, short-term momentum and long-term reversal of stock market prices are empirically robust stock price anomalies (see, for example, Jegadeesh, 1990, DeBondt and Thaler, 1985, Lo and MacKinlay, 1999, Campbell, 2000, and Hirshleifer, 2001).

is a commonly observed investment style. In contrast to the efficient market hypothesis put forward by Fama (1970), in the game considered here prices have information content as they are a signal for the achieved coordination in the market. Indeed, in the switch equilibrium the players understand this signal and base their trading on past prices.

Our paper relates to a large theoretical and experimental literature on coordination games. A well-known equilibrium selection theory is the one by Harsanyi and Selten (1988). Their theory is based on two criteria, payoff dominance and risk dominance. If these criteria have opposite implications, then Harsanyi and Selten (1988), as well as Anderlini (1990), favor payoff dominance, referring to the assumption of collective rationality. However, there is no general consensus on this issue. In Carlsson and van Damme (1993) and Harsanyi (1995) the risk-dominant equilibrium is selected for. Many experimental studies examine cases where predictions based on payoff and risk dominance conflict.<sup>3</sup> The major conclusion from these experiments is that the equilibrium selection depends on preplay communication, the number of players, the time horizon and the structure of the interaction.

The coordination problem is most severe when players are indifferent between all pure strategy equilibria of the stage game as is the case for the game in this paper. Crawford and Haller (1990) have characterized optimal strategies in repeated play of these pure coordination games. The strategies are such that players randomize in early stages of the game until they happen to achieve coordination in some stage. From that stage on players can maintain coordination forever since the actions of the precedent are now focal for everyone (cf. Schelling, 1960). As we have argued above, such strategies cannot be applied in our context of imperfect monitoring.

The experiment closest to ours is Hommes et al. (2005) who have studied expectation formation in an asset market game which is similar to the one we consider in this paper. In their experiment agents also have to predict the future price of an asset, but they have no information about how the equilibrium price

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<sup>3</sup>Among them are van Huyck et al. (1990, 1991, 1993), van Huyck et al. (1992), Cooper et al. (1992), Mehta et al. (1994), Brandts and MacLeod (1995), Cachon and Camerer (1996), Clark et al. (2001), Berninghaus and Ehrhart (1998, 2001) and Berninghaus et al. (1998).

is determined given the agents' forecasts, which makes it difficult to compare the experimental results with some rational benchmark. In accordance with our result Hommes et al. find that subjects coordinate on a common prediction strategy within each group. However, the observed prediction strategy varies widely across groups.

The outline of the paper is the following. In section 2 we give a formal description of the game considered in this paper. Thereafter, in section 3, we present the results from a laboratory experiment. Section 4 explains the equilibrium selection observed in the experiment. In section 5 we discuss the analogy of our experiment to the stock market. Section 6 concludes.

## 2 The Game

In the following we present the stochastic coordination game that we studied in a laboratory experiment.

### 2.1 The Stage Game

There are five players  $i = 1, \dots, 5$ , who simultaneously choose between two actions  $u$  (up) and  $d$  (down).<sup>4</sup> There is exogenous noise which we model by a random variable  $X$  that is uniformly distributed on  $\{0, 1, \dots, 6\}$ . We interpret  $X$  as the number of non-strategic players choosing action  $u$ . Correspondingly,  $6 - X$  is the number of non-strategic players choosing action  $d$ . For any strategy profile  $s = (s_1, \dots, s_5)$  with  $s_i \in \{u, d\}$  for all  $i$ , let  $R(s)$  be the random variable defined by

$$R(s) = \begin{cases} u, & \text{if } |\{i \mid s_i = u\}| + X > |\{i \mid s_i = d\}| + 6 - X \\ d, & \text{else} \end{cases} \quad .^5$$

$R$  is called *sentiment*, because  $R(s) = u$ , respectively  $R(s) = d$ , if the players' sentiment

$$|\{i \mid s_i = u\}| - |\{i \mid s_i = d\}|$$

<sup>4</sup>A more general version of the game involves an arbitrary odd number of players. Since there were five players in our laboratory experiment w.l.o.g. we focus our discussion on this case.

<sup>5</sup>By  $|A|$  we denote the cardinality of a set  $A$ .



plus the sentiment of the exogenous noise

$$X - (6 - X)$$

is positive, respectively negative. Observe that the exogenous noise can overrule the strategic players: If all players choose  $u$  and  $X = 0$ , then  $R(s) = d$ . Similarly,  $R(s) = u$  if all players choose  $d$  and  $X = 6$ .

Player  $i$ 's payoff is  $G > 0$ , if she correctly predicts the overall sentiment, and it is zero, otherwise. More precisely,  $i$ 's payoff at strategy profile  $s$  is a random variable  $\Pi^i(s)$  given by

$$\Pi^i(s) = \begin{cases} G, & \text{if } s_i = R(s) \\ 0, & \text{else} \end{cases}$$

As we have argued in the introduction, this game can be motivated by trading decisions on asset markets. We can interpret the players in our game as agents acting on behalf of some principals of an investment fund. The agents' actions are given by buying ( $u$ ) or selling ( $d$ ) one unit of an asset. At the end of a period the principals reward their agents according to the success of their action taken at the beginning of that period. Clearly, buying one extra unit of an asset at the beginning of a period is optimal if and only if prices increase during that period. Similarly, selling is optimal if and only if prices decrease. If the agent's action was optimal she receives a positive reward  $G > 0$  from the principal. Otherwise, she receives 0. In addition to the strategic traders there are agents who are trading for other reasons, for example in order to satisfy certain liquidity needs. The behavior of these "noise traders" is modelled by the exogenous random variable  $X$ . The actual asset price movement  $R$  is determined by the actions of all agents, strategic and noise traders. If market sentiment is positive, i.e. if there are more buying than selling orders, then prices go up. Conversely, if market sentiment is negative, prices go down. Observe that in this stylized model of an asset market the size of the sentiment does not play any role. Prices go up (down) by one tick only, independent of whether the sentiment was strongly or weakly positive (negative). Also, traders can only buy or sell one unit of the asset.

The stage game is a symmetric coordination game in expected payoffs. It is immediate to see that it has two Nash equilibria in pure strategies, namely  $s^U$

with  $s_i^U = u$  for all  $i = 1, \dots, 5$ , and  $s^D$  with  $s_i^D = d$  for all  $i = 1, \dots, 5$ . The expected payoff of player  $i$  at these equilibria is  $\frac{6}{7}G$ .

The stage game also has a unique Nash equilibrium in mixed strategies, where each player chooses  $u$  with probability 0.5. Observe that the expected payoff of a player in the mixed Nash equilibrium is smaller than her expected payoff in a pure strategy Nash equilibrium.

## 2.2 The Repeated Game

Consider now a finite repetition of the stage game over  $T$  periods. A player's payoff in the repeated game is given by the sum of the payoffs obtained in periods  $t = 1, \dots, T$ . After players have chosen their actions in any period  $t$  they observe the realized sentiment  $R_t$  in period  $t$  but neither the actions chosen by the other players nor the realization of the exogenous noise  $X_t$ . Hence, the repeated game is a stochastic coordination game with imperfect monitoring.

It is immediate to see that any sequence of pure strategy Nash equilibria of the stage game, i.e. any sequence of play that leads to coordination in all periods, is a pure strategy Nash equilibrium of the repeated game. Conversely, any pure strategy Nash equilibrium of the repeated game is given by a sequence of pure strategy Nash equilibria of the stage game. There are two stationary pure strategy Nash equilibria, where all players play  $u$  or all players play  $d$  in all periods. This is the simplest strategy players could use and we call it *stolid up*, resp. *stolid down* behavior. Under stolid behavior the sentiment process  $(R_t)_t$  is i.i.d. with

$$\text{Prob}(R_t = u) = \frac{6}{7}, \quad \text{Prob}(R_t = d) = \frac{1}{7}, \quad t = 1, \dots, T,$$

for stolid up and

$$\text{Prob}(R_t = d) = \frac{6}{7}, \quad \text{Prob}(R_t = u) = \frac{1}{7}, \quad t = 1, \dots, T,$$

for stolid down.

There are many other equilibria in pure strategies. Obviously any pure strategy Nash equilibrium is payoff equivalent to a Nash equilibrium in pure strategies

which depend on public information only, i.e. only on past realizations of the sentiment variable  $R_t$  and not on past actions taken by the player herself. One particularly simple Nash equilibrium in nontrivial public strategies is such that in all periods  $t \geq 2$  all players choose

$$\begin{cases} u, & \text{if } R_{t-1} = u \\ d, & \text{else} \end{cases}.$$

Here, the realization of the sentiment variable in the last period is taken as a signal on which players coordinate their action. We call this *switch behavior* since the players' sentiment changes from an extreme "up" to an extreme "down" mood if and only if the noise has overruled them in the last period. Under switch behavior  $(R_t)_t$  is a stationary Markov process with

$$\begin{aligned} \text{Prob}(R_{t+1} = u | R_t = u) &= \frac{6}{7}, \\ \text{and } \text{Prob}(R_{t+1} = u | R_t = d) &= \frac{1}{7}, \end{aligned}$$

for all  $t = 1, \dots, T$ .

The repeated game also has many (perfect Bayesian) Nash equilibria in mixed strategies. By Kuhn's (1953) theorem we can restrict to behavior strategies when characterizing mixed strategy Nash equilibria since the repeated game is a game with perfect recall. Obviously, any sequence of pure and mixed strategy Nash equilibria of the stage game gives rise to a mixed strategy Nash equilibrium of the repeated game. In particular, there is the stationary and symmetric mixed strategy Nash equilibrium, where in all periods all players choose  $u$  with probability 0.5. We call this *random behavior*. In this case  $(R_t)_t$  is a random walk with

$$\text{Prob}(R_t = u) = 0.5$$

for all  $t = 1, \dots, T$ . It is easy to see that any Nash equilibrium in strategies which depend on public information only must be given by a sequence of Nash equilibria (pure or mixed) of the stage game. In addition there is a plethora of (perfect Bayesian) Nash equilibria in mixed strategies that depend on private information.

Summarizing we see that the repeated game has a large number of Nash equilibria, even if we restrict to pure strategy equilibria which are all strict and all give rise to the same payoffs for the players. Moreover, the stochastic properties of the sentiment process  $(R_t)_t$ , which is the price process in our asset market interpretation of the game, critically depend on the equilibrium that is being played.

### 2.3 Equilibrium Selection

As we have seen in the last section the predictive power of Nash equilibrium is very limited so that the question of equilibrium selection arises. Different from most coordination games studied in the previous literature the pure strategy Nash equilibria in our stochastic coordination game are all payoff equivalent, so that no equilibrium appears to be particularly focal. Moreover, opponents' actions are not observed which further aggravates the coordination problem. Equilibrium selection theories based on payoff dominance or risk dominance (Harsanyi and Selten, 1988) clearly have no bite. The same is true for standard dynamic models of learning, adaptive play or evolution. The tracing procedure of Harsanyi and Selten (1988) is the only equilibrium selection theory that gives a reasonably narrow (and even unique) prediction for the game we are studying. It selects the inefficient equilibrium with random behavior, where in each period all players mix between  $u$  and  $d$  with probability 0.5. This is due to symmetry reasons and the fact that the tracing procedure always selects a unique equilibrium. Since our game is symmetric with respect to the actions  $u$  and  $d$  and since the selection must not depend on the labelling of these actions, there is only one equilibrium for which there does not exist a different equilibrium with the role of the actions  $u$  and  $d$  just reversed: the equilibrium with random behavior.

While there is no dynamic model that selects a unique equilibrium of our stage game, there are obviously dynamics that induce behavior which is consistent with some non-stationary equilibrium of the repeated game. Consider, for example, the adaptive dynamic, where in each period, all players choose a best-reply to a maximum-likelihood estimation of the other players' action in the previous period. This dynamic induces switching behavior as in the switch equilibrium we

described in section 2.2: If the sentiment is  $R_t$  in period  $t$ , then, independently of player  $i$ 's action in  $t$ , it is most likely that all players  $j \neq i$  have chosen action  $R_t$ . Hence, if  $i$  myopically expects all players to retain their action in period  $t + 1$ , then it is a best-reply to play  $R_t$  in period  $t + 1$ . Since this is true for all  $i$ , agents will play according to the switch equilibrium in all periods  $t \geq 2$ . We do not want to argue whether this is a reasonable behavioral model or not.<sup>6</sup> Instead we would like to point out that it is straightforward to construct similar adaptive models, which support other equilibria of the repeated game. So no equilibrium seems to be particularly prominent in this respect.<sup>7</sup>

### 3 The Experiment

In this section we present the results of an experiment where the game was played in a computer laboratory. Given our theoretical analysis and the negative results concerning equilibrium selection discussed in the previous section we would expect to observe a wide variety of play across groups, even if the subjects within one group learn to coordinate on one equilibrium. Surprisingly, however, we find that after an initial learning phase most groups in our experiment played according to the switch equilibrium.

#### 3.1 Experimental Design

We framed the experiment according to the financial market interpretation of the coordination game. In our view this is the most interesting economic application of our game and the framing allows us to compare the experimental results with stylized facts about stock prices that are observed on real markets. Accordingly, participants in the experiment were asked to bet on an increasing ( $u$ ) or decreasing ( $d$ ) asset price movement and they gained whenever their prediction matched the actual asset price movement.

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<sup>6</sup>It is clearly no learning model since players never learn the true strategies used by their opponents.

<sup>7</sup>For example, any player may choose a best-reply to a maximum-likelihood estimation of the other players' actions in the last three periods, assuming that the others choose the same action in all periods, etc.

Each group consisted of five subjects and played the game in two rounds with 100 periods each, where the first round was intended to be a learning phase. The details of the experimental procedure are presented below.

### 3.2 Experimental Procedure and Subject Pool

The computerized experiment was conducted in the computer laboratories of the University of Zurich in November and December 2001.<sup>8</sup> It lasted for approximately 90 minutes with the first 20 minutes consisting of orientation and instructions.<sup>9</sup> Before we started the experiment the subjects had to answer some control questions. In this way we made sure that everyone had understood the game. A total of 50 students from the University of Zurich participated in the experiment. They were recruited by announcements in the university promising a monetary reward contingent on performance in a group decision making experiment. The average payoff of a participant was 40 CHF (approximately \$25, at that time).<sup>10</sup>

We had 5 sessions with 2 groups each. Subjects were randomly assigned to a group and played the game via computer terminals. The computer terminals were fully separated from one another preventing any communication between the subjects. After the first round with 100 periods the experiment was restarted and the same groups played a second round with 100 periods. Subjects were informed about the fact that they would play a second round only after they finished the first round. Participants were not allowed to communicate at any time during the experiment and they were monitored to make sure that they observe this rule. In particular, they were not allowed to communicate during the short break between the two rounds of play. The subjects' payoffs were given in ECU (experimental currency units), where 100 ECU was converted to 0.25 CHF (approximately \$0.15).

The realization of the noise variable was determined by a 10-sided die, which

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<sup>8</sup>The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007).

<sup>9</sup>The instructions and supplementary material is available from the authors upon request.

<sup>10</sup>The payoff was the sum of a show-up fee and of the payoffs for the single plays and the strategy game, which are described in the following.

was thrown by the person supervising the experiments. The throw of the die was repeated until a number between 0 and 6 was realized. The realized number was invisible to the subjects. Both groups in each session played with the same die so that the exogenous noise was the same for these groups. The gain for a correct guess in any period was  $G = 20$  ECU. In each period the price movement and the subject's gain in the previous period were displayed on her computer terminal. The price changes in the last seven periods were also visible. Subjects could see the whole history of the given round by scrolling down in the field, in which the last seven periods were displayed. At no time the subjects received any information about the previous actions of the other members of their group or about the previous realizations of the die.

After the two rounds of the game subjects played a strategy game. They were informed about the play of the strategy game only after they finished the two rounds of single plays of the game. In this way we made sure that they did not use the previous two rounds of play of the game just to experiment with strategies to be used in the strategy game. In the latter, all participants had to define a strategy, which we coded in a computer program and with which the game was played afterwards. The subjects were asked to indicate when they change their strategy from  $u$  to  $d$  and when they change from  $d$  to  $u$ . They were free to write down their strategy as they wanted, e.g. they could choose a free text description of their strategy. One play of the strategy game over 100 periods was paid per person. For this play the subjects were randomly matched to each other. In the strategy game 100 ECU corresponded to 1 CHF, i.e. the payoff was four times the payoff in the single plays. We increased the payoffs in order to give subjects a high incentive to truthfully reveal the strategy they believe to be optimal. Finally, we asked the subjects to fill out a questionnaire concerning their own behavior during the experiment and their beliefs about the strategies chosen by their opponents.

After the experiment was completed the subjects were paid separately in cash contingent on their performance in the single plays of the game. The strategy game was paid out a few days later because the subjects' strategies had to be programmed first and then played by a computer.

### 3.3 Predictions

From the theoretical analysis in sections 2.2 and 2.3 we deduce the following main hypotheses which we will test with our experiment.

**Hypothesis 1:** Subjects play according to some pure strategy Nash equilibrium of the repeated game.

**Hypothesis 2:** There is no unique equilibrium selection.

### 3.4 Results

In the appendix we present some charts summarizing the data of our experiment for three selected groups.<sup>11</sup> The first series of charts displays the frequency of  $u$  choices among subjects in both rounds. As we see, the degree of coordination is quite low in the beginning of play in round 1 while it is very high during the second round of play, where at least 4 and in most periods 5 subjects choose the same action. Hence, the charts seem to confirm our first hypothesis, that subjects play according to some pure strategy Nash equilibrium of the repeated game. The following analysis will provide a rigorous test of this hypothesis.

The same charts, which display the subjects' degree of coordination, also display the price movement, so that we can analyze the strategies used by the subjects in the course of the game. Contradicting our hypothesis 2 we find much evidence for a unique equilibrium selection: all groups seem to play according to the switch equilibrium in the majority of periods during round 2, i.e. in each period they bet on the price movement in the previous period (group 9 shows this behavior in its purest form). Below we will present a rigorous analysis of the data which will lead to a rejection of hypothesis 2.

For the same groups the appendix also presents another series of charts which display the cumulated change in the noise traders' sentiment as determined by the die and the cumulated change in the price movement.<sup>12</sup> A first inspection of

<sup>11</sup>As we will see later, groups 2, 4 and 9 are representative for slow, moderate and fast learning groups, respectively.

<sup>12</sup>By definition the noise traders' sentiment is positive, if the die shows 4, 5, or 6 points, it is negative, if the die shows 0, 1, or 2 points, and it is zero, if the die shows exactly 3 points.



these charts shows that the volatility (measured in terms of variance) of the price movement is much higher than the volatility of the exogenous noise. Moreover, in round 2, there are long phases in which prices move in the same direction. These phases are broken every now and then and the price reverts to its long-term average. In the introduction we explained that these phenomena of excess volatility, short-term momentum and long-term reversal we identify in our experiment can also be observed on real financial markets. The finance literature classifies these price properties as “anomalies” and explains them by irrational behavior on the part of investors. By contrast, our experiment shows that they can also be obtained as the result of a particular equilibrium selection in a game, which captures the coordination game structure of stock markets. Below we will present a detailed analysis of the price process in our experiment.

### **Coordination**

For a detailed analysis of hypothesis 1 we determine the degree of coordination for all groups throughout round 2. The question is whether the degree of coordination is large enough such as to conclude that subjects play according to some pure strategy Nash equilibrium.<sup>13</sup> If we count the number of periods with full coordination, i.e. all subjects choosing the same action, we find that 4 out of 10 groups are fully coordinated in more than 90% of the periods, 7 groups are fully coordinated in more than 80% of the periods, and 9 groups are fully coordinated in at least 74% of the periods. The only exception is group 5 which shows full coordination in 14% of the periods only. Hence, 9 out of 10 groups are fully coordinated in a clear majority of periods in round 2 so that we can reject the hypotheses that subjects do not play according to a pure strategy Nash equilibrium of the repeated game at the 1% level in a one-sided binomial test.

### **Learning to Coordinate**

In order to examine whether subjects learn to coordinate over time we determine the cumulative frequency of coordinated play by at most 3 and 4 players within

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<sup>13</sup>Recall from section 2.2 that a strategy profile is a pure strategy Nash equilibrium of the repeated game if and only if it leads to full coordination in all periods.

each group and compare it across intervals with 50 periods in both rounds (see Table 1).<sup>14</sup> If subjects learn to coordinate over time, we should observe that the cumulative frequency distribution in periods 51 – 100 of round 2 first order stochastically dominates (FOSD) the distribution in periods 1 – 50 of round 2, which in turn FOSD the cumulative frequency distribution in periods 51 – 100 in round 1, which in turn FOSD the distribution in periods 1 – 50 in round 1. As can be seen from Table 1 this is indeed the case: For 5 out of 10 groups the cumulative frequency for coordinated play by at most 3 or 4 players is monotonically decreasing over time. For 4 groups there is only a slight violation of FOSD in round 2 between periods 1 – 50 and 51 – 100. Only for group 5 we do not find any monotonicity in the degree of coordination.

We apply a chi-square test in order to test the null hypothesis that the distribution of the degree of coordination, which has support  $\{3, 4, 5\}$ , is the same in the first 50 periods of round 1 and in the last 50 periods of round 2. For 6 groups (2, 3, 4, 6, 8, 10) we can reject this hypothesis at the 1% level, and for 2 groups (1 and 7) we can reject it at the 10% level (p-values are 0.068 for group 1 and 0.073 for group 7). For groups 5 and 9 the hypothesis cannot be rejected. These groups are either very well coordinated from the very beginning (group 9), so that learning in is not significant, or they do not show a high degree of coordination during the whole game (group 5).

Although almost all groups learn to coordinate over time, there is a large heterogeneity in the speed of learning across groups. For example, groups 1, 7, 9 and 10 are already well coordinated during the first 50 periods in round 1, while group 2 does not show a high degree of coordination till the beginning of round 2 (see Table 1). In order to further analyze the speed of learning we define the *learning phase* as follows. In accordance with our analysis above, we say that a group is *coordinated* in some period  $\tau$  if at least four members of the group choose the same action in  $\tau$ .<sup>15</sup> The end of the learning phase then is the first period  $t$  in round 1 such that the group is coordinated in all periods  $\tau$  with

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<sup>14</sup>Observe that there are always at least 3 players who choose the same action in any period. Hence, 3 subjects choosing the same action corresponds to no coordination.

<sup>15</sup>A group is *fully coordinated* in a period  $\tau$ , if five members of the group choose the same action in  $\tau$ .

$t + 1 \leq \tau \leq 100$ .<sup>16</sup> Table 2 shows the length of the learning phase for all groups which are coordinated<sup>17</sup> in round 2, i.e. for all groups except group 5. We see that there is a sizable learning phase (the mean is 50.11 and the median is 38 periods) and that there is a large heterogeneity in the speed of learning across groups. Groups 7, 9, and 10, can be classified as fast learners (coordination is achieved within the first third of the periods in round 1), groups 1, 3, 4, and 10, can be classified as moderate learners (coordination is achieved within the second third of the periods in round 1), and groups 2, 6, and 8, can be classified as slow learners (coordination is not achieved until the last third of the periods in round 1).

### Equilibrium Selection

Given that we observe equilibrium play during round 2 the next question is whether all groups play according to the same equilibrium which would lead to a rejection of hypothesis 2. A first inspection of the charts in the appendix has already shown that there is considerable evidence for the selection of the switch equilibrium. For a more detailed analysis we count for each subject the number of periods within the learning phase and within round 2, where the switch strategy is being played, i.e. where the action is identical to the observed price change in the previous period. Figure 1 displays the cumulative distribution over the relative frequency of play of the switch strategy. We have excluded the members of group 5 from the subject pool since this group does not play according to an equilibrium in round 2, so that the question of equilibrium selection is irrelevant for this group. Hence, there are 45 subjects left in the pool.

As can be seen in Figure 1 more than 90% of the subjects play the switch strategy in more than 90% of the periods in round 2. Already in the learning phase we observe a predominant play of the switch strategy: approximately 56% of the subjects choose the switch strategy in more than 80% of the periods within the learning phase and approximately 82% of the subjects choose the switch

<sup>16</sup>In case the length of the learning phase is close to 100 according to this definition we define it to be equal to 100.

<sup>17</sup>We consider group 1 as being coordinated in round 38 even though in the last round two players deviate because in the last period playful actions are often observed.

strategy in more than 70% of the periods within the learning phase. One may suspect that this predominant play of the switch strategies is mainly driven by subjects' betting on a trending price movement. Hence, we do the same analysis but restrict to those periods  $t$  that follow a change in the price movement, i.e. to those periods  $t$ , for which  $R_{t-1} \neq R_{t-2}$ . Still we find that the switch strategy is chosen much more frequently than any other strategy (see Figure 2): more than 75% of the subjects use the switch strategy, i.e. follow the change in the price movement, in more than 90% of the periods where such a change occurred during round 2. During the learning phase these numbers are smaller but even there more than 50% of the subjects use the switch strategy in more than 60% of the periods where such a change occurred during the learning phase. And approximately 33% of the subjects follow the change in the price movement in more than 70% of the periods in which this event occurred during the learning phase.

For each subject we then test the hypothesis that he or she follows the price movement with probability 0.5 or lower. We can reject this hypothesis at the 1% level for all 45 subjects in a one-sided binomial test. Within the learning phase we can reject it at the 1% level for 31 subjects, at the 5% level for 3 subjects and at the 10% level for 5 subjects. For 6 subjects we cannot reject the hypothesis at a reasonable level of significance within the learning phase. If we restrict to those periods in round 2, which follow a change in the price movement, we find that for 30 out of 45 subjects we can reject the hypothesis at the 1% level; for 7 subjects we can reject it at the 5% level and for 3 subjects we can reject it at the 10% level. Only for 5 subjects the hypothesis cannot be rejected at a reasonable level of significance. For the learning phase we get a different picture now: Only for 5 subjects we can reject the hypothesis at the 1% level, for 4 subjects at the 5% level and for 3 subjects at the 10% level, while for 33 subjects we cannot reject the hypothesis. This result suggests that in the course of play subjects mainly learn which action to take after there was a change in the price movement.

The main conclusion from our analysis above is that subjects play according to the switch strategy in round 2 whenever they belong to a group showing coordinated play in round 2. Hence, we can reject our hypothesis 2 that there is no unique equilibrium selection.

To give further evidence for the selection of the switch equilibrium we analyze the strategies specified by experienced subjects in the strategy game. In the first period all subjects choose  $u$ . For later periods they were asked to indicate under which conditions they switch their action from  $u$  to  $d$  and vice versa. It turns out that for no subject the decision to switch does depend on the direction of the switch (from  $u$  to  $d$  or from  $d$  to  $u$ ). As can be seen in Table 3 there are five types of strategies which we can reduce to two main types. The first three are the switch strategy and some variants, i.e. switch after being wrong once, twice, or three times. 41 out of 50 subjects choose the switch strategy and 4 choose a variant of it. 5 subjects also switch after being right for a certain number of periods.<sup>18</sup> One explanation for the latter type of strategy is that these subjects fall into gamblers' fallacy and believe in a reversal for no good reason. Summarizing, the strategy game confirms the equilibrium selection we observed in the actual play of the game: the majority of subjects plays according to the switch equilibrium strategy.

### Price Process

We close our analysis of the experimental data by studying some properties of the price process we observe in round 2. The first observation concerns the volatility of the exogenous noise and of the endogenous price movement. We take the volatility of the noise as a reference to determine excess volatility, because in our model the price movement would follow the exogenous noise if no strategic agents were present. For each group we compute the standard deviation of the cumulated price movement and the cumulated noise traders' sentiment. Here, we define

$$P_t := |\{\tau \mid \tau \leq t, R_\tau = u\}| - |\{\tau \mid \tau \leq t, R_\tau = d\}|$$

to be the cumulated price movement and

$$N_t := |\{\tau \mid \tau \leq t, X_\tau \geq 4\}| - |\{\tau \mid \tau \leq t, X_\tau \leq 2\}|$$

to be the cumulated noise-traders' sentiment in period  $t, t \geq 1$ , and we set  $P_0 = N_0 = 0$ . As we can see in Table 4, for all groups the standard deviation of the

<sup>18</sup>Group 5, which did not show equilibrium behavior, consisted mainly of players who selected strategies in the strategy game which were of this type.

cumulated price movement is higher than the standard deviation of the cumulated noise traders' sentiment. Hence, the hypothesis that the price and the exogenous noise show the same volatility can be rejected on a 1% level in a one-sided binomial test.

Our next observation concerns the momentum of the cumulated price movement. The empirical frequency of  $R_t = R_{t-1}$  is between 75% and 95% and hence is strictly larger than 0.5 for all groups. For all groups we can reject the hypothesis that the price movement does not show any momentum, i.e.  $\text{Prob}(R_t = R_{t-1}) \leq 0.5$ , at the 1% level in a chi-square test. Taking each group as an independent observation we can therefore reject the hypothesis that the price movement does not show any momentum at the 1% level in a binomial test.

Finally, we test for mean-reversion in the cumulated price movement. A commonly used test is the *variance ratio test*, which exploits the fact that under the random walk hypothesis the variance of the  $q$ th differences  $P_{t+q} - P_t$  is linear in  $q$ . Hence, if  $\hat{\sigma}^2(q)$  denotes the sample variance for the  $q$ th differences, then the variance ratio

$$\text{VR}(q) = \frac{\frac{1}{q}\hat{\sigma}^2(q)}{\hat{\sigma}^2(1)}$$

is close to 1 under the null hypothesis of a random walk.<sup>19</sup> If  $\text{VR}(q)$  is significantly greater than 1 we can reject the random walk hypothesis in favor of trending behavior, while if  $\text{VR}(q)$  is significantly smaller than 1 we can reject the random walk hypothesis in favor of mean reverting behavior. Table 5 reports the value of the variance-ratio test statistic for all groups in round 2. Due to the integer constraint we are restricted to  $q \in \{10, 20, 25, 50\}$ . As we see, we can clearly reject the random walk hypothesis at the 1% level. For all groups we observe highly significant trending behavior on a short horizon ( $q = 10$ ). On longer horizons ( $q = 25$ ) we observe mean reverting behavior for 6 groups, which, however, is significant for one group only.

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<sup>19</sup>Adjusting for finite samples and overlapping  $q$ th differences we define  $\hat{\sigma}^2(q) := \frac{1}{(nq - q + 1)(1 - \frac{1}{n})} \sum_{k=q}^{nq} (P_k - P_{k-q} - q\hat{\mu})^2$ , where  $\hat{\mu} = \frac{1}{nq}(P_{nq} - P_0)$  is the sample mean and  $nq = 100$  is the sample size.

## 4 Explaining the Equilibrium Selection

The main findings from our experiment are that

1. subjects are not coordinated in the beginning of play
2. after a sizable learning phase subjects learn to play according to a pure strategy Nash equilibrium of the repeated game,
3. the equilibrium selection is unique and is given by the switch equilibrium.

The unique equilibrium selection is particularly remarkable since all pure strategy equilibria of the game are payoff equivalent which is not the case for most coordination games previously studied in the experimental literature (cf. the introduction). As we have argued in section 2.3, Harsanyi and Selten (1988) is the only equilibrium selection theory which gives a clear prediction for the game we are considering. However, according to their theory the equilibrium with random behavior should be selected which is not confirmed by our experiment.

In the following we will argue that myopic best-reply cannot explain the observed equilibrium selection. Instead we will propose two behavioral explanations, one of which is based on overconfidence and the second utilizes focal-points together with probability matching.

### **Myopic Best-Reply or Overconfidence**

As we have seen in section 2.3 there is a simple adaptive model which supports the switch equilibrium and hence can potentially explain the equilibrium selection we observe. In this model, all players play a best-reply to a maximum-likelihood estimation of the other players' actions in the last period. While this adaptive model of myopic best-reply clearly is consistent with the switch equilibrium, it does not seem to explain well our experimental data. If subjects had played according to myopic best-reply, coordination would have been achieved within the first periods of play. By contrast, for most groups we observe a sizable phase of uncoordinated play (cf. Table 1) before the groups start playing according to the switch equilibrium. The questionnaire we conducted after the experiment also provides evidence against myopic best-reply behavior. In this questionnaire we

asked the subjects whether they think the die or the behavior of the other players in their group was responsible for a change in the price movement after long periods of increasing prices. 42 out of 48 subjects, who gave a clear answer to this question, stated that they think the die was responsible for the change in the price movement. Maximum-likelihood estimation of the other subjects' actions based on the last period's price movement is not consistent with this answer, because it requires subjects to attribute any outcome to the other subjects' choices and not to the exogenous noise.

Nevertheless, a player who attributes a change in the price movement to the exogenous noise may still behave in accordance with the adaptive model of myopic best-reply if she is overconfident, i.e. believes to have understood the situation better than the average participant. In our case, an overconfident player has understood that the die was responsible for breaking a trend but believes that her opponents are naive in that they attribute the change in the price movement not to the noise but to the others' behavior. Hence, an overconfident player will play the switch strategy since it is a best-reply to the belief that all others are playing a best-reply to a naive maximum-likelihood estimation.

### **Focal-Points and Probability Matching**

Games of pure coordination, like the one we are studying, have several equilibria which are indistinguishable from an abstract point of view, since they are all payoff equivalent. Nevertheless, we may observe a unique equilibrium selection in any actual play of the game. This situation was already illustrated by Schelling (1960) with his well known example about two strangers having to decide about a meeting point in New York without being able to communicate with each other. Schelling introduced the idea that persons coordinate on "focal points" (like the Grand Central Station in New York) if they have to solve such a problem. Obviously, such a coordination requires the existence of a common "frame," in particular actions must be labelled in the same way for all players. If there is no common frame the players are in a state of complete ignorance about how their opponents perceive the game, so that mixing uniformly between all actions seems to be the only reasonable thing to do.

Applied to our game, in the absence of a common frame we would expect



to observe the mixed equilibrium we named “random behavior.” However, since actions in our game are labelled “ $u$ ” and “ $d$ ,” there is a common frame and the notion of a focal point can, in principle, be applied. Both actions,  $u$  and  $d$ , could be focal leading to the stolid up, respectively stolid down equilibria. One may suspect that the action  $u$  is the focal one, which is also confirmed by the fact that all participants choose  $u$  in the first period of the strategy game. Nevertheless, in our experiment we neither observe the stolid up nor the stolid down equilibrium, which can be explained as follows. Even if players think that stolid up behavior is the most reasonable strategy (it is focal and it is simple), they may doubt that their opponents have arrived at the same conclusion. Hence, having no idea about their opponents’ behavior in the beginning of play, it seems reasonable to assume that others behave randomly, i.e. mix between  $u$  and  $d$  with probability 0.5. Consequently, players perceive the game as a single-person decision problem, namely the degenerate coordination game we obtain for  $n = 1$ . Clearly, given that the exogenous noise is unbiased, any strategy is optimal in this game. However, as it is known from many psychological studies (for a review see Fiorina, 1971, or Brackbill and Bravos, 1962) animals and human beings tend to perform probability matching in such single person decision situations. This kind of behavior was also regarded as important for decision making by Arrow (1958). In our case probability matching means that players select their strategy such that the frequency of  $u$  is equal to the probability that the sentiment of the exogenous noise is positive or non-negative which is equal to  $3/7$  or  $4/7$ . According to this reasoning we should observe switching between actions analogous to probability matching in the beginning of play. Looking at the data of our experiment we see that indeed there is considerable switching between  $u$  and  $d$  in the first 20 periods of round 1 before coordination is achieved in most groups (see Figure 3). The mean number of  $u$  choices is 13.52 and hence is only slightly above  $20 \times 4/7 = 11.43$ , which is the number of  $u$  choices we would expect to observe in a single person decision problem under probability matching. Probability matching behavior in the beginning of play rules out the occurrence of the stolid up (or stolid down) equilibrium.

Hence, given that there does not seem to be a “universal” focal action in our coordination game, the next question is whether in each period there exists a

(potentially different) action, which will be considered as focal by all players. And indeed, the sentiment variable (price movement) is an endogenous and publicly observable signal, which can be used to label an action so that it becomes the focal one. In principle, any history of past realizations of the sentiment variable can be used as a signal but we will argue that the last period's sentiment is the prominent one. Firstly, using the realized sentiment in more than one period requires a sophisticated rule about how to translate this multidimensional signal into an action. Hence, one coordination problem is replaced by another, making the use of a multidimensional signal very unreasonable.<sup>20</sup> A different argument in favor of using a one-dimensional signal, i.e. the sentiment (or price movement) in a single period, relies on costs (cf. Binmore and Samuelson, 2006). If the observation and processing of a signal is costly, for example because it causes disutility to interpret complex signals, then the players' payoffs are maximized if they use a one-dimensional signal only.<sup>21</sup> Assuming that the cost of observing and processing a signal is not too high the players' payoff is higher when they use the signal than it is when they don't, since in the latter case they are unable to identify focal points and will most likely fail to coordinate.

Secondly, using the last period's sentiment as a signal seems to be more prominent than using the sentiment in any other previous period. The time scale induces a common frame which makes the last period's sentiment a focal signal. Summarizing, in the switch equilibrium players overcome the coordination problem by choosing in each period the action that is focal according to the publicly observed signal, namely last period's sentiment.

The proposed explanations may be the reasons why the switch strategy is more frequently used than any other strategy already during the learning phase (see section 3.4). Given this bias towards switching behavior, during the learning phase subjects will necessarily learn that switching behavior is more successful than any other strategy, since switching is a best-reply to switching. As a consequence, subjects move from the rather uncoordinated behavior in the beginning

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<sup>20</sup>Of course, there is also not a unique way to translate the sentiment in one period into an action but choosing  $u$  and not  $d$  when the signal was  $u$  clearly is focal here.

<sup>21</sup>Provided, of course, they use the signal in the most efficient way, so that they achieve perfect coordination of their actions.

of play to the coordinated play of the switch strategy.

## 5 The Analogy to Financial Markets

In this section we argue that our simple experiment indeed has some analogy to the stock market. To do so recall the logical structure of our experiment. Exogenously given is the result of the die which generates a simple i.i.d. process without drift. Yet, the endogenous behavior of the players in the coordination game transforms the exogenous noise into price processes that show momentum, reversal and excess volatility. These three properties are apparent from the logic of the switch equilibrium: When the players are coordinated, the probability for the prices to continue in the same direction is  $5/6$ , leading to phases of prices moving in one direction for quite some time, i.e. leading to momentum. With probability  $1/6$  the die will break this coordination and a reversal, i.e. a change in the direction in which prices move, follows. Momentum and reversal imply that the price process is on average farther away from its mean than the exogenous noise, which is the excess volatility feature of our game. The charts in the appendix clearly show this feature of the prices and the analysis of the price process in Section 3.4 has given the statistical evidence for this.

To build the analogy to a stock market one first needs to decide which aspect of a stock market is similar to our die in the game. In this respect we interpret the die as the price process that would occur with random behavior in the game. In a stock market with random behavior the exogenous dividend process is the only systematic determinant of the price process. Also, referring to the standard economic valuation model, in any point in time stock prices should be equal to the expected discounted future dividends paid by the stock. Hence complete irrationality and complete rationality imply that the dividend process is the only exogenous determinant of stock prices. However, as mentioned before, empirical evidence has shown that stock prices are more volatile than their expected discounted future dividends (Shiller, 1981). Figure 4 in the appendix shows this finding for the S&P 500 from 1871 to 2008.<sup>22</sup> Discounting correctly anticipated

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<sup>22</sup>All our computations are based on the long-term data put online by Robert Shiller at <http://www.econ.yale.edu/shiller/data.htm>

future dividends by 5% results<sup>23</sup> in the solid line that is far less volatile than the actual price of the S&P 500. Our experiment would then suggest that the reason for excess volatility is a combination of momentum and reversal. On the same data we find quite some momentum in the short run. The average next month return after a month with positive return is about 0.11% while it is -0.06% after a negative month. Moreover, the probability of having a positive month following a positive month is 62.97% and the probability to have a negative month after a month with negative returns is 48.25%. These numbers are highly significant. Also, for monthly returns the autocorrelation of first order is 28.5% which is significant on the 95% level. Hence, analyzing previous price movements, as technical analysis does, helps to understand on which sentiment (bull or bear) the market has currently coordinated. As in the experiment, also in the S&P 500 data momentum and reversal can be concluded from a variance-ratio test. Figure 5 in the appendix shows the variance-ratios of the (log) S&P 500 for various time periods measured in months. For horizons up to 5 years the figure indicates momentum while for very long periods the variance-ratios are below 1 indicating reversal. The figure also shows the confidence intervals for the null-hypothesis that log-S&P 500 follows a random walk, i.e.  $\log P_t = \log P_{t-1} + \epsilon$ . Since there are naturally fewer periods with long horizons the confidence interval widens.

Finally, our experiment would suggest that a reversal does not come “out of the blue” but is more likely in periods where the exogenous factors give disconfirming facts for the endogenous equilibrium found in the financial market. Based on our experimental findings, we would expect that, for example, the price-dividend ratio is indicative for reversals. Extreme price-dividend ratios indicate that the endogenously found prices are at odds with the exogenous process of the dividends in the same way as from time to time the result of the die is at odds with the endogenously found coordination in the game. Indeed, as Figure 6 shows, this seems to be the case. A high (low) price-dividend ratio at the beginning of one year is indicative for smaller (higher) returns in that year. Of course, in a real stock market many more aspects than dividends and coordination on positive or negative momentum are important and thus the empirical results of

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<sup>23</sup>As Mankiw et al. (1991) and Scott(1990) have shown excess volatility is also obtained when discounting with market interest rates than with an exogenous discount factor.

the stock market are not as clear cut as those in the experiment. But it looks like that nevertheless the main findings of the experiment help to guide one's intuition for the much more complex stock market.

## 6 Conclusion

We have studied a repeated stochastic coordination game with imperfect public monitoring. In this game any pattern of coordinated play is a perfect Bayesian Nash equilibrium. Moreover, standard equilibrium selection arguments either have no bite or they select an equilibrium that is not observed in actual play of the game. We gave experimental evidence for a unique equilibrium selection in this game and explained this very robust finding by equilibrium selection based on behavioral arguments, in particular focal point analysis, probability matching and overconfidence.

Our results have interesting applications in finance because the observed equilibrium price process exhibits momentum, reversal and excess volatility. Indeed the behavioral arguments that we employed in the stylized asset market we considered are all well known from the behavioral finance literature (cf. Shefrin, 2000, Shiller, 1981, Shleifer, 2000, Barberis and Thaler, 2003). The behavioral finance literature has attributed large increases in investment activity on stock markets to the greater fool theory, i.e. to the idea that investors believe other investors are still willing to buy their stock at a higher price. This is a clear sign of overconfidence since it is impossible for the majority of investors to be on the winning side of this speculation. Moreover, the large amount of trading activity exhibited by the average investor is attributed in this literature to probability matching, i.e. to the desire to employ a trading strategy that somehow matches the frequent ups and downs of the random process one is trying to master. As Shefrin (2000) has shown, for most traders a stolid buy&hold strategy would have obtained much better results for their investments. Finally, observed asset prices are a focal point in the search for signals revealing which information the other investors have at a time. Indeed, as Treynor and Ferguson (1985) have argued, technical analysis can be profitable due to the information it may reveal about the other traders' state of information.

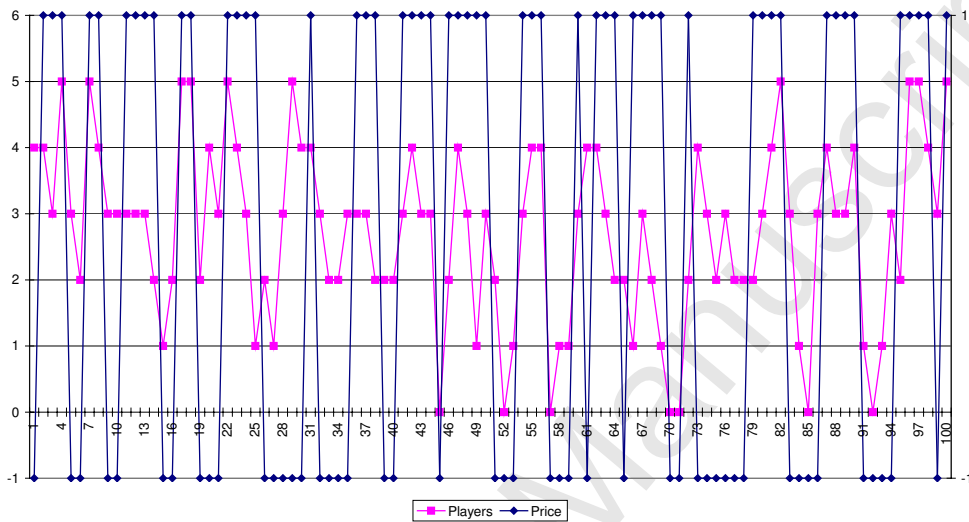
Of course, a real stock market is much more complex than the stylized asset market we considered in this paper. But on the other hand our much simpler setting has the advantage of being able to demonstrate more cleanly the role of focal point analysis, probability matching and overconfidence in the coordination of investors' actions on asset markets, that is shown to lead to momentum, reversal and excess volatility in asset prices. Further research will have to enrich our simple model with more realistic features like a dividend process with asymmetric information on the part of the investors.

## Appendix: Experimental Results

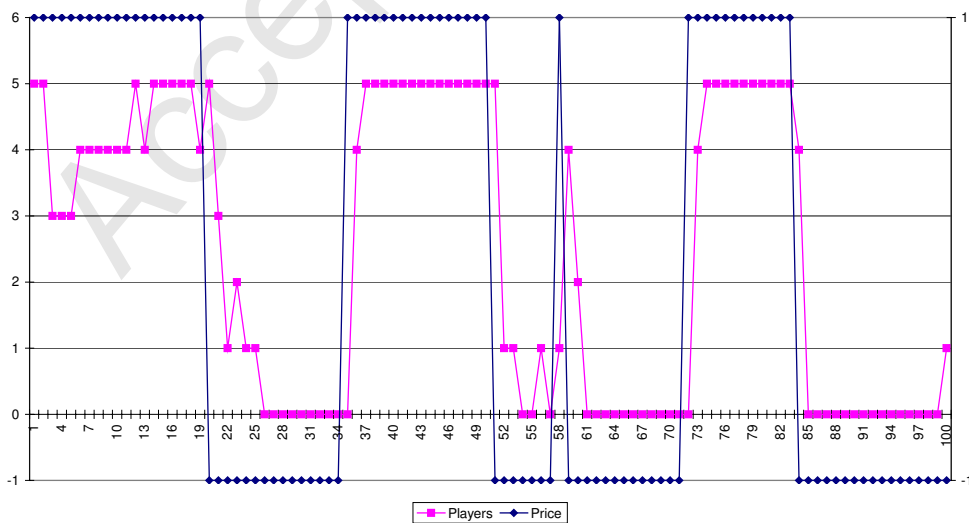
The following charts display the experimental results for some selected groups (2, 4, 9) in both rounds. The first series of charts displays the frequency of  $u$  among the subjects (left scale from 0 to 5) and the price movement (right scale, “-1” stands for a downward price movement and “1” stands for an upward price movement).

The second series of charts displays the cumulated change in the noise traders’ sentiment as determined by the die and the cumulated change in the price movement.

Group 2, Round 1:  
Frequency of  $u$  and price movement

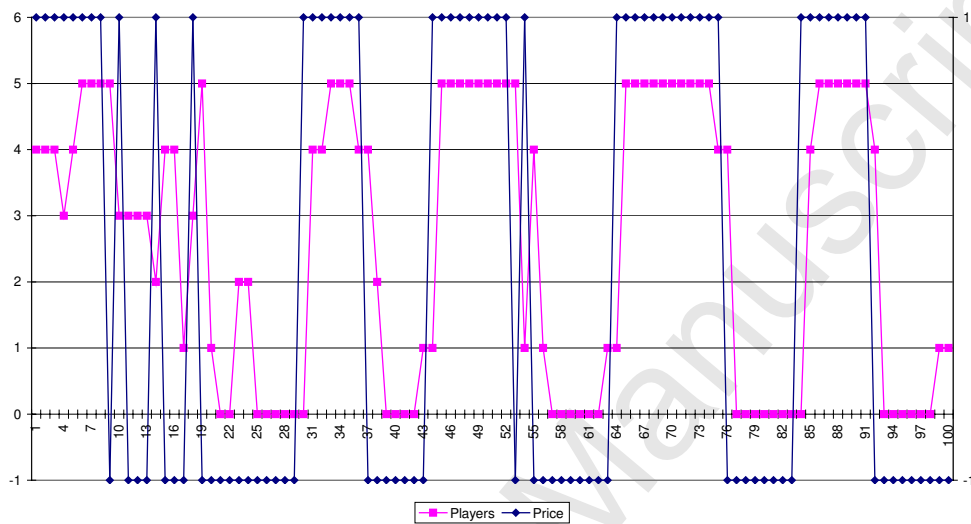


Group 2, Round 2:  
Frequency of  $u$  and price movement

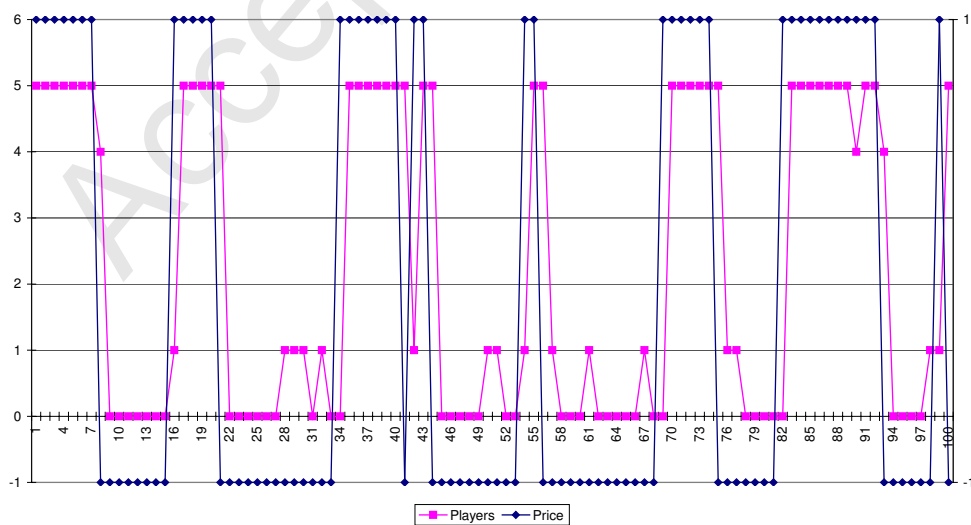




Group 4, Round 1:  
Frequency of  $u$  and price movement

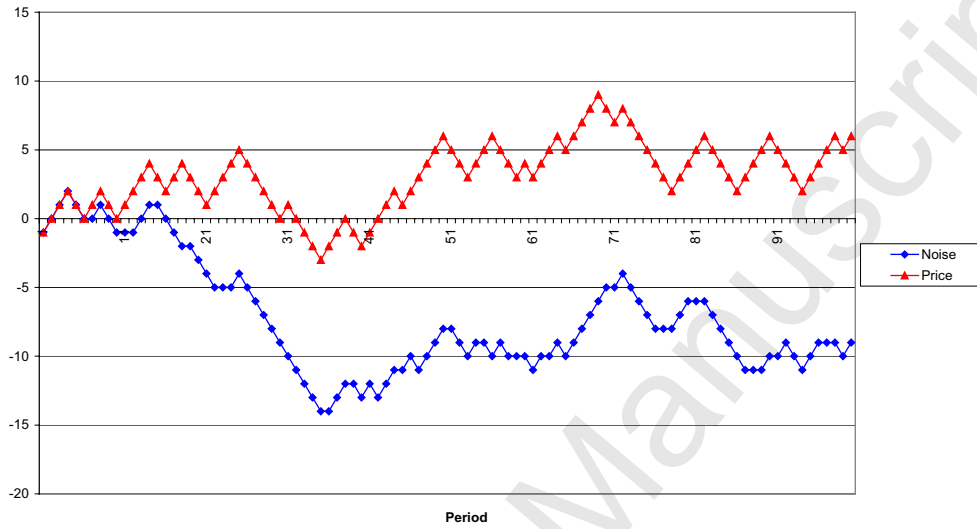


Group 4, Round 2:  
Frequency of  $u$  and price movement

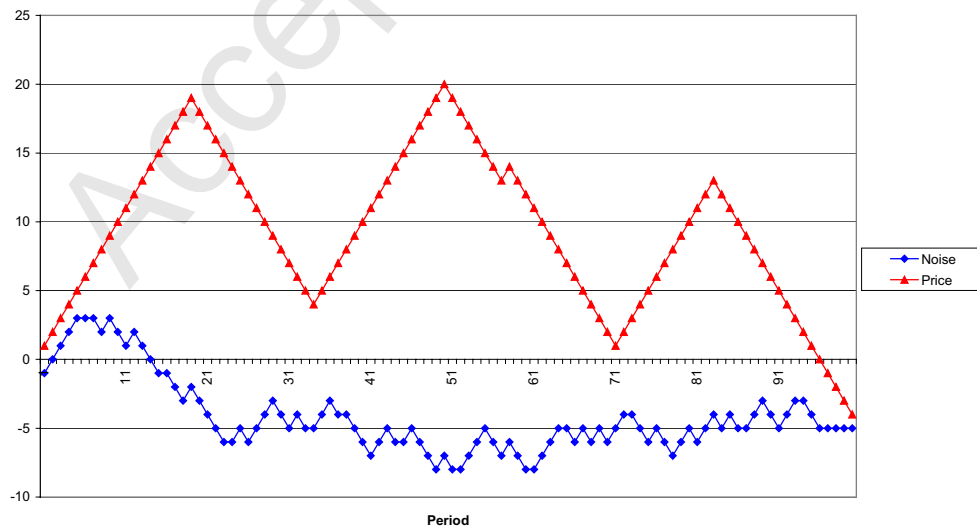


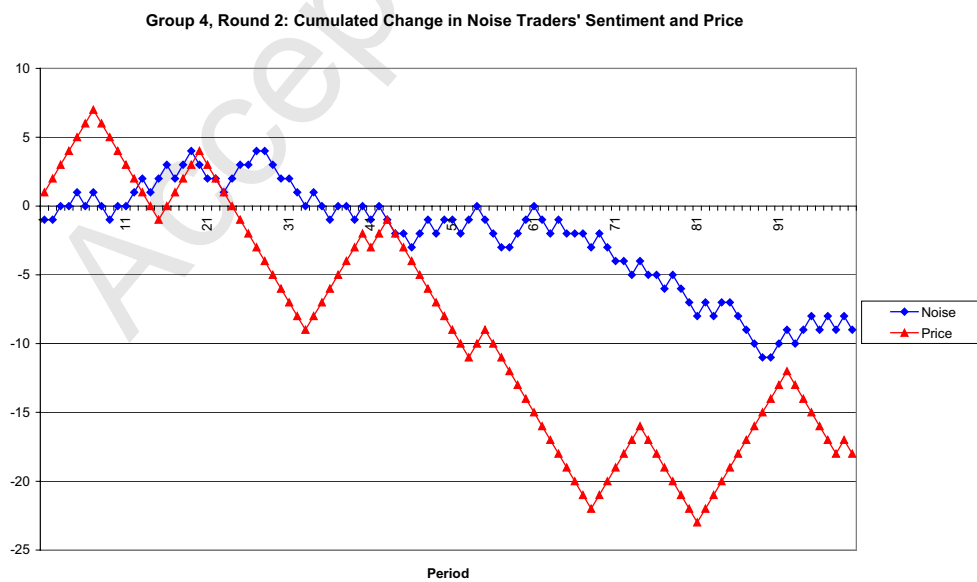
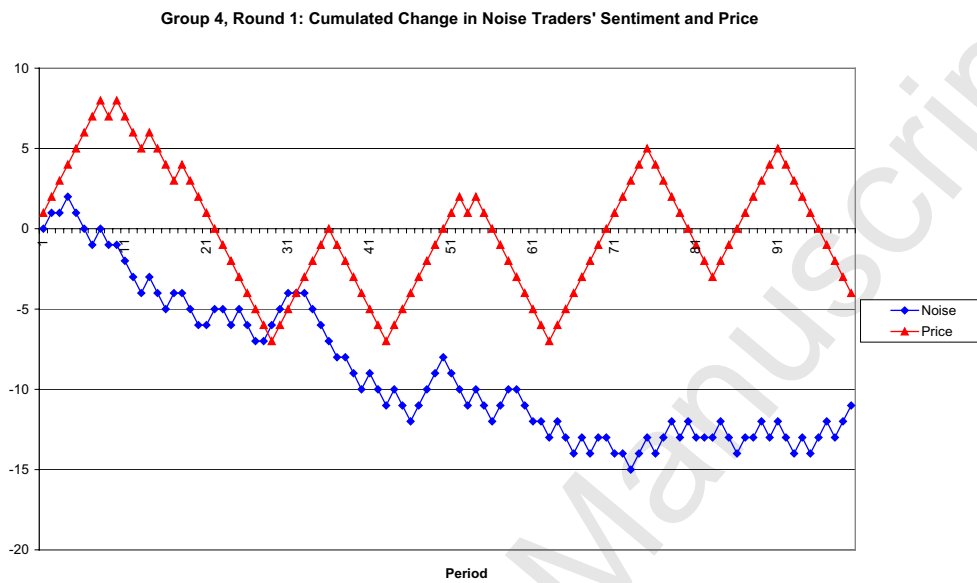


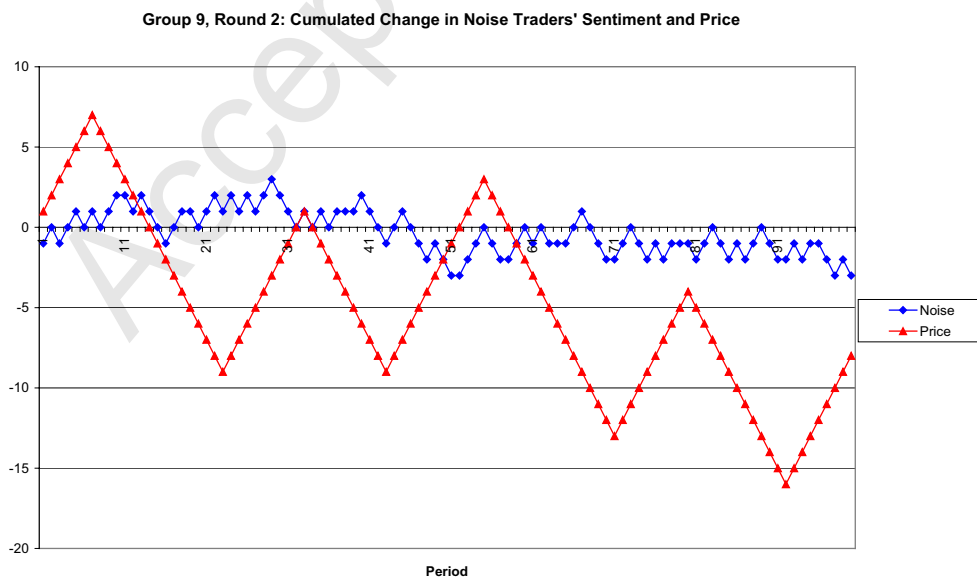
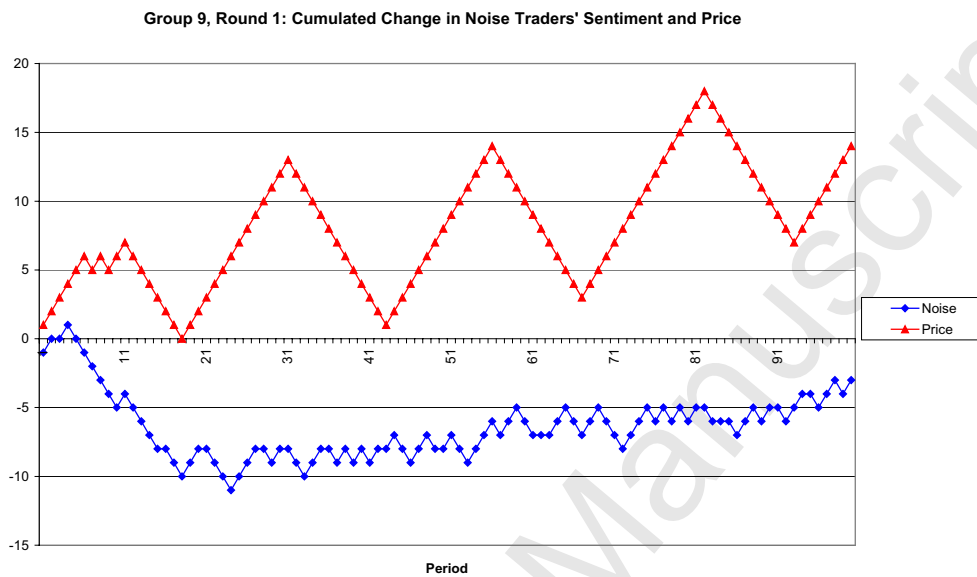
Group 2, Round 1: Cumulated Change in Noise Traders' Sentiment and Price



Group 2, Round 2: Cumulated Change in Noise Traders' Sentiment and Price







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| Group | Number of coordinated. subjects $\leq$ | Round 1      |                | Round 2      |                |
|-------|--|--------------|----------------|--------------|----------------|
|       |  | Periods 1-50 | Periods 51-100 | Periods 1-50 | Periods 51-100 |
| 1     | 3                                      | 2            | 1              | 1            | 0              |
|       | 4                                      | 16           | 10             | 5            | 7              |
| 2     | 3                                      | 30           | 23             | 5            | 1              |
|       | 4                                      | 43           | 40             | 17           | 9              |
| 3     | 3                                      | 14           | 8              | 7            | 0              |
|       | 4                                      | 27           | 8              | 7            | 1              |
| 4     | 3                                      | 10           | 0              | 0            | 0              |
|       | 4                                      | 24           | 11             | 8            | 11             |
| 5     | 3                                      | 22           | 25             | 19           | 21             |
|       | 4                                      | 43           | 46             | 44           | 42             |
| 6     | 3                                      | 20           | 2              | 0            | 0              |
|       | 4                                      | 37           | 25             | 1            | 0              |
| 7     | 3                                      | 1            | 0              | 0            | 0              |
|       | 4                                      | 15           | 8              | 8            | 6              |
| 8     | 3                                      | 22           | 5              | 1            | 0              |
|       | 4                                      | 35           | 18             | 10           | 15             |
| 9     | 3                                      | 2            | 0              | 0            | 0              |
|       | 4                                      | 4            | 4              | 1            | 0              |
| 10    | 3                                      | 5            | 0              | 0            | 0              |
|       | 4                                      | 14           | 6              | 0            | 3              |

Table 1: Cumulative frequency distribution: number of periods in which at most 3 or 4 subjects choose the same action.

| Group  | Period |
|--------|--------|
| 1      | 38     |
| 2      | 100    |
| 3      | 41     |
| 4      | 38     |
| 6      | 71     |
| 7      | 10     |
| 8      | 100    |
| 9      | 20     |
| 10     | 33     |
| Mean   | 50.11  |
| Median | 38     |

Table 2: Length of the learning phase in round 1 for all groups showing coordinated play in round 2.

| Strategies   | Number of Subjects |
|--|--------------------|
| Switch after being wrong once                                      | 41                 |
| Switch after being wrong twice                                     | 3                  |
| Switch after being wrong three times <sup>a</sup>                  | 1                  |
| Switch after being wrong twice and being right seven (eight) times | 3                  |
| Switch after being wrong twice and being right three times         | 2                  |

Table 3: Strategy game: choices for periods  $\geq 2$ .

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<sup>a</sup>This person added a complicate estimation about the future development of the price to this rule.

| Group | STDEV Price | STDEV Noise <sup>a</sup> |
|-------|-------------|--------------------------|
| 1     | 5.24        | 2.74                     |
| 2     | 5.59        | 2.74                     |
| 3     | 10.23       | 3.97                     |
| 4     | 8.70        | 3.97                     |
| 5     | 5.60        | 3.63                     |
| 6     | 4.20        | 3.63                     |
| 7     | 3.84        | 1.85                     |
| 8     | 4.25        | 1.85                     |
| 9     | 5.42        | 1.37                     |
| 10    | 5.42        | 1.37                     |

Table 4: Sample standard deviation of the cumulated price movement and the cumulated noise traders' sentiment.

<sup>a</sup>Observe that the standard deviation of the noise traders' sentiment is identical for groups 1 and 2, 3 and 4, 5 and 6, 7 and 8, 9 and 10, since these groups were in the same session and hence played with the same realization of the die.

| Group | VR(q) <sup>a</sup> |                 |                 |                 |
|-------|--------------------|-----------------|-----------------|-----------------|
|       | q = 10             | q = 20          | q = 25          | q = 50          |
| 1     | 7.01<br>(0.000)    | 5.26<br>(0.000) | 2.71<br>(0.001) | 4.50<br>(0.000) |
| 2     | 6.59<br>(0.000)    | 4.68<br>(0.000) | 2.41<br>(0.006) | 4.56<br>(0.000) |
| 3     | 2.98<br>(0.000)    | 2.39<br>(0.003) | 2.77<br>(0.001) | 2.50<br>(0.031) |
| 4     | 2.98<br>(0.000)    | 2.40<br>(0.002) | 2.61<br>(0.002) | 2.20<br>(0.067) |
| 5     | 3.82<br>(0.000)    | 1.96<br>(0.027) | 0.92<br>(0.445) | 0.78<br>(0.393) |
| 6     | 5.76<br>(0.000)    | 2.64<br>(0.000) | 0.96<br>(0.474) | 2.27<br>(0.057) |
| 7     | 5.07<br>(0.000)    | 1.07<br>(0.444) | 0.35<br>(0.124) | 0.43<br>(0.240) |
| 8     | 4.76<br>(0.000)    | 0.94<br>(0.451) | 0.19<br>(0.073) | 0.25<br>(0.176) |
| 9     | 4.57<br>(0.000)    | 1.57<br>(0.126) | 0.97<br>(0.479) | 0.24<br>(0.174) |
| 10    | 4.57<br>(0.000)    | 1.57<br>(0.126) | 0.97<br>(0.479) | 0.24<br>(0.174) |

Table 5: Variance ratio of the cumulated price movement for different  $q$ th differences (p-values in brackets).

<sup>a</sup>The test statistic  $z(q) = \sqrt{nq}(\text{VR}(q) - 1) \left( \frac{2(2q-1)(q-1)}{3q} \right)^{-1}$  is asymptotically  $N(0, 1)$  (see Lo and MacKinlay, 1988).

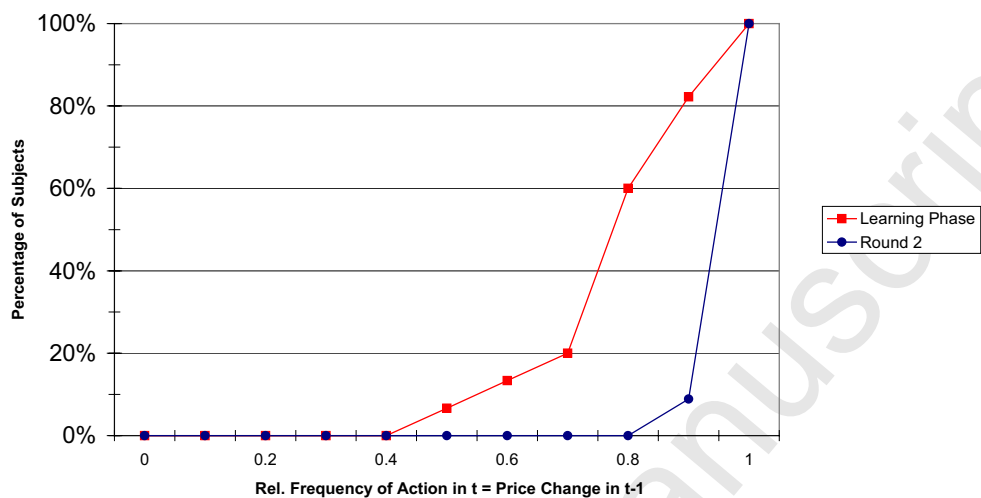


Figure 1: Cumulative distribution of the switch strategy.

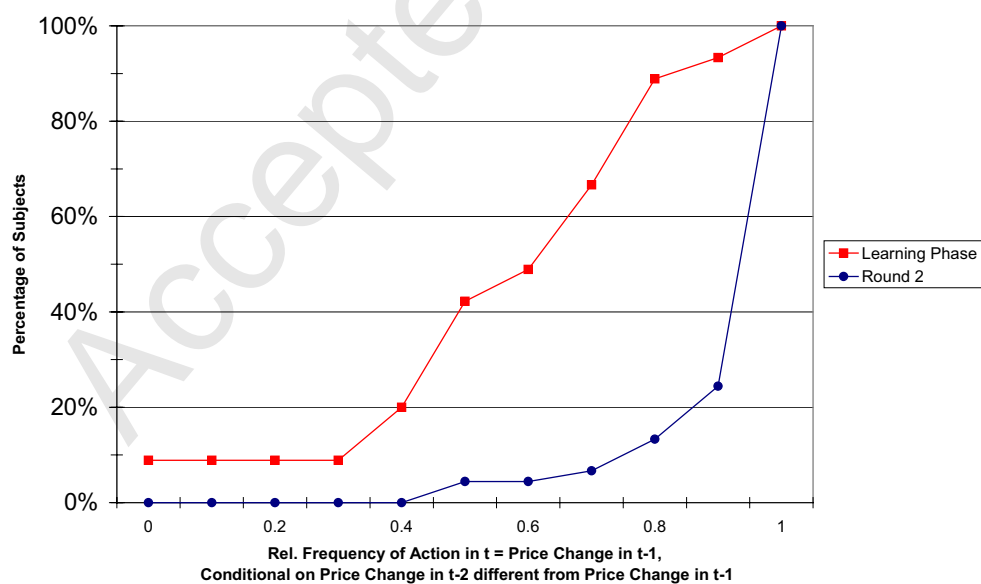


Figure 2: Cumulative distribution of the switch strategy in periods following a change in the price movement.

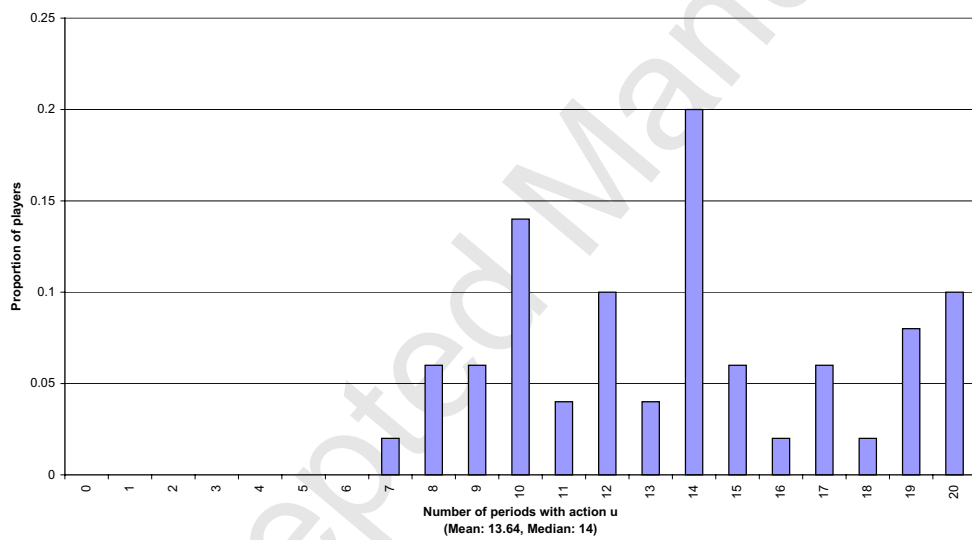


Figure 3: Distribution of the number of  $u$  choices for all players in the first 20 periods of round 1.



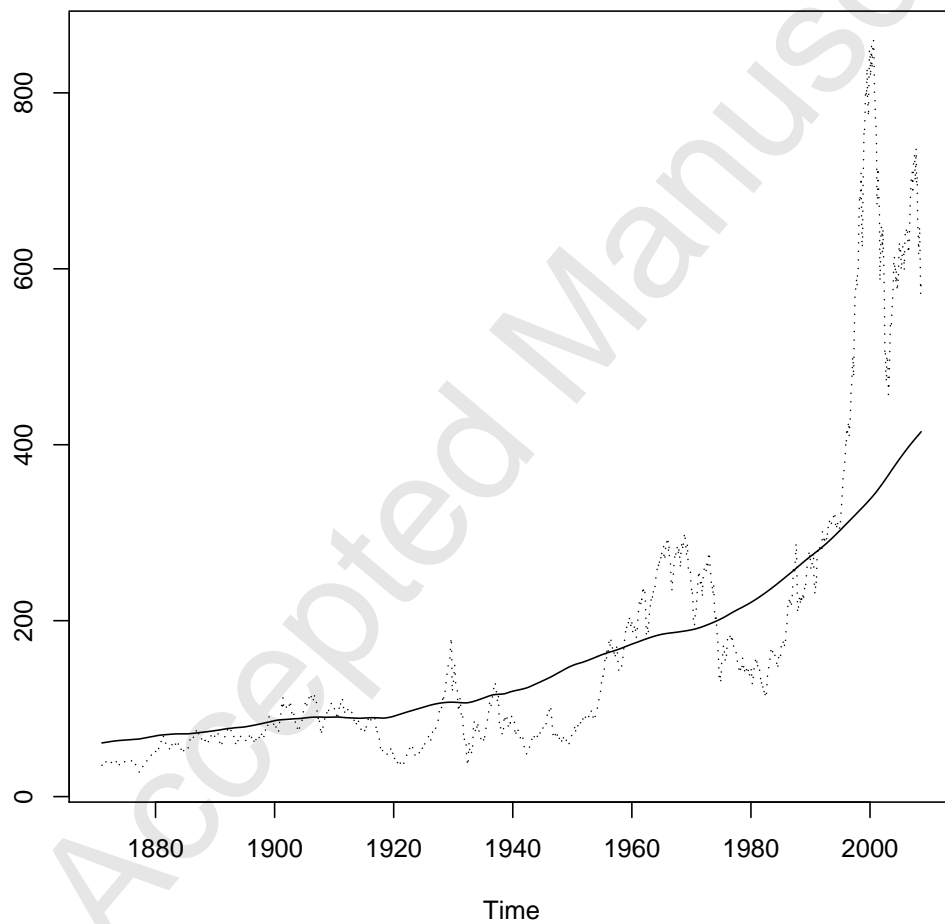


Figure 4: Actual real stock price (dotted line) and perfect foresight stock price (solid line) computed with a 5% discount rate (US data 1871-2008).

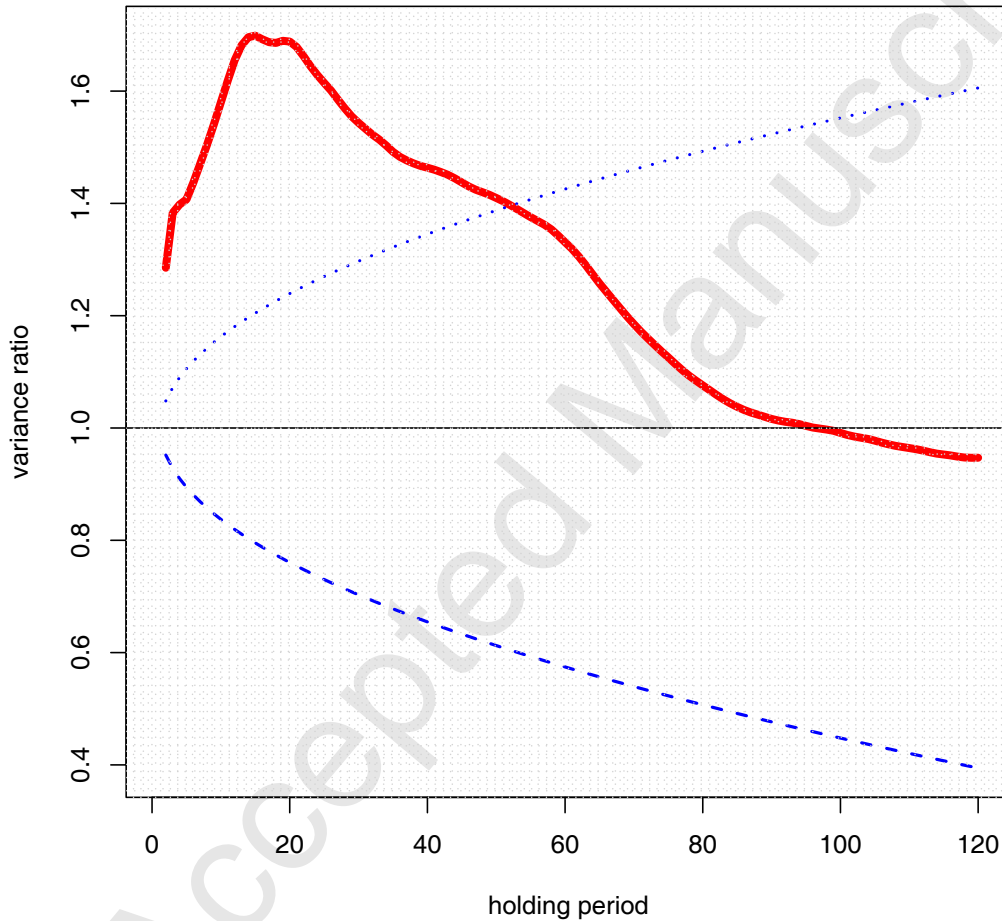


Figure 5: Variance ratios of the log-S&P 500 for various horizons (measured in months, solid line) and confidence band (dotted and dashed lines) for the null-hypothesis that log-S&P 500 follows a random walk, i.e.  $\log P_t = \log P_{t-1} + \epsilon$ .

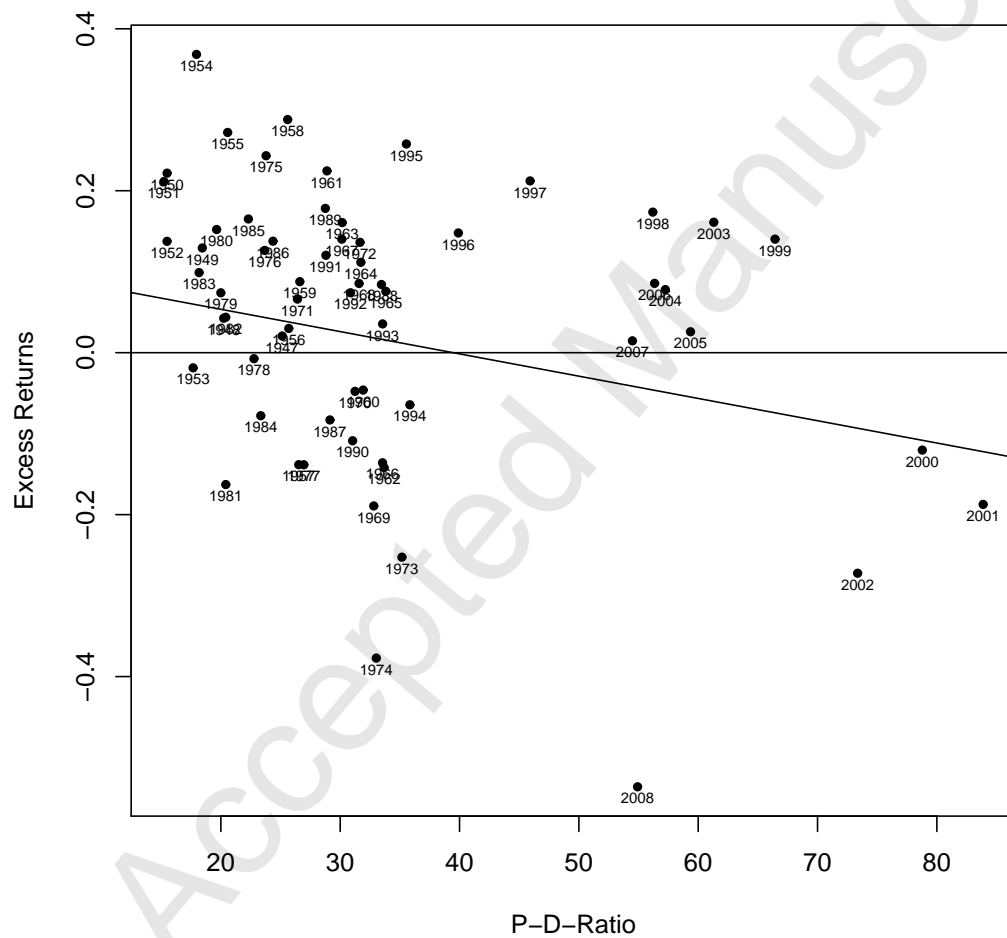


Figure 6: Annual real return of S&P 500 in excess of risk free rate as of price-dividend ratio at the end of the previous year (monthly US data 1871-2008).