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Active Robustness of a Milk Manufacturing Workshop with Time Constraints

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Active Robustness of a Milk Manufacturing Workshop with Time Constraints

Abstract

This paper deals with the active robustness of a milk manufacturing workshop including time interval constraints. Such systems have robustness properties towards time disturbances. The robustness property can be a static property, but using a dynamic control strategy may increase the robustness value.

Controlled P-time Petri nets are used as a modelling tool. Some definitions and a series of lemmas are cited in order to build a theory dealing with robustness problems. In order to avoid the death of marks at the synchronization transitions of the P-time Petri net model, a robust control strategy facing time disturbances is presented. The proposed approach tries to reject the disturbance as soon as it is observed. Furthermore, this approach is used by an algorithm computing an active robustness margin at a given node. It is illustrated step by step on an example of a milk manufacturing workshop. A comparison with the bound provided for the passive robustness is made.

1 Introduction

Manufacturing workshops with time constraints are Discrete Event Systems (DES), where a time validity interval is associated to each operation. Its lower limit indicates the minimum time necessary to execute the operation. The upper limit determines the maximum time not to exceed in order to ensure the quality of the manufactured products. Time windows constraints may be found in many industrial areas (Collart-Dutilleul et al. 2007, Kats et al. 2008, Dohn et al. 2009). For example in the food industry, there are minimum operating times which integrate the process limitations. These workshops are generally modeled by Controlled P-time Petri nets, to study the instants of beginning and ending for operations. Such systems have a robustness property towards time disturbances.

The robustness is defined as the ability of the system to preserve the specifications facing some expected or unexpected variations. The robustness is interpreted into different specializations (M'Halla 2010a, Jerbi 2006). The passive robustness is based upon variations included in validity time intervals. There is no control to preserve the required specifications. On the other hand, active robustness uses observed time disturbances to modify the control settings in order to keep on satisfying the specifications. Various strategies can be implemented to manage time disturbances in production systems (Jerbi et al. 2009, M'Halla et al. 2008, M'Halla et al. 2010b). In this paper, the control strategy corresponds to rejecting the disturbances by applying an inverse control loop: for a delay, sojourn times are set as small as possible; for an advance, sojourn times are as long as possible.

The work presented in this paper focuses on the robustness of a milk manufacturing workshop regarding time disturbances. This paper begins by modelling the workshop. Controlled P-time

Petri nets are used for this purpose. A functional decomposition of the P-time Petri net into three sets is done. The second section begins by giving some basic definitions concerning the robustness of a manufacturing workshop including time constraints. The passive robustness (local robustness) of a given path in the milk manufacturing workshop is analytically built up; an algorithm allowing computing a lower bound of the passive robustness margin is presented. Afterwards, the control problem in critical time manufacturing systems is tackled. An approach for the robust control is presented. The proposed strategy tries to reject the disturbance as soon as it is observed using the control in order to preserve the specifications facing some expected or unexpected variations (Collart-Dutilleul et al. 2007). This approach is used by another algorithm computing a bound for the active robustness margin. An illustrative example is outlined and the results are discussed. Finally, some conclusions of this work are given.

2 Modelling of a milk production unit

2.1 Modelling tools of DES integrating time constraints

2.1.1 P-time Petri nets

The following section presents the basic definitions concerning P-time Petri nets. The reader interested by a theoretical semantic analysis may read (Boyer and Roux 2010).

Definition 1. (Khansa et al. 1996): The formal definition of a P-time Petri net is given by a pair $\langle R; I \rangle$ where:

- R is a marked Petri net, defined by (P, T, I, O, M, M_0) where
 - P is a set of places
 - T is a set of transition
 - IN is the set of input arcs of transitions
 $IN: (P \times T) \rightarrow N$ (N is the set of natural numbers)
 - O is the set of output arcs of transitions
 $O: (T \times P) \rightarrow N$
 - M is the Marking application
 - M_0 is the initial value of the marking
- $IS: P \rightarrow Q^+ \times (Q^+ \cup \{+\infty\})$
 $p_i \rightarrow IS_i = [a_i, b_i]$ with $0 \leq a_i \leq b_i$.

IS_i defines the static interval of the staying time of a mark in the place p_i belonging to the set of places P (Q^+ is the set of positive rational numbers). A mark in the place p_i is taken into account in transition validation when it has stayed in p_i for a duration of at least a_i and at most b_i . After the duration b_i the token will be dead.

Définition 2 (Khansa et al. 1996):

At a given instant, a state is defined by a pair $\langle M, I \rangle$ such that:

- M is a marking application which assigns a given number of token to each place such that :
 $(\forall p \in P, M(p) \geq 0)$;
- I is an application "potential firing interval", which associates to each a token k in a place p_i an interval $[a_i^k, b_i^k]$. It is called dynamic interval in order to make a

difference with the static interval which is associated to each place. Dynamic intervals depend on arrival instants of tokens in the considered place. Let us assume that a token k arrives in the place p_i (which static interval is $[a_i, b_i]$) at a given instant c , at the instant $c+d$ ($a_i \leq d \leq b_i$) the dynamic interval of k becomes:

$$[a_i^k, b_i^k] = [\max(a_i - d, 0), b_i - d]$$

Firing condition of a transition from states

When the transition is enabled in the autonomous PN's sense, its firing interval depends first on the potential interval associated with the tokens which are in its input. Then according to these intervals, an interval of "potential" firing is associated to it. Afterwards, its "real" firing interval is established according to the potential intervals of the tokens whose places aren't input places of this transition.

The firing condition is formally expressed as follows:

A transition t_l is potentially firable from a state $S \langle M, IP \rangle$ in the interval $[a_l, b_l]$, if and only if:

- 1- it is enabled in autonomous PN's sense in this state : $\square \forall p \in {}^\circ t_l : m(p) \geq IN(p, t_l)$
- 2- $\forall p_i \in {}^\circ t_l$ there is a number of token greater than $IN(p_i, t_l)$ verifying $\bigcap_k [a_i^k, b_i^k] = [a_i^l, b_i^l]$ and $[a_i^l, b_i^l] \neq \emptyset$ where $k = 1, 2, \dots, I(p_i, t_l)$.
- 3- There must be no token in this place whose upper bound of the dynamic interval is strictly smaller than the lower bound of interval $[a_i^l, b_i^l]$. Otherwise there is a token that dies and consequently the net is said dead-tokens.

2.1.2 Controlled P-time Petri net

Definition 3. (Jerbi et al. 2004): The Controlled P-time Petri net is defined by a quadruplet $R_{pc} = (R_p, U, U_0)$ such that:

- R_p is a P-time Petri net which describes the opened loop system,
- U is the external control of the transitions (T) of R_p built on the predicates using the occurrence of internal or external observable events of the system: $U: T \rightarrow \{0, 1\}$,
- U_0 is the initial value of the predicate vector.

NOTATIONS

- p_i° (respectively ${}^\circ p_i$): the output transitions of the place p_i (the input transitions of the place p_i),
- q_{ie} : the expected sojourn time of the token in the place p_i ,
- q_i : the effective sojourn time of the token in the place p_i ,
- $St_e(n)$: the n^{th} expected firing instant of the transition t ,
- $St(n)$: the n^{th} effective firing instant of the transition t ,
- T_C : the set of controllable transitions.
- T_S : the set of synchronization transitions

2.2 Presentation of the workshop

Figure 1 shows a milk manufacturing unit composed of five machines (M_1, M_2, M_3, M_4, M_5) and six conveyors ($T_1, T_2, T_3, T_4, T_5, T_6$), where (M'Halla et al. 2010c):

– M_1 is a bottle filling machine,

- M_2 is a milk bottle capper,
- M_3 is a time/date stamp,
- M_4 is a labeling machine,
- M_5 is a packaging machine.

For simplicity, we disregard the nature of the precise operations performed in the milk production unit, and therefore we represent a model of a generic workshop.

To manufacture the products (bottles of 1000 ml), empty bottles are placed on the conveyor T_1 to supply the bottle filling machine M_1 . The filled bottles are transported towards the capping machine M_2 by the conveyor T_2 . After capping, the bottles arrive directly on T_3 . This conveyor carries the bottles to the machine M_3 (time/date stamp) to print the manufacturing date and end date of consumption. Once this task is completed, the bottles move towards the labelling machine M_4 via the conveyor T_4 .

The bottles are then transferred to the packaging machine M_5 , where they will be wrapped by welding in a group of 6. Lastly, the finished products are deposited on the conveyor T_6 towards the stock of finished products SA.

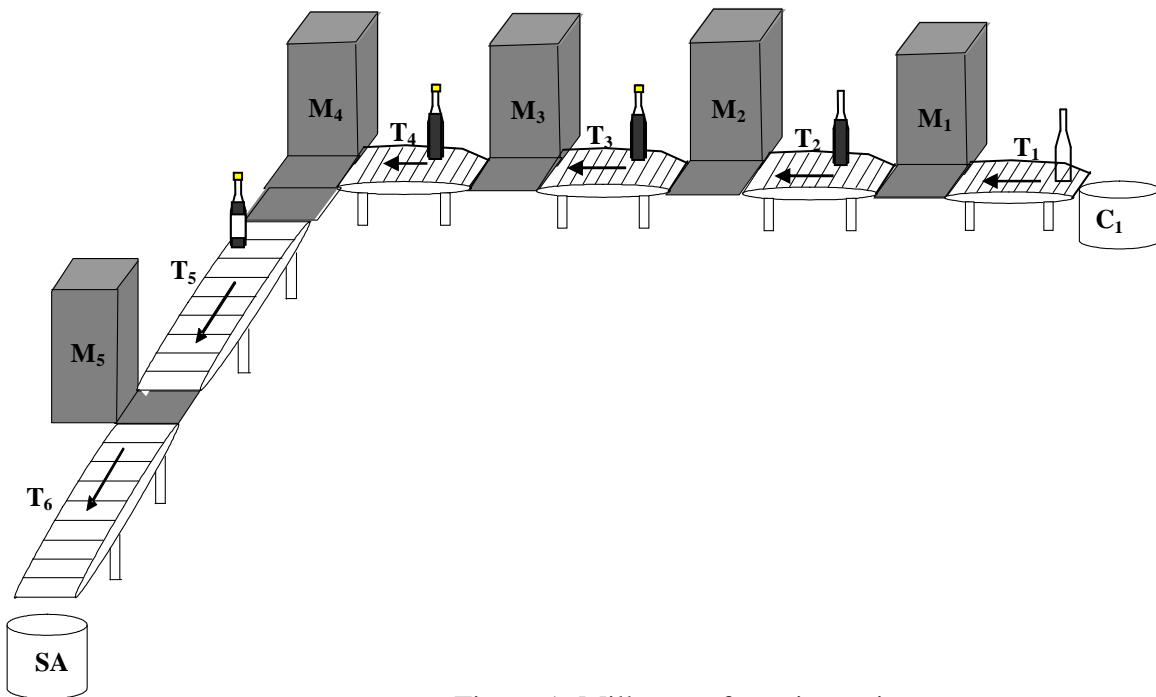


Figure 1: Milk manufacturing unit

2.3 Functional decomposition

As the sojourn times in places have not the same functional signification when they are included in the sequential process of a product or when they are associated to a free resource, a decomposition of the Petri net model into three sets is made using (Long et al. 93):

- R_U is the set of places representing the machines used,
- $Trans_C$ is the set of places representing the loaded transport resources,
- $Trans_{NC}$ is the set of places representing the unloaded transport resources (or the interconnected buffers).

This functional decomposition (or a similar one) has been used many times in the scientific literature in order to study control and regulation problems in manufacturing workshops. This is not a proof that it can be applied on all workshop topologies. We can only say that it corresponds to a good decomposition for many manufacturing systems.

Figure 2 shows a P- time PN (G) modelling a milk production workshop and its functional decomposition. The obtained G is used to study the robustness of the considered manufacturing workshop (M'Halla et al. 2010b).

3 Robustness of a milk manufacturing workshop

In this milk manufacturing workshop, a time interval is associated to each operation. Its lower bound indicates the minimum time needed to execute the operation. The upper bound sets the maximum time to not exceed otherwise the quality of the product is deteriorated (M'Halla et al. 2010a). When time constraints are violated in a workshop which is manufacturing food products, it can affect the health of consumers. Thus, the detection of a constraint violation must automatically cause the stop of the production and an elimination of dangerous products (an overheating of milk bottles in a hydromat for example).

On the other hand, when taking into account the system robustness, it can be proven that this type of violation has not occurred despite an incident happened in the production line. In this case, we plan to maintain the production while describing it as production in a degraded mode. Of course, the product is not degraded, but the production mode is degraded because the deliveries moments of the products are not those planned initially. In this context, the robustness of a given path in the milk manufacturing workshop is analytically built up. Two algorithms for building respectively, an upper bound of the passive robustness margin and the active robustness margin at a given node, are presented. The proposed algorithms allow increasing the ability of the system to fulfil the specifications facing some expected or unexpected variations.

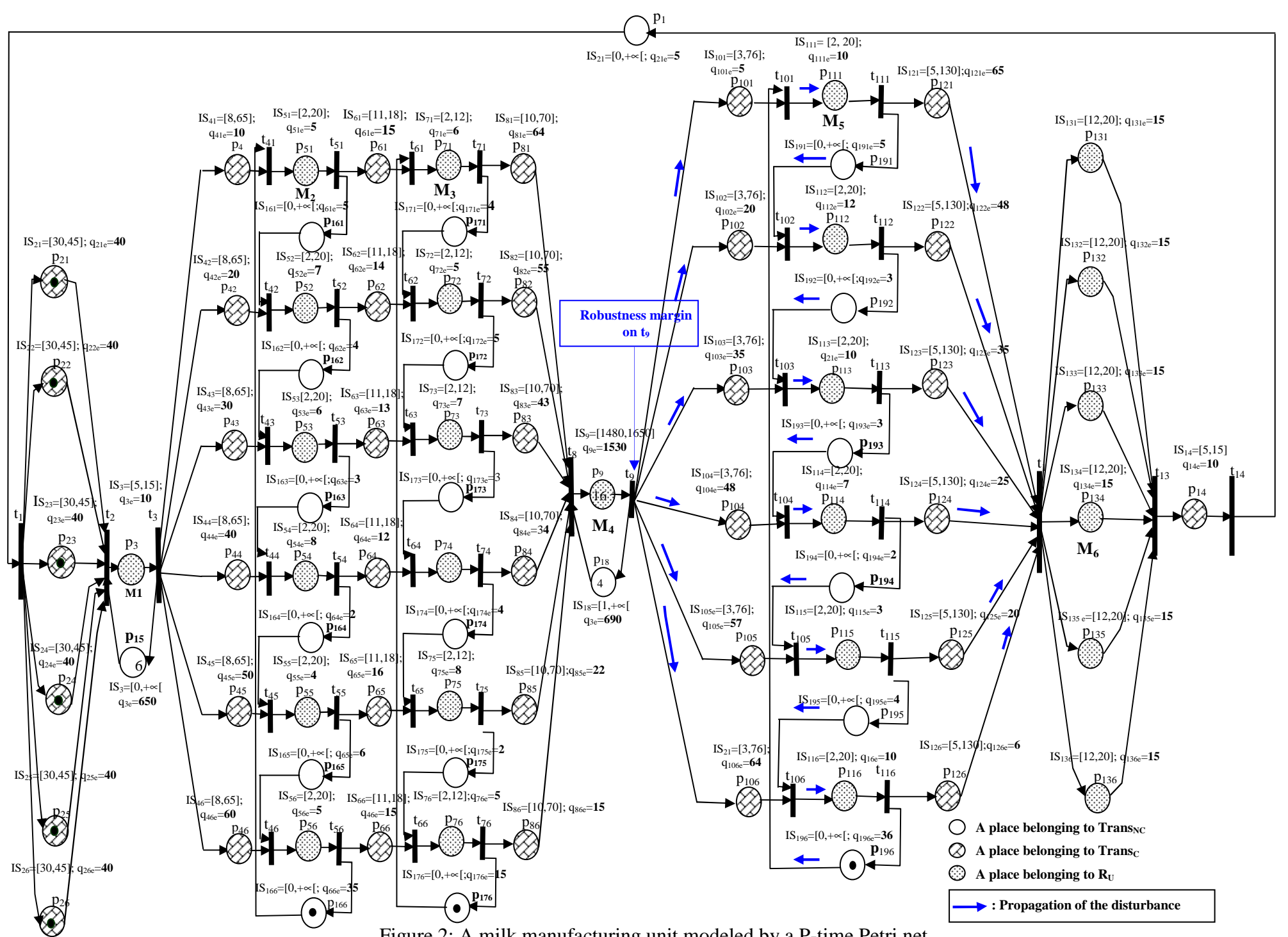


Figure 2: A milk manufacturing unit modeled by a P-time Petri net

3.1 Basic definitions

Preliminary definitions, useful for the rest of this paper, are given in order to study the robustness of a milk manufacturing workshop.

Definition 3. (Collart-Dutilleul et al. 2003): The robustness is defined as the ability of the system to remain valid with respect to the specifications when facing some expected or unexpected variations. The robustness characterizes the global capacity to deal with disturbances.

Definition 4 (Jerbi et al. 2004): A mono-synchronized subpath L_p is a path containing one and only one synchronization which is its last node.

Definition 5. (Jerbi et al. 2004): An elementary mono-synchronized subpath is a mono-synchronized subpath beginning with a place p such that $^{\circ}p$ is a synchronization transition.

3.1 Local passive robustness computations

3.1.1 Preliminary definitions

Definition 6. : Let us consider a discrete event system and G the associated Petri net model. Let us call $B(G)$ the behavior of G corresponding to the trajectory of states successively reached. Let $C(B(G))$ be the schedule of conditions established on the system behavior $B(G)$. $C(B(G))$ is materialized by a series of constraints which must be checked by $B(G)$. A non respect of $B(G)$ corresponds to a violation of $C(B(G))$. It is said that a subset SG of G is robust to a disturbance δ if and only if $\forall n \in SG$, the occurrence of the disturbance δ at the node n does not involve a violation of $C(B(G))$.

Definition 7. (Collart-Dutilleul et al. 2007): Passive robustness is when the required specifications are met without any need for a change in the control settings, even in the presence of variations.

Definition 8. (Jerbi et al. 2004): It is said that a path L_p has a local passive robustness on $[\delta_{\min}, \delta_{\max}]$ if the occurrence of a disturbance $\delta \in [\delta_{\min}, \delta_{\max}]$ at any place $p \in L_p$ does not involve a token death at the synchronization transitions of L_p .

3.1.2 Passive Robustness computation

To compute the local passive robustness interval, the concepts of compensable and transmissible margins are introduced. Let us denote by: Δr_{c_k} and Δr_{t_k} respectively the compensable margin and the transmissible margin on the mono-synchronized subpath L_{p_k} . The local passive robustness delay $\Delta r_{L_{p_k}}$ can be calculated using formulas (1), (2) and (3) (Jerbi et al. 2004).

With:

$$\Delta r_{L_{p_k}} = \Delta r_{c_k} + \Delta r_{t_k} \quad (1)$$

$$\Delta r_{c_k} = \sum_{p_i \in L_{p_k} \cap (R_N \cup \text{Trans}_{NC})} (q_{ie} - a_i) \quad (2)$$

$$\Delta r_{t_k} = \min_{\substack{p_i^o = \text{OUT}(L_{p_k}) \\ p_i \notin L_{p_k}}} (b_i - q_{ie}) \quad (3)$$

In table 1, we can highlight examples of the compensable margin and the transmissible margin associated to some mono-synchronized subpaths Lp_k of the P-time Petri net model (figure 2).

Table 1: Compensable margin and transmissible margin associated to some mono-synchronized subpaths of G

Path	Δt_k	Δc_k
$Lp_0=(p_9, t_9, p_{18}, t_8)$	6	689
$Lp_1=(p_9, t_9, p_{101}, t_{101})$	$+\infty$	0
$Lp_2=(p_9, t_9, p_{102}, t_{102})$	$+\infty$	0
$Lp_3=(p_9, t_9, p_{103}, t_{103})$	$+\infty$	0
$Lp_4=(p_9, t_9, p_{104}, t_{104})$	$+\infty$	0
$Lp_5=(p_9, t_9, p_{105}, t_{105})$	$+\infty$	0
$Lp_6=(p_9, t_9, p_{106}, t_{106})$	$+\infty$	0
$Lp_7=(p_{111}, t_{111}, p_{191}, t_{102})$	56	5
$Lp_8=(p_{112}, t_{112}, p_{192}, t_{103})$	41	3
$Lp_9=(p_{113}, t_{113}, p_{193}, t_{104})$	28	3
$Lp_{10}=(p_{114}, t_{114}, p_{194}, t_{105})$	19	2
$Lp_{11}=(p_{115}, t_{115}, p_{195}, t_{106})$	12	4
$Lp_{12}=(p_{116}, t_{116}, p_{196}, t_{101})$	71	36
$Lp_{13}=(p_{71}, t_{71}, p_{81}, t_8)$	15	0

A/ Computation algorithm of the exact value of passive robustness for a delay in a milk manufacturing workshop

In order to avoid the violation of schedule conditions, a recursive algorithm allowing computing an upper bound of the maximal time disturbances allowed in a given point of the milk manufacturing workshop is presented. The purpose of this algorithm is to decrease the number of false alarms.

A-1/ Algorithm 1

Let us denote:

C_{ms} : the set of mono-synchronized subpaths,

C_{se} : the set of elementary mono-synchronized subpaths,

$L = \bigcup_i Lp_i$: the union of mono-synchronized subpaths

Δc_j : the compensable margin on the mono-synchronized subpath Lp_j

$Res(L, Lp)$: the maximal residue of disturbance on $OUT(Lp_i)$

δ_i : the disturbance that propagates along a path (Lp_i) parallel to the disturbance propagation path Lp_j

$$E(L, Lp) = \{L / (OUT(L) = OUT(Lp)) \wedge (\sum_{p \in L} M_0(p) = \sum_{p \in Lp} M_0(p)) \wedge (\delta \in L \wedge \delta = IN(L))\}$$

$$\varphi = \{Lp_j / (n^\circ = IN(Lp_j)) \wedge (Lp_j \in C_{ms}) \wedge (Lp_j \in G)\}$$

$$Margin \leftarrow \min_j [\Delta c_j + F(G \setminus Lp_j, OUT(Lp_j)^\circ, \min(b_i - q_{ie}) + \delta_i)]$$

$$p_i^\circ = OUT(Lp_j) \quad \text{Max}(Res(L, Lp_i))$$

$$p_i^\circ \notin Lp_j \quad L / (p_i \notin L) \wedge (p_i^\circ = OUT(Lp_j))$$

$$F(G^*, p^*, \Delta t)$$

```

{
   $\varphi^* = \{Lp_j / (p^* \in Lp_j) \wedge (Lp_j \in C_{se}) \wedge (Lp_j \in G^*)\}$ 
  If ( $\varphi^* == \Phi$  ou  $\Delta t == 0$ ) alors ( $F \leftarrow \Delta t$ )
  Else
  {
     $F \leftarrow \min_j \{ \min [\Delta t, (\Delta c_j + F(G^* \setminus Lp_j, OUT(Lp_j)^\circ, \min (b_i - q_{ie}) + \delta_i))] \}$ 
     $p_i^\circ = OUT(Lp_j)$  Max (Res(L,  $Lp_i$ )
     $p_i^\circ \notin Lp_j$   $p_i \notin L \wedge p_i^\circ = OUT(Lp_j)$ 
  }
}

```

A-2/ Description of the algorithm

The presented algorithm computes an upper bound of the maximal time disturbances allowed at a node n. It involves:

- Selecting the node where we need to calculate the passive robustness margin,
- Building the set of mono-synchronized subpaths (φ) defined as follows:

$$\varphi = \{Lp_j / (n^\circ = IN(Lp_j)) \wedge (Lp_j \in C_{ms}) \wedge (Lp_j \in G)\}$$

- Calculating the passive robustness margin associated to the set of subpaths φ ,

$$\text{Margin} \leftarrow \min_j [\Delta c_j + F(G \setminus Lp_j, OUT(Lp_j)^\circ, \min (b_i - q_{ie}) + \delta_i)]$$

$$p_i^\circ = OUT(Lp_j) \quad \text{Max (Res(L, } Lp_i))$$

$$p_i^\circ \notin Lp_j \quad L / (p_i \notin L) \wedge (p_i^\circ = OUT(Lp_j))$$

(δ_i is the maximum disturbance that propagates along a path parallel to the disturbance propagation path Lp_j).

- Removing the elementary mono-synchronized subpath for the construction of the whole φ^* defined as follows:

$$\varphi^* = \{Lp_j / (p^* \in Lp_j) \wedge (Lp_j \in C_{se}) \wedge (Lp_j \in G^*)\}$$

- Computing for each mono-synchronized subpath of φ^* , the passive robustness margin F defined as follows:

$$F \leftarrow \min_j \{ \min [\Delta t, (\Delta c_j + F(G^* \setminus Lp_j, OUT(Lp_j)^\circ, \min (b_i - q_{ie}) + \delta_i))] \}$$

$$p_i^\circ = OUT(Lp_j) \quad \text{Max (Res(L, } Lp_i))$$

$$p_i^\circ \notin Lp_j \quad p_i \notin L \wedge p_i^\circ = OUT(Lp_j)$$

- Stopping the algorithm if the following condition is satisfied:

$$(\varphi^* == \Phi \text{ or } \Delta t == 0) \text{ then } (F \leftarrow \Delta t)$$

Note: When we remove a mono-synchronized subpath of G, this is done only for the construction of φ^* . The places for which we seek mono-synchronized subpaths belong simply to G. The places adding constraints on a synchronization transition are taken into account even if they belong to an eliminated mono synchronized subpath.

A-3/ Justification of the algorithm

The above algorithm applies the passive robustness formula on all sequences of mono-synchronized sub-path. This formula was proved in (Jerbi et al. 2004).

The algorithm is based on a function F which is called recursively using a part of the net which is more and more restricted. When the robustness of a mono-synchronized sub-path is computed, it is removed from the net to be treated in the next call of the function F . As the number of mono-synchronized sub-paths is finite, the convergence of the considered algorithm is proved, because the function stops when the sub-net to be studied becomes empty: $\varphi^* == \Phi$.

At the level of a given mono-synchronized subpath L_p , a part of the disturbance is rejected and the other is transmitted. The transmitted value computed by Algorithm 1 is lower than the real maximal value, because it does not take into account the compensation which is provided by a time disturbance of the same nature. The proof of this property can be found in (Jerbi et al. 2009). As a consequence, the margin calculated by this algorithm is a lower bound of the effective margin.

The polynomial computation time of this algorithm can be proved such a way:

- The computation of the extreme firing instants for a given mono-synchronized subpath is less complex than the same computation on the whole Petri net structure which can be performed using an $O(n^3)$ time computing algorithm, where n correspond to the number of transition of the graph (Khansa 1997). In the works of Khansa, the P-Time Petri nets running in repetitive mode are transformed into equivalent co-graphs. These co-graphs are some bi-valued graph using negative values.
- The number of mono-synchronized subpath is smaller than the number of transitions. Consequently, the complexity of the computing time of all margins is smaller than $O(n^4)$.

All the margin values are compared (3 subtractions for a given path) to a given disturbance value less than n times (n corresponds to the number of transitions): this last effort is not significant with regards to the cost of a $O(n^4)$ time computation. The worst case computation time is smaller than $O(n^4)$.

B/ Illustrative example

Let us take the P-time Petri net example, figure 2, associated to a milk manufacturing unit. The application of algorithm 1 at the node t_9 , figure 4, allows computing a lower bound of passive robustness margin equal to 16 (Margin=16).

In fact, according to the recursive algorithm, the set of mono-synchronized subpaths (φ) is defined as follows, figure 4:

$$\varphi = \{Lp_j / (n^\circ = IN(Lp_j)) \wedge (Lp_j \in C_{ms}) \wedge (Lp_j \in G)\}$$

$$\varphi = \{\mathbf{Lp}_0=(p_9, t_9, p_{18}, t_8); \quad \mathbf{Lp}_1=(p_9, t_9, p_{101}, t_{101}); \quad \mathbf{Lp}_2=(p_9, t_9, p_{102}, t_{102}); \quad \mathbf{Lp}_3=(p_9, t_9, p_{103}, t_{103}); \\ \mathbf{Lp}_4=(p_9, t_9, p_{104}, t_{104}); \mathbf{Lp}_5=(p_9, t_9, p_{105}, t_{105}); \mathbf{Lp}_6=(p_9, t_9, p_{106}, t_{106})\}$$

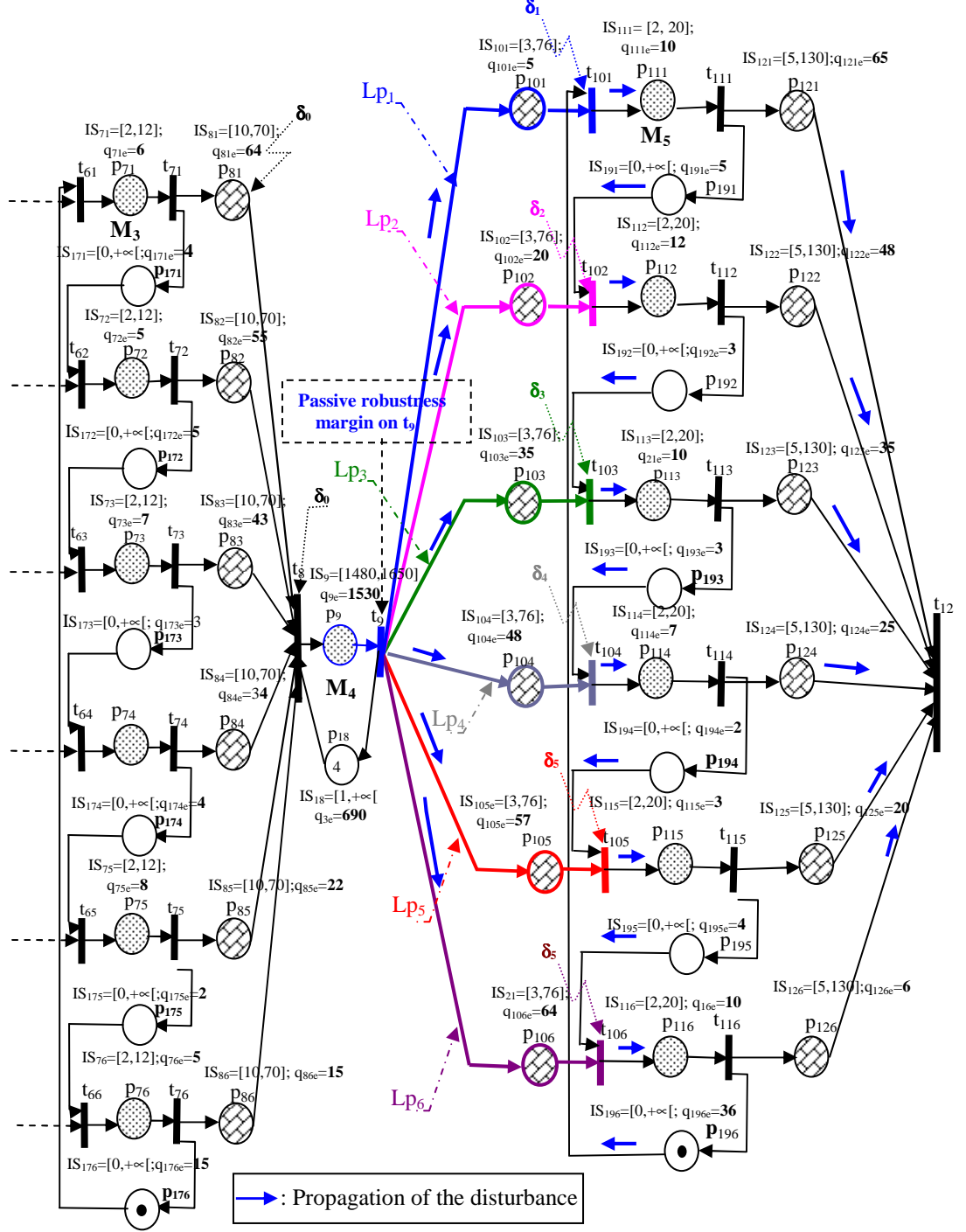


Figure 4: Example of disturbance propagation: case of passive robustness

The full set of margin associated to each mono-synchronized subpath is summarized in table 2. The application of algorithm 1 at the node t_9 , figure 4, allows to compute an upper bound of passive robustness margin equal to 16 ($F \leftarrow \min [\text{Margin } 0, \text{Margin } 1, \text{Margin } 2, \text{Margin } 3, \text{Margin } 4, \text{Margin } 5, \text{Margin } 6]$).

Table 2: Passive margin associated to propagation through mono-synchronized subpaths

Margin	Expression	Value
Margin 0	$\text{Margin } 0 \leftarrow \min [\Delta r_{c0} + F(G \setminus Lp_0, \text{OUT}(Lp_0)^o, \min(b_{81} - q_{81e})) + \delta_0]$	Margin0=695

	$j \quad p_{81}^o = \text{OUT}(Lp_0) \quad \text{Max}(\text{Res}(L, Lp_{13}))$ $p_{81}^o \notin Lp_0 \quad L/(p_{81}^o \notin L) \wedge (p_{81}^o = \text{OUT}(Lp_0))$ <p>(δ_1 is the residue of a disturbance that propagates along the path Lp_{13}) (see table 1)</p> $\text{Margin } 0 \Leftarrow \min [689 + F(G \setminus Lp_0, p_9, 6)]$	
Margin 1	$j \quad \text{Margin } 1 \Leftarrow \min [\Delta c_1 + F(G \setminus Lp_1, \text{OUT}(Lp_1)^o, \min(b_{196} - q_{196e})) + \delta_1]$ $p_{196}^o = \text{OUT}(Lp_1) \quad \text{Max}(\text{Res}(L, Lp_{12}))$ $p_{196}^o \notin Lp_1 \quad L/(p_{196}^o \notin L) \wedge (p_{196}^o = \text{OUT}(Lp_1))$ <p>(δ_1 is the residue of a disturbance that propagates along the path Lp_{12}) (see table 1)</p> $\text{Margin } 1 \Leftarrow \min [0 + F(G \setminus Lp_1, p_{111}, +\infty)]$	Margin 1=29
Margin 2	$j \quad \text{Margin } 2 \Leftarrow \min [\Delta c_2 + F(G \setminus Lp_2, \text{OUT}(Lp_2)^o, \min(b_{191} - q_{191e})) + \delta_2]$ $p_{191}^o = \text{OUT}(Lp_2) \quad \text{Max}(\text{Res}(L, Lp_7))$ $p_{191}^o \notin Lp_2 \quad L/(p_{191}^o \notin L) \wedge (p_{191}^o = \text{OUT}(Lp_2))$ <p>(δ_2 is the residue of a disturbance that propagates along the path Lp_7) (see table 1)</p> $\text{Margin } 2 \Leftarrow \min [0 + F(G \setminus Lp_2, p_{112}, +\infty)]$	Margin 2=24
Margin 3	$j \quad \text{Margin } 3 \Leftarrow \min [\Delta c_3 + F(G \setminus Lp_3, \text{OUT}(Lp_3)^o, \min(b_{192} - q_{192e})) + \delta_3]$ $p_{192}^o = \text{OUT}(Lp_3) \quad \text{Max}(\text{Res}(L, Lp_8))$ $p_{192}^o \notin Lp_3 \quad L/(p_{192}^o \notin L) \wedge (p_{192}^o = \text{OUT}(Lp_3))$ <p>(δ_3 is the residue of a disturbance that propagates along the path Lp_8) (see table 1)</p> $\text{Margin } 3 \Leftarrow \min [0 + F(G \setminus Lp_3, p_{113}, +\infty)]$	Margin 3=21
Margin 4	$j \quad \text{Margin } 4 \Leftarrow \min [\Delta c_4 + F(G \setminus Lp_4, \text{OUT}(Lp_4)^o, \min(b_{193} - q_{193e})) + \delta_4]$ $p_{193}^o = \text{OUT}(Lp_4) \quad \text{Max}(\text{Res}(L, Lp_9))$ $p_{193}^o \notin Lp_4 \quad L/(p_{193}^o \notin L) \wedge (p_{193}^o = \text{OUT}(Lp_4))$ <p>(δ_4 is the residue of a disturbance that propagates along the path Lp_9) (see table 1)</p> $\text{Margin } 4 \Leftarrow \min [0 + F(G \setminus Lp_4, p_{114}, +\infty)]$	Margin 4=18
Margin 5	$j \quad \text{Margin } 5 \Leftarrow \min [\Delta c_5 + F(G \setminus Lp_5, \text{OUT}(Lp_5)^o, \min(b_{194} - q_{194e})) + \delta_5]$ $p_{194}^o = \text{OUT}(Lp_5) \quad \text{Max}(\text{Res}(L, Lp_{10}))$ $p_{194}^o \notin Lp_5 \quad L/(p_{194}^o \notin L) \wedge (p_{194}^o = \text{OUT}(Lp_5))$ <p>(δ_5 is the residue of a disturbance that propagates along the path Lp_{10}) (see table 1)</p> $\text{Margin } 5 \Leftarrow \min [0 + F(G \setminus Lp_5, p_{115}, +\infty)]$	Margin 5=16
Margin 6	$j \quad \text{Margin } 6 \Leftarrow \min [\Delta c_6 + F(G \setminus Lp_6, \text{OUT}(Lp_6)^o, \min(b_{195} - q_{195e})) + \delta_6]$ $p_{195}^o = \text{OUT}(Lp_6) \quad \text{Max}(\text{Res}(L, Lp_{11}))$ $p_{195}^o \notin Lp_6 \quad L/(p_{195}^o \notin L) \wedge (p_{195}^o = \text{OUT}(Lp_6))$ <p>(δ_6 is the residue of a disturbance that propagates along the path Lp_{11}) (see table 1)</p> $\text{Margin } 6 \Leftarrow \min [0 + F(G \setminus Lp_6, p_{116}, +\infty)]$	Margin 6=106

C/ Discussion

The computed bound can be used to filter some alarms generated by the workshop during production. However, as the exact value is not known, some false alarms may go through the filtering system.

It may be interesting to evaluate the quality of the computed bound with regard to the exact static robustness value. For now, it is proposed to extend the robustness range using a dynamic control.

Actually, when a disturbance is going over the static robustness bound, changing the control may produce a change in the system which increases its tolerance.

Therefore, various strategies are implemented to manage time disturbances in production systems (Jerbi et al. 2004), (Collart-Dutilleul et al. 2007).

For this application, the chosen control policy tries to reject the disturbance as soon as it has been observed. The suggested strategy allows building a global control providing an active global robustness towards a given disturbance. A recursive algorithm is proposed to compute a bound of the active robustness towards a delay at a given node of the graph.

3.2 Active robustness computation

3.2.1 Definitions

Definition 9. : A temporal control is the modification of transitions firing instants using controlled P-time Petri net.

Definition 10. : An active robustness is a robustness ensured by the temporal control of the process transitions.

Definition 11. : A time disturbance δ is locally rejected by a path Lp if the effective firing instant of its last transition (t) is equal to the expected one: $St(n)=St_e(n)$.

Definition 12. (Jerbi et al. 2004): The time interval of the passive rejection capacity of a path Lp is $RC(Lp)=[Ca(Lp), Cr(Lp)]$ where:

$$Ca(Lp) = \sum_{p_i \in Lp \cap (R_N \cup Trans_{NC})} (q_{ie} - b_i), \quad (4)$$

$$Cr(Lp) = \sum_{p_i \in Lp \cap (R_N \cup Trans_{NC})} (q_{ie} - a_i). \quad (5)$$

$Ca(Lp)$ (respectively $Cr(Lp)$) is called the time interval of the passive rejection capacity for an advance (respectively a delay) time disturbance occurrence.

Definition 13. (M'Halla et al. 2010d) : The available control margin for an advance, $f_a(p_i)$, and the available control margin for a delay, $f_r(p_i)$, associated to the place p_i are defined as:

$$f_a : P \rightarrow Q^-$$

$$p_i \mapsto f_a(p_i) = \begin{cases} a_i - q_{ie} & \text{si } q_i \leq a_i \\ q_i - q_{ie} & \text{si } a_i < q_i < q_{ie} \\ 0 & \text{si } q_{ie} \leq q_i \leq b_i \end{cases} \quad (6)$$

$$f_r : P \rightarrow Q^+ \cup \{+\infty\}$$

$$p_i \mapsto f_r(p_i) = \begin{cases} b_i - q_{ie} & \text{si } q_i \leq q_{ie} \\ b_i - q_i & \text{si } q_{ie} < q_i \leq b_i \end{cases} \quad (7)$$

Definition 14. : A transition t constitutes an elementary subpath locally controllable on $[\max(f_a(p_i)), \min(f_r(p_i))]$ if $t \in T_C$.

$$p_i^\circ = t \quad p_i^\circ = t$$

Example 1

Let us suppose that the transition t_{102} , figure 5, is controllable ($t_{102} \in T_C$). The transition t_{102} is a synchronization transition ($t_{102} \in T_S$).

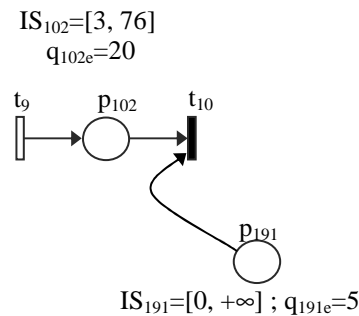


Figure 5: Case of controllable synchronization transition

Indeed

$$p_{191}^\circ = t_{102} \Rightarrow f_a(p_{191}) = 0 - 5 = -5 \text{ et } f_r(p_{191}) = +\infty$$

$$\begin{aligned}
p_{102}^{\circ} &= t_{102} \Rightarrow f_a(p_{102}) = 3 - 20 = -17 \text{ et } f_r(p_{102}) = 56 \\
\Delta_{\min} &= \max_{p_i = t_{102}}(f_a(p_i)) = \max(-17, -5) = -5 \\
\Delta_{\max} &= \min_{p_i = t_{102}}(f_r(p_i)) = \min(56, +\infty) = 56
\end{aligned}$$

Consequently, the transition t_{102} constitutes an elementary subpath locally controllable on $[-5, 56]$.

Definition 15. : A path Lp is locally controllable on the interval $[\Delta_{\min}, \Delta_{\max}]$ if one can generate by the control a variation $\delta \in [\Delta_{\min}, \Delta_{\max}]$ on its last node without causing any token death on the levels of its synchronization transitions

3.2.2 Robust control strategy facing time disturbances

In manufacturing workshops with time constraints, the determining parameter for quality and cost is the time, which must belong to a very strict validity interval. The control guarantees the respect of these specifications in order to avoid the violation of the constraint intervals associated to the various states.

The active robustness is based on the control of transition firing instants using controlled P-time Petri nets. This temporal control makes it possible, in certain cases, to avoid the death of tokens if the time disturbances exceed the bounds of the intervals associated to operations (Jerbi et al. 2009), (Declerck and Guezzi 2009).

In the case of active robustness (the temporal disturbances exceed the bounds of the intervals associated to the operations) a robust control approach is proposed. The adopted strategy consists in rejecting the disturbance as soon as it has been observed in order to avoid the death of marks on the levels of synchronization transitions. Therefore, constraints violations are avoided. If the disturbance is a delay (respectively an advance), we generate advances (respectively delays) on the controlled transitions firing of the propagation path of the disturbance in order to avoid the constraints violation of the schedule conditions. Furthermore, this approach is consolidated by a computing algorithm of the robustness margin at various graph nodes.

The control strategy which is applied in this paper is not the only one. Moreover, there is no proof that, the applied control is optimal. Some optimal control proposition may be found in the state of the art, but they still have to be adapted to the industrial context where transitions may be not controllable (Declerck 2011).

A/ Algorithm 2

This is a recursive algorithm allowing applying the approach of the rejection of the disturbance as soon as its observation, in a node n . In order to preserve schedule conditions, the presented algorithm makes it possible to compute a total delay margin available to be injected on a node n .

Let us denote:

- G the P-time Petri net model of the workshop,
- δ a delay time disturbance in p_i (node n), observed in a transition t ($t \in T_O$),
- $Cr(Lp_j)$: the time passive rejection capacity for a delay of the mono-synchronized subpath Lp_j ,
- $\delta_C \in [\Delta_{\min}, \Delta_{\max}]$ (resp δ'_C): advances (respectively delays) generated on the controlled transition firings on the propagation path of the disturbance (for example if the disturbance δ is a delay, $\delta_C = \Delta_{\min} = \max(f_a(p_i))$)

Using the above notations, the algorithm is specified as follows:

$$\begin{aligned} \Phi &= \{Lp_j / (n = \text{IN}(Lp_j)) \wedge (Lp_j \in C_{ms}) \wedge (Lp_j \in G)\} \\ \text{Active Margin} &\leftarrow \min_j [\text{Cr}(Lp_j) + E(G \setminus Lp_j, \text{OUT}(Lp_j)^\circ, \min(b_i - q_{ie}) + \delta'_c)] \\ &\quad p_i^\circ = \text{OUT}(Lp_j) \\ &\quad p_i \notin Lp_j \end{aligned}$$

$F(G^*, p^*, \Delta t)$

{

$\Phi^* = \{Lp_j / (p^* \in Lp_j) \wedge (Lp_j \in C_{se}) \wedge (Lp_j \in G^*)\}$

If $(\Phi^* == \Phi \text{ or } \Delta t == 0)$ then $(F \leftarrow \Delta t)$

Else

{

$F \leftarrow \min_j \{\min[\Delta t, \text{Cr}(Lp_j) + E(G^* \setminus Lp_j, \text{OUT}(Lp_j)^\circ, \min(b_i - q_{ie}) + \delta_c)]\}$

$p_i^\circ = \text{OUT}(Lp_j)$

$p_i \notin Lp_j$

}

}

B/ Description of the algorithm

E_{p_i} (respectively $E_{\text{OUT}(Lp)^\circ}$) is the set of existing mono-synchronized subpaths containing the place p_i (respectively $\text{OUT}(Lp)^\circ$).

δr the residue of the disturbance δ .

{

$E = \{\delta r, \text{OUT}(Lp)\}$ (F is a doublet composed by the residue of the disturbance and the output node of the path Lp)

$\forall Lp \in E_{p_i} \Rightarrow \delta r = \delta - \text{Cr}(Lp)$

If

$\delta r > (b_i - q_{ie}) / \{p_i^\circ \in T_S \wedge p_i \notin Lp \wedge p_i^\circ \in Lp\}$, there is a control problem on p_i° (we apply the robust control approach, which allows to generate a temporal shift, through the set of controlled transitions of the propagation path of the disturbance)

Else

$\delta r < (b_i - q_{ie}) / \{p_i^\circ \in T_S \wedge p_i \notin Lp \wedge p_i^\circ \in Lp\}$, we apply the same procedure for each element of $E_{\text{OUT}(Lp)^\circ}$

}

Example 2

Let us suppose that the transition t_{C1} and t_{C2} , figure 6, are controllable. Let δ a delay time disturbance at the input of the mono-synchronized subpath Lp_1 and δr its residue.

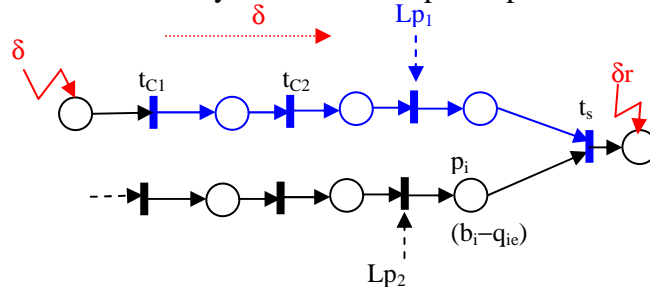


Figure 6: Example of algorithm application

Two cases arise:

1st case:

If $(b_i - q_{ie}) \geq \delta_r \geq \delta$ then there is no death of a mark in p_i .

2nd case:

If $(b_i - q_{ie}) < \delta_r < \delta$ then we should generate a temporal shift through the set of controlled transitions (t_{C1} and t_{C2}) of the propagation path in order to avoid the death of mark on the level of transition t_s ($t_s \in T_S$).

C/ Discussion

Concerning the program convergence, the behaviour of the second algorithm is so similar to the first one that a new convergence proof is not useful. The proof of the formula which is recursively applied by the algorithm can be consulted in (Jerbi et al. 2009).

Concerning the complexity analysis, things may be different, because of the added control loop. The control strategy tries to create a similar disturbance on the parallel path...

The number of parallel path is smaller than the number of transition.

On a given parallel path, the computation of the margins which may be used on a given mono-synchronized sub-path is reduced by an $O(n^3)$ time computing algorithm as in (Collart-Dutilleul et al. 2003). There are less mono-synchronized sub-paths than transitions in the net. The global control loop computation spend a $O(n^5)$ time for a given execution of the function F. This function has to be performed on all the mono-synchronized sub-paths, so the global time complexity of the algorithm 2 is less than $O(n^6)$ considering the number of transitions.

The above analysis uses some rough approximations which aim to show that the computing time is reasonable. The choice of the upper-bound can be refined. The main contribution of this paper is to illustrate that the integration of the control loop may considerably increase the robustness value on a real industrial topology.

D/ Illustrative example

D-1/Active robustness computing

The application of the algorithm at the node t_9 , figure 7, allows computing an active robustness margin. This P-time PN allows verifying the active robustness margin on t_9 computed by algorithm 2 and the application of the robust control strategy to the milk production unit. Similarly, table 3 shows the full set of active margins associated to each mono-synchronized subpath. The application of the algorithm at the node t_9 , figure 7, allows computing an active robustness margin equal to 63.

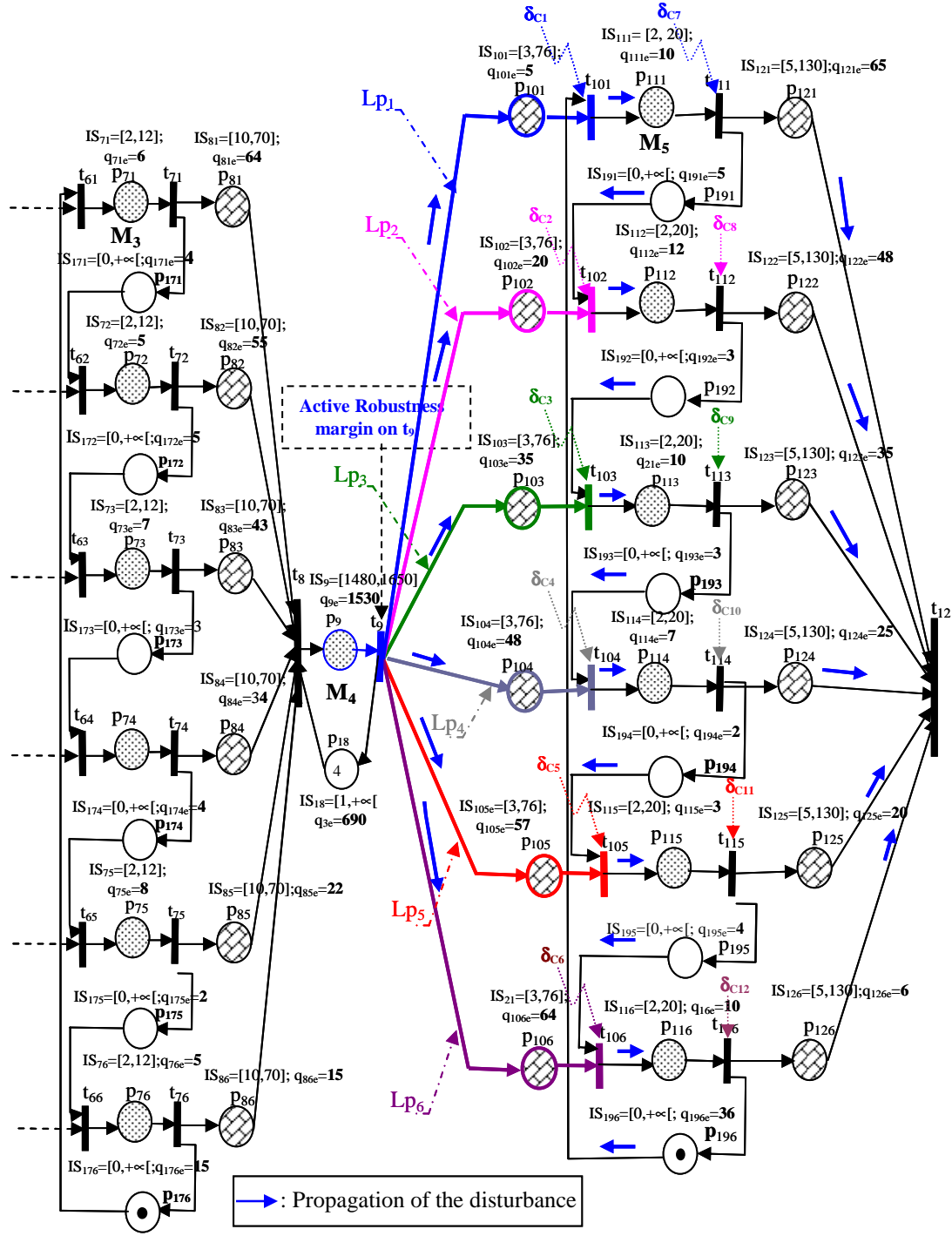


Figure 7: Example of computing active robustness margin

Table 3: Active margins associated to mono-synchronized sub-paths

<p>Active Margin 1</p>	<p>Active Margin 1 $\leftarrow \min_j [\text{Cr}(Lp_1) + F(G \setminus Lp_1, \text{OUT}(Lp_1)^\circ, \min(b_{196} - q_{196e})) + \delta'_{C1}]$ $p_{196}^\circ = \text{OUT}(Lp_1)$ $p_{196}^\circ \notin Lp_1$ $(\delta'_{C1} : \text{advances (respectively delays) generated on the controlled transition firings of the propagation path } Lp_1)$ Active Margin 1 $\leftarrow \min [0 + F(G \setminus Lp_1, p_{111}, +\infty)]$</p>	<p>Active Margin 1=63</p>
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Active Margin 2	$\text{Active Margin 2} \leftarrow \min_j [\text{Cr}(\text{Lp}_2) + F(\text{G} \setminus \text{Lp}_2, \text{OUT}(\text{Lp}_2)^\circ, \min(b_{191} - q_{191e})) + \delta'_{c2}]$ $p_{191}^\circ = \text{OUT}(\text{Lp}_2)$ $p_{191}^\circ \notin \text{Lp}_2$ <p>(δ'_{c2} : advances (respectively delays) generated on the controlled transition firings of the propagation path Lp_2)</p> $\text{Active Margin 2} \leftarrow \min [0 + F(\text{G} \setminus \text{Lp}_2, p_{112}, +\infty)]$	Active Margin 2=65
Active Margin 3	$\text{Active Margin 3} \leftarrow \min_j [\text{Cr}(\text{Lp}_3) + F(\text{G} \setminus \text{Lp}_3, \text{OUT}(\text{Lp}_3)^\circ, \min(b_{192} - q_{192e})) + \delta'_{c3}]$ $p_{192}^\circ = \text{OUT}(\text{Lp}_3)$ $p_{192}^\circ \notin \text{Lp}_3$ <p>(δ'_{c3} : advances (respectively delays) generated on the controlled transition firings of the propagation path Lp_3)</p> $\text{Active Margin 3} \leftarrow \min [0 + F(\text{G} \setminus \text{Lp}_3, p_{113}, +\infty)]$	Active Margin 3=67
Active Margin 4	$\text{Active Margin 4} \leftarrow \min_j [\text{Cr}(\text{Lp}_4) + F(\text{G} \setminus \text{Lp}_4, \text{OUT}(\text{Lp}_4)^\circ, \min(b_{193} - q_{193e})) + \delta'_{c4}]$ $p_{193}^\circ = \text{OUT}(\text{Lp}_4)$ $p_{193}^\circ \notin \text{Lp}_4$ <p>(δ'_{c4} : advances (respectively delays) generated on the controlled transition firings of the propagation path Lp_4)</p> $\text{Active Margin 4} \leftarrow \min [0 + F(\text{G} \setminus \text{Lp}_4, p_{114}, +\infty)]$	Active Margin 4=69
Active Margin 5	$\text{Active Margin 5} \leftarrow \min_j [\text{Cr}(\text{Lp}_5) + F(\text{G} \setminus \text{Lp}_5, \text{OUT}(\text{Lp}_5)^\circ, \min(b_{194} - q_{194e})) + \delta'_{c5}]$ $p_{194}^\circ = \text{OUT}(\text{Lp}_5)$ $p_{194}^\circ \notin \text{Lp}_5$ <p>(δ'_{c5} : advances (respectively delays) generated on the controlled transition firings of the propagation path Lp_5)</p> $\text{Active Margin 5} \leftarrow \min [0 + F(\text{G} \setminus \text{Lp}_5, p_{115}, +\infty)]$	Active Margin 5=71
Active Margin 6	$\text{Active Margin 6} \leftarrow \min_j [\text{Cr}(\text{Lp}_6) + F(\text{G} \setminus \text{Lp}_6, \text{OUT}(\text{Lp}_6)^\circ, \min(b_{195} - q_{195e})) + \delta'_{c6}]$ $p_{195}^\circ = \text{OUT}(\text{Lp}_6)$ $p_{195}^\circ \notin \text{Lp}_6$ <p>(δ'_{c6} : advances (respectively delays) generated on the controlled transition firings of the propagation path Lp_6)</p> $\text{Active Margin 6} \leftarrow \min [0 + F(\text{G} \setminus \text{Lp}_6, p_{116}, +\infty)]$	Active Margin 6=176

D-2/ Illustration of the robust control strategy

Let $\delta=63$ a time disturbance in p_9 (Hydromat) observed in t_9 , figure 7. The disturbance δ is propagated towards the six paths Lp_1 , Lp_2 , Lp_3 , Lp_4 , Lp_5 and Lp_6 .

($\varphi = \{\mathbf{Lp}_0 = (p_9, t_9, p_{18}, t_8) ; \mathbf{Lp}_1 = (p_9, t_9, p_{101}, t_{101}) ; \mathbf{Lp}_2 = (p_9, t_9, p_{102}, t_{102}) ; \mathbf{Lp}_3 = (p_9, t_9, p_{103}, t_{103}) ; \mathbf{Lp}_4 = (p_9, t_9, p_{104}, t_{104}) ; \mathbf{Lp}_5 = (p_9, t_9, p_{105}, t_{105}) ; \mathbf{Lp}_6 = (p_9, t_9, p_{106}, t_{106})\}$)

Let us suppose that the transitions t_{101} , t_{111} , t_{102} , t_{112} , t_{103} , t_{113} , t_{104} , t_{114} , t_{105} , t_{115} , t_{106} and t_{116} are controllable. According to definition 14, each transition constitutes a subpath locally controllable on $[\max(f_a(p_i)), \min(f_r(p_i))]$.

$$p_i^\circ = t \quad p_i^\circ = t$$

Table 4 gives the available control margin for an advance, $f_a(p_i)$, and the available control margin for a delay, $f_r(p_i)$, associated to each place p_i .

Table 4: Available control margin for an advance and for a delay associated to place p_i

Path	$f_a(p_i)$	$f_r(p_i)$
$Lp_1=(p_9, t_9, p_{101}, t_{101})$	$f_a(p_{101}) = -2$	$f_r(p_{101})=71$
$Lp_2=(p_9, t_9, p_{102}, t_{102})$	$f_a(p_{102}) = -17$	$f_r(p_{102})=56$
$Lp_3=(p_9, t_9, p_{103}, t_{103})$	$f_a(p_{103}) = -32$	$f_r(p_{103})=41$
$Lp_4=(p_9, t_9, p_{104}, t_{104})$	$f_a(p_{104}) = -45$	$f_r(p_{104})=28$
$Lp_5=(p_9, t_9, p_{105}, t_{105})$	$f_a(p_{105}) = -54$	$f_r(p_{105})=19$
$Lp_6=(p_9, t_9, p_{106}, t_{106})$	$f_a(p_{106}) = -61$	$f_r(p_{106})=12$
$Lp_7=(p_{111}, t_{111}, p_{191}, t_{102})$	$f_a(p_{111}) = -8$	$f_r(p_{111})=10$
$Lp_8=(p_{112}, t_{112}, p_{192}, t_{103})$	$f_a(p_{112}) = -10$	$f_r(p_{112})=8$
$Lp_9=(p_{113}, t_{113}, p_{193}, t_{104})$	$f_a(p_{113}) = -8$	$f_r(p_{113})=10$
$Lp_{10}=(p_{114}, t_{114}, p_{194}, t_{105})$	$f_a(p_{114}) = -5$	$f_r(p_{114})=13$
$Lp_{11}=(p_{115}, t_{115}, p_{195}, t_{106})$	$f_a(p_{115}) = -1$	$f_r(p_{115})=17$
$Lp_{12}=(p_{116}, t_{116}, p_{196}, t_{101})$	$f_a(p_{116}) = -8$	$f_r(p_{116})=10$

- On the path Lp_1 , the disturbance changes passively the firing instant of the transition t_9 and also the sojourn time in the place p_9 : $St_9(n) = St_{9e}(n) + 63$ and $q_9 = q_{9e} + 63 = 1593$. (**Active Margin 1** $\leftarrow \min [0 + F(G \setminus Lp_1, p_{111}, +\infty)]$)
- After the crossing of the transition t_{101} , the disturbance is transmitted to two paths, $Lp_7=(p_{111}, t_{111}, p_{191}, t_{102})$ and $Lp_{13}=(p_{111}, t_{111}, p_{121}, t_{12})$ through the starting place p_{111} . ($\Phi^*_1 = \{Lp_7, Lp_{13}\}$)

On the path Lp_7 , the disturbance is partially rejected in p_{191} ($Cr(Lp_7) = 5$) and the mark is available in p_{191} with a delay equal to 58. ($E_1 \leftarrow \min [+ \infty, 5 + F(G \setminus Lp_1 \setminus Lp_7, p_{112}, \min(b_{102} - q_{102}) + \delta_{C1} + \delta_{C7})]$)

If the transition t_{101} and t_{111} are not controllable, there is a death mark in p_{102} since the available control margin for a delay accepted is equal to 56. By injecting by the control an advance on the firing instant of the two transitions t_{101} ($St_{101}(n) = St_{101e}(n) + \delta_{C1} = St_{101e}(n) - 2$) and t_{111} ($St_{111}(n) = St_{111e}(n) + \delta_{C7} = St_{111e}(n) - 8$), the death of the mark in p_{102} is then avoided and the residue $\delta r = 48$ is transmitted to the two paths $Lp_8=(p_{112}, t_{112}, p_{192}, t_{103})$ and $Lp_{14}=(p_{112}, t_{112}, p_{122}, t_{12})$ through the place p_{112} . The same control methodology is then applied on Lp_8 and Lp_{14} and their downstream mono-synchronized paths in order to deal with the disturbance propagation.

The paths Lp_2 , Lp_3 , Lp_4 , Lp_5 and Lp_6 are locally controllable (see table 4). The token deaths are avoided on all their disturbance propagation paths using the same method.

To sum up, the active robustness margin computed by the recursive algorithm 2 is superior to the upper bound of passive robustness margin computed by algorithm 1. It is to be noted that the first robustness value is 16 when the second is equal to 63. Consequently the range $[16, 63]$ corresponds to the set of delay values where an alarm does not have to be generated. When a delay is bigger than 63, nothing can be claimed about its correctness. Roughly speaking, integrating a control policy in the robustness range computation allows multiplying by 3.9375 the range of alarm filtering.

4 Conclusion

This paper deals with the robustness of manufacturing workshops with time constraints. The Controlled P-time Petri Net model of a given workshop topology is used to elaborate several results regarding the robustness problem of manufacturing milk unit.

One of the main contributions of this work is a robust control strategy facing time disturbances. This strategy consists in generating by the control a temporal shift which compensates the disturbance on its propagation path in order to avoid the death of marks at the level of synchronization transitions of the P-time Petri net model. The suggested approach is consolidated by a recursive algorithm allowing computing an active robustness margin allowed at a given node. The results obtained in the illustrative example show that this algorithm is effective when temporal disturbances exceed the intervals of passive robustness. In this case, the computed active robustness margin allows increasing the system's ability to preserve the specifications facing some expected or unexpected variations.

Let us remember that the active robustness interval is really larger than the passive robustness interval and provides more possibilities to continue the production in a degraded mode. On the considered milk manufacturing workshop analysed in this paper, the active robustness value is about four times the passive one.

This degraded functioning mode makes it possible to keep on producing while providing an acceptable quality for the products. However, when the disturbance is outside of the active robustness range, the quality of the workshop products is not guaranteed any more. In this case, a production alarm is generated.

In future works, the state of this art may be used to improve the efficiency of the control, because there are some optimal control propositions (Declerck 2011).

The control proposed in this paper assumed that all transition can be observed. There is a need for an algorithm, built upon the lemmas results, providing localisation or partial localisation of time disturbances in order to extend the application range of this approach.

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