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# Survival at the Center – The Stability of Minimum Differentiation\*

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## Abstract

We model a Hotelling market with multidimensional product differentiation in an evolutionary framework. Both evolutionary stability (in the sense of Schaffer, 1989) and stochastic stability (following Kandori et al., 1993, and Young, 1993) are analysed. It is shown that firms move towards the center in product space, i.e. a “principle of minimum differentiation” on all dimensions of the product space applies.

**Keywords:** Hotelling; evolutionary game theory; product differentiation; location choice

**JEL Codes:** C73; L1; R3

## 1 Introduction

Since the seminal paper by Hotelling (1929) on horizontal product differentiation, there has been a long debate in the literature about just how much competitive firms will differentiate their products. While Hotelling argued (later shown to be wrong) that a "principle of minimum differentiation" applies, D'Aspremont et al. (1979) proved that under some assumptions

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firms will differentiate fully. Irmen and Thisse (1998) extended the standard model from one to some arbitrary but finite number of product characteristics. Whenever, from the consumers' point of view, a salient characteristic exists, firms will differentiate maximally in this characteristic and choose minimum differentiation in the others. We contribute to this literature by showing that economic evolutionary forces will lead to minimum product differentiation with regard to all characteristics, independent of the salience of product characteristics.

By now there exists a small literature which investigates economic evolutionary approaches to industrial organization. This literature was pioneered by Hansen and Samuelson (1988) and Schaffer (1989), who were the first to illustrate the economic evolutionary advantage of certain types of non-profit maximizing behavior in Cournot oligopoly. In evolutionary equilibrium, firms produce the Walrasian equilibrium quantity (Schaffer, 1989). Later on, Vega-Redondo (1997), Alós-Ferrer et al. (1999), and Schenk-Hoppé (2000) provided dynamic underpinnings for the emergence of Walrasian behavior in economic evolutionary models of Cournot oligopoly. The empirical relevance of this finding was confirmed through experiments (see e.g. Huck et al., 1999, 2000). Subsequent contributions examined economic evolutionary models of Bertrand oligopoly (Alós-Ferrer et al., 2000), insurance markets (Ania et al., 2002), and Bertrand oligopoly with consumer sluggishness (Hehenkamp, 2002). What the literature on economic evolutionary approaches to industrial organization has in common is that underperforming firms revise and adjust their behavior (e.g. by copying more successful firms) and that, occasionally, firms experiment with new products, quantities or prices.

In the context of product differentiation, the basic trade-off from a firm's point of view between more and less product differentiation is that more differentiation lowers price competition while moving in product space closer to the competitor leads to an increase in demand, as a firm "steals" customers away from its competitor. This trade-off is usually analyzed in the basic Hotelling model where customers are uniformly distributed along a line, two firms are located on this line, and customers face transportation costs. D'Aspremont et al. (1979) show that if these transportation costs are quadratic in distance, then firms will differentiate fully (in a game where firms choose location first and then enter into price competition). Economides (1986) generalizes the degree of convexity of the transportation costs. If transportation costs are not too convex, then firms will locate more towards the middle of the line, although minimum product differentiation never occurs. In another paper, Economides (1984) shows that if customers' willingness to pay is relatively small, then even with linear transportation costs firms will tend to move away from each other to achieve 'local monopolistic

positions'. Bester (1989) modifies the basic model by assuming that firms negotiate with customers over prices. In the limit of small transportation costs, firms will again try to differentiate as far as possible. A more recent literature (Tabuchi and Thisse, 1995; Anderson et al., 1997, see also Neven, 1986) relaxes the assumption of a uniform distribution. If the distribution is non-uniform but still symmetric, depending on the degree of concavity of the distribution density, either symmetric or asymmetric equilibria exist, which neither show maximum nor minimum product differentiation. However, as shown by Anderson et al., product differentiation is always excessive compared to the social optimum. Thus models with price competition seem to induce too much product differentiation, at least as long as transportation costs are sufficiently convex.

In the present paper, we investigate an economic evolutionary model of horizontal product differentiation, where the firms' choice of product characteristics is subject to economic evolution. *Economic evolution* consists of two parts, economic selection and economic mutation. *Economic selection* captures that badly performing firms withdraw their product from the market and replace it by a new one; firms with above-average performance leave their product on the market. *Economic mutation* reflects random factors that, independent of economic performance, lead to new products every once in a while. This random influence can be direct (at the product level) or indirect, e.g. via the appointment of new product managers.

Introducing an economic evolutionary mechanism into Irlen and Thisse's version of the Hotelling model has bearings on the trade-off between more and less product differentiation. What matters for economic (and biological) evolution is *relative* payoff. Moving closer to the competitor in product space has two effects on sales, both of which are beneficial for relative payoff: On the one hand, own sales are enhanced and, on the other hand, one's competitor's sales are reduced. Clearly, price competition gets more intense. The competitor will reduce his price to gain back (at least some) part of his lost demand. One's own price has to be lowered as well. It follows that the overall effect on relative profit will depend on the *relative* size of prices (after the product change).

It will be shown that it is always the firm closest to the center that will charge the highest price, while still facing the larger demand. This implies that the firm closest to the center earns higher relative payoff. From this it follows that minimum differentiation along all characteristics constitutes the unique economic evolutionary equilibrium.

What lies at heart of our observation is the so-called spite effect (Hamilton, 1970). Consider a Nash equilibrium with maximum differentiation along the salient product characteristic and minimum differentiation along the oth-

ers. Starting from there, a firm moving closer to its competitor reduces its own absolute profit, but reduces its competitor's profit even more. In consequence, relative payoff is increased giving the evolutionary advantage of higher fitness (as evolutionary biologists call it).

The spite effect has first been identified in the biological context (Hamilton, 1970). Hansen and Samuelson (1988) and Schaffer (1988, 1989) were the first to exploit the spite effect in economic models, viz. in oligopolistic models of quantity competition. What connects the two seemingly disparate fields is the interaction of few individuals (e.g. animals, plants or firms). To allow for investigation of these types of interaction, Schaffer generalizes the concept of evolutionary stability – developed by evolutionary biologists Maynard Smith and Price – from infinite to small and finite populations.<sup>1</sup> In contrast to Maynard Smith and Price's notion of evolutionary stability, Schaffer's (1988) concept allows to address interactions by few individuals who all interact with each other simultaneously. This is the relevant case for oligopolistic competition. Observe that in environments of small and finite populations relative performance will be determined by *relative payoff*.

The insight gained by using the concept of evolutionary stability even applies for the case of two firms only, which is our focus in this paper. In effect, Schaffer (1988) used the two firms example to introduce his concept. One economic interpretation he provides is that players in the game correspond to managers of the firms. Each firm is in the industry permanently, but managers can be hired and fired. Now a manager pursuing a strategy which maximizes the relative payoff of the firm is more likely to survive.

As the basis of our analysis we use Irmen and Thisse's generalization of the Hotelling model in Section 2. Following the approach by Schaffer (1989), in Section 3, we first establish *evolutionary stability* of minimum differentiation along all characteristics at the center. In Section 4, we pursue the dynamic analysis following Vega-Redondo (1996, 1997) and show that the center product is *stochastically stable* for payoff-monotonic processes of economic selection. Section 5 concludes.

## 2 The model

Our setup follows Irmen and Thisse (1998). Two firms compete in products and prices. Competition is in two stages. At stage one, both firms choose a

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<sup>1</sup>See Schaffer (1988) as well as Maynard Smith (1974, 1982) and Maynard Smith and Price (1973). Neill (2004) proposes a concept for the intermediate case of "large, but finite populations" and explores its connections to the concepts by Schaffer and by Maynard Smith and Price, respectively.

product simultaneously. At stage two, firms pick prices, given the products from stage one. The survival of firms is assumed to depend on their choice of products. Accordingly, we investigate the economic evolution of product choices.

More specifically, the two firms, labelled  $A$  and  $B$ , choose their products,  $a$  and  $b$ , from the space of feasible products  $X = [0, 1]^n$ , i.e.  $a, b \in X$ . Each product is characterized by its  $n$  product characteristics, indexed by  $i \in \mathcal{N} = \{1, \dots, n\}$ . Correspondingly, firm  $A$ 's (resp.  $B$ 's) choice of product characteristics is summarized by  $a = (a_1, \dots, a_n)$  (resp.  $b = (b_1, \dots, b_n)$ ) and  $c = (\frac{1}{2}, \dots, \frac{1}{2})$  denotes the center of the product space. Any product variant can be produced at the same constant marginal cost, which we normalize to zero without loss of generality.

A continuous mass of consumers is distributed over the unit hypercube  $X$  according to some nonnegative continuous probability density function  $g(x)$ . Following Irmen and Thisse (1998), we assume  $g(x)$  to represent the uniform distribution on  $X$ . The mass of consumers is normalized to 1. A consumer located at  $x \in X$  draws utility

$$V_A(x) = S - p_A - \sum_{i=1}^n t_i (x_i - a_i)^2$$

from consuming product  $a$  given its price  $p_A$  and given the salience coefficients  $t_i > 0$ ,  $i \in \mathcal{N}$ , of the different product characteristics.

A similar expression applies if consumer  $x$  buys from firm  $B$ . The gross surplus from consuming any product is assumed sufficiently high so that every consumer buys the product either from firm  $A$  or from firm  $B$ .

All this being said, the demand of firm  $A$  can be expressed as

$$D_A = \int_{\{x \in X: V_A(x) \geq V_B(x)\}} g(x) dx.$$

To simplify this expression, we determine the set of indifferent consumers, which are characterized by

$$p_A + \sum_{i=1}^n t_i (x_i - a_i)^2 = p_B + \sum_{i=1}^n t_i (x_i - b_i)^2. \quad (1)$$

Substituting  $K_i = 2t_i(b_i - a_i)$  for  $i \in \mathcal{N}$ ,  $K = (K_1, \dots, K_n)$ , and  $L = p_B - p_A + \sum_{i=1}^n t_i (b_i^2 - a_i^2)$ , we rewrite equation (1):

$$K \cdot x = L,$$

where " $\cdot$ " denotes the inner product of  $\mathbb{R}^n$ . Assuming  $a_i \leq b_i$  for all  $i \in \mathcal{N}$ , we can express the demand of firm  $A$  as

$$D_A = \int_{\{x \in X: K \cdot x \leq L\}} dx.$$

Geometrically, this demand represents the volume of the intersection of a linear half-space with the unit hypercube. A formula to compute this volume has been derived by Kendall (1961, pp. 48-50) or Barrow and Smith (1979). For our sake the version by Marichal and Mossinghoff (2008) is most convenient. Three cases have to be distinguished. First, in case of  $K_i \neq 0$  for all  $i \in \mathcal{N}$ , one obtains

$$D_A = \frac{1}{n! \prod_{i=1}^n K_i} \sum_{V \subseteq \mathcal{N}} (-1)^{|V|} (L - K \cdot 1_V)_+^n, \quad (2)$$

where  $|V|$  denotes the number of elements in  $V \subseteq \mathcal{N}$ , where  $1_V$  represents the characteristic vector of  $V$  in  $\{0, 1\}^n$ , and where  $(\alpha)_+^n := (\max\{\alpha, 0\})^n$  is the positive part of the functional expression  $\alpha \in \mathbb{R}$  raised to the power  $n$ . The right hand side of (2) has  $n - 1$  continuous derivatives as a function of  $L$ . It follows from (2) that  $\partial L / \partial p_A = -\partial L / \partial p_B$  implies  $\partial D_A / \partial p_A = -\partial D_A / \partial p_B$ . For  $n = 1$ , equation (2) reduces to  $D_A = \max\{0, \min\{L/K_1, 1\}\}$ , which is differentiable with respect to  $L$  on the interior of  $[0, K_1]$ . Second, if  $K_i = 0$  for some, but not all  $i$ , then equation (2) has to be adapted so as to ignore these dimensions (see Barrow and Smith, 1979). Finally, if  $K_i = 0$  for all  $i \in \mathcal{N}$ , then both firms choose identical locations and Bertrand competition results: The firm charging the strictly lowest price faces full demand; if both firms charge identical prices, the market is shared equally.

For any given pair of products,  $(a, b)$ , there exists a unique Nash equilibrium in prices,  $(p_A^*(a, b), p_B^*(a, b))$  (see Irlen and Thisse, p. 80). Equilibrium prices determine (2nd stage) equilibrium demand  $D_j(a, b)$  and (2nd stage) equilibrium profit  $\pi_j(a, b) = D_j(a, b) p_j^*(a, b)$ , for each of the two firms  $j = A, B$ .<sup>2</sup> These are the profit functions relevant at stage one. Notice that, by symmetry of the market, equilibrium prices and hence stage-one profits are symmetric between firms, i.e.  $p_A^*(a, b) = p_B^*(b, a)$  and  $\pi_A(a, b) = \pi_B(b, a)$  for all  $(a, b) \in X^2$ . Moreover, it can be easily shown that  $D_A > D_B$  if and only if  $p_A^* > p_B^*$ .

<sup>2</sup>In an earlier working paper version of this article, we additionally show that our main results would not change if we used evolutionary equilibrium prices instead (Hehenkamp and Wambach, 2009, available upon request).



### 3 Evolutionary stability of minimum differentiation

Our first result identifies the center product as the unique evolutionarily stable product in the Hotelling framework. In defining evolutionary stability, we extend Schaffer's (1988) concept, allowing for asymmetric product profiles to be evolutionarily stable. Relying on Schaffer's original concept, we would otherwise restrict attention to symmetric profiles so that minimum differentiation would be implicitly imposed.

**Definition 1 (Evolutionary stability)** *Define the relative payoffs of firms A and B by  $\Phi_A(x, y) = \pi_A(x, y) - \pi_B(x, y) = -\Phi_B(x, y)$ , where  $x \in X$  and  $y \in X$  denote the products sold by firms A and B, respectively. Then we call a product profile  $(x^*, y^*)$  (strictly) evolutionarily stable if and only if*

$$\begin{aligned} \Phi_A(x, y^*) &\leq (<) \Phi_A(x^*, y^*) && \text{for all } x \neq x^* \text{ and} \\ \Phi_B(x^*, y) &\leq (<) \Phi_B(x^*, y^*) && \text{for all } y \neq y^*. \end{aligned}$$

We denote the set of all evolutionarily stable product profiles by  $X^{FESS}$ .

A product profile  $(x^*, y^*)$  is evolutionarily stable if neither firm A nor firm B can unilaterally generate strictly higher *relative* profit with some alternative product  $x$  (resp.  $y$ ) given the other firm sticks to product  $y^*$  (resp.  $x^*$ ). Any situation with non-maximal relative profits is inherently unstable as it gives the firm with higher relative profit an economic evolutionary advantage over the other that will eventually have consequences (e.g. better access to capital that ensures survival in an economic downturn). Expressed in evolutionary terms, no mutant product  $x$  (resp.  $y$ ) can successfully invade the population  $(x^*, y^*)$  consisting of incumbent products  $x^*$  and  $y^*$ . By symmetry of the stage-one profit functions, the second inequality is equivalent to the first for symmetric profiles. Moreover, for symmetric profiles, the above concept and Schaffer's original concept are equivalent in the sense that the same product is identified as evolutionarily stable.

**Theorem 1** *The center  $(c, c)$  represents the unique evolutionarily stable product profile, i.e.  $X^{FESS} = \{(c, c)\}$ . Moreover, the center  $(c, c)$  is strictly evolutionarily stable.*

In the evolutionary equilibrium of product choices thus both firms differentiate minimally, producing and selling center products. The intuition runs as follows: Consider a product profile of maximum differentiation, e.g.

$(a, b) = ((0, \dots, 0), (1, \dots, 1))$ , and suppose firm A moves closer towards firm B. This has two potentially opposite effects. First, taking prices as given, firm A steals away demand of firm B. This increases firm A's relative profit. Second, price competition becomes fiercer and leads to a reduction in prices for both firms. The key is now to see that firm A, moving closer to the center, will cause a stronger price reduction on part of firm B and that this lower price does not compensate for the first effect. Accordingly, firm A is left with the larger share of demand so that firm A's relative profit is increased in the total of the two effects.

**Proof.** By symmetry, we have  $\Phi_A(c, c) = \Phi_B(c, c) = 0$ . We establish that (i)  $c$  is the unique maximizer of  $\max_z \Phi_A(z, c)$  (which shows strictness) and that (ii) no other product profile  $(a, b) \neq (c, c)$  is evolutionarily stable (which proves uniqueness).

**Part (i):** *Profile  $(c, c)$  is strictly evolutionarily stable.*

Rewrite the relative profit of firm A as

$$\Phi_A = p_A(D_A - D_B) + (p_A - p_B)D_B.$$

and recall that  $D_A > D_B$  if and only if  $p_A^* > p_B^*$ . Accordingly, to establish  $\Phi_A(z, c) < \Phi_A(c, c)$ , it is sufficient to show that  $p_B^*(z, c) > p_A^*(z, c)$  holds for any  $(z, c)$  with  $z \neq c$ .

Consider any product  $a \neq c$  and suppose to the contrary that  $p_B^*(a, c) \leq p_A^*(a, c)$ . Without loss of generality assume that  $a \leq c$ , i.e.  $a_i \leq 1/2$  for all  $i \in \mathcal{N}$ , with strict inequality for at least one  $i_0$ . Set  $L(a, b; p_A, p_B) \equiv p_B - p_A + \sum_{i=1}^n t_i (b_i^2 - a_i^2)$  and  $L(a, b) \equiv \sum_{i=1}^n t_i (b_i^2 - a_i^2)$  to obtain  $L(a, b) = (\sum_{i=1}^n (b_i + a_i) K_i) / 2$  and hence  $L(a, c) < K \cdot c$ . It follows that

$$L(a, c; p_A^*(a, c), p_B^*(a, c)) \leq L(a, c) < K \cdot c$$

and hence

$$\begin{aligned} X_1 &:= \{x \in X : K \cdot x \leq L(a, b; p_A^*(a, c), p_B^*(a, c))\} \\ &\subset X_2 := \{x \in X : K \cdot x \leq K \cdot c\}, \end{aligned}$$

where the inclusion is proper because of  $c \notin X_1$ . Moreover, the difference set  $X_2 \setminus X_1$  has positive measure, which implies that

$$D_A(a, c) = \int_{X_1} dx < \int_{X_2} dx = D_A(c, c) = \frac{1}{2}.$$

Notice that the second and third equality hold because the hyperplane  $K \cdot x = K \cdot c$  runs through the center of the unit hypercube and divides it into two

parts of equal volume. Because of  $D_B = 1 - D_A$ , we obtain  $D_A(a, c) < D_B(a, c)$  in contradiction to  $p_B^*(a, c) \leq p_A^*(a, c)$ . Thus, the center profile  $(c, c)$  is strictly evolutionarily stable.

**Part (ii):** *No other profile than  $(c, c)$  is evolutionarily stable.*

Observe that, by the zero-sum nature of relative payoff, strict evolutionary stability of  $(c, c)$  implies

$$\Phi_A(c, z) > \Phi_A(c, c) \quad \forall z \neq c. \quad (3)$$

Now, suppose some other profile  $(a, b) \neq (c, c)$  were evolutionarily stable. First, consider the case  $b \neq a = c$ . By part (i), we have  $\Phi_B(a, b) < \Phi_B(c, c) = 0$ . That is, firm B can strictly increase relative payoff by switching to the center product, which violates the evolutionary stability of  $(a, b)$ . The case  $a \neq b = c$  can be handled similarly.

Second, consider  $a \neq c$  and  $b \neq c$ . On the one hand, if  $\Phi_A(a, b) \leq 0$  then  $\Phi_A(c, b) > \Phi_A(c, c) = 0 \geq \Phi_A(a, b)$ , where the first inequality follows from (3). On the other hand, if  $\Phi_A(a, b) = -\Phi_B(a, b) > 0$ , then  $\Phi_B(a, c) > \Phi_B(c, c) = 0 > \Phi_B(a, b)$ . In both cases, a firm strictly raises relative payoff by unilaterally switching to the center product and, therefore, the profile  $(a, b)$  is not evolutionarily stable. ■

## 4 Stochastic stability of minimum differentiation

The Hotelling game of product choices is now played recurrently between the two firms. The outcome of each period is characterized by a state  $\omega$  that collects the product choices of the two firms, i.e.  $\omega = (a, b)$ . For technical convenience, we consider a discretized set of feasible products,  $X_\delta = \{0, \frac{1}{m}, \frac{2}{m}, \dots, \frac{m-1}{m}, 1\}^n$ , where  $\delta = 1/m$  represents the grid size of each product characteristic. The corresponding state space is given by  $\Omega = X_\delta^2$ . To ensure that minimum differentiation at the product center is feasible,  $(c, c) \in \Omega$ , we assume  $m/2 \in \mathbb{N}$ .<sup>3</sup>

Our process of economic evolution consists of two parts, the process of economic selection and the process of (economic) mutation. To define the former, let  $\omega = (a, b) \in \Omega$  be the current state, let  $s(\omega) \in \Omega$  be the direct successor to  $\omega$  and let  $\pi_j(\omega)$  denote the equilibrium profit of firm  $j$  in state  $\omega$ . Then we assume: (1) If  $\pi_A(\omega) = \pi_B(\omega)$  then  $s(\omega) = \omega$ . (2) If  $\pi_A(\omega) <$

<sup>3</sup>We could let the grid size differ between product characteristics, i.e. make  $m$  contingent on  $i$ . This would have no effect on our result as long as the center of each characteristic belongs to the grid, i.e.  $m_i/2 \in \mathbb{N}$ .

$\pi_B(\omega)$  then all transitions from  $\omega$  to  $\omega' = (a', b)$ , where  $a' \in X_\delta$  is arbitrary, have positive probability and all transitions from  $\omega$  to  $\omega' = (a', b')$  such that  $b' \neq b$  have zero probability. (3) If  $\pi_A(\omega) > \pi_B(\omega)$  a symmetric definition applies. That is, any firm earning highest profit sticks to its product in the following period; any firm earning strictly lower profit will experiment and pick an arbitrary product with positive probability (with full support on the set of feasily products).<sup>4</sup>

The process of mutation requires each firm to pick some arbitrary product  $x \in X_\delta$ , with small, but positive probability  $\varepsilon > 0$ . This probability is assumed to be independent across firms, states, and periods.

The overall process of economic selection and mutation has a unique stationary distribution,  $\mu(\varepsilon)$ , which describes the long-run frequencies with which every state is observed along any sample path, with probability one. A state  $\omega \in \Omega$  is called *stochastically stable* if the corresponding component of the limit distribution  $\mu^* = \lim_{\varepsilon \rightarrow 0} \mu(\varepsilon)$  is positive, i.e. if  $\mu_\omega^* > 0$ .

Our second result establishes that any process of economic evolution covered by our assumptions selects a unique stochastically stable state where both firms differentiate minimally and pick the center product.

**Theorem 2** *The unique stochastically stable state exhibits minimum differentiation with regard to all characteristics, i.e.  $a = b$ . Both firms locate their product at the center of the product space, i.e.  $a = b = c$ .*

**Proof.** First, the center  $(c, c)$  can be reached by at most one mutation from every other action pair  $(a, b)$ . This follows from part (ii) in the proof of Theorem 1. Second, at least two mutations are required to leave  $(c, c)$ , since one mutation alone would always bring the system back to  $(c, c)$ . This follows from part (i) in the proof of Theorem 1. ■

Theorem 2 establishes a link between strict evolutionary stability and stochastic stability in the technically asymmetric Hotelling model. Alós-Ferrer and Ania (2005) find a similar link for general symmetric  $N$ -player games with finite strategy space when players' actions evolve according to imitation and mutation.

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<sup>4</sup>This definition is in line with customary formulations of economic evolutionary selection (see e.g. Kandori et al., 1993; Rhode and Stegeman, 1996; Robson & Vega-Redondo, 1996; or Vega-Redondo, 1997). It is most closely related to Sloth and Whitta-Jacobsen (2006, p. 8) albeit their Assumption 2 does not allow to investigate the Hotelling framework. Notice that we abstain from investigating processes of imitation, since our main result of minimum differentiation would not come as a surprise for this important class of selection processes. We also do not consider introspection (Bergin and Bernhardt, 2004) where agents compare their payoff from their own current and past actions.

## 5 Conclusion

We analyze a Hotelling market with product differentiation in an economic evolutionary framework. In a dynamic model, firms act by offering a different product when they realize that they obtained less profit than their competitor. Occasionally, they experiment by offering new products. It is shown that under these assumptions firms move to the center of product space, i.e. a “principle of minimum differentiation” as stated by Hotelling in his original work applies.

This result holds because in an economic evolutionary context firms concentrate on relative profits rather than absolute profits. Now, by moving closer in product space towards their competitors, firms steal business away from their competitor which makes them relatively better off, although the decrease in prices due to more competition harms both firms.

For our analysis we assumed that in stage two, after making their product choice, firms compete in prices a la Nash. In a pure economic evolutionary approach to horizontal product differentiation one might also consider price competition as subject to economic evolution. In an earlier working paper version of this article, we analyze this case (see Hehenkamp & Wambach, 2009, available upon request). It turns out that, independently of the firms’ products, economic evolution leads to prices that divide the market equally between the two firms. Then again, moving to the center is the unique economic evolutionary outcome.

A possible extension would be to allow for  $N$  firms in the market. If all firms are located on the Salop circle (Salop, 1979), then the evolutionarily stable outcome will be that all firms locate at equidistance, at least if firms compare themselves to their competitor with the highest payoff. This again implies some form of minimum differentiation, as any other distribution of locations on the circle has at least two neighboring firms being further away from each other. Equidistance is evolutionarily stable, as a firm moving away from its position will have a relatively smaller payoff compared to its neighbor from which it moved away, thus making any change in position unattractive.

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