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Simultaneous determination of porosity, tortuosity, viscous and thermal characteristic lengths of rigid porous materials.

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running title : Ultrasonic characterization of porous materials.

ABSTRACT

We present an improved method for the characterization of air-saturated porous materials by simultaneous measurement of porosity, tortuosity, viscous and thermal characteristic lengths via ultrasonic transmission only. The proposed method is based on a temporal model of the direct and inverse scattering problem for the transient ultrasonic waves in a homogeneous isotropic slab of rigid porous material. The advantage of the proposed method is that the four parameters are determined simultaneously using just transmitted experimental wave from a porous material saturated by one gas (air). In addition, no relationship is assumed between the two characteristic lengths.

I. INTRODUCTION

Acoustic damping in air-saturated porous materials is described by the inertial, viscous, and thermal interactions between the fluid and the structure[1−3]. These materials are mainly used to reduce noise and vibration pollution. The physical parameters[1−4] describing the ultrasonic propagation in these media are : porosity, tortuosity, viscous and thermal characteristic lengths. These parameters play an important role in the attenuation and dispersion of acoustic waves in a porous medium at the high frequencies[4]. The high frequency domain[4] corresponds to the range of frequencies such that the viscous boundary layer thickness $\delta = (2\eta/\omega\rho)^{1/2}$ is smaller than the radius r of the pores (η and ρ are respectively the viscosity and density of the saturating fluid and ω represents the pulsation frequency).

The transmitted waves[5−9] are often used for the ultrasonic characterization of air-saturated porous materials in the frequency[5−7] and time[8,9] domains. When the structure of the porous materials is rigid, two independent parameters are generally measured in transmitted mode using ultrasonic waves ; the tortuosity and the viscous characteristic length. The thermal characteristic length is deduced from a fixed ratio with the viscous characteristic length. When the porous medium is subsequently saturated by two gases (air and helium), the determination of the thermal characteristic length independently of the viscous length is possible[5]. In the case of a porous material having a structure which vibrates[10,11] the ultrasonic waves transmitted, allow measurement of the porosity, and mechanical parameters.

The reflected waves by the first interface[12,13] of a slab of rigid porous material permit the measurement of the tortuosity and porosity. When the reflected wave by the second interface is detected experimentally, the determination of the characteristic lengths becomes possible[14]. The use of both transmitted and reflected waves simultaneously[15,16] gives a good estimation of porosity, tortuosity, viscous and thermal characteristic lengths. Other methods[17−21], not using ultrasonic waves, have been developed for the characterization of rigid porous materials, mea-

suring some parameters mentioned above.

In this work, we introduce an improved method to measure the porosity, tortuosity, the viscous and thermal characteristic lengths simultaneously, by solving the inverse problem in the time domain, without the use of reflected waves. Experimental data are used in the inversion process from waves transmitted by the porous material in the high frequency range. No relationship is assumed between the characteristic lengths such that both lengths are determined independently. This work shows that it is now possible to measure the porosity using ultrasonic transmitted waves only. In addition, the thermal characteristic length can be obtained regardless of the viscous length, without saturating the porous medium by another gas, or the use data reflected wave. Thus, the method is reliable, rapid and presents advantages over the classic techniques used to date. These results open perspectives yet to be explored for the ultrasonic characterization techniques of air-saturated porous materials.

II. MODEL

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the initial case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory[22]. In air-saturated porous media, the vibrations of the solid frame can often be neglected in absence of direct contact with the sound source, so that the waves can be considered to propagate only in fluid. This case is described by the equivalent-fluid model, which is a particular case of the Biot model, in which fluid-structure interactions are taken into account by two frequency response factors : dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al*[4], and the dynamic compressibility of the air in the porous material $\beta(\omega)$ given by Allard *et al*[1]. In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency increases. Consider a homogeneous

porous material that occupies the region $0 \leq x \leq L$. A sound pulse impinges normally on the medium. It generates an acoustic pressure field p and an acoustic velocity field v within the material. The acoustic fields satisfy the following equivalent-fluid macroscopic equations (along the $x-$axis)[1] :

$$\rho\alpha(\omega)j\omega v = \frac{\partial p}{\partial x}, \qquad \frac{\beta(\omega)}{K_a} j\omega p = \frac{\partial v}{\partial x}, \tag{1}$$

where, $j^2 = -1$, ρ the fluid density and K_a is the compressibility modulus of the fluid. In the high frequency domain, the viscous effects are concentrated in a small volume near the frame and the compression/dilatation cycle is faster than the heat transfer between the air and the structure, and it is a good approximation to consider that the compression is adiabatic. The high-frequency approximation of the responses factors $\alpha(\omega)$ and $\beta(\omega)$ when $\omega \to \infty$ are given by the relations :

$$\alpha(\omega) = \alpha_\infty \left(1 + \frac{2}{\Lambda}\sqrt{\frac{\eta}{j\omega\rho}}\right), \qquad \beta(\omega) = 1 + \frac{2(\gamma-1)}{\Lambda'}\sqrt{\frac{\eta}{j\omega P_r \rho}}, \tag{2}$$

where $j^2 = -1$, α_∞ is the tortuosity, Λ the viscous characteristic length, Λ' the thermal characteristic length, η the fluid viscosity, γ the adiabatic constant, P_r the Prandtl number. The expression, in frequency domain, of the transmission coefficient $T(\omega)$ of a slab of porous material is given by :

$$T(\omega) = \frac{2Y(\omega)}{2Y(\omega)\coth\left(k(\omega)L\right) + (1 + Y^2(\omega))\sinh\left(k(\omega)L\right)}, \tag{3}$$

where :

$$Y(\omega) = \phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}, \qquad \text{and} \qquad k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}},$$

ϕ is the porosity of the material. In the time domain, $\alpha(\omega)$ and $\beta(\omega)$ act as operators and in the high frequency approximation their expressions are given by [21] :

$$\tilde{\alpha}(t) = \alpha_\infty \left(\delta(t) + \frac{2}{\Lambda}\left(\frac{\eta}{\pi\rho}\right)^{1/2} t^{-1/2}\right), \quad \tilde{\beta}(t) = \left(\delta(t) + \frac{2(\gamma-1)}{\Lambda'}\left(\frac{\eta}{\pi Pr\rho}\right)^{1/2} t^{-1/2}\right), \tag{4}$$

in these equations, $\delta(t)$ is the Dirac function. In this model the time convolution of $t^{-1/2}$ with a function is interpreted as a semi derivative operator following the definition of the fractional derivative of order ν given in Samko and coll [24],

$$D^{\nu}[x(t)] = \frac{1}{\Gamma(-\nu)} \int_0^t (t-u)^{-\nu-1} x(u) du, \tag{5}$$

where $\Gamma(x)$ is the gamma function. Using equations (1) and (4) in the time domain, it follows the fractional propagation equation :

$$\frac{\partial^2 p}{\partial x^2} - \left(\frac{\alpha_\infty}{c_0^2}\right) \frac{\partial^2 p}{\partial t^2} - B \frac{\partial^{3/2} p}{\partial t^{3/2}} - C \frac{\partial p}{\partial t} = 0, \tag{6}$$

where the coefficients c_0, B and C are constants respectively given by ;

$$c_0 = \sqrt{\frac{K_a}{\rho}}, \quad B = \frac{2\alpha_\infty}{K_a} \sqrt{\frac{\rho\eta}{\pi}} \left(\frac{1}{\Lambda} + \frac{\gamma - 1}{\sqrt{Pr}\Lambda'}\right) \quad C = \frac{4\alpha_\infty(\gamma-1)\eta}{K_a \Lambda \Lambda' \sqrt{Pr}}. \tag{7}$$

The incident $p^i(t)$ and transmitted $p^t(t)$ fields are related in time domain by the transmission scattering operator[9] T :

$$p^t(x,t) = \int_0^t \tilde{T}(\tau) p^i \left(t - \tau - \frac{(x-L)}{c_0}\right) d\tau, \quad \text{where :} \quad \tilde{T}(t) = \frac{4\phi\sqrt{\alpha_\infty}}{(\phi + \sqrt{\alpha_\infty})^2} G\left(t + \frac{L}{c}, \frac{L}{c}\right). \tag{8}$$

As in most porous materials saturated air, the multiple reflections are negligible because of the high attenuation of the sound wave, the expression of the transmission operator $\tilde{T}$ takes into account only the reflections at interfaces $x = 0$ and $x = L$. G is the Green function of the medium given by[25] :

$$G(t,k) = \begin{cases} 0 \quad \text{if} \quad 0 \le t \le k, \\ \\ \frac{b'}{4\sqrt{\pi}} \frac{k}{(t-k)^{3/2}} \exp\left(-\frac{b'^2 k^2}{16(t-k)}\right) + \Delta \int_0^{t-k} h(t,\xi) d\xi \quad \text{if} \quad t \ge k, \end{cases} \tag{9}$$

where : $h(\xi,\tau) = -\frac{1}{4\pi^{3/2}} \frac{1}{\sqrt{(\tau-\xi)^2 - k^2}} \frac{1}{\xi^{3/2}} \int_{-1}^{1} exp\left(-\frac{\chi(\mu,\tau,\xi)}{2}\right) (\chi(\mu,\tau,\xi) - 1) \frac{\mu d\mu}{\sqrt{1-\mu^2}}$, and :
$\chi(\mu,\tau,\xi) = \left(\Delta\mu\sqrt{(\tau-\xi)^2 - k^2} + b'(\tau-\xi)\right)^2 / 8\xi$, $b' = Bc_0^2\sqrt{\pi}$, $c' = C.c_0^2$, $\Delta^2 = b'^2 - 4c'$.

III. INVERSION OF EXPERIMENTAL DATA

The inverse problem is to find the parameters α_∞, ϕ, Λ and Λ' which minimize numerically the discrepancy function $U(\alpha_\infty, \phi, \Lambda, \Lambda') = \sum_{i=1}^{i=N}(p^t_{exp}(x,t_i) - p^t(x,t_i))^2$, wherein $p^t_{exp}(x,t_i)_{i=1,2,...n}$ is the discrete set of values of the experimental transmitted signal and $p^t(x,t_i)_{i=1,2,...n}$ the discrete set of values of the simulated transmitted signal predicted from Eq. (8). The inverse problem is solved numerically by the least-square method. For its iterative solution, we used the simplex search method (Nedler Mead)[26] which does not require numerical or analytic gradients. Experiments are performed in air using a broadband Ultran NCT202 transducer with a central frequency of 190 kHz in air and a bandwith of 6 dB extending from 150 to 230 kHz. Pulses of 400V are provided by a 5058PR Panametrics pulser/receiver. The received signals are filtered above 1MHz to avoid high-frequency noise. Electronic interference is eliminated by 1000 acquisition averages. The experimental setup is shown in Fig. 1. Consider a sample of plastic polyurethane foam M1, of thicknesses $0.8 \pm 0.01cm$. Sample M1 was characterized using classic methods[3,6,9,12,13,27] and gave the following physical parameters $\phi = 0.85 \pm 0.05$, $\alpha_\infty = 1.45 \pm 0.05$, $\Lambda = (30 \pm 1)\mu m$, $\Lambda' = (60 \pm 3)\mu.m$. The porosity is measured by the direct[27] and ultrasonic methods[12−14], giving the same results. Tortuosity is related to the formation factor[28] used to describe the electrical conductivity of a porous solid saturated with conducting fluid. The direct method for the measurement of tortuosity, is based on the measurement of formation factor. For air-saturated plastic foams[1−3], it is not possible to saturate the material by a conducting fluid. The ultrasonic methods[3,5,6,9,12−14] are best suited for the measurement of tortuosity. The standard methods used for measuring the viscous and thermal characteristic lengths of the porous sample are the ultrasonic method[4−6].

Fig. 2 gives a scanning Electron Microscope image of a polyurethane foam sample showing the microstructure of the pores. Fig. 3 shows the experimental incident signal (dashed line) generated by the transducer, and the experimental transmitted signal (solid line). The amplitude

is represented by an a.u. and the point number represented in the abscissa is proportional to time. After solving the inverse problem simultaneously for the porosity ϕ, tortuosity α_∞, viscous and thermal characteristic lengths Λ and Λ', we find the following optimized values : $\phi = 0.87 \pm 0.01$, $\alpha_\infty = 1.45 \pm 0.01$, $\Lambda = (32.6 \pm 0.5)\mu m$ and $\Lambda' = (60 \pm 0.5)\mu m$. The values of the inverted parameters are close to those obtained by conventional methods[3,6,9,12,13,27]. We present in Figs. 4(a)-4(b), the variation of the minimization function U with the porosity, tortuosity, viscous characteristic length, and the ratio between Λ' and Λ. In Fig. 5, we show a comparison between an experimental transmitted signal and simulated transmitted signal for the optimized values of ϕ, α_∞, Λ and Λ'. The difference between the two curves is small, which leads us to conclude that the optimized values of the physical parameters are correct. We studied the impact of $(\Lambda'/\Lambda \neq 2)$ on the determination of other parameters, and we arrived at the following result : By increasing the ratio (Λ'/Λ) of 2 to 3, the porosity increases from 13% of its initial value, while the viscous characteristic length decreases by 9% of its initial value. The value of the tortuosity remains unchanged. We can therefore conclude that the porosity and viscous characteristic length, are the parameters that were most affected by the change in the ratio between Λ' and Λ.

The use of the expression of the transmission operator in the time domain (Eq. 8), or that the transmission coefficient in the frequency domain (Eq. 3), then taking the inverse transform numerically, gives exactly the same result for the inversion process. The time and frequency approaches are complementary. For transient signals in very short time, the temporal model is best suited. However, for signals with a wide temporal content (narrow frequency content), the frequency approach is preferred. For the experimental signals used in this work, the two approaches are equivalent and give the same results.

Given the low sensitivity of the porosity and the thermal length, relative to that of the tortuosity and the viscous length of the transmitted waves, we thought[9,12,13] it was not possible to solve the inverse problem with respect to porosity and thermal length. However, this study came to contradict this earlier conclusion. The measurement of these two quantities (ϕ and Λ')

is now possible using only the transmitted ultrasonic waves. This promising result, open horizons yet to be explored for the ultrasonic characterization of porous materials, thus limiting the experimental data. We recall that until now, the measurement of porosity and thermal characteristic length required the combination of data in transmission and reflection. The alternative ultrasonic method[5] for measuring Λ' using only the transmitted waves, is to saturate the porous material by another gas, which is not always easy to do experimentally, without damaging the internal structure of the material porous. The proposed method has the advantage of being simple, without any intervention on the fluid saturating the porous material. Increasingly limiting the experimental data to the transmitted signals, the inversion is faster to perform. Solve the inverse problem in the time domain for transient signals has the advantage of treating the full information of the experimental signal, acting simultaneously on speed, attenuation and dispersion of the ultrasonic wave.

IV. CONCLUSION

An inverse scattering estimate of the porosity, tortuosity, viscous and thermal characteristic lengths was given by solving the inverse problem in time domain for waves transmitted by a slab of air-saturated porous material. The inverse problem is solved numerically by the least-square method. The reconstructed values of these parameters are in agreement with those obtained using classical methods. The proposed experimental method has the advantage of being simple, rapid, and efficient for estimating those parameters and further characterizing porous materials. This study shows that it is not necessary to use the reflected waves for a complete ultrasonic characterization of air-saturated porous material.

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FIGURE CAPTIONS

Fig. 1. Experimental set-up of the ultrasonic measurements.

Fig. 2. A scanning Electron Microscope image of a polyurethane foam showing the microstructure of the pores

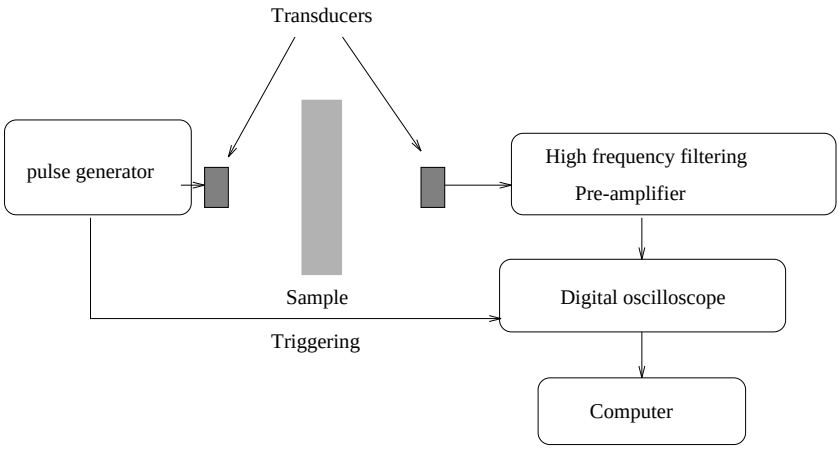
Fig. 3. Experimental incident signal (solid line) and experimental transmitted signal (dashed line).

Fig. 4-a. Variation of the minimization function U with porosity and tortuosity

Fig. 4-b. Variation of the cost function U with the viscous characteristic length Λ and the ratio Λ'/Λ

Fig. 5. Comparison between the experimental transmitted signal (black dashed line) and the simulated transmitted signals (red line) using the reconstructed values of ϕ, α_∞, Λ and Λ'.

Fig. 1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High frequency filtering
Pre-amplifier
Sample
Digital oscilloscope
Triggering
Computer

Fig. 2

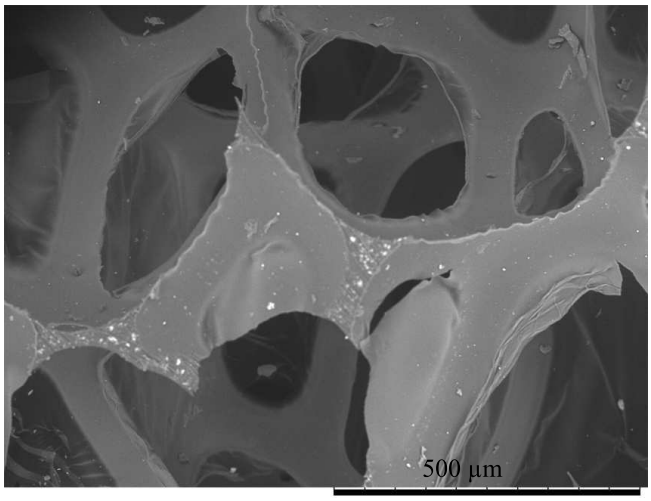


Fig. 3

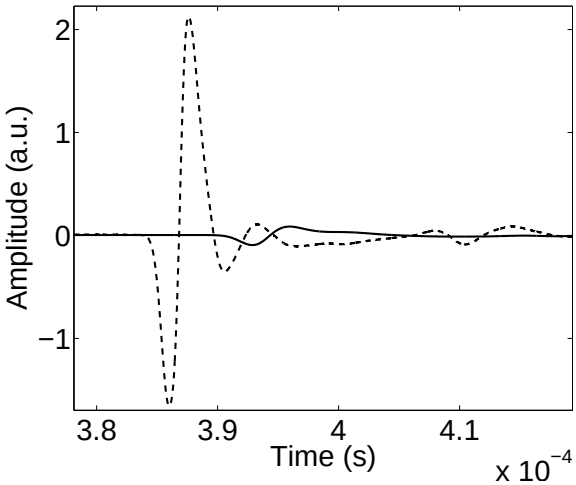
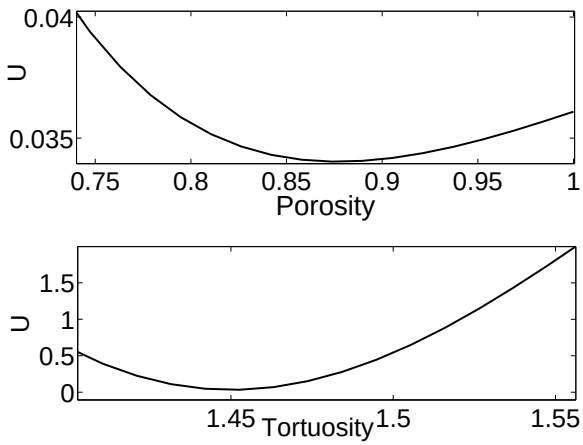


Fig. 4-a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0.9
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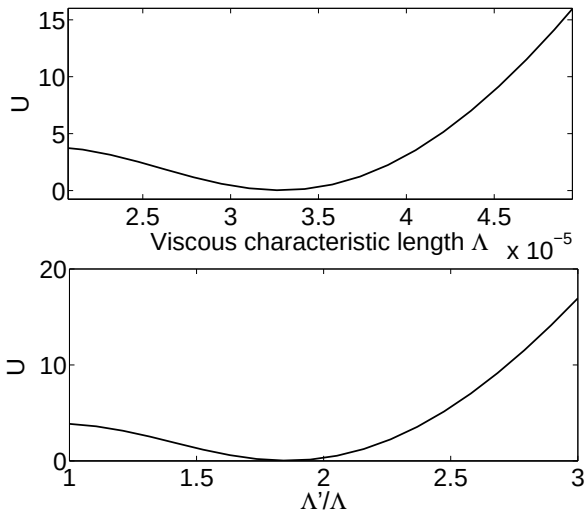
0
1.45
Tortuosity
1.5
1.55

Fig. 4-b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3
3.5
4
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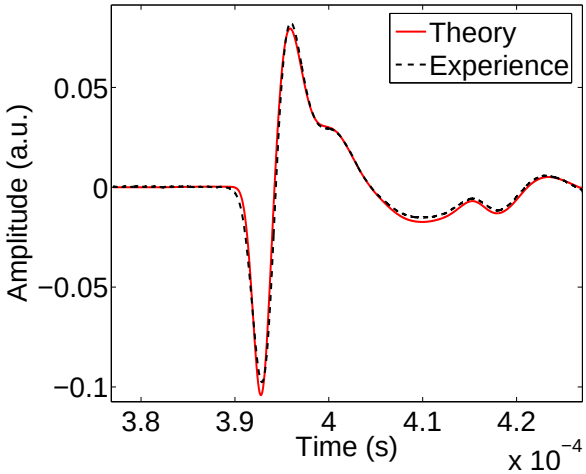
1
1.5
2
2.5
3
Λ'/Λ

Fig. 5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Time (s)
x 10^−4