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Domain wall tilting in the presence of the Dzyaloshinskii-Moriya interaction in out-of-plane magnetized magnetic nanotracks

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Abstract

We show that the Dzyaloshinskii-Moriya interaction (DMI) can lead to a tilting of the domain wall (DW) surface in perpendicularly magnetized magnetic nanotracks when DW dynamics is driven by an easy axis magnetic field or a spin polarized current. The DW tilting affects the DW dynamics for large DMI and the tilting relaxation time can be very large as it scales with the square of the track width. The results are well explained by an analytical model based on a Lagrangian approach where the DMI and the DW tilting are included. We propose a simple way to estimate the DMI in a magnetic multilayers by measuring the dependence of the DW tilt angle on a transverse static magnetic field. Our results shed light on the current induced DW tilting observed recently in Co/Ni multilayers with inversion asymmetry, and further support the presence of DMI in these systems.
The effect of inversion asymmetry on the magnetic and electronic transport properties at interfaces of low dimensional magnetic film is currently attracting a growing attention. In the presence of spin-orbit coupling, inversion asymmetry leads to an additional term in the exchange interaction, namely the Dzyaloshinskii-Moriya interaction (DMI) \[1, 2\], which tends to make the magnetization rotate around a local characteristic vector $D$. This can destabilize the uniformly magnetized states leading to novel chiral magnetic order, such as spin spiral \[3, 4\]. Novel out-of-equilibrium transport phenomena have also been demonstrated, such as current induced spin orbit torques induced by the Rashba spin orbit coupling and/or the spin Hall effect, leading to current induced magnetization reversal \[5–7\]. A recent striking example of the impact of the inversion asymmetry in ultrathin magnetic films is the current induced domain wall motion (CIDM) in perpendicularly magnetized nanotacks. This was first outlined by Miron et al. who reported very efficient CIDM in asymmetric Pt/Co(0.6 nm)/AlOx multilayers whereas symmetric Pt/Co/Pt multilayers showed no effects \[8, 9\]. The high perpendicular anisotropy in this material leads to very narrow DW ($\sim 5$ nm), so that in typical experiments, the nanotrack width ($\sim 100$ nm) is much larger than the DW width. It is thus expected that the magnetization rotates parallel to the DW surface (Bloch DWs) to minimize the magnetostatic energy. Whereas these experiments were first interpreted in terms of a high non-adiabatic torque induced by the Rashba spin orbit coupling, we recently proposed that the high efficiency arises from two key features resulting from the inversion asymmetry and the high spin orbit coupling in this material \[10\]: First, a change of the DW equilibrium structure from Bloch to Néel induced by the DMI. This leads to chiral DWs where the DW magnetization rotates perpendicular to the DW surface with a unique sense of rotation \[11, 12\]; Second, a large Slonczewski-like spin orbit torque which is maximal in the Néel configuration \[5, 13\]. Recent CIDM experimental results in Pt/Co/Ni \[9\] and Pt/CoFeB/MgO multilayers \[15\] seem to support this scheme.

In this Letter, we show that the inversion asymmetry not only affects the DW dynamics through a change of the internal DW structure but also through a modification of its shape. In perpendicular magnetized nanotacks, the DW surface is expected to be perpendicular to the nanotrack axis to minimize the DW length and thus its energy. However, in the presence of DMI, when driving the DW dynamics, micromagnetic simulations reveal that a large DMI can lead to a sizable tilting of the DW surface which can strongly affect the DW dynamics.
This DW tilting is a dynamical effect which occurs whatever the driving mechanism, e.g. an external magnetic field or a spin polarized current, and is thus intrinsically different from the previously reported current induced DW tilting \[1,3,5,30\]. The results are well explained using an analytical model based on a Lagrangian approach where the DMI and the DW tilting are included. We also show that the DW tilting can be controlled using a static transverse magnetic field, and this provides a simple way to measure directly the DMI experimentally. Our results shed light on the unexplained current induced DW tilting recently observed in Co/Ni asymmetric multilayer nanotracks \[4\] and further support the presence of DMI in these materials \[9\].

We consider a magnetic ultrathin film grown on a substrate with a capping layer in a different material so that the inversion symmetry is broken along the vertical axis (z). The magnetization is supposed oriented out-of-plane with a strong perpendicular anisotropy. In addition to the standard micromagnetic energy density which includes the exchange, anisotropy, Zeeman and demagnetizing energy, we add the following DMI that reads in a continuous form \[10\]:

\[
E_{DM} = D \left[ m_z \frac{\partial m_x}{\partial x} - m_x \frac{\partial m_z}{\partial x} + id.(x \rightarrow y) \right].
\]

This form corresponds to a sample isotropic in the plane, where the Dzyaloshinskii vector for any in-plane direction \( u \) is \( Dz \times u \) with \( D \) a uniform constant, originating from the symmetry breaking at the z surface. Micromagnetic simulations are based on the Landau-Lifshitz-Gilbert equation:

\[
\frac{\partial \mathbf{m}}{\partial t} = -\frac{\gamma_0}{\mu_0 M_s} \delta E \times \mathbf{m} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} - \gamma_0 H_{SO} J \mathbf{m} \times (\mathbf{m} \times \mathbf{u}_y)
\]

where \( \gamma_0 = \mu_0 \gamma \) with \( \gamma \) the gyromagnetic ratio, \( E \) the energy density and \( M_s \) the saturation magnetization. We assume that the injection of a current density \( J \) in the nanotrack leads to a Slonczewski-like torque \(-\gamma_0 H_{SO} J \mathbf{m} \times (\mathbf{m} \times \mathbf{u}_y)\) \[5,7,20\]. To simplify, we do not consider the effect of the adiabatic and non-adiabatic spin transfer torque as well as the field like part of the spin orbit torque \[21\]. In the following, we consider sufficiently large values of D \((D > 0.12 \text{ mJ/m}^2 \text{ for our simulation parameters})\) so that the Néel configuration is stable at equilibrium \[10\]. 2D micromagnetic simulations are performed using modified homemade micromagnetic solvers \[10,22\]. The following parameters have been used \[9\]: exchange parameter \( A=10^{-11} \text{ J/m} \), saturation magnetization \( M_s = 1.09 \times 10^6 \text{ A/m} \), uniaxial anisotropy constant \( K = 1.25 \times 10^6 \text{ J/m}^3 \), Gilbert damping parameter \( \alpha = 0.5 \), thickness of \( t_m = 0.6 \text{ nm} \).

The DW tilting induced by the DMI can simply be introduced by considering the effect
of a static in-plane magnetic field $H_y$ transverse to the magnetic track (Fig 1(b)). In the presence of $H_y$, the Zeeman interaction leads to a rotation of the DW magnetization away from the Néel configuration. To recover the Néel configuration energetically favored by the DMI, the DW surface tilts by an angle $\chi$ at the cost of a higher DW energy due to the larger DW surface. Fig 1(b) shows the resulting DW tilting for $\mu_0 H_y = 100$ mT and a large value $D = 2$ mJ/m². The tilt angle as a function of $H_y$ and $D$ is plotted on Fig. 1(c,d). As expected, the DW tilting increases with $H_y$ and, for a fixed $H_y$, increases with $D$. The tilt angle can be roughly estimated from energetic considerations assuming that the DW always stays in a Néel configuration with an energy per surface unit $\sigma_0$. On the one hand, for a DW tilted by an angle $\chi$, the DW surface and thus the total energy is increased by a factor $1/\cos \chi$; on the other hand, the Zeeman energy per unit surface scales as $\sigma_Z \sin \chi$ with $\sigma_Z = -\pi \mu_0 H_y M_s \Delta$ ($\Delta$ is the DW width). This leads to a total DW energy $E_{DW} \approx w t_m (\sigma_0 - \sigma_Z \sin \chi)/\cos \chi$, where $w$ is the track width. The minimization of this energy leads to $\sin \chi = \sigma_Z/\sigma_0$. Importantly, the slope of the DW tilting as a function of $H_y$ on Fig. 1(c) depends directly on the value of $D$. This provides a direct way to measure $D$ experimentally, from the dependence of the DW equilibrium tilt angle on $H_y$.

In the presence of DMI, a tilting of the DW surface can also be induced dynamically by applying an easy axis external magnetic field $H_z$. The magnetization distribution in the track for different magnetic fields (Fig. 2(a), $D = 2$ mJ/m²) reveals that the DW tilts significantly when driven by $H_z$ in the steady state regime. As shown on Fig. 2(b), the steady-state tilt angle rapidly increases with $H_z$ and $D$, although a saturation is observed for large $H_z$. Fig 2(d) shows the DW velocity $v$ along the track direction as a function of $H_z$ for different values of $D$. As expected, the DMI leads to an increase of the Walker field [10]. For large values of $H_z$, the DW velocity significantly deviates from the expected linear scaling as $D$ increases. This deviation is the direct result of the DW tilting: the propagation of the tilted DW at a velocity $v_n$ normal to its surface leads to a velocity $v = v_n/\cos \chi$ along the track direction. When considering $v_n$ instead of $v$ (Fig. 2(d), inset)), the expected linear scaling is recovered and the velocity in the steady state regime does not depend anymore on $D$. The time dependence of the DW tilt angle is shown on Fig. 2(c) for several $w$ when applying $\mu_0 H_z = 100$ mT at $t=0$ [31].

To describe the dynamics of tilted DWs induced by the DMI, we consider an extended collective coordinate model (CCM) [23] where the DW is described by three variables: its
position $q$ in the track, the DW magnetization angle $\psi$ and the tilt angle of the DW surface $\chi$ (cf Fig 1(a)). The DW profile is described by the following Ansatz for the azimuthal $\theta$ and polar angle $\phi$:

$$\phi(x,y,t) = \psi(t) - \pi/2$$

$$\theta(x,y,t) = 2 \arctan\left(\frac{x \cos \chi + y \sin \chi - q \cos \chi}{\Delta}\right)$$

(\(\Delta = \sqrt{\frac{A}{K - \mu_0 M_s^2}}\)). The effect of the DMI on the DW profile and dynamics is taken into account by an additional term in the DW energy (see below) \[10\]. To derive the dynamical equations, a Lagrangian approach is considered \[24–26\]. The LLG equation can be derived by writing the Lagrange-Rayleigh equations for the Lagrangian

$$L = E + \frac{M_s}{\gamma} \dot{\phi} \sin \theta$$

with $E$ the micromagnetic energy density and $m = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$. The effect of the damping and spin orbit torques is included in the dissipative function $F = \alpha M_s/(2\gamma)[d\mathbf{m}/dt - (\gamma_0/\alpha)H_{SO}J \mathbf{m} \times \mathbf{u}_y]^2$.

The Lagrange-Rayleigh equations then leads to the following CCM equations:

$$\dot{\psi} + \frac{\alpha \cos \chi}{\Delta} \dot{q} = \gamma_0 H_z + \frac{\pi}{2} \gamma_0 H_{SO} J \sin \psi,$$

$$\frac{\dot{q} \cos \chi}{\Delta} - \alpha \dot{\psi} = \gamma_0 \frac{H_k}{2} \sin 2(\psi - \chi) + \frac{\pi D \gamma_0}{2\mu_0 M_s \Delta} \cos(\psi - \chi) - \frac{\pi}{2} \gamma_0 H_y \sin \psi,$$

$$\ddot{\chi} \alpha \mu_0 M_s \Delta \pi^2 \left(\frac{\tan^2 \chi + \left(\frac{w}{\pi \Delta}\right)^2}{\cos^2 \chi}\right) =$$

$$-\sigma \tan \chi + \pi D \cos(\psi - \chi) + \mu_0 H_k M_s \Delta \sin 2(\psi - \chi)$$

(\(\sigma\) is the wall energy per unit area with $\sigma = 4\sqrt{AK} + \pi D \sin(\psi - \chi) + \mu_0 H_k M_s \Delta \sin^2(\psi - \chi) + \pi \Delta M_s H_y \cos(\psi)$, with $H_k$ the DW demagnetizing field \[32\].

Assuming that $\alpha w \gg \Delta$, these equations lead to a typical time scale for the tilting to settle $\tau = \alpha \mu_0 M_s w^2/(6\sigma \gamma_0 \Delta)$. The $w^2$ dependence is explained by the time to reverse the spins in the nanotrack surface swept by the DW when the tilting takes place. On the other hand, the magnetization angle in the DW frame relaxes on a shorter time scale $\tau_\phi = \frac{1 + \alpha^2}{\alpha \gamma} \frac{1}{\pi D/(2M_s \Delta) - H_k}$ which does not depend on $w$. In the steady state regime ($\dot{\chi} = 0$, $\dot{\psi} = 0$) and $H_y = 0$, the tilt angle is directly related to the DW velocity $v$ as:

$$\tan \chi = \frac{2M_s}{\gamma \sigma} v \cos \chi$$

(5)

with the DW velocity $v = \frac{\gamma_0 \Delta}{\alpha \cos \chi} (H_z + \frac{\pi}{2} H_{SO} J \sin \psi)$. This points to the dynamical origin of the DW tilting. A more physical picture can be obtained from the expression of the Lagrangian integrated over the nanotrack $L_{DW}/(t_m w) = \frac{\sigma}{\cos \chi} - \frac{2M_s(\Phi + \chi)}{\gamma} \dot{q}$, where $\Phi$ is the magnetization angle in the DW frame ($\Phi = \psi - \chi$). The first term is the DW internal energy proportional to the DW surface and thus scaling as $1/\cos \chi$. The second term can be seen
as a kinetic potential \[\Phi\], which contrary to a kinetic energy, is linear with the DW velocity and the DW angle. For the field driven case in the steady state regime, \(\Phi\) is defined only by the in-plane torques due to \(H_z, D\) and \(H_k\) (see Eq. 2) and does not depend on \(\chi\). The tilt angle in the steady state regime can thus be deduced from the minimization of \(L_{DW}\) with \(\chi\) at fixed \(\Phi\), which leads to Eq. 5. The tilt angle is thus the result of a balance between the gain in the kinetic potential resulting from the DW tilting and the cost in the increased DW energy due to the larger surface.

We now compare the predictions of this model with the results of the micromagnetic simulations. The continuous lines on Fig. 1(c,d) show the DW tilting induced by \(H_y\) predicted by the CCM whereas the DW tilt angle, time dependence and DW velocity driven by \(H_z\) are plotted in continuous lines on Fig. 2(b-d) : a general good agreement is obtained with the micromagnetic simulations despite the simplicity of the model. We also plotted the results of the standard \((q,\psi)\) model on Fig. 2(d) (dashed line). The model does not reproduce the nonlinear increase of the DW velocity, but a good agreement is obtained when considering the DW velocity in the direction perpendicular to its surface \(v_n\) (inset). The DW tilting thus does not affect the DW velocity perpendicular to its surface.

We now consider the current driven DW dynamics induced by the Slonczewski-like spin orbit torque in the presence of a large DMI. This torque is expected for structure with inversion asymmetry such as Pt/Co/AlOx trilayers \[5, 9, 21\]. It may arise from the spin Hall effect due to the current flowing in the non magnetic layer and/or from the Rashba spin orbit interaction \[5, 7\]. It leads to an effective easy-axis magnetic field on the DW \(H_{SO}\) proportional to \(\sin \psi\) (see Eq. 2), which is thus maximal for a Néel DW configuration \((\psi = \pm \pi/2)\) obtained for sufficiently high \(D\). The field \(H_{SO}\) can be very large \(\sim 0.07\) T/(10^{12} A/m^2) in Pt/Co/AlOx \[27\] (see also Ref. \[6, 15, 28, 29\]). Similarly to the action of \(H_y\), the spin orbit torque tends to rotate the DW magnetization along the y direction away from the Néel configuration, providing an additional source for the DW tilting.

The results of micromagnetic simulations of the DW dynamics driven by the spin orbit torque with \(\mu_0 H_{SO} = 0.1\) T/(10^{12} A/m^2) are shown on Fig. 3. When injecting a current in the track, a fast DW motion is observed against the electron flow and the velocity increases with \(J\) and \(D\) (see Fig. 3(d)). At the same time, a significant tilting of the DW occurs (see Fig. 3(a) for \(D = 2\) mJ/m^2), which increases with \(J\) and \(D\) (Fig. 3(b)). The DW velocity and the tilting predicted by the CCM are shown on Fig. 3(b,d), continuous lines. An
excellent agreement is obtained with the micromagnetic simulations except at higher current densities for the tilt angle due to the onset of a more complex DW structure (see Fig. 3(a) for $J = 2.5 \times 10^{12} \text{ A/m}^2$). The DW velocity in the direction perpendicular to the DW surface $v_n$ for $D = 2 \text{ mJ/m}^2$ is plotted Fig. 3(d), inset. Contrary to the field driven case, the standard $(q,\psi)$ model strongly overestimates the DW velocity (continuous line). As expected, the DW tilting leads to an additional rotation of the DW angle $\psi$ away from $\pi/2$ where the torque is maximal. The DW tilting thus leads to a large decrease of the DW velocity. This clearly illustrates the importance of the DW tilting on the CIDWM for large DMIs. Fig. 3(c) shows the time dependence of the tilting for a current pulse of $J = 0.25 \times 10^{12} \text{ A/m}^2$ applied at $t=0$. The CCM (continuous lines) reproduces well the time scale for the tilting to take place which scales as $w^2$.

Experimentally, Ryu et al. recently reported fast current induced DW motion associated with a significant DW tilting in asymmetric perpendicularly magnetized (Pt/Co/Ni/Co/TaN) nanotracks [4, 9]. By studying the dependence of the current induced DW velocity on an in-plane longitudinal magnetic field, they present evidence of chiral DWs driven by the Slonczewski like spin-orbit torque in agreement with the presence of a DMI. The DW tilting is reversed for up/down and down/up DW which is well explained by Néel DWs pointing in opposite directions due to the DMI. From the longitudinal magnetic field required to suppress the CIDM and using the magnetic and transport parameters of Ref. [4, 9], one can deduce a DMI of $D = 0.8 \text{ mJ/m}^2$ for $A=1 \times 10^{-11} \text{ J/m}$. Using this value, micromagnetic simulations predict a steady state tilt angle of about $18^\circ$ for $J = 1 \times 10^{12} \text{ A/m}^2$ close to the one measured experimentally ($\sim 20^\circ$) [34]. Smaller additional contributions may also arise from the anomalous Hall effect and the Oersted field [35]. Our model thus accounts for the DW tilting reported by Ryu et al. which further supports the presence of DMI in these inversion asymmetric multilayers.

To conclude, we have shown that the DMI can lead to a tilting of the DW surface in perpendicularly magnetized nanotracks when DW dynamics is driven by an easy axis magnetic field or a spin polarized current. The DW tilting is the result of the balance between the gain in the kinetic potential of the moving DW when tilted and the increased DW energy due to the larger DW surface. The DW tilting affects the DW dynamics for large DMI and the tilting relaxation time can be very large as it scales with the square of the track width. We propose a simple way to estimate the DMI in magnetic multilayers by
measuring the dependence of the DW tilt angle on a transverse static magnetic field. Our results shed light on the current induced DW tilting observed recently in perpendicularly magnetized Co/Ni multilayers with inversion asymmetry and further support the presence of DMI in these materials. This work was supported by project Agence Nationale de la Recherche, project ANR 11 BS10 008 ESPERADO.


[31] See also the corresponding movie for $w = 100$ nm in the Supplementary Materials.

[32] Note that the magnetization angle in the DW frame $\Phi = \psi - \chi$ is the relevant one for the demagnetizing field and DMI terms, whereas the absolute angle $\psi$ is involved for the $H_{SO}$ and $H_y$ terms.

[33] Note that this expression is general. Very small DW tilts are also predicted in the absence of DMI, but the DMI leads to large tilt angle as it allows fast steady state DW motion and it decreases the internal DW energy.

[34] The following parameters have been used : $M_s = 0.6 \times 10^6$ A/m, $K_u = 0.59 \times 10^6$ J/m$^3$, $\mu_0H_{SO} = 4.8 \times 10^{-14}$ T/(A/m$^2$) corresponding to a spin Hall angle of 0.1, a damping constant $\alpha = 0.05$, a spin polarization $P = 0$.

Supplementary Materials: Alternative mechanisms for current induced domain wall tilting in magnetic nanotracks

In this section, we discuss other mechanisms that may lead to current induced DW tilting in magnetic nanotracks.

**Anomalous Hall effect**

When injecting a current in a perpendicularly magnetized nanotrack, the Anomalous Hall effect (AHE) leads to charge accumulations on the edges of the track whose sign depends on the out-of-plane orientation of the magnetization. Around a magnetic domain wall (DW), the abrupt change in the charge polarity leads to a deviation of the current lines around the DW which results in an additional current density $+\Delta J$ (resp. $-\Delta J$) on the right (resp. left) edge of the track. This effect was first described by Partin et al. \[1\] in 1974 and is at the basis of the DW drag effect introduced by Berger \[2\]: The current loop around the DW induced by the AHE creates an additional out-of-plane magnetic field which tends to move the DW in the direction of the charge carrier. This effect is mostly prevalent in narrow and thick nanotracks. For nm thin magnetic nanotracks, the DW drag effect is generally very small compared to the spin transfer/spin orbit torque \[3\]. However, the additional current densities $\pm\Delta J$ on the edges translate into different DW velocities $\nu = \nu_0 \pm \Delta \nu$ which can create DW tilting. Assuming a constant velocity with time, the tilting angle $\chi$ increases with the pulse length $\tau$ as:

$$\tan \chi = \frac{2\tau}{w}\Delta \nu \approx \frac{2\tau}{w} \left( \frac{d \nu}{dJ} \right)_J \Delta J$$ \hspace{1cm} (6)

where $w$ is the track width. Partin et al. \[1\] calculated the current density distribution in the nanotrack for a non-tilted DW. To first order in the Hall angle, the additional current density $\Delta J$ at the right and left edge of the track reads:

$$\frac{\Delta J}{J} = \pm \frac{40 \tan \theta_H}{9\pi} \hspace{1cm} (7)$$

where $J$ is the injected current density.

For a Hall angle of 1% typically observed in perpendicularly magnetized ultrathin multilayers, this leads to an additional current density of about $\pm 1.5\%$ at the right and
left edge of the track. Ryu et al. reported in Ref. [4] current induced DW tilting in Pt(1.5)/Co(0.3)/Ni(0.7)/Co(0.15)/TaN(5) (thickness in nm). Assuming an Hall angle < 2%, one obtains $\Delta J/J < 3\%$. For a DW velocity of 100 m/s for a current density of $1 \times 10^{12} \text{A/m}^2$, Eq. 6 leads to a DW tilting angle < 3° for the 100 ns pulses used in their experiments. This is about an order of magnitude lower than the one observed experimentally ($\sim 20^\circ$). Reciprocally, the DW tilting observed experimentally would need an unrealistically large Hall angle of about 16%. Thus the anomalous Hall effect cannot account for the reported DW tilting but may add a small contribution.

Current induced DW tilting was also reported by Yamanouchi et al. in (Ga,Mn)As diluted magnetic semiconductor nanotracks [5, 6]. The nanotracks were typically 5 µm wide and 30 nm thick. In Ref. [5], the authors report a large DW tilting (up to 60°) for $J < 1 \times 10^{10} \text{A/m}^2$ which increases with the pulse width. In these experiments, the large thickness of the magnetic films makes unlikely the presence of Dzyaloshinskii-Moriya (DMI). However, (Ga,Mn)As may have very large Hall angles (5 to 10 % \[5, 7\]) and the authors used long current pulses (up to 20 µs). Assuming an Hall angle of 0.1, the additional current circulating around the DW due to the anomalous Hall effect has a density $\Delta J/J = 15\%$. Using Eq. 6 and for a DW velocity of about 1.75 m/s for $J = 4 \times 10^9 \text{A/m}^2$, one can estimate a DW tilting of about 60° in agreement with the experimental results. In the case of (Ga,Mn)As, the anomalous Hall effect seems thus to be the main driving force which creates the DW tilting [19].

**Oersted field effect**

The Oersted field created by the current circulating in the nanotrack has a perpendicular component which is antisymmetric with respect to the center of the track and can reach relatively large values on the edges (up to 5.2 mT at $J = 1 \times 10^{12} \text{A/m}^2$ for the 10 µm wide and 2.65 nm thick track used in the experiments of Ryu et al. [4, 5, 8]). It may thus favor a tilting of the DW. However, in the experiments of Ryu et al. [4], the Oersted field should tilt the DW in a direction opposite to what is observed experimentally and thus can not explained the reported DW tilting. It may however decrease the steady state DW angle as it goes against the effect of the DMI.

To study the effect of the Oersted field on the DW tilting, we carried out micromagnetic
simulations taking into account the Oersted field for a 300 nm wide and 50 µm long track. The following parameters have been used to mimic the experiments of Ryu et al. [4, 9]: $A = 1 \times 10^{11}$ J/m, $D = 0.8$ mJ/m$^2$, $M_s = 0.6 \times 10^6$ A/m, $K_u = 0.59 \times 10^6$ J/m$^3$, a spin orbit torque effective field $\mu_0 H_{SO} = 4.8 \times 10^{-14}$ T/(A/m$^2$) corresponding to a spin Hall angle of 0.1, a damping constant $\alpha = 0.05$, a spin polarization $P = 0$, a buffer layer of Pt of 1.5 nm and a Co/Ni thickness of 1.15 nm. The current density is assumed homogeneous in the track. In the absence of Oersted field, the steady state angle is $18.3^\circ$ for $J = 1 \times 10^{12}$ A/m$^2$, close to the predictions of the collective coordinate model ($17.3^\circ$). For this wire width, the Oersted field leads only to a slight decrease of the steady state angle ($17.6^\circ$). For a given current density, larger effects of the Oersted field are expected for larger width [8]. To study the effect of the track width, we carried out micromagnetic simulations for a 2 µm wide nanotrack ($J = 1 \times 10^{12}$ A/m$^2$). Due to the much larger numbers of cells and the larger DW tilting relaxation time, the steady state was not fully reached in this case. However, one can estimate that the Oersted field leads to a decrease of the DW tilting by about $14 - 15\%$, which leads to a steady angle of $15 - 16^\circ$. Experimentally, Ryu et al. reported tilt angles up to $\sim 13^\circ$ in 2 µm wide wires ($J = 1.1 \times 10^{12}$ A/m$^2$).

For larger width ($w = 5, 10$ and $20$ µm), tilt angles of around $20^\circ$ ($J = 1.1 \times 10^{12}$ A/m$^2$) have been reported by Ryu et al. and the tilt angle depends little on the track width. Had the Oersted a large impact on the tilt angle, a large decrease of the tilt angle as $w$ increases would be expected [8], which is not observed. We can thus conclude that the Oersted field has little impact on the steady state tilt angle in these experiments.

**Domain wall deformation in soft in-plane magnetized nanotracks**

The DW deformation of moving DWs in soft in-plane magnetized nanotracks is a well established phenomenon [10–16]. In the absence of magnetocrystalline anisotropy, the equilibrium internal DW structure in soft nanotrack is defined by the demagnetizing and the exchange energy and thus depends only on the track aspect ratio for a given material. This leads to wide DWs whose internal structure can be easily deformed as compared to DWs in materials with strong magnetocrystalline anisotropy. Two types of DWs are typically found in such tracks, head-to-head vortex DWs (VDW) and transverse DWs (tDW) [15]. When driving a DW dynamics by an external magnetic field or a spin polarized current,
complex DW deformations have been predicted \cite{10, 12, 16}. For example, in the viscous regime, the VDW dynamics is characterized by a shift of the transverse position of the vortex core, whereas in the precessional regime, periodic DW transformations from tDWs to vortex/antivortex DWs occur \cite{10, 12, 16}. Such periodic transformations of tDW into VDW were observed experimentally in permalloy nanotrack tracks, induced by the injection of consecutive current pulses \cite{14}. However, these changes are in fact a modification of the internal DW structure induced by the precessional torque and not a tilting of the DW surface. For a standard Bloch DW, this is physically equivalent to the in-plane rotation of the DW magnetization. In terms of our collective coordinate model, the DW deformations in VDW/tDW is equivalent to a change of the variable $\psi$ which describes the DW magnetization (see Ref. \cite{16, 17} for a more detailed discussion) but not a change of the inclination of the DW surface $\chi$. A tilting of the DW surface in soft magnetic nanotrack tracks were predicted in the special case of a DW moving in the presence of a transverse in-plane magnetic field \cite{18}, which may relate closer to the present study as the DMI acts similarly to a constant external magnetic field exerted on the DW.


[19] The Oersted field may also contribute to the DW tilting but the resulting DW tilting is expected to be much lower (a few degree) due to the low injected current density [5, 8].
FIG. 1: (a) Schematic of the tilted DW. (b) Micromagnetic configuration of a 100 nm wide track with $D=2 \text{ mJ/m}^2$ and a transverse magnetic field $\mu_0 H_y = 100 \text{ mT}$. The color scale is the same as Fig. 2(a). (c) DW tilt angle as a function of $\mu_0 H_y$ for $D = 2 \text{ mJ/m}^2$ and (d) as a function of $D$ for $\mu_0 H_y = 100 \text{ mT}$. Dots are the results of micromagnetic simulations whereas the continuous lines are the results of the CCM.
FIG. 2: Dynamics of the DW driven by an external magnetic field $H_z$ for a 100 nm wide nanotrack. 
(a) Magnetization pattern of the DW for different values of $H_z$ with $D=2$ mJ/m$^2$. Tilt angle (b) 
and velocity (d) of the DW as a function of $H_z$ for different values of $D$. The inset in (d) shows 
the DW velocity $v_n$ in the direction perpendicular to the DW surface ($v_n = v \cos \chi$). (c) Time 
dependence of the tilt angle for $\mu_0 H_z = 100$ mT and different track widths $w$ for $D = 2$ mJ/m$^2$. In 
(b,c,d), the results of the micromagnetic simulation (resp. CCM) are plotted in coloured dots (resp. 
continuous lines). The dashed (resp. continuous) black line in (d) (resp (d), inset) corresponds to 
the prediction of the standard $(q, \psi)$ model for $D = 2$ mJ/m$^2$. 
FIG. 3: Dynamics of the DW driven by the spin orbit torque ($\mu_0 H_{SO} = 0.1 \text{ T}/(10^{12} \text{ A/m}^2)$) for a 100 nm wide nanotrack. The results of the micromagnetic simulation (resp. CCM) are plotted in coloured dots (resp. continuous lines). (a) Magnetization pattern of the DW for $D = 2 \text{ mJ/m}^2$ and different values of $J$. The white arrow indicates the current direction. (b,d) Tilting (b) and velocity (d) of the DW as a function of $J$ for different values of $D$. The inset in (d) shows the DW velocity in the direction perpendicular to the DW surface $v_n$ for $D = 2 \text{ mJ/m}^2$. The continuous black line is the result of the standard ($q,\psi$) model. (c) Time dependence of the DW tilt angle for different track widths $w$ for a current of density $0.25 \times 10^{12} \text{ A/m}^2$ applied at $t=0$ ($D=2 \text{ mJ/m}^2$).