NON-LOCAL FILTER FOR REMOVING A MIXTURE OF GAUSSIAN AND IMPULSE NOISES

Haijuan Hu, Bing Li, Quansheng Liu

NON-LOCAL FILTER FOR REMOVING A MIXTURE OF GAUSSIAN AND IMPULSE NOISES

Haijuan Hu[1], Bing Li[2] and Quansheng Liu[1]
[1]*UMR 6205, Laboratoire de Mathématiques de Bretagne Atlantique, Université de Bretagne-Sud, Campus de Tohannic, 56017 Vannes, France*
[2]*Department of Math.,Zhongshan Polytechnic, 528404 Zhongshan, P.R.China*
haijuan.hu@univ-ubs.fr, libcs@263.net, Quansheng.Liu@univ-ubs.fr

Keywords: Gaussian Noise, Impulse Noise, Mixed Noise, Image Restoration, Denoising, Trilateral Filter, Non-local Means Filter, Law of Large Numbers, Central Limit Theorem.

Abstract: In this paper we first present two convergence theorems which give a theoretical justification of the Non-Local Means Filter. Based on these theorems, we propose a new filter, called Non-Local Mixed Filter, to remove a mixture of Gaussian and random impulse noises. This filter combines the essential ideas of the Trilateral Filter and the Non-Local Means Filter. It improves the Trilateral Filter and extends the Non-Local Means Filter. Our experiments show that the new filter generally outperforms two other recent proposed methods. A careful discussion and simple formulas are given for the choice of parameters for the proposed filter.

1 INTRODUCTION

The main objective of this paper is to extend the Non-Local Means Filter (Buades et al., 2005) for removing Gaussian noise to the case where the image is contaminated by a mixture of Gaussian and random impulse noises, based on two convergence theorems for the Non-Local Means Filter that we will present.

Let us first introduce the Gaussian and impulse noise models. As usual, we denote a digital image by a $N \times N$ matrix $u = \{u(i) : i \in I\}$, where $I = \{0,1,...,N-1\}^2$ and $0 \leq u(i) \leq 255$. The additive Gaussian noise model is: $v(i) = u(i) + \eta(i)$, where $u = \{u(i) : i \in I\}$ is the original image, $v = \{v(i) : i \in I\}$ is the noisy one, and η is the Gaussian noise: $\eta(i)$ are independent and identically distributed Gaussian random variables with mean 0 and standard deviation $\sigma > 0$. We always denote by u the original image, v the noisy one. The random impulse noise model is:

$$v(i) = \begin{cases} \eta(i) & \text{with probability } p, \\ u(i) & \text{with probability } (1-p), \end{cases}$$

where p is the impulse probability (the proportion of the occurrence of impulse noise), and $\eta(i)$ are independent random variables uniformly distributed on the interval $[\min\{u(i) : i \in I\}, \max\{u(i) : i \in I\}]$.

There is a large literature for removing Gaussian noise. A very important progress in this classical research field was marked by the proposition of the Non-Local Means Filter (NL-means) by Buades, Coll and Morel. The key idea of this filter is to estimate the original image by weighted means along similar local patches. Since then a series of important works have been done by many authors in various contexts using this interesting idea, see e.g. the optimal spatial adaptive patch-based filter in (Kervrann and Boulanger, 2006), the K-SVD (Elad and Aharon, 2006) and BM3D (Dabov et al., 2007) algorithms. There are also many methods to remove impulse noise, see e.g. the variational methods in (Nikolova, 2004; Chan et al., 2004; Dong et al., 2007).

However, few filters are known to remove a mixture of Gaussian and impulse noises, although such noises can take place quite often. On this subject, in (Garnett et al., 2005) an interesting statistic called ROAD is introduced to detect impulse noisy pixels; this statistic is combined with the Bilateral Filter (Smith and Brady, 1997; Tomasi and Manduchi, 1998) leading to the so-called Trilateral Filter (TriF). The performance of TriF is related to the efficiency of the ROAD statistic for detecting impulse noise and the performance of the Bilateral Filter for removing Gaussian noise. A slightly different version of the ROAD statistic is proposed in (Dong et al., 2007).

In this paper, we first (cf. Section 2) present two convergence theorems, which gives a good theoretical justification for NL-means with a probabilistic interpretation of the similarity phenomenon which ex-

ists very often in natural images. We then (cf. Section 3) propose a new filter called Non-Local Mixed Filter (NLMixF) to remove mixed noises, using the presented convergence theorems in an adaptive way. This filter improves the Trilateral Filter and extends NL-means. Our experimental results (cf. Section 4) also show that for removing mixed noise, our filter NLMixF outperforms the two algorithms recently proposed in (Yang and Wu, 2009) and (Xiao et al., 2011) which are based respectively on the ideas of BM3D (Dabov et al., 2007) and K-SVD (Elad and Aharon, 2006).

2 CONVERGENCE THEOREMS FOR NON-LOCAL MEANS

The Non-Local Means Filter (NL-means) (Buades et al., 2005) is mainly based on the similarity of local patches. For $i \in I$ and d an odd integer, let $\mathcal{N}_i(d) = \{j \in I : |j-i| \leq (d-1)/2\}$ be the window with center i and size $d \times d$, where $|j-i| = \max(|j_1-i_1|, |j_2-i_2|)$ for $i=(i_1,i_2)$ and $j=(j_1,j_2)$. Set $\mathcal{N}_i{}^0(d) = \mathcal{N}_i(d)\backslash\{i\}$. We sometimes simply write $\mathcal{N}_i$ and $\mathcal{N}_i{}^0$ for $\mathcal{N}_i(d)$ and $\mathcal{N}_i{}^0(d)$, respectively. Denote $v(\mathcal{N}_i) = \{v(k) : k \in \mathcal{N}_i\}$ as the vector composed of the gray values of v in the window $\mathcal{N}_i$ arranged lexicographically.

The denoised image by NL-means is given by

$$\overline{v}(i) = \frac{\sum_{j\in\mathcal{N}_i(D)} w(i,j)v(j)}{\sum_{j\in\mathcal{N}_i(D)} w(i,j)}$$

with

$$w(i,j) = e^{-||v(\mathcal{N}_i)-v(\mathcal{N}_j)||_a^2/(2\sigma_r^2)} \quad (j \neq i), \qquad (1)$$

where $\sigma_r > 0$ is a control parameter,

$$||v(\mathcal{N}_i)-v(\mathcal{N}_j)||_a^2 = \frac{\sum_{k\in\mathcal{N}_i(d)} a(i,k)|v(k)-v(\mathcal{T}(k))|^2}{\sum_{k\in\mathcal{N}_i(d)} a(i,k)}, \qquad (2)$$

$a(i,k) > 0$ being some fixed weights usually chosen to be a decreasing function of the Euclidean norm $\|i-k\|$ or $|i-k|$, and $\mathcal{T} = \mathcal{T}_{ij}$ is the translation mapping of $\mathcal{N}_i$ onto $\mathcal{N}_j$: $\mathcal{T}(k) = k-i+j, k \in \mathcal{N}_i$. Originally, $\mathcal{N}_i(D)$ in (1) is chosen as the whole image I, but in practice, it is better to choose $\mathcal{N}_i(D)$ with an appropriate number D. We call $\mathcal{N}_i(D)$ searches windows, and $\mathcal{N}_i = \mathcal{N}_i(d)$ local patches.

We now present some convergence theorems for NL-means via probability theory. For simplicity, we use the same notation $v(\mathcal{N}_i)$ to denote both the observed image patches and the corresponding random variables (in fact the observed image is just a realization of the corresponding variable). Therefore the distribution of the observed image $v(\mathcal{N}_i)$ is just that of the corresponding random variable.

Definition 2.1. *Two patches $v(\mathcal{N}_i)$ and $v(\mathcal{N}_j)$ are called similar if they have the same probability distribution.*

We sometimes simply say that the two windows $\mathcal{N}_i$ and $\mathcal{N}_j$ are similar in the same sense. Definition 2.1 is a probabilistic interpretation of the similarity phenomenon that occurs very often in natural images. According to this definition, two observed patches $v(\mathcal{N}_i)$ and $v(\mathcal{N}_j)$ are similar if they are issued from the same probability distribution. In practice, we consider that two patches $v(\mathcal{N}_i)$ and $v(\mathcal{N}_j)$ are similar if their Euclidean distance is small enough, say $\|v(\mathcal{N}_i)-v(\mathcal{N}_j)\| < T$ for some threshold T.

The following theorem is a kind of Marcinkiewicz law of large numbers. It gives an estimation of the almost sure convergence rate of the estimator to the real image in NL-Means.

Theorem 2.1. *Let $i \in I$ and let I_i be the set of $j \in I$ such that the patches $\mathcal{N}_i$ and $\mathcal{N}_j$ are similar (in the sense of Definition 2.1). Set*

$$v^0(i) = \frac{\sum_{j\in I_i} w^0(i,j)v(j)}{\sum_{j\in I_i} w^0(i,j)},$$

where

$$w^0(i,j) = e^{-\|v(\mathcal{N}_i{}^0)-v(\mathcal{N}_j{}^0)\|_a^2/(2\sigma_r^2)}. \qquad (3)$$

Then for any $\varepsilon \in (0, \frac{1}{2}]$, as $|I_i| \to \infty$,

$$v^0(i) - u(i) = o(|I_i|^{-(\frac{1}{2}-\varepsilon)}) \quad \textit{almost surely}, \qquad (4)$$

where $|I_i|$ denotes the cardinality of I_i.

Theorem 2.1 improves the similarity principle in (Li et al., 2011) which is just (4) with $\varepsilon = 1/2$. It shows that $v^0(i)$ is a good estimator of the original image $u(i)$ if the number of similar patches $|I_i|$ is sufficiently large. Here we use the weight $w^0(i,j)$ instead of $w(i,j)$, as $w^0(i,j)$ has the nice property that it is independent of $v(j)$ if $j \notin \mathcal{N}_i$. This property is used in the proof, and makes the estimator $v^0(i)$ to be "almost" non-biased: in fact, if the family $\{v(j)\}_j$ is independent of the family $\{w^0(i,j)\}_j$ (e.g. this is the case when the similar windows are disjoint), then it is evident that $\mathbb{E}v^0(i) = u_i$. We can consider that this non-biased property holds approximately as for each j there are few pixels k such that $w^0(i,k)$ are dependent of $v(j)$. A different explanation about the biased estimation of NL-means can be found in (Xu et al., 2008).

Notice that when $v(\mathcal{N}_j)$ is not similar to $v(\mathcal{N}_i)$, then the weight $w^0(i,j)$ is small and negligible. Therefore in practice we can take all windows. But

selecting only similar windows can slightly improve the restoration result, and can also speed up the algorithm. The difference between $\overline{v}(i)$ and $v^0(i)$ is also small, so that Theorem 2.1 shows that $\overline{v}(i)$ is also a good estimator of $u(i)$. But very often $v^0(i)$ gives better restoration result.

The following result is a generalized central limit theorem; it states that $v^0_{|I_i|}(i)$ tends to $u(i)$ just like $1/\sqrt{|I_i|}$ in the sense of probability distribution.

Theorem 2.2. *Under the condition of Theorem* 2.1, *assume additionally that* $\{v(\mathcal{N}_j) : j \in I_i\}$ *with the lexicographical order is a stationary sequence of random vectors. Then as* $|I_i| \to \infty$,

$$\sqrt{|I_i|}(v^0(i) - u(i)) \xrightarrow{d} \mathcal{L},$$

where $\xrightarrow{d}$ *means the convergence in distribution,* $\mathcal{L}$ *is a mixture of centered Gaussian laws in the sense that it has a density of the form*

$$f(t) = \int_{\mathbb{R}^{|\mathcal{N}_i^0|}} \frac{1}{\sqrt{2\pi c_x}} e^{-\frac{t^2}{2c_x^2}} \nu(dx),$$

ν *being the law of* $v(\mathcal{N}_i^0)$ *and* $c_x > 0$.

By Theorems 2.1 and 2.2, the larger the value of $|I_i|$, the better the approximation of $v^0(i)$ to $u(i)$. This will be confirmed in another paper where we shall introduce the notion of degree of similarity for images, showing that the larger the degree of similarity, the better the quality of restoration. Due to the limitation of space, the proofs of theorems will be given elsewhere.

3 NON-LOCAL MIXED FILTER

In this section, we will define our new filter. Before this we first recall the Trilateral Filter (Garnett et al., 2005). This filter is based on the statistic ROAD (Rank of Ordered Absolute Differences) defined by

$$\mathrm{ROAD}(i) = r_1(i) + \cdots + r_m(i), \tag{5}$$

$r_k(i)$ being the k-th smallest term in $\{|u(i) - u(j)| : j \in \mathcal{N}_i(d)\backslash\{i\}\}$, m a constant taken as $m = 4$ in (Garnett et al., 2005). The ROAD statistic serves to detect noisy points: in fact, if i is an impulse noisy point, then $\mathrm{ROAD}(i)$ is large; otherwise it is small. The Trilateral Filter (TriF) is by definition

$$\mathrm{TriF}(v)(i) = \frac{\sum_{j\in\mathcal{N}_i(D)} w(i,j)v(j)}{\sum_{j\in\mathcal{N}_i(D)} w(i,j)}, \tag{6}$$

where

$$w(i,j) = w_S(i,j)w_R(i,j)^{J_I(i,j)}w_I(j)^{1-J_I(i,j)}$$

contains the spatial factor $w_S(i,j) = e^{-|i-j|^2/(2\sigma_S^2)}$, the radiometric factor $w_R(i,j) = e^{-(v(i)-v(j))^2/(2\sigma_R^2)}$ (which measure the similarity between the pixels i and j), the impulse factor $w_I(i)$ and the joint impulse factor $J_I(i,j)$ defined by

$$w_I(i) = e^{-\frac{\mathrm{ROAD}(i)^2}{2\sigma_I^2}}, \tag{7}$$

$$J_I(i,j) = e^{-\frac{((\mathrm{ROAD}(i)+\mathrm{ROAD}(j))/2)^2}{2\sigma_J^2}}, \tag{8}$$

$\sigma_S, \sigma_R, \sigma_I$ and σ_J being control parameters. (In fact, Garnett et al. (2005) initially defined the joint impulse factor as $J(i,j) = 1 - J_I(i,j)$. We found that it is more convenient to use $J_I(i,j)$ instead of $J(i,j)$.) Notice that if either i or j is an impulse noisy point, then the value of $J_I(i,j)$ is close to 0; otherwise it is close to 1. Similarly, $w_I(i)$ is close to 0 if i is an impulse noisy point, and to 1 otherwise.

Our new filter will be based on the following weighted norm that we call *mixed norm*:

$$||v(\mathcal{N}_i^0) - v(\mathcal{N}_j^0)||_M^2 \tag{9}$$
$$= \frac{\sum_{k\in\mathcal{N}_i^0} w_{S,M}(i,k)J_I(k,\mathcal{T}(k))\,|v(k) - v(\mathcal{T}(k))|^2}{\sum_{k\in\mathcal{N}_i^0} w_{S,M}(i,k)J_I(k,\mathcal{T}(k))},$$

where $w_{S,M}(i,k) = e^{-|i-k|^2/(2\sigma_{S,M}^2)}$, and $J_I(k,\mathcal{T}(k))$ is defined in (8). Recall that if k or $\mathcal{T}(k)$ is an impulse noisy point, then $J_I(k,\mathcal{T}(k))$ is close to 0, so that the concerned point contributes little to the weighted norm (9). Therefore the mixed norm (9) filters impulse noisy points. Clearly, it also measures the similarity between the patches $v(\mathcal{N}_i)$ and $v(\mathcal{N}_j)$ and takes into account the spatial factor. Our new filter that we call *Non-Local Mixed Filter* (NLMixF) is by definition

$$\mathrm{NLMixF}(v)(i) = \frac{\sum_{j\in\mathcal{N}_i(D)} w(i,j)v(j)}{\sum_{j\in\mathcal{N}_i(D)} w(i,j)},$$

where

$$w(i,j) = w_S(i,j)w_I(j)w_M(i,j)$$

contains the spatial factor $w_S(i,j) = e^{-|i-j|^2/(2\sigma_S^2)}$, the similarity factor $w_M(i,j) = e^{-||v(\mathcal{N}_i)-v(\mathcal{N}_j)||_M^2/(2\sigma_M^2)}$, and the impulse factor $w_I(j)$ defined in (7), with σ_S and σ_M being parameters. Notice that NLMixF reduces to NL-means when $\sigma_I = \sigma_J = \sigma_S = \infty$. This filter NLmixF is an improved version of the filter introduced in (Li et al., 2011). Compared to the filter of (Li et al., 2011), it improves the quality of restoration and contains entirely NL-means filter thanks to the added spatial factor $w_{S,M}$ in the mixed norm (9). Notice that for each impulse noisy point j in $\mathcal{N}_i(D)$, the weight $w(i,j)$ is close to 0. Hence our new filter can be regarded as an application of Theorems 2.1 and

2.2 to the remained image (which can be considered to contain only Gaussian noise) obtained after filtering the impulse noisy points by the mixed norm (9).

4 SIMULATIONS AND CHOICES OF PARAMETERS

In this section, we present some experimental results to compare the new filter NLMixF with NL-means, TriF, and two recently algorithms proposed in (Yang and Wu, 2009) and (Xiao et al., 2011). As usual we use PSNR (Peak Signal-to-Noise Ratio) defined by

$$\text{PSNR}\,(\bar{v}) = 10\log_{10}\frac{255^2|I|}{\sum_{i\in I}(\bar{v}(i)-u(i))^2}$$

to measure the quality of a restored image, where u is the original image, $\bar{v}$ the restored one. In our experiments we use the 512×512 images Lena, Bridge, Boats and the 256×256 image Peppers. They are all available on line.[1]

In our implementations, image boundaries are handled by assuming symmetric boundary conditions. In the original image Peppers, there are black boundaries of width of one pixel, we therefore compute the PSNR value for the image of size 254×254 obtained after removing the four boundaries.

There are several parameters to be tuned in NLMixF. Recall that NLMixF reduces to NL-means when $\sigma_I=\sigma_J=\sigma_S=\infty$. So for removing Gaussian noise, a reasonable choice is to take σ_I,σ_J and σ_S large enough (though this choice is not necessarily optimal). To apply our filter easily in practice, we look for a simple and uniform formula in terms of p and σ. We first look for a linear relation; when this does not seem possible we test some slightly more complicated functions. To obtain the formulas, we consider Gaussian noise with $\sigma=10,20,30$, impulse noise with $p=0.2,0.3,0.4,0.5$ and their mixture. We have done our best to choose the formulas, but we can not guarantee that our formulas are always optimal due to the complexity of the subject. Our choices of parameters for NLMixF are shown in the following, where Gaussian noise, impulse noise and mixed noise are abbreviated respectively as Gau, Imp and Mix:

$$\sigma_I = 60+2\sigma-50p,\quad \sigma_J = 45+0.5\sigma-50p,$$

$$\sigma_M = 4+0.4\sigma+30p-\sqrt{2\sigma p},$$

$$\sigma_S = \begin{cases} 0.6+p & \sigma=0 \quad \text{(Imp)}\\ 15 & \sigma>0 \quad \text{(Gau or Mix)}\end{cases}$$

$$\sigma_{S,M} = \begin{cases} 15 & \sigma=0 \quad p>0 \quad \text{(Imp)}\\ 1.5 & \sigma>0 \quad p=0 \quad \text{(Gau)}\\ 2 & \sigma>0 \quad p>0 \quad \text{(Mix)}\end{cases}$$

$$d = \begin{cases} 9 & \sigma=0 \quad p>0 \quad \text{(Imp or Mix)}\\ \left.\begin{array}{lll} 5 & \sigma=10 & p=0\\ 7 & \sigma=20 & p=0\\ & 30 & \end{array}\right\} & \text{(Gau)}\end{cases}$$

$$D = \begin{cases} 7 & \sigma=0 \quad p>0 \quad \text{(Imp)}\\ \left.\begin{array}{lll} 9 & \sigma=10 & p=0\\ 13 & \sigma=20 & p=0\\ 15 & \sigma=30 & p=0\end{array}\right\} & \text{(Gau)}\\ \left.\begin{array}{lll} 7 & \sigma=10 & p>0\\ 11 & \sigma=20 & p>0\\ 15 & \sigma=30 & p>0\end{array}\right\} & \text{(Mix)}\end{cases}$$

In the calculation of ROAD, we choose 3×3 neighborhoods and $m=4$. For impulse noise or mixed noise with $p=0.4,0.5$, to further improve the restoration results, we use 5×5 neighborhoods and $m=12$ to calculate ROAD (5). Consistently, the choice of σ_I,σ_J depend on m, thus they should be multiplied by a factor empirically chosen as 4.2. Evidently, our choice of parameters is not restricted to $\sigma=10,20,30$ and $p=0.2,0.3,0.4,0.5$. This choice can also be applied to any value of σ in the interval [10,30] and p in the interval [0.2, 0.5], or even larger intervals. Note that when $\sigma_S=15$ or $\sigma_{S,M}=15$, we get $w_S(i,j)\approx1$ or $w_{S,M}(i,j)\approx1$. This means that for impulse noise we can omit the factor $w_{S,M}(i,j)$, and for Gaussian noise and mixed noise we can omit the factor $w_S(i,j)$. A full discussion of the roles of the different choices of parameters goes beyond of the scope of this paper. The problem of choice of parameters for NL-means has been considered in the literature, see for example (Xu et al., 2008) and (Duval et al., 2011).

For TriF, we choose parameters and apply the filter according to the suggestion of (Garnett et al., 2005). We use $\sigma_I=40,\sigma_J=50,\sigma_S=0.5,\sigma_R=2\sigma_{QGN}$, where σ_{QGN} is an estimator for the standard deviation of "quasi-Gaussian" noise defined in (Garnett et al., 2005). For impulse noise, when $p>0.25$, it was proposed in (Garnett et al., 2005) to apply the filter with two to five iterations. We apply two iterations for $p=0.3,0.4$, and four iterations for $p=0.5$. For mixed noise, we apply TriF twice with different values of σ_S as suggested in (Garnett et al., 2005): with all impulse noise levels p, for $\sigma=10$, we use first $\sigma_a=0.3$, then $\sigma_S=1$; for $\sigma=20$, first $\sigma_S=0.3$, then $\sigma_{Sb}=15$; for $\sigma=30$, first $\sigma_S=15$, then $\sigma_S=15$. Note that when $\sigma_S=15$, we can omit the spatial factor.

[1] for Lena, Peppers and Boats, cf. http://decsai.ugr.es/∼javier/denoise/test_images/index.htm; for Bridge, cf. www.math.cuhk.edu.hk/∼rchan/paper/dcx/.

Table 1: Choice of parameters for NL-means.

σ	10	20	30
d (size of local patches $\mathcal{N}_i(d)$)	7	9	13
D (size of search windows $\mathcal{N}_i(D)$)	9	9	11

Table 2: PSNR values for removing random impulse noise.

p		0.2	0.3	0.4	0.5
Lena	TriF	34.44	32.55	31.27	29.14
	NLMixF	**35.35**	**33.09**	**31.52**	**29.85**
Bridge	TriF	26.70	25.20	24.43	**23.31**
	NLMixF	**27.65**	**25.46**	**24.45**	**23.31**
Peppers	TriF	30.85	28.64	27.74	26.14
	NLMixF	**31.94**	**29.40**	**28.34**	**26.47**
Boats	TriF	30.19	28.57	**27.64**	**26.13**
	NLMixF	**31.24**	**29.05**	27.50	26.10

For NL-means, we use $\sigma_r = 4 + 0.4\sigma$ in accordance with the choice of σ_M in NLMixF (with $p = 0$). This choice is different from the proposed one in the original NL-means algorithm, and generally gives better restoration results. The values for d and D are shown in Table 1.

We now present some experimental results. Tables 2 and 3 show the performances of NLMixF for removing impulse noise and Gaussian noise by comparing it with TriF and NL-means (for which we use $w(i,i) = \max\{w(i,j) : j \neq i, j \in \mathcal{N}_i(D)\}$ and $a(i,k) = \frac{1}{(d-1)/2}\sum_{l=|i-k|}^{(d-1)/2} \frac{1}{(2l+1)^2}$ in (2)). Table 4 compares NLMixF with TriF for removing mixed noise. We add Gaussian noise and then impulse noise for simulation of mixed noise. Since NL-means is not suitable for removing impulse noise, we do not include it in Tables 2 and 4. We can see that NLMixF improves TriF in almost all the cases, especially when p is small ($p = 0.2$), or σ is large ($\sigma = 20, 30$). Some examples are shown in Figs. 1. In Table 5, we compare the PSNR values with the two algorithms in (Yang and Wu, 2009) and (Xiao et al., 2011), where we show the reported PSNR values for these two algorithms. In Fig. 2, we show the denoised images by NLMixF and IPAMF+BM in (Yang and Wu, 2009), using the same noisy image.

Table 3: PSNR values to remove Gaussian noise.

	Lena			Bridge		
σ	10	20	30	10	20	30
TriF	33.21	29.48	26.51	30.62	27.42	25.05
NLMixF	34.92	31.73	29.81	32.65	29.33	27.56
NL-means	**35.03**	**31.78**	**29.89**	**32.72**	**29.84**	**27.93**

Table 4: PNSR values for removing mixed noise by NLMixF and TriF.

p	0.2	0.3	0.4	0.2	0.3	0.4
	Lena $\sigma = 10$			Bridge $\sigma = 10$		
TriF	31.60	30.88	29.66	25.14	24.59	23.93
NLMix	**32.78**	**31.47**	**29.94**	**26.06**	**24.71**	**23.80**
	Lena $\sigma = 20$			Bridge $\sigma = 20$		
TriF	28.75	28.11	27.26	23.73	23.32	22.84
NLMixF	**30.54**	**29.65**	**28.44**	**24.44**	**23.61**	**22.86**
	Lena $\sigma = 30$			Bridge $\sigma = 30$		
TriF	26.48	25.77	25.02	22.40	22.06	21.52
NLMixF	**28.87**	**28.07**	**27.34**	**23.38**	**22.79**	**22.24**
	Peppers $\sigma = 10$			Boats $\sigma = 10$		
TriF	29.05	27.95	26.63	28.32	27.60	**26.79**
NLMixF	**30.49**	**28.56**	**27.47**	**29.73**	**28.20**	26.76
	Peppers $\sigma = 20$			Boats $\sigma = 20$		
TriF	26.70	25.87	25.29	26.35	25.74	25.07
NLMixF	**28.54**	**27.37**	**26.51**	**27.66**	**26.61**	**25.60**
	Peppers $\sigma = 30$			Boats $\sigma = 30$		
TriF	24.71	23.91	23.27	24.54	24.01	23.37
NLMixF	**27.04**	**25.91**	**25.14**	**26.34**	**25.56**	**24.69**

Table 5: Compare PNSR values for mixed noise.

Lena $\sigma = 10$	$p = 0.1$	$p = 0.2$	$p = 0.3$
(Xiao et al., 2011)	32.75	31.66	30.42
(Yang and Wu, 2009)	33.61	32.12	30.69
NLMixF	**34.10**	**32.78**	**31.47**

5 CONCLUSIONS

We have first presented two convergence theorems for the Non-Local Means Filter (Buades et al., 2005). Based on these convergence theorems and the idea of Trilateral Filter (TriF) (Garnett et al., 2005) we have then given a filter called Non-Local Mixed Filter (NLMixF) to remove Gaussian noise, impulse noise and their mixture. To make easy the application of our filter, we have also given empirical formulas for the choice of parameters which can at least be used for Gaussian noise with $\sigma \in [10, 30]$, impulse noise with $p \in [0.2, 0.5]$, and their mixture. Our experiments show that NLMixF outperforms TriF, as well as the more recent methods proposed in (Yang and Wu, 2009),and (Xiao et al., 2011) based respectively on the ideas of BM3D (Dabov et al., 2007) and K-SVD (Elad and Aharon, 2006).

ACKNOWLEDGEMENTS

The authors are very grateful to the reviewers for helpful comments and remarks. The work has been partially supported by the National Natural Science Foundation of China (Grant No. 10871035, No. 11101039 and No. 11171044), and by Zhong-

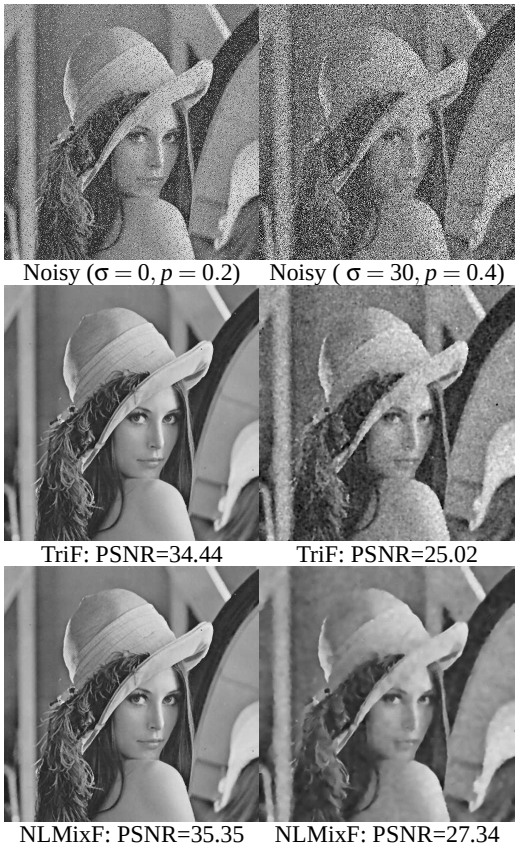

Figure 1: Images corrupted by impulse noise (left) and mixed noise (right).

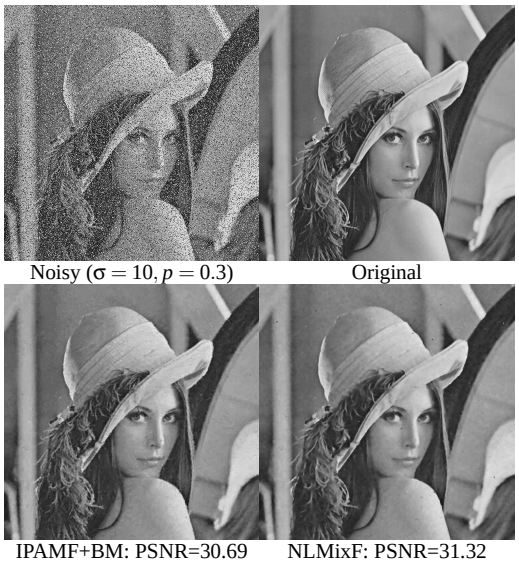

Figure 2: Restored images by NLmixF and IPAMF+BM (Yang and Wu, 2009).

shan Science and Technology Department (Grant No. 2008A245).

REFERENCES

Buades, A., Coll, B., and Morel, J. (2005). A review of image denoising algorithms, with a new one. *Multiscale Model. Simul.*, 4(2):490–530.

Chan, R., Hu, C., and Nikolova, M. (2004). An iterative procedure for removing random-valued impulse noise. *Signal Processing Letters, IEEE*, 11(12):921–924.

Dabov, K., Foi, A., Katkovnik, V., and Egiazarian, K. (2007). Image denoising by sparse 3-D transform-domain collaborative filtering. *IEEE Trans. Image Process.*, 16(8):2080–2095.

Dong, Y., Chan, R., and Xu, S. (2007). A detection statistic for random-valued impulse noise. *IEEE Trans. Image Process.*,, 16(4):1112–1120.

Duval, V., Aujol, J., and Gousseau, Y. (2011). A bias-variance approach for the nonlocal means. *SIAM J. Imaging Sciences*, 4:760–788.

Elad, M. and Aharon, M. (2006). Image denoising via sparse and redundant representations over learned dictionaries. *IEEE Trans. Image Process.*, 15(12):3736–3745.

Garnett, R., Huegerich, T., Chui, C., and He, W. (2005). A universal noise removal algorithm with an impulse detector. *IEEE Trans. Image Process.*, 14(11):1747–1754.

Kervrann, C. and Boulanger, J. (2006). Optimal spatial adaptation for patch-based image denoising. *IEEE Trans. Image Process.*, 15(10):2866–2878.

Li, B., Liu, Q., Xu, J., and Luo, X. (2011). A new method for removing mixed noises. *SCIENCE CHINA Information Sciences*, 54:51–59.

Nikolova, M. (2004). A variational approach to remove outliers and impulse noise. *J. Math. Imag. Vis.*, 20(1):99–120.

Smith, S. and Brady, J. (1997). SUSANA new approach to low level image processing. *Int. J. Com. Vis.*, 23(1):45–78.

Tomasi, C. and Manduchi, R. (1998). Bilateral filtering for gray and color images. In *Sixth International Conference on Computer Vision*, pages 839–846. IEEE.

Xiao, Y., Zeng, T., Yu, J., and Ng, M. (2011). Restoration of images corrupted by mixed gaussian-impulse noise via l sub (1-l) sub (0) minimization. *Pattern Recognition*, 44(8):1708–1720.

Xu, H., Xu, J., and Wu, F. (2008). On the biased estimation of nonlocal means filter. In *International Conference on Multimedia Computing and Systems/International Conference on Multimedia and Expo*, pages 1149–1152.

Yang, J.-X. and Wu, H.-R. (2009). Mixed gaussian and uniform impulse noise analysis using robust estimation for digital images. *Proceedings of the 16th International Conference on Digital Signal Processing*, pages 468–472.