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INTERVAL APPROACH APPLIED TO BLADES OF WINDSCREEN WIPER

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Abstract : Friction-induced vibration due to mode coupling is a major concern in a wide variety of mechanical systems. Stability analysis and the associated non-linear amplitudes around a steady-state equilibrium point actually are two of the most important points in the study of non-linear dynamical systems depending on given control parameters. Determining stable and unstable regions is only one aspect of the problem of friction-induced flutter instability. The instability magnitude is a more significant design factor than the instability region. In this paper, we propose to apply interval analysis to obtain the maximum and minimum amplitudes of a complete non-linear mechanical system. This approach allows us to avoid numerical procedures that are both time and cost consuming to perform when parametric design studies are needed.

AMS Subj. Classification : 70K50

Key Words : Mechanical system, friction induced vibration, stability analysis, interval approach

1 Introduction

A great number of studies have been carried out on the dynamic behaviour of dry friction systems, such as braking systems or windscreen-wipers from [1] and [2]. Previous works have shown that such systems can be subject to instabilities of the sprag-slip type [3] and that the dynamic behaviour is highly sensitive to design parameters as well as physical parameters, such as the friction coefficient. In this context, interval arithmetic is used in order to simulate the non-linear behaviour of a dry friction system, taking into account the uncertainty of the dry friction coefficient.

2 Studied system

The dry friction system model consists of a point mass m and of two rigid stems T_1 and T_2 whose respective lengths are l_1 and l_2 and masses m_1 and m_2 , as illustrated in figure 1. It is assumed that the contact at point C is permanent and it is modelled by using the Coulomb's law $T = \mu N$.

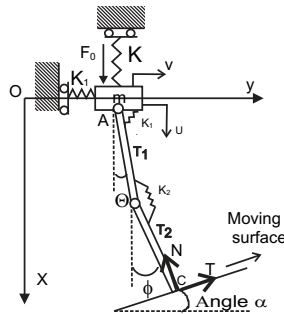


FIGURE 1 – dry friction system

3 Modelisation

The principle of virtual work helps to obtain a system of four non-linear equations with five unknowns u , v , θ , ϕ and N . Taking into account that contact at point C is supposed to be permanent, the linking equation can be reduced into a system of three non-linear differential equations. A change of variables $z = [v, \dot{v}, \theta, \dot{\theta}, \phi, \dot{\phi}]^T$ is used to write the second order non-linear differential equations system into a first order one :

$$M_{CL}\dot{z} = A(z)z + B(z)F_o + G(z)$$

Matrix G contains all non-linear terms, with in particular all the quadratic terms and the non-linear terms (trigonometric kind).

4 Interval analysis

4.1 Introduction

The aim of this study is to take into account the uncertainties of the friction coefficient, to determine the dynamical behaviour of the system and to obtain the high and the low limits of the non-linear amplitudes of vibrations. Because the uncertainties of the friction coefficient could only be bounded (no information of the behaviour between the bounds), we have to use an interval approach. The toolbox Intlab¹ and Matlab software are used to perform calculation. The main difficulty is that this toolbox provides only basic functions for interval calculation and offers no differential equation solvers. Therefore, we have to program the numerical integrations in order to determine the behaviour of the system. We have used the Euler method. Graphs on figure 2 show the evolution of intervals for angles θ et ϕ versus time taking account of the uncertainties of

1. <http://www.ti3.tu-harburg.de/rump/intlab/>

the friction coefficient ($\pm 10\%$). The intervals over-estimate the real solution so we need to find a way to decrease this phenomenon.

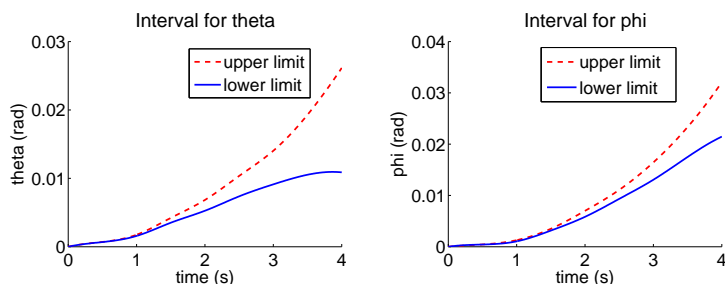


FIGURE 2 – Interval for angles θ and ϕ versus time

4.2 Solutions to decrease the over-estimates

The simple application of interval arithmetic which consists in replacing all the arithmetic operations and the functions by their equivalents in intervals can lead to significant over-estimates. These over-estimates can be due to several phenomena. First, interval calculation considers that each occurrence in an equation or a system of equations of a parameter modelled by an interval is independent. This independence of the occurrences does not correspond to the studied system's physical reality and so pessimism due to the multiple occurrences may be obtained. Second, interval arithmetic cannot correctly represent complex fields of the state variables in the state space, while using interval boxes whose sides are parallel to the axes. For example, the interval calculation algorithm considers that the image of an interval is a single interval, which is not always the case. It is possible that the image is made up of various distinct zones, whereas the found solution is a single interval including all these various zones. In this case, the wrapping effect gives an interval which over-estimates in an excessive way the exact

image. Finally, according to the form under which the state equations are written the intervals representing the state variables can be different. Thus, the form under which the state equations are written is of primary significance. In order to limit the pessimism phenomenon, various occurrences of the friction coefficient were factorized as often as possible. When it is not possible to factorize the uncertain parameter in certain components of the equations, some studies of functions were carried out. The equations can be also integrated with the Runge Kutta 4 methods. It provides better results, because for a same precision, a bigger step could be used, so the number of numerical integrations is reduced.

4.3 Results

Results from several simulations of the double pendulum's dynamic behaviour around one of its positions of balance are presented. The friction coefficient is defined by the interval $[0.11, 0.13]$, the other parameters are consider reals. The table 1 shows the details of each study. The table 2 presents the amplitudes of the intervals for the state variable ϕ versus time. The results of the 4 studies are compared, the amplitude of the interval of study 1 is being considered as reference 100%.

	Integration		Optimisation		time (sec)
	Euler	RK4	coefficient	variable	
Study 1	X				197
Study 2	X		X		201
Study 3	X		X	X	1129
Study 4		X	X	X	989

TABLE 1 – Details of each study

These results show that the methods used to reduce the over-estimation decrease indeed the amplitudes. These reductions can

Amplitude of $\dot{\phi}$	1 sec	2 sec	3 sec	4 sec
Study 1	100%	100%	100%	100%
Study 2	99,6%	99,6%	99,6%	99,6%
Study 3	94,7%	84%	72,9%	63,6%
Study 4	93,6%	83,3%	72,3%	62,8%

TABLE 2 – Amplitude of $\dot{\phi}$ interval for each study

reach nearly 40% for $\dot{\phi}$ (at the time $t=4s$, the amplitude of the interval of $\dot{\phi}$ in the study 4 is equal to 62.8% of the reference amplitude).

5 Conclusion

The dynamical behaviour of a dry friction system has been studied, into account the uncertainties of the friction coefficient with an interval approach. The resolution of the system was carried out with several numerical integration's methods, and some solutions have been used in order to decrease the over-estimate of the results. The decrease is about 30 to 40%.

Références

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