

2D analytical airgap field model of an inset permanent magnet synchronous machine, taking into account the slotting effect

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In this paper, an analytical method for the computation of the electromagnetic field in the airgap of an inset permanent magnet synchronous machine is proposed. The originality lies in the fact that both the stator and rotor saliencies have been taken into account in the analytical procedure. The purpose is to compute both quickly and accurately the magnetic fluxes and instantaneous torque of the machine, in order to minimize torque ripples in an optimal pre-design process. It is shown that the instantaneous torque is in good agreement with finite element results, for a shorter computation time. The model presented here can be used when the 2D approximation of the machine is valid, which is the case for most of machines' topologies.

Index Terms—analytical modeling, torque calculation, airgap flux, finite element verification.

I. INTRODUCTION

ELECTRIC MOTORS for electric and hybrid traction applications have received a lot of interest in recent years [1][2][3][4], due to the necessity of reducing fuel consumption, both for economic and environmental reasons. Permanent magnet synchronous machines are promising in these demanding applications, because of their potential high torque density and efficiency [5]. The design of the electric machine for such embedded application can be difficult, due to the necessity of reaching both a high torque density and a compact size, which are contradictory optimization criteria [6] [7]. The torque quality, i.e. small torque ripple, is also an important parameter in such applications [8].

The torque ripple computation is by no way an obvious problem. Most authors use the well known finite element method [9] [10]. This method presents the advantage of taking into account all the geometrical complexity of the machine, as well as magnetic saturation. However, the computation time can be prohibitive in a first design optimization process, since the instantaneous torque computation requires a time-stepping analysis [11]. Therefore, to include the cogging torque minimization criteria in the pre-design optimization process, it is better to develop an analytical approach [12]. Most of the analytical approaches proposed in the literature are done for surface mounted permanent magnet machines [13]. However, more complex permanent magnet rotor topologies, such as the inset permanent magnet structure, can be more suitable for high speed and flux weakening purpose [14].

This article proposes an exact analytical computation of the magnetic field in the airgap of a synchronous permanent magnet machine having an inset permanent magnet topology. The double saliency of the machine (i.e. the stator slotting and the rotor ferromagnetic teeth) is taken into account analytically. In a first part, a literature review on the subject is proposed. Then the analytical model is presented, with the detailed mathematical procedure. To finish, some results about the airgap induction and instantaneous torques are given and compared with finite element analysis. A good agreement, for a much smaller computation time, has been found.

II. OVERVIEW OF THE PREVIOUS WORK ON THE TOPIC

The problem of modeling the stator slotting effect of electrical permanent magnet machines has been dealt with in several references of the literature. In [15], the stator slotting effect has been computed in a radial flux machine by derivation of a relative modulation function. This modulation function has been computed using the same conformal map than the one of Carter in his well-known paper [16]. However, this method only allows computing the airgap induction radial component, which is a problem for the tangential force computation by the Maxwell stress tensor method [17]. Moreover, the magnetic interaction between the slots is not taken into account. Other authors [18][19] have corrected these drawbacks of the conformal mapping method by using the numerical Schwartz-Christoffel Matlab toolbox [20], which permits to take into account rigorously the whole magnetic domain. However, this method requires the numerical computation of the conformal map coefficients, which is time consuming, because non linear transcendental equations must be solved numerically.

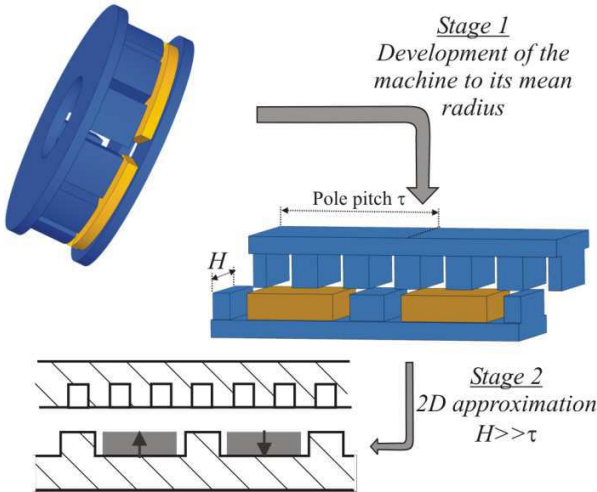
As an alternative to conformal mapping, some authors prefer the subdomain method [21] [22] [23], which permits to find an exact solution of the potential in the airgap of the machine, under the form of Fourier series. The idea of this method is to decompose the machine airgap into elementary rectangular domains. The Laplace or Poisson equation is solved in each domain, and the domains are linked using classical boundary conditions in electromagnetism. This method generally requires a numerical inversion of a linear Cramer system, which can be done quite easily and fastly compared to the transcendental equations of the conformal map. Some papers [21][22] apply this method to the rotor saliency for inserted permanent magnet machines. Other authors [13], on the contrary, use this method to compute the stator slotting effect of surface mounted permanent magnet machines, which can be with a double excitation winding [24]. Other authors have even managed to take into account analytically a more complex shape for the slots than a

56 simple rectangular [25].
 57 In each case, some work remains to be done to take into account both the stator slotting effect, and the salient pole rotor topology
 58 with inset magnets. This is the purpose of the further analytical development.
 59 Thanks to the model presented in this work, it will be possible to include the torque ripple minimization criterion from early pre-
 60 design stages, without increasing too much the computation time. Indeed, the pre-design of hybrid vehicles is a hard task [26], since
 61 all the traction chain has to be taken into consideration. Thus, for computation time purposes, the model used for the actuator is often
 62 simplified. For example, in [27], the slotting effect of the machine has been neglected. This can be a problem, because the torque
 63 quality (i.e. low torque ripples) is also an important point in such applications.

64 III. SYMPLIFYING HYPOTHESIS FOR THE ANALYTICAL MODELLING

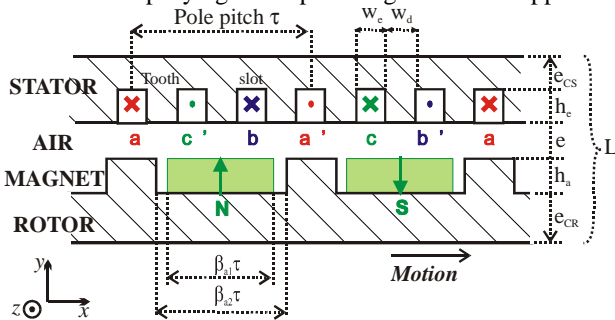
65 A. 2D assumption and machine's topology

66 As done in all the previous references about analytical modeling, the machine is considered as bi-dimensional (2D), i.e. the edges
 67 effects are neglected. This means that the machine is considered as infinite perpendicularly to the plane of representation.
 68 This is typically the case in conventional radial flux machines [19] in which the axial length H is important compared to the pole
 69 pitch characteristic dimension τ .
 70 Notice that this 2D assumption can be also valid for others topologies than conventional radial flux machines. For example, for axial
 71 flux machines [28], the 2D assumption is also possible, provided the difference H between the external and internal radii is important
 72 compared to the pole pitch τ at the mean radius. More precisely, for an axial flux machine, it is first assumed that the electromagnetic
 73 phenomena happening at the mean radius are sufficient to describe the behavior of the machine. The machine is made equivalent to a
 74 fictitious linear actuator having the same geometry than the real axial flux machine at its mean radius. Then the thickness H of this
 75 fictitious linear actuator is assumed to be very important compared to its pole pitch τ . This 2D approximation for the axial flux
 76 machine is summed up in Fig. 1.



77 Fig. 1: 2D approximation of the axial flux machine

80 Then, under the 2D assumption, for both a radial and axial flux machine, the problem to study can be represented in Fig. 2 (the main
 81 geometrical parameters are given in this figure, and are recalled in Table I). For the radial flux machine, such a representation
 82 implies that the curvature of the machine has been neglected, which is valid if the mean airgap radius (i.e. the mid airgap radius) is
 83 important compared to the dimensions along the y axis. This is assumed to be the case in the following of this paper. A discussion
 84 about this simplifying assumption is given in the Appendix 1.



85 Fig. 2: 2D problem to study for both a radial and axial flux machine, and main geometrical parameters

87
88

TABLE I
GEOMETRICAL PARAMETERS OF THE MACHINE

Description	Notation
Dimensions along the x direction	
Pole pitch	τ
Magnet relative opening with regard to the pole pitch	$\beta_{a1} (0 < \beta_{a1} < 1)$
Rotor inter-teeth space relative opening with regard to the pole pitch	$\beta_{a2} (\beta_{a1} < \beta_{a2} < 1)$
Stator slot width	w_e
Stator tooth width	w_d
Dimensions along the y direction	
Magnet thickness	h_a
Airgap width	e
Rotor yoke thickness	e_{cr}
Stator yoke thickness	e_{cs}
Slot dept	h_e
Total width of the machine	$L = e_{cr} + h_a + e + h_e + e_{cs}$
Dimensions along the z direction	
Active thickness of the machine along the z axis	H
Physical data	
Remanent induction of the magnets (NdFeB) at room temperature	B_r

89

90 *B. Other hypothesis*

91 Other classical simplifying assumptions are made:

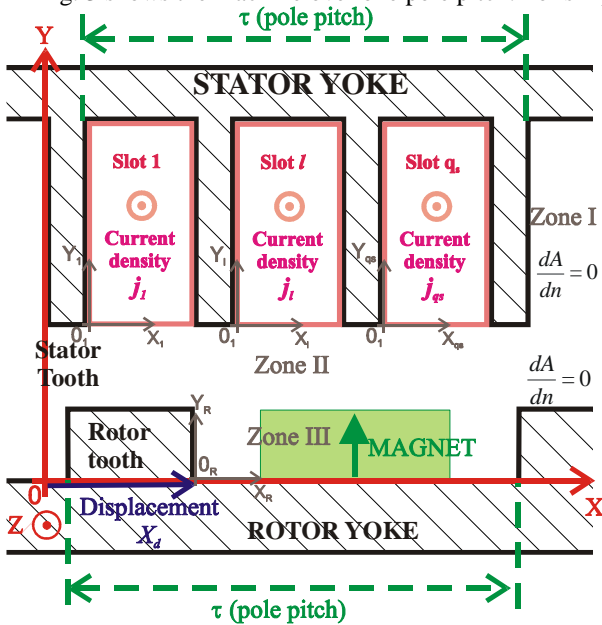
- 92 – No saturation effect is taken into account. The magnetic permeability of the ferromagnetic parts is assumed to be infinite. This
- 93 assumption is also discussed in Appendix I.
- 94 – It is assumed that continuous rate has been reached, and that the machine is driven with sinusoidal currents waves.
- 95 – Finally, the current density in each slot is assumed to be uniform in the entire slot surface.

96

IV. THE CALCULATION METHOD

97 *A. Definition of the problem*

98 Fig. 3 shows the machine over one pole pitch. For simplicity, the number of stator slots per pole is chosen to equal to 3.



99
100

Fig. 3: Representation of the problem to solve, with double (i.e. rotor and stator) saliencies

101
102

Due to the 2D approximation, the problem can be reduced to the computation of the following vector potential:

$$\vec{A}(x, y) = A(x, y) \cdot \vec{u}_z \quad (1)$$

103 Each slot is characterized by a number l , where l can evolve between 1 and 3. The current density in the slot l is called j_l , and is
 104 assumed to be uniform over the slot. A first difficulty arises from the fact that the geometry of the problem and its solution depend
 105 on the rotor displacement X_d . The stator slots zone is called Zone I, the airgap is called Zone II, and Zone III is the magnet zone (Fig.
 106 3).

107 B. Vector potential mathematical expressions

108 In Zone III, due to the magnet (whose magnetization function is called $M(x)$), the potential is given by a Poisson equation:

$$\Delta A^{(III)} = \mu_0 \frac{\partial M}{\partial x} \quad (2)$$

109 As done in [22], the magnetization can be developed into a Fourier series:

$$M(X_R) = \sum_{n=1}^{+\infty} M_n \cdot \cos\left(n \frac{\pi}{\beta_{a2} \tau} X_R\right) \quad (3)$$

110 The coefficients M_n are given by:

$$M_n = \frac{4 B_r}{\pi \mu_0} \frac{1}{n} \sin\left(n \frac{\beta_{a1}}{\beta_{a2}} \frac{\pi}{2}\right) \quad (4)$$

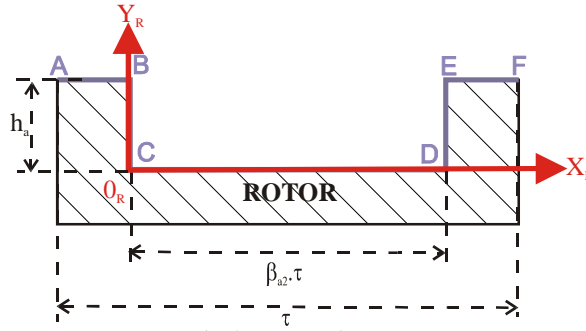


Fig. 4: Zoom on the rotor

111 It can be shown that the Poisson equation (2), as well as the homogeneous Neumann boundary conditions $dA/dn=0$ on the segments
 112 [BC], [DE] and [CD] (Fig. 4) are satisfied if the following expression for the potential is chosen, in the main coordinate system
 113 (0xy):
 114
 115

$$A^{(III)}(x, y) = c_0 + \sum_{n \geq 1} \left[c_n \cosh\left(n \frac{\pi}{\beta_{a2} \tau} y\right) - \mu_0 \frac{\beta_{a2} \tau}{\pi} \sum_{n \geq 1} \frac{M_n}{n} \cos\left(n \frac{\pi}{\beta_{a2} \tau} (x - X_d)\right) \right] \cdot \cos\left(n \frac{\pi}{\beta_{a2} \tau} (x - X_d)\right) \quad (5)$$

116 The unknown coefficients c_n ($n \geq 0$) are determined in the next sections.

117 For Zone II, a Laplace Equation must be solved:

$$\Delta A^{(II)} = 0 \quad (6)$$

118 Moreover, in this zone, in the airgap (Zone II), an anti-periodicity condition has to be taken into account:

$$A^{(II)}(x + \tau, y) = -A^{(II)}(x, y) \quad (7)$$

119 This implies that the Fourier series only present odd terms:

$$A^{(II)}(x, y) = \sum_{k \geq 1} b_k(y) \cos\left((2k-1) \frac{\pi}{\tau} x\right) + \sum_{k \geq 1} b_k'(y) \sin\left((2k-1) \frac{\pi}{\tau} x\right) \quad (8)$$

120 The Laplace equation (6) implies:

$$b_k(y) = b_k^{(1)} \cosh\left((2k-1)\frac{\pi}{\tau}y\right) + b_k^{(2)} \sinh\left((2k-1)\frac{\pi}{\tau}y\right) \quad (9)$$

$$b_k'(y) = b_k^{(3)} \cosh\left((2k-1)\frac{\pi}{\tau}y\right) + b_k^{(4)} \sinh\left((2k-1)\frac{\pi}{\tau}y\right)$$

121 For Zone I, a generic slot l is considered, with its local coordinate system (O_l, X_l, Y_l) (Fig. 5).
 122 Due to the currents, the equation in the slot is a Poisson equation:

$$\Delta A^{(l,l)} = -\mu_0 \cdot j_l \quad (10)$$

123 The homogeneous solution can be found by solving the corresponding Laplace equation ($\Delta A_h^{(l,l)} = 0$):

$$A_h^{(l,l)}(X_l, Y_l) = a_0^{(l)} + \sum_{m \geq 1} a_m^{(l)} \cosh\left(m \frac{\pi}{w_e}(Y_l - h_e)\right) \cos\left(m \frac{\pi}{w_e} X_l\right) \quad (11)$$

124 A particular solution to the Poisson equation $\Delta A_p^{(l,l)} = -\mu_0 j_l$ must be found. The following quadratic form can be proposed as a
 125 particular solution in the slot:

$$A_p^{(l,l)}(X_l, Y_l) = -\frac{1}{2} \mu_0 j_l (Y_l - h_e)^2 \quad (12)$$

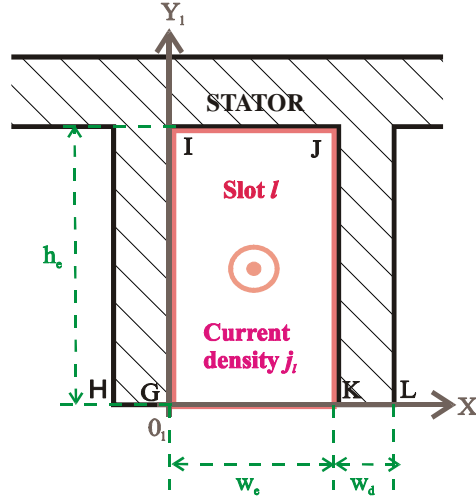


Fig. 5: Zoom on a stator slot

126
 127
 128 In the main (Oxy) axis system, it can be found:

$$A^{(l,l)}(x, y) = a_0^{(l)} + \sum_{m \geq 1} a_m^{(l)} \cosh\left(m \frac{\pi}{w_e}(y - (h_a + e + h_e))\right) \cos\left(m \frac{\pi}{w_e}(x - (l-1)(w_e + w_d))\right) - \frac{1}{2} \mu_0 j_l (y - (h_a + e + h_e))^2 \quad (13)$$

130
 131
 132 **C. Boundary conditions between all the different domains**

133 Starting from the mathematical expressions of the previous section, it is possible to write the boundary conditions between all the
 134 zones, and determine the series' coefficients. As known in electromagnetic theory, the boundary conditions to satisfy are:

- 135 – the continuity of the vector potential
- 136 – the continuity of its normal derivative

137 The following notations are introduced:

- 138 – $K_{c_k}^{(II)} = \cosh\left((2k-1)\frac{\pi}{\tau}(h_a + e)\right)$
- 139 – $K_{s_k}^{(II)} = \sinh\left((2k-1)\frac{\pi}{\tau}(h_a + e)\right)$

$$140 \quad - \quad K_{c_k}^{(II)'} = \cosh\left(\left(2k-1\right)\frac{\pi}{\tau}h_a\right)$$

$$141 \quad - \quad K_{s_k}^{(II)'} = \sinh\left(\left(2k-1\right)\frac{\pi}{\tau}h_a\right)$$

$$142 \quad - \quad K_{c_m}^{(I)} = \cosh\left(m\frac{\pi}{w_e}h_e\right)$$

$$143 \quad - \quad K_{c_n}^{(III)} = \cosh\left(n\frac{\pi}{\beta_{a2}\tau}h_a\right)$$

144 *1) Boundary conditions between Zone I and Zone II.*

145 The continuity of the potential between Zone I and Zone II can be written, in mathematical terms, as:

$$\forall x \in \bigcup_{l=1}^{q_s} [(l-1)(w_e + w_d), (l-1)(w_e + w_d) + w_e], \quad (14)$$

$$A^{(I,l)}(x, y = h_a + e) = A^{(II)}(x, y = h_a + e)$$

147 Using the boundary integral method explained in [29][30], the following equation can be obtained:

$$a_0^{(l)} - \frac{1}{2}\mu_0 j_l h_e^2 = \sum_{k \geq 1} [b_k^{(1)} K_{c_k} + b_k^{(2)} K_{s_k}] I_{C,k}^{(l)} + \sum_{k \geq 1} [b_k^{(3)} K_{c_k} + b_k^{(4)} K_{s_k}] I_{S,k}^{(l)} \quad (15)$$

148 And, for each slot l :

$$\forall m \geq 1,$$

$$a_m^{(l)} = \frac{1}{K_{c_m}^{(I)}}.$$

$$\left\{ \sum_{k \geq 1} [b_k^{(1)} K_{c_k} + b_k^{(2)} K_{s_k}] \alpha_{C,(m,k)}^{(l)} + \sum_{k \geq 1} [b_k^{(3)} K_{c_k} + b_k^{(4)} K_{s_k}] \alpha_{S,(m,k)}^{(l)} \right\} \quad (16)$$

149 The normalized coefficients $I_{C,k}^{(l)}$, $I_{S,k}^{(l)}$, $\alpha_{C,(m,k)}^{(l)}$, and $\alpha_{S,(m,k)}^{(l)}$ are given in Appendix II.

150 Concerning the normal derivative of the potential between Zone I and Zone II, it is possible to write:

$$\begin{aligned} \forall x \in \bigcup_{l=1}^{q_s} [(l-1)(w_e + w_d), (l-1)(w_e + w_d) + w_e], \\ \left(\frac{dA^{(II)}}{dy} \right)_{y=h_a+e} = \left(\frac{dA^{(I)}}{dy} \right)_{y=h_a+e} \\ \forall x \in [0, \tau] - \left\{ \bigcup_{l=1}^{q_s} [(l-1)(w_e + w_d), (l-1)(w_e + w_d) + w_e] \right\}, \\ \left(\frac{dA^{(II)}}{dy} \right)_{y=h_a+e} = 0 \end{aligned} \quad (17)$$

151 Using again the boundary integral method [29][30], it can be found:

$$\begin{aligned} \forall k \geq 1 \\ b_k^{(1)} K_{c_k} - b_k^{(2)} K_{s_k} \\ = \frac{1}{2k-1} \sum_{l=1}^{q_s} \left\{ - \sum_{m \geq 1} m a_m^{(l)} K_{c_m}^{(I)} \alpha_{C,(m,k)}^{(l)} + 2 \frac{\mu_0 h_e w_e j_l}{\pi} I_{C,k}^{(l)} \right\} \end{aligned} \quad (18)$$

152 And also:

$$\begin{aligned} & \forall k \geq 1 \\ & b_k^{(3)} K_{c_k} - b_k^{(4)} K_{s_k} \\ & = \frac{1}{2k-1} \sum_{l=1}^{q_s} \left\{ -\sum_{m \geq 1} m a_m^{(l)} K_{c_m}^{(l)} \alpha_{S,(m,k)}^{(l)} + 2 \frac{\mu_0 h_e w_e j_l}{\pi} I_{S,k}^{(l)} \right\} \end{aligned} \quad (19)$$

153 2) *Boundary conditions between Zone II and Zone III.*

154 Using the same method as in the previous section for taking into account the boundary conditions, it is possible to get the following
155 relation, for each rotor displacement X_d , for what concerns the potential continuity between Zone II and Zone III:

$$c_0 = \sum_{k \geq 1} \left[b_k^{(1)} K_{c_k}^{(II')} + b_k^{(2)} K_{s_k}^{(II')} \right] J_{C,k} + \sum_{k \geq 1} \left[b_k^{(3)} K_{c_k}^{(II')} + b_k^{(4)} K_{s_k}^{(II')} \right] J_{S,k} \quad (20)$$

156 And also:

$$\begin{aligned} & \forall n \geq 1, \\ & c_n = \frac{1}{K_{c_n}^{(III)}} \left\{ \sum_{k \geq 1} \left[b_k^{(1)} K_{c_k}^{(II')} + b_k^{(2)} K_{s_k}^{(II')} \right] \beta_{C,(n,k)} \right. \\ & \quad \left. + \sum_{k \geq 1} \left[b_k^{(3)} K_{c_k}^{(II')} + b_k^{(4)} K_{s_k}^{(II')} \right] \beta_{S,(n,k)} - \mu_0 \frac{\beta_{a2} \tau M_n}{\pi n} \right\} \end{aligned} \quad (21)$$

157 The definitions and expressions of the normalized numbers $J_{C,k}$, $J_{S,k}$, $\beta_{C,(m,k)}$, and $\beta_{S,(m,k)}$ are given in appendix II.

158 For the continuity of the normal derivative of the potential between Zones II and III, the two following equations are obtained:

$$\begin{aligned} & \forall k \geq 1 \\ & b_k^{(1)} K_{c_k}^{(II')} - b_k^{(2)} K_{s_k}^{(II')} = \frac{1}{2k-1} \sum_{n \geq 1} n c_n K_{c_n}^{(III)} \beta_{C,(n,k)} \end{aligned} \quad (22)$$

$$\begin{aligned} & \forall k \geq 1 \\ & b_k^{(3)} K_{c_k}^{(II')} - b_k^{(4)} K_{s_k}^{(II')} = \frac{1}{2k-1} \sum_{n \geq 1} n c_n K_{c_n}^{(III)} \beta_{S,(n,k)} \end{aligned} \quad (23)$$

159 *D. Solution of the problem.*

160 1) *Matrix formulation*

161 All the series are limited to a finite number of terms, called N . This number is computed in the next part, so as to offer the best
162 compromise between the computation time, and the accuracy.

163 The unknown parameters are, using the vector notation:

$$164 \forall l \in \llbracket 1, q_s \rrbracket, \left[a^{(l)} \right] = \begin{bmatrix} a_1^{(l)} \\ \vdots \\ a_N^{(l)} \end{bmatrix}, \left[b^{(1)} \right] = \begin{bmatrix} b_1^{(1)} \\ \vdots \\ b_N^{(1)} \end{bmatrix}, \left[b^{(2)} \right] = \begin{bmatrix} b_1^{(2)} \\ \vdots \\ b_N^{(2)} \end{bmatrix}, \left[b^{(3)} \right] = \begin{bmatrix} b_1^{(3)} \\ \vdots \\ b_N^{(3)} \end{bmatrix}, \left[b^{(4)} \right] = \begin{bmatrix} b_1^{(4)} \\ \vdots \\ b_N^{(4)} \end{bmatrix}, \text{ and } \left[c \right] = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix}.$$

165 Equation (16) can be written for all the numbers m between 1 and N . So the following matrix equation is obtained, for all slot number l
166 between 1 and q_s :

$$\forall l \in \llbracket 1, q_s \rrbracket, \left[a^{(l)} \right] = \left[D_1 \right] \left\{ \begin{array}{l} \left[\alpha_C^{(l)} \right] \left(\left[D_2 \right] \left[b^{(1)} \right] + \left[D_3 \right] \left[b^{(2)} \right] \right) \\ + \left[\alpha_S^{(l)} \right] \left(\left[D_2 \right] \left[b^{(3)} \right] + \left[D_3 \right] \left[b^{(4)} \right] \right) \end{array} \right\} \quad (24)$$

167 The following square matrixes for each slot are introduced (see appendix II):

$$168 \left[\alpha_C^{(l)} \right] = \left[\alpha_{C,(m,k)}^{(l)} \right]_{(m,k) \in \llbracket 1, N \rrbracket^2} \text{ and } \left[\alpha_S^{(l)} \right] = \left[\alpha_{S,(m,k)}^{(l)} \right]_{(m,k) \in \llbracket 1, N \rrbracket^2}.$$

169 The other diagonal matrixes are also defined in Appendix II.

170 Equations (18) and (19), for each k between 1 and N , lead to the following matrix relations:

$$\begin{aligned} [D_2] \cdot [b^{(1)}] - [D_3] \cdot [b^{(2)}] &= -[INV] \sum_{l=1}^{q_s} [\alpha_C^{(l)}]^T [D_4] [a^{(l)}] \\ &+ 2 \frac{\mu_0 h_e w_e}{\pi} \sum_{l=1}^{q_s} j_l [I_C^{(l)}] \end{aligned} \quad (25)$$

$$\begin{aligned} [D_2] \cdot [b^{(3)}] - [D_3] \cdot [b^{(4)}] &= -[INV] \sum_{l=1}^{q_s} [\alpha_S^{(l)}]^T [D_4] [a^{(l)}] \\ &+ 2 \frac{\mu_0 h_e w_e}{\pi} \sum_{l=1}^{q_s} j_l [I_S^{(l)}] \end{aligned} \quad (26)$$

171 The matrix $[INV]$, as well as the following matrixes, are also defined in Appendix II: $[I_C^{(l)}] = [I_{C,k}^{(l)}]_{k \in [1,N]}$, and $[I_S^{(l)}] = [I_{S,k}^{(l)}]_{k \in [1,N]}$

172 Thanks to (16), it can be obtained:

$$\begin{aligned} \forall l \in [1, q_s], [a^{(l)}] &= [D_1] \left\{ [\alpha_C^{(l)}] \left([D_2] [b^{(1)}] + [D_3] [b^{(2)}] \right) \right. \\ &\left. + [\alpha_S^{(l)}] \left([D_2] [b^{(3)}] + [D_3] [b^{(4)}] \right) \right\} \end{aligned} \quad (27)$$

173 And, from (21):

$$\begin{aligned} [c] &= [D_5] \left\{ [\beta_C] \left([D_6] [b^{(1)}] + [D_7] [b^{(2)}] \right) \right. \\ &\left. + [\beta_S] \left([D_6] [b^{(3)}] + [D_7] [b^{(4)}] \right) - [V] \right\} \end{aligned} \quad (28)$$

174 The matrix $[\beta_C] = [\beta_{C,(n,k)}]_{(n,k) \in [1,N]^2}$ and $[\beta_S] = [\beta_{S,(n,k)}]_{(n,k) \in [1,N]^2}$ are also given in Appendix II, as well as the vector

175 $[V] = [V_n]_{n \in [1,N]}$ and the other diagonal matrixes.

176 Equations (22) and (23) lead to the following matrix equations:

$$[D_6] \cdot [b^{(1)}] - [D_7] \cdot [b^{(2)}] = [INV] [\beta_C]^T [D_8] [c] \quad (29)$$

$$[D_6] \cdot [b^{(3)}] - [D_7] \cdot [b^{(4)}] = [INV] [\beta_S]^T [D_8] [c] \quad (30)$$

177 2) *Numerical computation of the problem.*

178 Recalling the equations' system to solve:

$$\begin{cases} \forall l \in [1, q_s], [a^{(l)}] = [D_1] \left\{ [\alpha_C^{(l)}] \left([D_2] [b^{(1)}] + [D_3] [b^{(2)}] \right) \right. \\ \left. + [\alpha_S^{(l)}] \left([D_2] [b^{(3)}] + [D_3] [b^{(4)}] \right) \right\} \\ [D_2] \cdot [b^{(1)}] - [D_3] \cdot [b^{(2)}] = -[INV] \sum_{l=1}^{q_s} [\alpha_C^{(l)}]^T [D_4] [a^{(l)}] + 2 \frac{\mu_0 h_e w_e}{\pi} \sum_{l=1}^{q_s} j_l [I_C^{(l)}] \\ [D_2] \cdot [b^{(3)}] - [D_3] \cdot [b^{(4)}] = -[INV] \sum_{l=1}^{q_s} [\alpha_S^{(l)}]^T [D_4] [a^{(l)}] + 2 \frac{\mu_0 h_e w_e}{\pi} \sum_{l=1}^{q_s} j_l [I_S^{(l)}] \\ [c] = [D_5] \left\{ [\beta_C] \left([D_6] [b^{(1)}] + [D_7] [b^{(2)}] \right) + [\beta_S] \left([D_6] [b^{(3)}] + [D_7] [b^{(4)}] \right) - [V] \right\} \\ [D_6] \cdot [b^{(1)}] - [D_7] \cdot [b^{(2)}] = [INV] [\beta_C]^T [D_8] [c] \\ [D_6] \cdot [b^{(3)}] - [D_7] \cdot [b^{(4)}] = [INV] [\beta_S]^T [D_8] [c] \end{cases} \quad (31)$$

179 If $q_s=3$, a system of 8 equations with 8 unknown vectors must be solved. Equations (25) and (26) are substituted in Equation (24).
 180 In order to reduce the number of equations, the following process is proposed: in a first stage, by an analytical substitution, we can
 181 express this system with only the unknown series' coefficients vectors: $[b^{(1)}][b^{(2)}][b^{(3)}][b^{(4)}]$.
 182 After algebraic substitutions, the system can be written under the form:

$$[M] \cdot [b] = [P] \quad (32)$$

183 Where the unknown vector $[b]$ is $[b] = \begin{bmatrix} b^{(1)} \\ b^{(2)} \\ b^{(3)} \\ b^{(4)} \end{bmatrix}$. The matrix $[M]$ and the vector $[P]$ are obtained by linear combinations of the matrixes

184 introduced in the previous section.

185 To solve the linear system (32), it is necessary to perform a numerical inversion of the matrix $[M]$. It is also necessary to choose the
 186 number N of harmonics. For that, the following numerical procedure may be proposed: firstly, it is possible to compute the inversion
 187 of the system for $N=1$, which is very fast, since the size of the matrix $[M]$ is just 4. Then the following algorithm is applied.

188 Suppose that we have just computed the solution for a certain number N of harmonics. For this number, the matrix $[M]$ is called $[M_N]$,
 189 $[V]$ is called $[V_N]$, and the solution $[b_N]$ is:

$$190 [b_N] = \begin{bmatrix} [b_N^{(1)}]^T & [b_N^{(2)}]^T & [b_N^{(3)}]^T & [b_N^{(4)}]^T \end{bmatrix}^T$$

191 To compute the vector solution $[b_{N+1}]$ for $N+1$ harmonics, which is solution of the equation $[M_{N+1}] \cdot [b_{N+1}] = [V_{N+1}]$, a Gauss-Siedel
 192 algorithm can be used. In this inversion method, a first initialisation of the solution is required. Instead of taking the zero vector for the
 193 initialisation, we use in the initial step the solution of the previous system with N harmonics, more precisely:

$$194 [b_{N+1,0}] = \begin{bmatrix} [b_N^{(1)}]^T & 0 & [b_N^{(2)}]^T & 0 & [b_N^{(3)}]^T & 0 & [b_N^{(4)}]^T & 0 \end{bmatrix}^T$$

195 So the number of iterations of the Gauss-Seidel process is reduced in order to get the solution $[b_{N+1}]$.

196 We stop increasing the number N when the difference of the solutions between two iterations becomes as small as wanted (in practice
 197 less than 1% in relative value). So the number N of harmonics is not fixed, but is optimized for each machine's geometry.

198 V. COMPARISON WITH FINITE ELEMENTS

199 For the finite element verification, as well as for the torque computations, the geometry presented in Table II is used. Moreover, for the
 200 torque computations, the mean airgap radius (i.e. the mid airgap radius R_m of the radial flux machine, or the mean active radius of the
 201 axial flux machine) is assumed to be 0.1m (see Appendix I for a detailed discussion about this point). In the finite element code, in
 202 order to be consistent with the assumption of infinite iron permeability made for the analytical model, a high relative permeability is
 203 assigned to the ferromagnetic parts (in practice, this relative permeability is taken equal to 1000). A discussion about the consequences
 204 of this linear model is provided in Appendix I: the results are compared with ones obtained using non linear B-H experimental
 205 relationships.
 206

207 TABLE II
 208 GEOMETRY OF THE MACHINE CHOSEN FOR THE INDUCTION AND TORQUE COMPUTATIONS (ANALYTICAL AND FINITE ELEMENT VERIFICATION)

Notation	Numerical values
h_a	5mm
e	2mm
e_{cr}	10mm
e_{cs}	10mm
h_e	30mm
τ	60mm
β_{a1}	0.6
β_{a2}	0.8
w_e	10mm
w_d	10mm
H	100mm
B_r	1.2T

209 A. The magnets' field.

210 The no-load airgap magnetic field (created by the magnets, when all the stator currents are zero) has been computed, both
 211 analytically and by finite elements. On Fig. 6, a contour [AB] in the middle of the airgap is chosen, on which we shall plot the
 212 induction caused by the magnets (all the current densities $j_1=j_2=j_3=0$). This segment length is equal to the pole pitch τ . The results are
 213 shown on Fig. 7, Fig. 8 and Fig. 9, respectively for the potential, the B_x and the B_y induction components. The agreement is satisfying.
 214

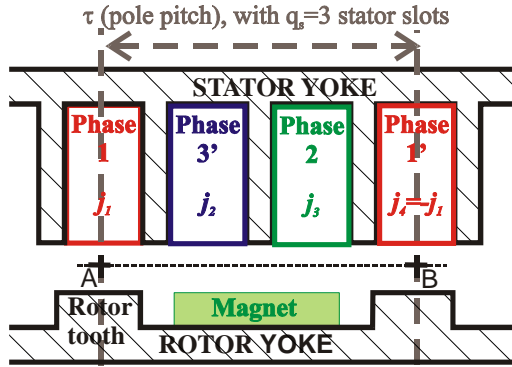


Fig. 6: The plot contour [AB] in the airgap

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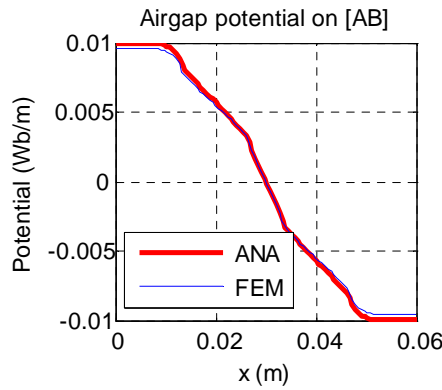


Fig. 7: The potential caused by the magnets on the segment [AB]

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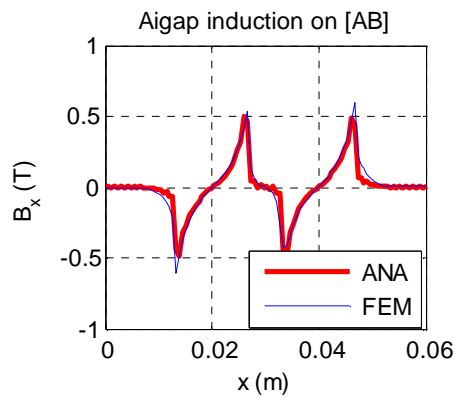


Fig. 8: The B_x induction caused by the magnets on the segment [AB]

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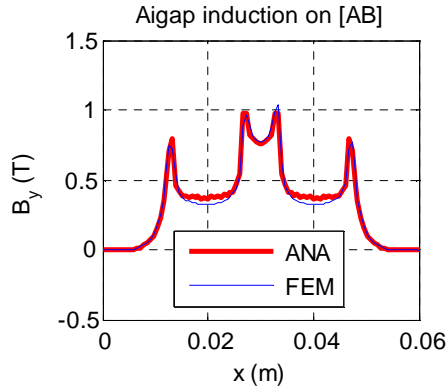


Fig. 9: The B_y induction caused by the magnets on the segment [AB]

224
225

226 **B. The stator currents' field.**

227 The results of the field caused by the stator currents (in the computation procedure, $B_r=0$) are now analyzed. According to the
 228 Park theory, a d axis current is applied. Since the system is linear, the calculus is performed for a current of just 1A in each slot. So,
 229 on Fig. 6, current densities of $j_1=1A/S_{slot}$ (S_{slot} is the slot surface), $j_2=j_1/2$, and $j_3=-j_1/2$ have been applied. For the q axis current, the
 230 rotor undergoes a translation of a half pole pitch in comparison with the d axis calculation. The results for the d axis currents are
 231 shown on Fig. 10, Fig. 11 and Fig. 12, in good accordance with finite elements.

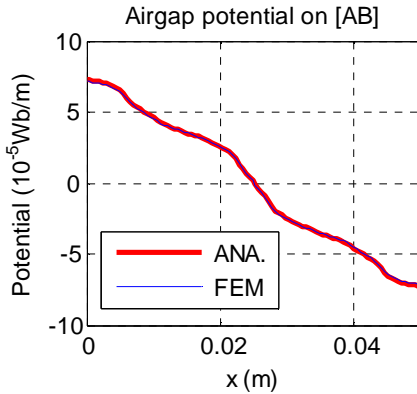


Fig. 10: The potential caused by the d axis currents on the segment [AB]

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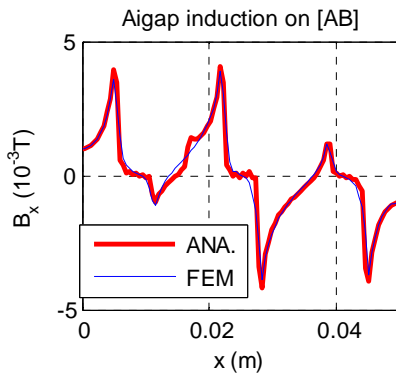
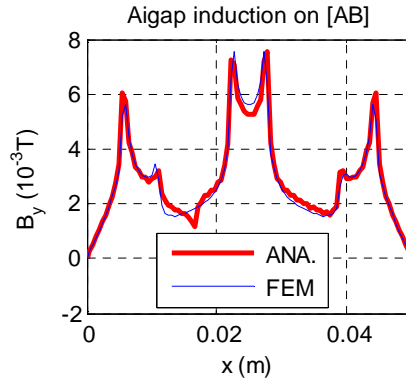


Fig. 11: The B_x induction caused by the d axis currents on [AB]

236
237
238

Fig. 12: The B_y induction caused by the d axis currents on [AB]

The results obtained using the two methods are in good agreement. The time necessary for the analytical computation is about 30% less than the time necessary for the finite element calculation.

VI. THE INSTANTANEOUS TORQUE COMPUTATION

A. The calculation method

The analytical model can be used for the computation of the instantaneous torque. The analytical computation is performed as the following: the Maxwell stress tensor formula is applied on the [AB] segment of Fig. 6 located in the middle of the airgap, giving the tangential effort on this segment as:

$$F_t = \int_{[AB]} H_x B_y dx \quad (33)$$

This analytical calculation is repeated for each elementary position of the rotor.

The torque calculation with finite elements is based on the same principles, but the field and induction and then the Maxwell stress tensor are computed numerically on the [AB] segment. The spatial period of the torque is a tooth pitch. The tooth pitch has been decomposed in ten elementary positions (both for finite element or analytical computation). For each of them, the calculation has been carried out, and a cubic interpolation has been applied between the ten points. This strategy permits to save computation time, without decreasing the model accuracy.

In the Park formalism, a q current of 800A per slot and a d current per slot of -100A are applied. The total torque is shown on Fig. 13. The analytical and finite elements calculations are in good agreement: the mean torque is around 170Nm for the two methods.

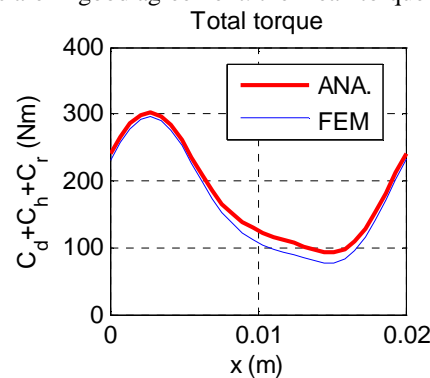


Fig. 13: The total torque over a tooth pitch

The cogging torque is given on Fig. 14, (it is computed with zero currents). Its mean value is zero (as expected), although its variations are quite high (50Nm). The discrepancy of 8% on the maximum value can be explained by the fact that the instantaneous torque computation is very sensitive to the number N of harmonics chosen for the analytical computation. Indeed, the torque is obtained by multiplying two derivatives of the potential: the normal induction B_y and the tangential field $H_x = B_x / \mu_0$. These derivatives are quite irregular signals (see the previous section), and an important number of harmonics can be required to represent them with a good precision. Here, as explained in Section IV.D.2, we stopped increasing the number N of harmonics if the [b] vectors between two iterations do not present more than 1% of difference in relative value. If we choose a lowest error, the torques computed by finite elements and analytically become closer (a difference of 4% is observed for a relative error of 0.3%). However, this solution does not appear as optimal, because it increases much the computation time, and then cannot be used for a pre-design process. We believe that a

269 difference of about 10% is acceptable in a pre-design process stage. The most important is however to get an exact mean value, which
 270 is the case, even for a small number of harmonics.
 271

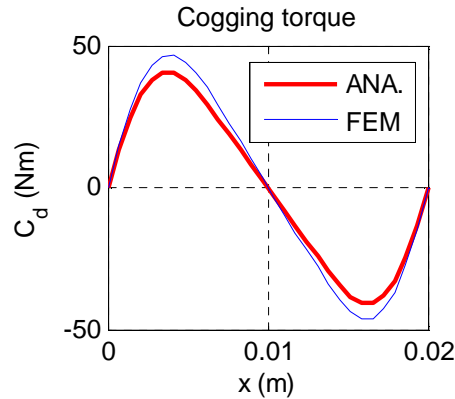


Fig. 14: The cogging torque over a tooth pitch

272
 273
 274
 275 The reluctant torque, which is due only to the stator currents (the remanent induction of the magnets is put to zero), is presented in Fig.
 276 15. The fluctuation is quite important, but the mean value remain weak (10Nm) compared to the total torque, mainly due to the hybrid
 277 torque component. This one is obtained by subtracting to the total torque the cogging and the reluctant torque (Fig. 16).

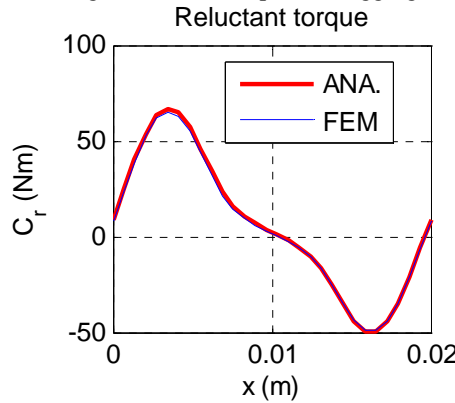


Fig. 15: The reluctant torque over a tooth pitch

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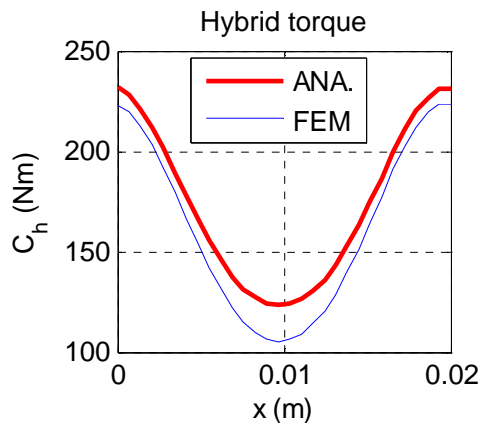


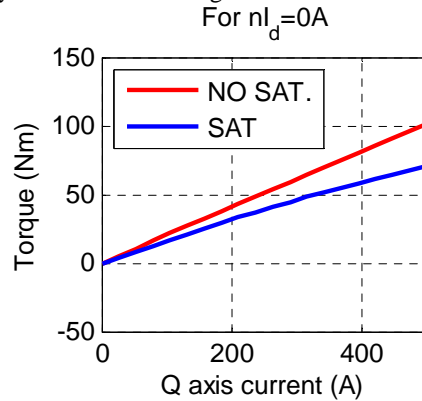
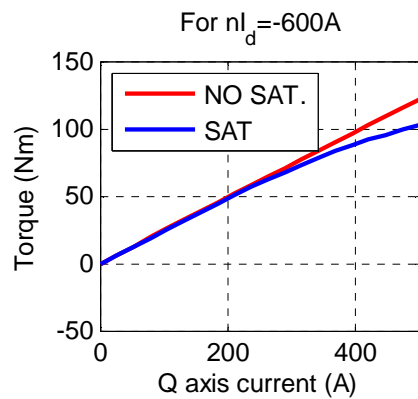
Fig. 16: The hybrid torque over a tooth pitch

281
 282
 283 **B. Discussion**

284 It has been shown that the analytical method gives results in good agreement with finite elements. Nevertheless, it could be
 285 interesting to illustrate the limit of this method.

286 The main limit of the method consists in the fact that saturation of the machine's ferromagnetic parts is not taken into account. So
 287 torque calculations are performed in this section using the finite element method including ferromagnetic material saturation (the FeSi
 288 B-H curve given in Appendix I), and the results are compared with the analytical model. This calculation permits to understand the
 289 consequences of this limitation on the torque computation.

290 So a high q axis current (up to 500A) is applied, so as to create saturation, and this case is studied using finite elements
 291 computations. The mean torque is computed over a tooth pitch, as in the previous section. The torque curve taking into account
 292 saturation is compared with the one derived from our analytical linear model. This work has been done for two values of d axis
 293 current. The first is for a zero d axis current (Fig. 17), and the second is for a d axis current per slot of $nI_d=-600A$ (Fig. 18).
 294 The first fact to notice is that the saturation phenomena reduces the total flux in the machine, especially when the q axis current is high
 295 Therefore the torque is not as high as in the non saturated case.
 296 Another point is the effect of the d axis current. A strong d axis current permits to increase the torque significantly, since it allows a
 297 reluctant torque component. But it also permits a reduction of the effect of saturation (Fig. 18), because the total flux of the machine is
 298 weakened. It could be shown in the same way that flux weakening also decreases the iron losses.

Fig. 17: Mean torque with and without saturation ($nI_d=0$)Fig. 18: Mean torque with and without saturation ($nI_d=-600A$)

VII. CONCLUSION

304 An analytical method was developed in order to take into account the stator and rotor saliency of the machine, for the computation of
 305 the magnetic induction in the machine airgap. This permits to compute precisely the machine's torque. In a coming paper, we are
 306 going to use this model for the computation of the inductions in the ferromagnetic parts of the machine, so as to be able to evaluate the
 307 iron losses, taking into account the stator leakage flux.
 308

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311

APPENDIX 1

312 In this appendix, the two fundamental simplifying assumptions of this work (no curvature effect, no saturation) are discussed.

313 A. *The curvature effect*

314 It is possible to use the model presented in this article for a radial flux machine, on the condition of neglecting the curvature effect. It
 315 means that the real machine is made equivalent to a linear actuator, as shown in Fig. 19. The pole pitch τ of the linear actuator is
 316 chosen equal to the pole pitch of the radial flux machine at its mean airgap radius R_m (i.e. the mid airgap radius). All the other radial
 317 dimensions (magnet thickness h_a , airgap width e , slot depth h_s ,...) are the same for both actuators. The relative magnet opening on the
 318 pole pitch and the relative slot opening on the tooth pitch are also identical.

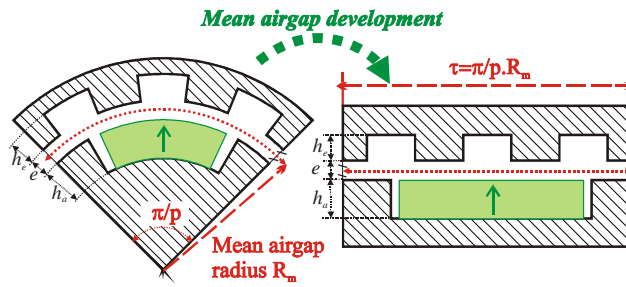


Fig. 19: Development of the radial flux machine to its mean radius to obtain the equivalent linear actuator

This approximation is justified if the mean airgap radius R_m is important compared to the other radial dimensions (magnet thickness, airgap width, slot dept...). To understand to what extent the approximation can be done, finite element calculations have been carried out with the radial flux machine, and with the equivalent linear actuator, in order to compare the results. The machine geometry used in the comparison is given in Table III. The ferromagnetic parts are taken into account using a linear material of relative magnetic permeability equal to 1000.

TABLE III
GEOMETRICAL PARAMETERS OF THE RADIAL FLUX MACHINE

Description	Notation	Numerical value
Dimensions along the radial direction		
Mean airgap radius	R_m	0.076m
Airgap width	e	0.002m
Slot dept	h_e	0.03m
Magnet thickness	h_a	Interval [0.001m;0.01m]
Dimensions along the orthoradial direction		
Pole pair number	p	4
Pole pitch	τ	0.06m
Magnet relative opening with regard to the pole pitch	β_{a1} ($0 < \beta_{a1} < 1$)	0.6
Rotor inter-teeth space relative opening with regard to the pole pitch	β_{a2} ($\beta_{a1} < \beta_{a2} < 1$)	0.8
Stator relative slot opening with regard to the tooth pitch	β_e ($0 < \beta_e < 1$)	0.5
Dimensions along the z direction		
Active thickness of the machine along the z axis	H	0.1m
Physical data		
Remanent induction of the magnets (NdFeB) at room temperature	B_r	1.2T

For example, the no-load flux is studied. The parameter of importance is the value of the ratio h_a/R_m between the magnet radial thickness and the mean airgap radius. The magnet thickness varies between 0.001m and 0.01m. The ratio between the no-load flux of the two actuators is shown in Fig. 20.

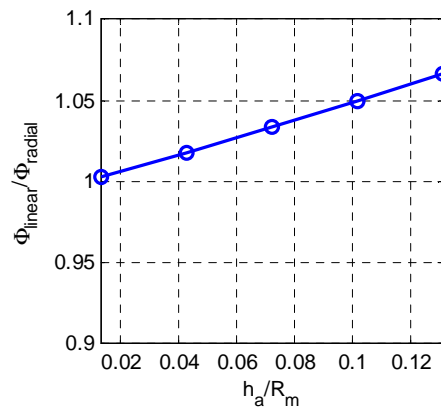
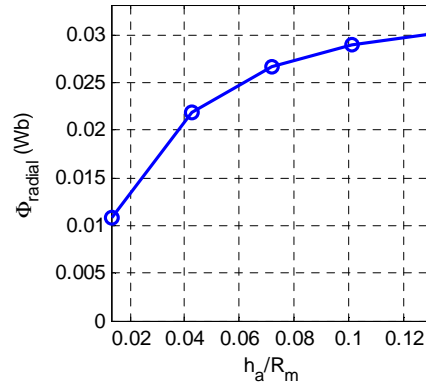


Fig. 20: Ratio between the no-load fluxes obtained in the radial flux machine and the equivalent linear machine

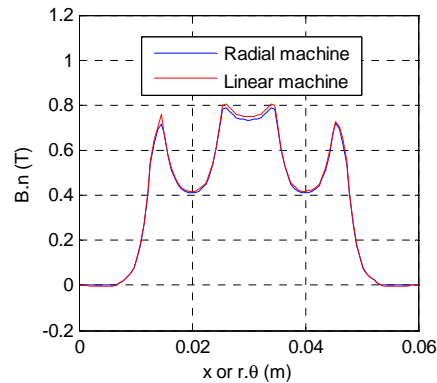
As expected, the two fluxes are nearly the same for small magnet thicknesses with regard to the pole pitch, whereas some discrepancy appears by increasing the magnet thickness. However, the difference is never more than 7% in relative value. Moreover, a magnet thickness of 0.01m (for which the difference is the highest) is not useful, because, as shown in Fig. 21, no significant increase of the

337 no-load flux appears between the two last points of the magnet thicknesses. Thus it can be concluded that the error on the no-load flux
 338 made by the development of the radial flux machine to the equivalent linear one is restricted to a few percent in relative value.
 339 However, if this error has to be corrected, some authors have proposed methods based on conformal mapping [29], in order to
 340 rigorously transform the radial flux machine to an equivalent linear actuator. This conclusion allows us to compute the magnetic field
 341 in the linear equivalent actuator (in Cartesian coordinates), and then apply the results to the case of the radial flux machine.

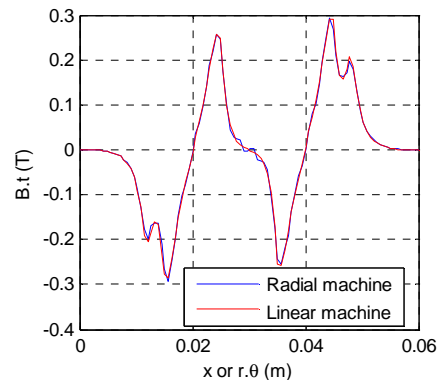


342
 343 Fig. 21: No-load flux in the radial flux machine (in function of the magnet thickness h_a)

344 It can also be useful to check if the inductions are locally also the same for the two kinds of machines. For that, it is proposed to plot in
 345 Fig. 22 and Fig. 23 the normal and the tangential inductions in the middle of the airgap (dotted red line in Fig. 19). Indeed these
 346 inductions are involved in the cogging torque computation, and of course in the no-load flux calculation (the no-load flux is the
 347 integral of the normal induction component). It can be seen that the waveforms are nearly identical, which demonstrates the validity of
 348 the mean radius development also for the local induction computation.



349
 350 Fig. 22: No-load induction (normal component) in the radial flux machine and in the equivalent linear one (for a magnet thickness $h_a=0.0035$ m) on a line located in the
 351 middle of the airgap (dotted red line of Fig. 19)

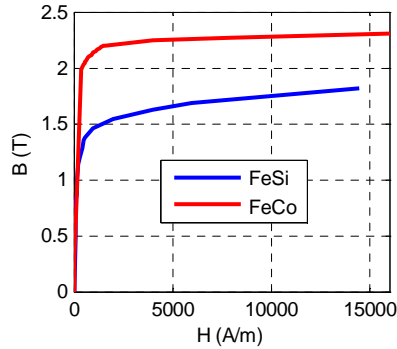


353
 354 Fig. 23: No-load induction (tangential component) in the radial flux machine and in the equivalent linear one (for a magnet thickness $h_a=0.0035$ m) on a line located in
 355 the middle of the airgap (dotted red line of Fig. 19)

356 **B. The saturation effect**

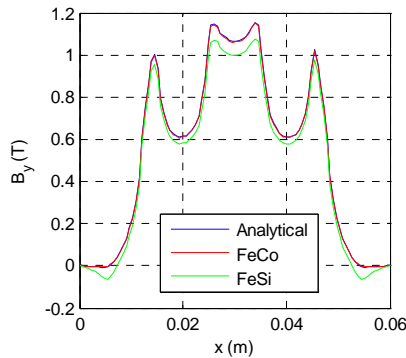
357 The analytical model implies that the ferromagnetic parts are assumed to have an infinite magnetic permeability. It is appropriate to
 358 compare the results provided by this model with the ones obtained with real non-linear B-H anhysteretic relationships experimentally
 359 measured in Iron-Silicon (FeSi) and Iron-Cobalt (FeCo) laminations (Fig. 24). The main advantage of Iron-Cobalt material is its high

360 saturation magnetization (nearly 2.3T), and high initial permeability. Its drawback is its high cost and low resistivity, which implies
 361 high dynamic loss. The computations are then carried out using a non-linear finite element code.



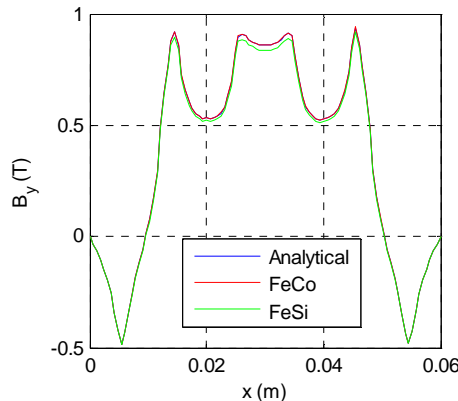
362
 363 Fig. 24: Non-linear B-H anhyysteretic relationships experimentally obtained in FeSi and FeCo laminations

364 In this study, the same geometry as in Table II is adopted. The B_y inductions obtained on the [AB] segment (Fig. 6), using the
 365 analytical and the non-linear finite element models, are plotted in Fig. 25 for no-load excitation conditions. The analytical model and
 366 the non-linear finite element model with the FeCo curve provide identical results, while a decrease of the airgap induction can be seen
 367 with the FeSi material. This decrease is responsible of an error of 8% on the total no-load flux between the analytical model, and the
 368 model using the FeSi curve.



369
 370 Fig. 25: B_y induction obtained on the [AB] segment (Fig. 6) for the analytical model, and the non-linear finite element models using the FeCo material and the FeSi
 371 material (no-load excitation conditions)

372 In Fig. 26, the same analysis has been repeated, being this time under field weakening conditions: in addition to the magnets' field, a d
 373 axis current has been added ($nI_d = -600A/m$, see Section VI.B). The level and the waveform shape has been changed compared to the
 374 previous case in which only the no-load flux was considered. It is interesting to notice that the field weakening permits to reduce the
 375 saturation level of the machine, and then justify the use of the linear analytical model. Indeed the difference on the fluxes between the
 376 linear model and the FeSi saturated model is now 3% in relative value, whereas it was 8% under no-load condition.



377
 378 Fig. 26: B_y induction obtained on the [AB] segment for the analytical model, and the non-linear finite element models using the FeCo material and the FeSi material
 379 (field weakening condition: superposition of the magnets' field and d axis current $nI_d = -600A/m$)

380 To conclude, it must be said that the saturation effect can decrease the airgap induction level, and then the torque (see Section VI for a
 381 detailed discussion). The flux weakening reduces the saturation effect. However, for the studied geometry, this effect is limited to a
 382 decrease of 10% of the airgap induction between the analytical model and the non-linear model using FeSi laminations. For FeCo
 383 laminations, due to the very high saturation level of this material, the saturation do not need to be taken into account, justifying the use
 384 of the analytical model.

385 It must be noticed that the saturation effect can be taken into account using the model proposed by the author of [31]. The decrease of
 386 the airgap induction due to the saturation effect is modeled by a fictitious increase of the airgap thickness in the analytical
 387 computations.

388
 389

390 APPENDIX 2

391 The numbers α introduced in for the continuity conditions were defined the formulas:

$$\alpha_{C,(m,k)}^{(l)} = \sin_C \left(\pi \frac{m\tau - (2k-1)w_e}{2\tau} \right) \cos \left(\pi \frac{m\tau - (2k-1)w_e}{w_e\tau} (l-1)(w_e + w_d) - \frac{m\pi(l-1)(w_e + w_d)}{w_e} + \pi \frac{m\tau - (2k-1)w_e}{2\tau} \right) \\ + \sin_C \left(\pi \frac{m\tau + (2k-1)w_e}{2\tau} \right) \cos \left(\pi \frac{m\tau + (2k-1)w_e}{w_e\tau} (l-1)(w_e + w_d) - \frac{m\pi(l-1)(w_e + w_d)}{w_e} + \pi \frac{m\tau + (2k-1)w_e}{2\tau} \right) \quad (34)$$

$$\alpha_{S,(m,k)}^{(l)} = \sin_C \left(\pi \frac{m\tau + (2k-1)w_e}{2\tau} \right) \sin \left(\pi \frac{m\tau + (2k-1)w_e}{w_e\tau} (l-1)(w_e + w_d) - \frac{m\pi(l-1)(w_e + w_d)}{w_e} + \pi \frac{m\tau + (2k-1)w_e}{2\tau} \right) \\ - \sin_C \left(\pi \frac{m\tau - (2k-1)w_e}{2\tau} \right) \sin \left(\pi \frac{m\tau - (2k-1)w_e}{w_e\tau} (l-1)(w_e + w_d) - \frac{m\pi(l-1)(w_e + w_d)}{w_e} + \pi \frac{m\tau - (2k-1)w_e}{2\tau} \right) \quad (35)$$

$$I_{S,k}^{(l)} = \sin_C \left((2k-1) \frac{\pi}{2\tau} w_e \right) \sin \left((2k-1) \frac{\pi}{\tau} (l-1)(w_e + w_d) + (2k-1) \frac{\pi}{2\tau} w_e \right) \quad (36)$$

$$I_{C,k}^{(l)} = \sin_C \left((2k-1) \frac{\pi}{2\tau} w_e \right) \cos \left((2k-1) \frac{\pi}{\tau} (l-1)(w_e + w_d) + (2k-1) \frac{\pi}{2\tau} w_e \right) \quad (37)$$

392 The different normalized coefficients for the potential continuity equalities between Zone I and Zone II are given by:

393

$$J_{C,k} = \sin_C \left((2k-1) \beta_{a2} \frac{\pi}{2} \right) \cos \left((2k-1) \frac{\pi}{\tau} X_d + (2k-1) \beta_{a2} \frac{\pi}{2} \right) \quad (38)$$

$$J_{S,k} = \sin_C \left((2k-1) \beta_{a2} \frac{\pi}{2} \right) \sin \left((2k-1) \frac{\pi}{\tau} X_d + (2k-1) \beta_{a2} \frac{\pi}{2} \right) \quad (39)$$

$$\beta_{C,(n,k)} = \sin_C \left[(n - (2k-1) \beta_{a2}) \frac{\pi}{2} \right] \cos \left[\frac{\pi \frac{n - (2k-1) \beta_{a2}}{\beta_{a2}\tau} X_d - \frac{n\pi X_d}{\beta_{a2}\tau}}{+ (n - (2k-1) \beta_{a2}) \frac{\pi}{2}} \right] + \sin_C \left[(n + (2k-1) \beta_{a2}) \frac{\pi}{2} \right] \cos \left[\frac{\pi \frac{n + (2k-1) \beta_{a2}}{\beta_{a2}\tau} X_d - \frac{n\pi X_d}{\beta_{a2}\tau}}{+ (n + (2k-1) \beta_{a2}) \frac{\pi}{2}} \right] \quad (40)$$

$$\beta_{S,(n,k)} = \sin_C \left[(n + (2k-1) \beta_{a2}) \frac{\pi}{2} \right] \sin \left[\frac{\pi \frac{n + (2k-1) \beta_{a2}}{\beta_{a2}\tau} X_d - \frac{n\pi X_d}{\beta_{a2}\tau}}{+ (n + (2k-1) \beta_{a2}) \frac{\pi}{2}} \right] - \sin_C \left[(n - (2k-1) \beta_{a2}) \frac{\pi}{2} \right] \sin \left[\frac{\pi \frac{n - (2k-1) \beta_{a2}}{\beta_{a2}\tau} X_d - \frac{n\pi X_d}{\beta_{a2}\tau}}{+ (n - (2k-1) \beta_{a2}) \frac{\pi}{2}} \right] \quad (41)$$

$$V_n = \mu_0 \frac{\beta_{a2}\tau}{\pi} \frac{M_n}{n} \quad (42)$$

394 When the equations are put under a matrix formulation, it is convenient to introduce the following diagonal matrix:

$$395 [D_1] = \begin{bmatrix} \ddots & & & & \\ & \frac{1}{\cosh \left(m \frac{\pi}{w_e} h_e \right)} & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots \end{bmatrix}, [D_2] = \begin{bmatrix} \ddots & & & & \\ & \cosh \left((2k-1) \frac{\pi}{\tau} (h_a + e) \right) & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots \end{bmatrix}, [D_3] = \begin{bmatrix} \ddots & & & & \\ & \sinh \left((2k-1) \frac{\pi}{\tau} (h_a + e) \right) & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots \end{bmatrix},$$

$$396 \quad [D_4] = \begin{bmatrix} \ddots & & & \\ & m \cosh\left(m \frac{\pi}{w_e} h_e\right) & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}, [INV] = \begin{bmatrix} \ddots & & & \\ & \frac{1}{2k-1} & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}, [D_5] = \begin{bmatrix} \ddots & & & \\ & \frac{1}{\cosh\left(n \frac{\pi}{\beta_{a2}\tau} h_a\right)} & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix},$$

$$397 \quad [D_6] = \begin{bmatrix} \ddots & & & \\ & \cosh\left((2k-1) \frac{\pi}{\tau} h_a\right) & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}, \text{ and } [D_7] = \begin{bmatrix} \ddots & & & \\ & \sinh\left((2k-1) \frac{\pi}{\tau} h_a\right) & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}, [D_8] = \begin{bmatrix} \ddots & & & \\ & n \cosh\left(n \frac{\pi}{\beta_{a2}\tau} h_a\right) & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}.$$