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# Quantum Oscillations and $\pi$ -States in Multiply Connected Ferromagnet-Superconductor Hybrids

A.V. Samokhvalov · A.S. Mel'nikov · A.I. Buzdin

**Abstract** On the basis of Usadel equations, we consider superconductivity nucleation and Josephson current in multiply connected mesoscopic superconductor/ferromagnet (S/F) hybrids. We demonstrate that the exchange field can provoke an increase in the critical temperature  $T_c$  of the superconducting transition in the magnetic field. We study the Josephson effect in S/F composites and demonstrate that the negative sign of the critical current ( $\pi$  state) can be realized in such structures despite a dispersion of the distances between different segments of superconducting electrodes.

**Keywords** SF hybrids · Usadel equations · Little–Parks oscillations ·  $\pi$  state

The particularity of the proximity effect in superconductor/ferromagnet (S/F) hybrid structures is the damped oscillatory behavior of the Cooper pair wave function inside the ferromagnet [1, 2] (for the reviews, see [3]). This special type of the proximity effect results in the  $\pi$  Josephson S/F/S junction [4], which has at the ground state the opposite sign of the superconducting order parameter in the electrodes. Both the damped oscillatory S/F proximity effect and the  $\pi$  states are proven to be very robust vs. different types of the impurities scattering (magnetic and nonmagnetic), interface transparency, and exist in the diffusive (dirty) limit.

Naturally, at the first stage, the S/F systems with planar (layered) geometry and well-controlled layers thickness have been considered [3]. However, it may be of interest to address a question how the unusual proximity effect and the  $\pi$  states could manifest itself in multiply connected geometry and/or in S/F structures with a poorly defined thickness of ferromagnetic (F) spacer between superconducting (S) electrodes. The goal of this paper is to study the hallmarks of the  $\pi$ -superconductivity in two model hybrid S/F systems (see Fig. 1). The first system consists of thin-walled hollow S cylinder placed in electrical contact with a F core. The second one is two S rod-shaped electrodes embedded in a ferromagnet.

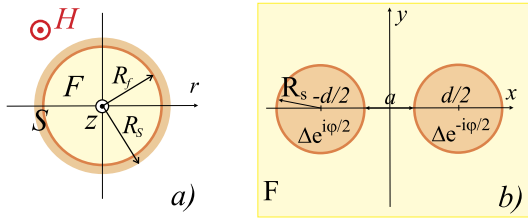
*Vortex States in Thin-Walled S Cylinder* The Little–Parks effect [5] is known to be a sensitive experimental tool for observation of interference phenomena in multiply connected systems, and thus it is natural to use it for the study of the peculiarities of superconductivity nucleation in mesoscopic S/F hybrids. We consider a generic example of hybrid S/F systems with a cylindrical symmetry: F cylindrical filament (core) surrounded by a thin-walled S shell (Fig. 1a).

The calculations of the second-order superconducting phase transition temperature  $T_c$  were based on the linearized Usadel equations [6] for the averaged anomalous Green's functions  $F_f$  and  $F_s$  for the F and S regions, respectively (see [7, 8] for details). We look for a homogeneous along core solution characterized by certain angular momentum  $L$ . Figure 2 shows examples of dependences of the critical temperature  $T_c$  on the external magnetic field  $H$  for different values of the S/F interface resistance. The phase boundary exhibits Little–Parks oscillations, indicating transitions between the states with different angular momenta  $L \rightarrow L \pm 1$  of the S order parameter. The interplay between the oscillations of  $T_c$  due to the orbital effect and the oscillations due to

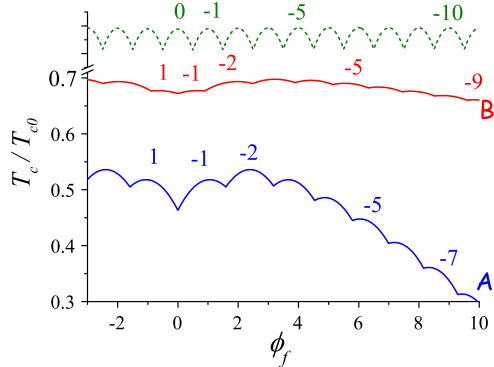
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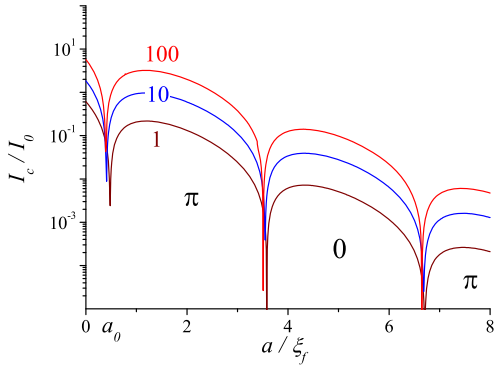
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**Fig. 1** The cross section of the hybrid S/F systems under consideration: (a) thin-walled S shell around a F cylinder; (b) two identical S cylindrical rod-shaped electrodes of radius  $R_s$  surrounded by a F metal



**Fig. 2** The typical dependences of the critical temperature  $T_c$  on magnetic flux  $\phi_f = \pi R_f^2 H / \Phi_0$  enclosed in F cylinder for different values of the S/F interface resistance  $R_A < R_B$ . The numbers near the curves denote the corresponding values of vorticity  $L$ . The dashed line shows the Little–Parks oscillations, when the exchange interaction is canceled



**Fig. 3** Influence of the electrode radius  $R_s$  on the dependence of the critical current  $I_c$  on the distance  $a$  between two S rod-shaped electrodes embedded in F-metal. The numbers near the curves denote the values of the radius  $R_s$  in the units of  $\xi_f$

the exchange field results in breaking of the strict periodicity of the  $T_c(H)$  dependence. We have also observed a slow modulation of the amplitude of the quasiperiodic  $T_c(H)$  oscillations and a shift of the main  $T_c$  maximum to finite external magnetic field values.

*S/F/S Junction Between Two Superconducting Rod* Now we proceed with calculation of the Josephson critical cur-

rent between two rod-shaped S electrodes of a radius  $R_s$  surrounded by a F metal (Fig. 1b). The supercurrent  $I_s(\varphi) = I_c \sin(\varphi)$  flowing across this S/F/S weak link depends on the phase difference  $\varphi$  between the order parameters of the rods:  $\Delta_{1,2} = \Delta e^{\pm i\varphi/2}$ . In the dirty limit and for large enough distance between the S cylinders ( $a = d - 2R_s > 2\xi_f$ ), the strong exchange field ( $h \gg \pi T_c$ ) and  $R_s \gg \xi_f$  the expression for the critical current  $I_c$  reads (see [9] for details):

$$I_c(a, R_s)/I_0 = \frac{2dR_s}{\xi_f} \int_0^\infty dy \frac{e^{-2(\sqrt{y^2+d^2/4}-R_s)/\xi_f}}{y^2+d^2/4} \times \cos\left(2\frac{\sqrt{y^2+d^2/4}-R_s}{\xi_f} + \frac{\pi}{4}\right), \quad (1)$$

$$I_0 = \frac{\pi\sigma_n\Delta\xi_f^2}{2\sqrt{2}e\gamma_b^2\xi_n^2} \tanh\left(\frac{\Delta}{2T}\right).$$

Here,  $\xi_f = \sqrt{D_f/h}$ ,  $\xi_n = \sqrt{D_f/2\pi T_c}$ ,  $D_f$ , and  $\sigma_n$  are the diffusion constant and the normal state conductivity of the F-metal, and the parameter  $\gamma_b = R_b\sigma_n/\xi_n$  related to the boundary resistance per unit area  $R_b$ . In Fig. 3 we present some typical plots of the critical current  $I_c$  vs. the distance  $a$  calculated from Eq. (1). The  $0-\pi$  transitions are observed to be very robust with respect to a geometry of the S/F/S junction and are determined rather by the thickness of the F spacer between S electrodes than by a shape of the electrodes. For fixed thickness of the F spacer, these transitions can be triggered by temperature variation. Note that a set of superconducting particles embedded in a ferromagnetic matrix realizes an intrinsically frustrated Josephson network, which may reveal a spontaneous current.

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