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# Optimising the management of uneven-aged *Pinus sylvestris* L. and *Pinus nigra* Arn. mixed stands in Catalonia, north-east Spain

Antoni TRASOBARES<sup>a\*</sup>, Timo PUKKALA<sup>b</sup>

<sup>a</sup> Centre Tecnològic Forestal de Catalunya, Pujada del seminari s/n, 25280, Solsona, Spain

<sup>b</sup> University of Joensuu, Faculty of Forestry, PO Box 111, 80101 Joensuu, Finland

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**Abstract** – This study uses a simulation-optimisation system, PINUSMIX, to optimise the structure and management of uneven-aged mixtures of *Pinus sylvestris* L. and *Pinus nigra* Arn. in Catalonia (north-east Spain). The simulation sub-system consists of a method for drawing the initial tree diameters from a Weibull distribution and a stand growth and yield simulator based on individual-tree growth, height, ingrowth and survival models. The simulation sub-system was combined with the optimisation algorithm of Hooke and Jeeves. The system was used to optimise the management of uneven-aged mixtures of *P. sylvestris* and *P. nigra* on medium site characteristics in the region. When the land expectation value with a 20-year cutting cycle and a 2% discounting rate was maximized, the land expectation value was 1331 euro ha<sup>-1</sup>, and the optimal prior-thinning stand volume was 86 m<sup>3</sup> ha<sup>-1</sup>. In the optimal stand structure *P. sylvestris* dominated. The effects of discounting rate, cutting cycle length, objective function, site, timber prices, type of diameter distribution and biodiversity considerations were logical. Increasing discounting rate and shortening the cutting cycle decreased the optimal prior-thinning stand densities. Maximising wood production or net income resulted in higher volumes of growing stock than did maximising profitability. Forcing the inclusion of large trees in the stand, for biodiversity reasons, clearly decreased profitability but had no effect on wood production.

**simulation / continuous cover forestry / investment efficiency / biodiversity**

**Résumé** – Optimisation de l'aménagement des peuplements irréguliers et mélangés de *Pinus sylvestris* L. et *Pinus nigra* Arn. en Catalogne (nord-est de l'Espagne). L'article décrit un système de simulation et optimisation, PINUSMIX, pour l'aménagement des peuplements irréguliers et mélangés de *Pinus sylvestris* L. et *Pinus nigra* Arn. en Catalogne (nord-est de l'Espagne). La combinaison des systèmes simulation-optimisation permet de trouver la structure et le scénario sylvicole optimal pour une fonction objective donnée. Le sous-système de simulation est basé sur une méthode pour dessiner les diamètres initiaux des arbres à partir d'une distribution de Weibull et un simulateur de la croissance et de la production du peuplement basé sur des modèles de croissance en diamètre, hauteur, développement, et survie. Le sous-système de simulation était combiné avec l'algorithme d'optimisation développé par Hooke et Jeeves. Le système était utilisé pour optimiser l'aménagement des peuplements irréguliers et mélangés de *P. sylvestris* et *P. nigra* en conditions moyennes de site dans la région. Quand la valeur espérée du fond, avec un cycle de coupe de vingt ans, pour un taux d'escompte de 2 %, était maximisée, la valeur espérée du fond atteignait 1331 euro ha<sup>-1</sup>, et le volume optimal avant-coupe du peuplement était 86 m<sup>3</sup> ha<sup>-1</sup>. Dans la structure optimale du peuplement *P. sylvestris* était dominant. Les effets de taux d'escompte, durée du cycle de coupe, fonction objectif, site, prix du bois, type de distribution des diamètres, et considérations de biodiversité, étaient logiques. Augmenter le taux d'escompte et écourter la durée du cycle de coupe diminuaient les densités optimales d'avant-coupe. La maximalisation de la production de bois ou du revenu net se traduisait en niveaux de volume sur pied plus élevés que ceux obtenus avec la maximalisation de la rentabilité. L'inclusion pour des raisons de biodiversité d'arbres de grandes dimensions avait pour effet de diminuer clairement la rentabilité mais n'affectait pas la production de bois.

**simulation / gestion de forêts irrégulières / efficacité d'investissement / biodiversité**

## 1. INTRODUCTION

*Pinus sylvestris* L. and *Pinus nigra* Arn. ssp. *salmannii* var. *pyrenaica* mixtures form large forests in the *Montane-Mediterranean* vegetation zones of Catalonia (from 600 to 1600 m a.s.l.), occupying an area of 267 000 ha [12, 13]. Both species supply important products such as poles, saw logs and con-

struction timber. Most of the stands in the region are rather small private woodlots. Often the only management conducted on this type of property is selection felling [5], which leads to considerable within-stand variation in tree age. However, trees for felling are frequently selected without much attention to the future yields of the residual stand, due to good prices for poles and saw logs. The thinnings often remove the most vigorous

\* Corresponding author: antoni.trasobares@ctfc.es

trees, which may decrease the potential of residual growing stocks [10]. To improve the current management practices, efficient planning tools are required that help managers to analyse the economic, ecological and social effects of alternative management options of these forests.

Traditionally, decisions about the optimum management schedule for uneven-aged stands in Spain have been based on “balanced” diameter distributions characterized by de Liocourt’s [22] constant “q” [23]. A balanced diameter distribution – one that can produce a sustained yield while maintaining an essentially constant structure and volume [21, 25] – is expected to show a smooth geometric progression of the number of trees in successive diameter classes, with the ratio of the number of trees in a given diameter class to those in the next larger class defined as “q” [2]. However, the “q” parameter alone does not tell how the stand should be managed, since it does not show the stand density and how harvests should be conducted. Selection of the best treatment is a complex task that involves the optimal stand structure as well as the harvest proportions for every species and diameter class in the stand. The optimum combination of many variables is most easily sought using a set of models and a simulator that is able to predict stand development under any set of management parameters. This search can be automated by using optimisation [1, 16, 27, 28, 41].

In this study a system for stand management support, PINUSMIX, was used to optimise the management of *P. sylvestris* and *P. nigra* mixtures on medium site characteristics in Catalonia. The system consists of a method for drawing the initial tree diameters from a Weibull distribution, a stand growth and yield simulator based on individual-tree growth, height, ingrowth and survival models, and an optimisation algorithm, which finds the optimum stand structure and management schedule for a given objective function. The effects of discounting rate, length of the cutting cycle, objective function, site, timber prices, type of diameter distribution and biodiversity considerations were analysed.

## 2. MATERIALS AND METHODS

### 2.1. Decision variables

Optimising management means finding the optimal values for a set of decision variables (*DVs*). In uneven-aged forests, management consists of finding a proper diameter distribution and selection thinning method for the stand; the diameter distribution, stand density, and thinning intensity are selected so that a specified objective function, for instance land expectation value, is maximised; and thinning is conducted so that the stand structure always reaches the same pre-thinning state just prior to the next thinning.

In this study, the diameter distribution of the stand in the beginning of a cutting cycle was described with the Weibull distribution function [2, 11, 24, 27]. The probability density distribution of the three-parameter Weibull distribution for a random variable  $d$  (tree diameter) is:

$$f(d) = \frac{c}{b} \left( \frac{d-a}{b} \right)^{c-1} \exp \left\{ - \left( \frac{d-a}{b} \right)^c \right\}, \quad ad \leq \infty \quad (1)$$

where  $d$  is tree diameter at breast height (cm); and  $a$ ,  $b$  and  $c$  are parameters which define the location (minimum), scale (range) and shape (skewness) of the distribution, respectively. To avoid optimal diameter distributions resembling even-aged stands, in optimisations the minimum value for the  $b$  parameter was 10. The minimum diameter

(parameter  $a$ ) was fixed to 7.5 cm, which was the minimum tree size in the inventory data used to develop the simulation system used in this study.

When stand structure is described with the Weibull distribution and its parameter  $a$  is fixed, the whole set of *DVs* in management of uneven-aged stands is as follows:

1. Total number of trees per hectare for species  $s$  prior to selection thinning ( $N_s$ ).
2. Parameter  $b$  of the Weibull distribution for species  $s$  prior to selection thinning ( $b_s$ ).
3. Parameter  $c$  of the Weibull distribution for species  $s$  prior to selection thinning ( $c_s$ ).
4. Percentage of trees harvested from different diameter classes for species  $s$  ( $H_{1s}$ ,  $H_{2s}$ ,  $H_{3s}$ , ...,  $H_{ms}$ ).

The percentage of harvested trees was specified separately for the following four diameter classes: < 20 cm, 20–29.99 cm, 30–39.99 cm and  $\geq$  40 cm, leading to four *DVs* per species to describe the harvest. The selected diameter classes corresponded to the different timber-product and unit-price categories used in Catalonia. Because each variable was specified separately for every species, the total number of *DVs* needed for specifying the management system was fourteen ( $N_s$ ,  $b_s$ ,  $c_s$ ,  $H_{1s}$ ,  $H_{2s}$ ,  $H_{3s}$ , and  $H_{4s}$  for two species).

The optimal combination of *DVs*, i.e. the optimal management, was found with the combined use of an iterative optimisation algorithm [19] and a simulation model. A given combination of *DVs* was fed into the simulator, which simulated cuttings and the development of the stand, and from these results the value of the objective function was calculated. The objective function value was passed back to the optimisation algorithm, which made alterations in the values of *DVs*, based on the feed-back from the simulation program. This process was repeated many times, until a user-specified stopping criterion was met. The best solution found was assumed to be the optimal one, or at least close to it.

### 2.2. The simulation sub-system

The simulation proceeded as follows:

1. Generate an uneven-aged prior-thinning stand (initial stand).
2. Simulate a partial cutting.
3. Simulate tree growth, tree survival, and ingrowth to the end of the cutting cycle.

The tree diameters of the initial stand were drawn from the Weibull distribution specified by the scale and shape parameters ( $b$  and  $c$  in Eq. (1)). A diameter range of 7.5–50 cm was divided into 50 classes of equal width. Then the frequency of each *dbh* class was calculated from equation (1) using the midpoint diameter of the class. The frequencies were scaled so that their sum equalled the total number of trees per hectare, after which the height of the midpoint tree in the class was calculated. These steps resulted in a set of representative trees (also called a tree list), one for every diameter class. Once the tree list was defined, the simulator used the growth and yield model for uneven-aged mixtures of *P. sylvestris* and *P. nigra* in Catalonia developed by Trasobares et al. [37]. This model, prepared using data from the Spanish National Forest Inventory, consists of an individual tree diameter growth model, a tree survival model, an ingrowth model, and a static individual tree height model, for simulating stand development. Separate models were used for *P. sylvestris* and *P. nigra*. The growth and yield model provides unbiased estimates for different combinations of predictors, and for a 10-year period, permits explaining about 65% of variation in stand basal area change (see more details in [37]).

The simulation of a 10-year time step consisted of the following steps:

1. For each tree, add the 10-year diameter increment and the predicted plot factor (a measure of site fertility) to the diameter, using the models developed by Trasobares et al. [37].

**Table I.** Stumpage prices of products and the percentage of product in different diameter classes.

Product	Stumpage price, euro m <sup>-3</sup>	Diameter class, cm			
		< 20	20–30	30–40	> 40
<i>P. sylvestris</i>		Percentage			
Particle board wood	7.51	75	30	20	20
Poles	36.06	5	20	5	0
Sawlog	20.43	20	50	75	70
High quality sawlog	27.05	0	0	0	10
<i>P. nigra</i>					
Particle board wood	7.51	75	30	20	20
Poles	36.06	5	20	5	–
Sawlog	20.43	20	50	75	80
High quality sawlog	–	–	–	–	–

**Table II.** Mean stumpage prices (euro m<sup>-3</sup>) per diameter class and species.

Species	Diameter class, cm			
	< 20	20–30	30–40	> 40
<i>P. sylvestris</i>	11.52	19.68	18.63	18.51
<i>P. nigra</i>	11.52	19.68	18.63	17.85

2. Multiply the frequency of each tree (number of trees per hectare represented by a given tree) by the density-independent mortality rate of 0.962 [37] and a density-dependent 10-year probability of survival.

3. Calculate the number of trees per hectare that enter the first *dbh*-class via ingrowth and the mean *dbh* of ingrowth at the end of a 10-year growth period [37].

4. Calculate tree heights using static height models [37].

5. Calculate tree volumes using the following formulas provided by the Spanish National Forest Inventory [20]:

$$v_{syl} = 30.83 + 0.0003210 \times dbh^2 \times h \quad (2)$$

$$v_{nig} = 14.46 + 0.0003435 \times dbh^2 \times h \quad (3)$$

where  $v_{syl}$  and  $v_{nig}$  are *P. sylvestris* and *P. nigra* tree volumes (dm<sup>3</sup>), respectively, for the province of Lleida; *dbh* is diameter at breast height (dm); *h* is tree height (m). These volume functions are based partly on the same permanent plots that were used to develop the stand simulator [37].

### 2.3. Economic data

The study used stumpage prices specified separately by four diameter classes [10]. These values were modified slightly according to tree species and recent surveys by forest industries, forest owners, and forest managers in the region. The official prices at industry park, provided by the Consorci Forestal de Catalunya [8], were also taken into account when the final stumpage prices for *P. sylvestris* and *P. nigra* were determined (Tabs. I and II).

### 2.4. Objective functions

The management schedule for a stand was optimised using the land expectation value (*LEV*) as the objective function. *LEV* was calculated

using the following formula [1, 18]:

$$LEV_t = \frac{VG_t}{((1+i)^t - 1)} - VGS_t \quad (4)$$

where  $LEV_t$  is the land expectation value for the *t*-year cutting cycle (euro ha<sup>-1</sup>);  $VG_t$  is value growth harvested every *t* years (euro ha<sup>-1</sup>);  $VGS_t$  is the value of the residual growing stock for the *t*-year cutting cycle (euro ha<sup>-1</sup>); *i* is the rate of interest (percentage divided by 100); *t* is length of the cutting cycle.

The valuation formulae utilized in the above equation are:

$$VGS_t = \sum_{s=1}^2 \sum_{i=1}^{50} m_{is} P_{is} v_{is} \quad (5)$$

$$VG_t = \sum_{s=1}^2 \sum_{i=1}^{50} n_{is} P_{is} v_{is} - VGS_t \quad (6)$$

where  $m_{is}$  is post-cutting frequency of the *i*th diameter class of species *s*;  $n_{is}$  is prior-cutting frequency of the *i*th diameter class of species *s*;  $P_{is}$  is stumpage price (euro m<sup>-3</sup>) of the *i*th diameter class of species *s*; and  $v_{is}$  is tree volume (m<sup>3</sup>) of the *i*th diameter class of species *s*.

The *LEV* is an appropriate criterion for determining sustainable equilibrium diameter distributions if economic efficiency is the objective of management [1, 4, 7, 11, 14, 16, 18]. Optimal management with the *LEV* goal was calculated for medium site characteristics – mean values for elevation, slope, latitude, and continentality in the modeling data [37] – using a discounting rate of 2%. Díaz Balteiro and Prieto Rodríguez [9] proposed this discounting rate based on the fact that 2% is very close to the rate of return of the public debt in Spain [28]. Discounting rates of 1%, 3%, and 4% were also used to study the effect of this parameter on the optimal management.

Because the idea of uneven-aged forestry is to keep the stand structure unchanged, a penalty function was added to the objective function

[3], the value of which increased as a function of the difference between the initial and final stand characteristics. The penalty function was as follows:

$$Penalty = p1 |V_{(end)} - V_{(ini)}|^{p2} + p3 \sum_{s=1}^2 \sum_{i=1}^4 |N_{(end)is} - N_{(ini)is}|^{p2} \quad (7)$$

where  $V_{(end)}$  and  $V_{(ini)}$  are stand volumes at the end and beginning of the cutting cycle, respectively;  $N_{(end)is}$  and  $N_{(ini)is}$  are the number of trees per hectare of species  $s$  and the  $i$ th diameter class at the end and beginning of the cutting cycle. The diameter classes used in the penalty function were: < 20 cm, 20–29.99 cm, 30–39.99 cm, and > 40 cm. The objective function ( $OF$ ), which was maximised when the management of an uneven-aged *P. sylvestris* and *P. nigra* mixed stand was optimised, was

$$OF = LEV - Penalty. \quad (8)$$

After preliminary analyses,  $p2$  was set to 2. For  $p1$  and  $p3$ , suitable initial values were found using trial and error; and these values were gradually increased during optimisation using a sequential unconstrained minimization technique [3], as described in Haight and Getz [15], Haight et al. [17], and Wikström and Eriksson [41]. This technique was used to avoid too high penalties, since very large penalty parameters may lead to solutions that are far from optimal, as the search procedure becomes too constrained.

In addition to the  $LEV$ , the profit ( $PRF$ ) (Eq. (9)), the mean annual harvested volume ( $WP$ ) (Eq. (10)), the mean annual net income ( $NET$ ) (Eq. (11)) and the managed forest value ( $MFV$ ) [1, 14, 31] (Eq. (12)) – the value of land and trees – were used as objective functions to study the effect of type of objective function on the optimal management.

$$PRF_t = NET_t - i \times (VGS_t + VBL_t) \quad (9)$$

$$WP_t = \frac{HV_t}{t} \quad (10)$$

$$NET_t = \frac{VG_t}{t} \quad (11)$$

$$MFV_t = VG_t + \frac{VG_t}{((1+i)^t - 1)} \quad (12)$$

where  $PRF_t$  is mean annual profit for a  $t$ -year cutting cycle (euro  $ha^{-1} a^{-1}$ );  $NET_t$  is the mean annual net income for a  $t$ -year cutting cycle (euro  $ha^{-1} a^{-1}$ ),  $VBL_t$  is the value of bare land for a  $t$ -year cutting cycle (euro  $ha^{-1}$ ),  $WP_t$  is the mean annual harvested volume for a  $t$ -year cutting cycle ( $m^3 ha^{-1} a^{-1}$ ),  $HV_t$  is total harvested volume for a  $t$ -year cutting cycle ( $m^3$ ), and  $MFV_t$  is the managed forest value for a  $t$ -year cutting cycle (euro  $ha^{-1}$ ). The bare land values used to calculate  $PRF$  were obtained using a simulation-optimisation system for management of even-aged *P. sylvestris* L. stands in Spain [28]. The site index used was such that the mean annual timber production of the even-aged stand nearby equalled the average growth rate of an uneven-aged stand in Catalonia (about  $2.5 m^3 ha^{-1} a^{-1}$ ). The bare land values for different discounting rates differed as follows: 1% = 4169 euro  $ha^{-1}$ , 2% = 627 euro  $ha^{-1}$ , 3% = 0 euro  $ha^{-1}$  and 4% = 0 euro  $ha^{-1}$ .

Of the five objective functions,  $LEV$  and  $PRF$  describe profitability because the opportunity cost is taken into account. The other objective functions maximise production or income without paying attention to the amount of capital tied to production.

## 2.5. Optimisation method

Optimising the management of a *P. sylvestris* and *P. nigra* mixture, when the simulation sub-system described here is used, requires a non-

linear optimisation algorithm, because the model is not convex and differentiable in the  $DVs$ , and the model has constraints requiring a non-linear penalty function. Therefore the simulator was linked with the direct search method of Hooke and Jeeves [19], an algorithm previously used for optimising the management of both even-aged [26, 28, 30, 32, 38] and uneven-aged stands [16, 27]. As the name implies, the direct search method uses only objective function values, retaining a certain number of combinations of  $DVs$ , which are improved iteratively. The direct search starts from an initial point. A better solution is sought both along the  $DVs$ , to discover the pattern of the objective function, and along the direction of the discovered pattern [3]. If the search does not provide a better solution, the search step is reduced.

Because the Hooke and Jeeves method is sensitive and is inconsistent with respect to the number of  $DVs$ , the starting points, the feasible range of  $DVs$ , etc., it is common to solve the problem for several starting points and compare these solutions. In this study, each optimisation problem was first solved for 11 different initial vectors of  $DVs$ , each run starting from the best of 5000 random search trials, except the first one, which started from a user-defined starting point. To find an approximate optimum, rather low penalty parameters ( $p1$  and  $p3$  in Eq. (7)) were used initially. The set of 11 direct runs was repeated 2 more times with five (2nd round) and 25 (3rd round) times larger penalty parameters. In the second and third rounds, the first direct search was started from the best solution of the previous round. The other 10 direct searches began from the best of 5000 random combinations of decision variables.

The step of changing the values of  $DVs$  was initially 10% of a user-specified range, and the step was gradually decreased during the direct search. The search was terminated when the step size of every  $DV$  was less than 0.01 times the initial step size. The user-specified ranges of  $DVs$  were:  $N$ , 990 (10–1000);  $b$ , 15 (10–25);  $c$ , 2.5 (0.5–3.0);  $H_1, \dots, H_4$ , 100 (0–100). The 5000 random values of  $DVs$  that were generated in the beginning of a direct search were limited to these ranges, but the direct search was free to move outside the range (except that the minimum value of  $b$  was set to 10).

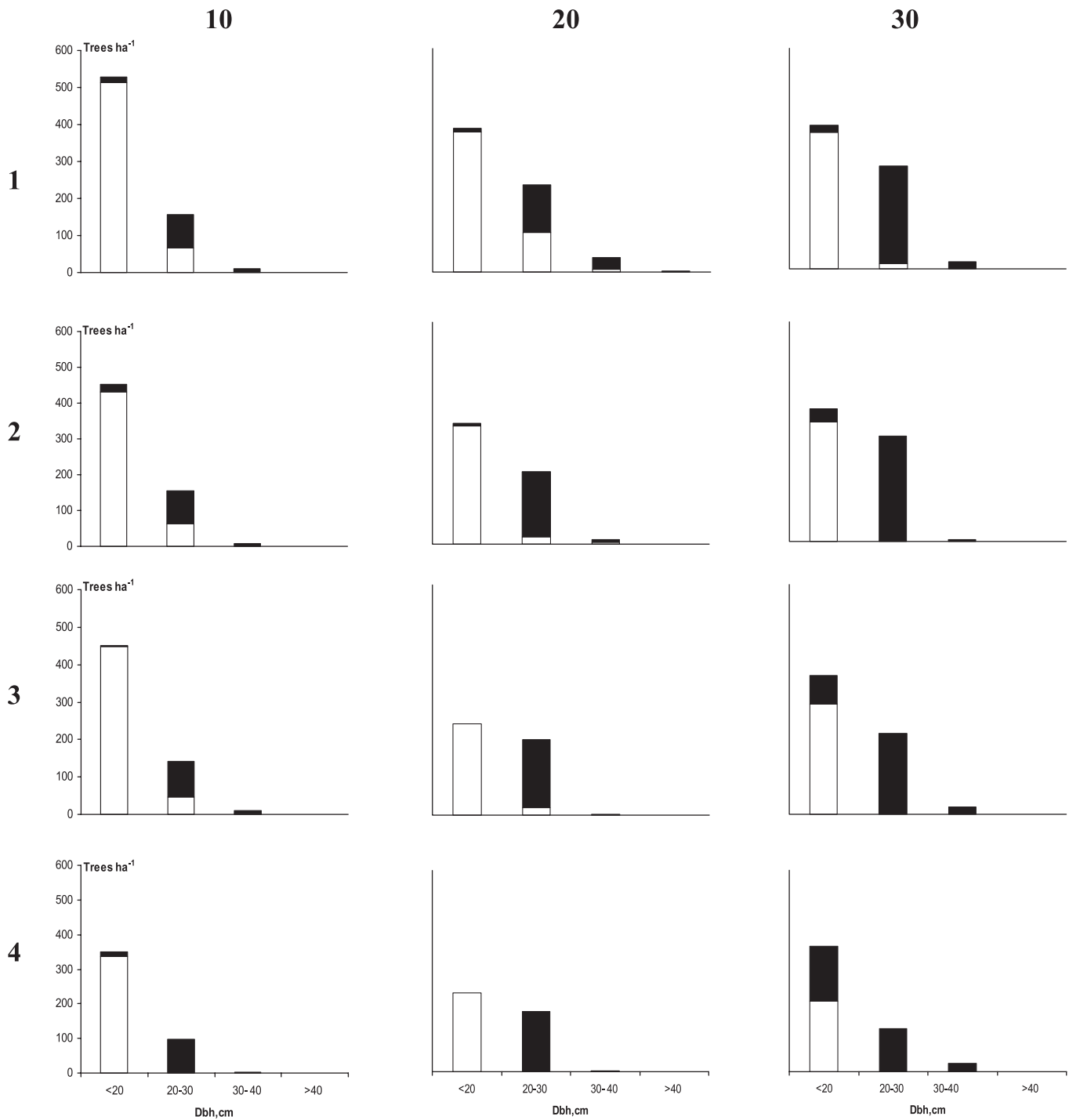
## 3. RESULTS

### 3.1. Optimal management with LEV goal

The highest  $LEV$ s and yield values were achieved with a 10-year cutting cycle (Tab. III). However, the optimal diameter distributions with a 10-year cutting cycle corresponded to rather young even-aged stands (the “ $b$ ” parameter was equal or close to 10). Cutting cycles of 20 and 30 years provided lower economic and production results, but the related optimal diameter distributions were wider and the stands resembled an uneven-aged stand more than with the 10-year cycle. The  $LEV$  values were fairly similar for 20- and 30-year cutting cycles. The longer the cutting cycle was and the higher the discounting rate was, the higher were the harvest percentages in the four diameter classes. The optimal number of trees per hectare prior to selection thinning in the stand was clearly dominated by *P. sylvestris*. The higher the discounting rate was, the higher was the opportunity cost of holding trees in the stand; and therefore the initial stand volumes were lower (Tab. III and Fig. 1).

### 3.2. Effect of the objective function

The optimal management and the maximized  $LEV$ ,  $PRF$ ,  $WP$ ,  $NET$  and  $MFV$  values for medium site characteristics, 20-year cutting cycle and 2% discounting rate were compared in



**Figure 1.** Harvested (black) and residual (white) number of trees per hectare in the four diameter classes used in the study for optimal management with the *LEV* objective function, different cutting cycles in years (columns) and discounting rates as percentage (rows).

order to investigate the effect of objective function on the optimal stand structure and management (Tab. IV and Fig. 2). The maximization of *LEV* and *PRF* resulted in similar optimal values for decision values and initial stand volume. Maximization of *WP*, *NET* or *MFV* resulted in rather similar stand structure and management, differing only in the number of young trees.

The *WP*, *NET* and *MFV* goals resulted in higher stand densities than were obtained with the profitability goals. The presence of large trees in the stand was smaller when profitability was included in the objective function. With all objective functions, the optimal number of trees per hectare prior to selection thinning in the stand was dominated by *P. sylvestris*.

**Table III.** Optimal combination of *DVs*, land expectation value (*LEV*, euro ha<sup>-1</sup>), mean annual harvest (*WP*, m<sup>3</sup> ha<sup>-1</sup> a<sup>-1</sup>) and stand volume with *LEV* goal, for medium site characteristics, different cutting cycles and discounting rates.

Variable <sup>a</sup>	Cutting cycle (years)											
	10				20				30			
	discounting rate (%)				discounting rate (%)				discounting rate (%)			
	1	2	3	4	1	2	3	4	1	2	3	4
<i>N<sub>s</sub></i>	681	595	540	436	606	439	438	416	626	613	537	421
<i>N<sub>n</sub></i>	22	28	66	19	63	98	43	0	70	45	58	113
<i>b<sub>s</sub></i>	10.00	10.67	10.00	10.00	13.59	12.95	13.21	12.96	13.48	13.38	12.94	11.24
<i>b<sub>n</sub></i>	10.00	10.00	15.23	10.00	10.00	10.00	10.00	–	10.00	10.00	10.00	10.00
<i>c<sub>s</sub></i>	1.68	1.91	1.98	1.97	1.94	2.32	3.30	3.00	2.40	2.87	2.11	1.60
<i>c<sub>n</sub></i>	0.01	0.55	1.45	0.01	0.08	1.38	0	–	1.06	2.18	3.41	0.01
<i>LEV</i>	4327.8	1550.5	747.9	576.1	3401.5	1331.2	731.6	380.2	3756.5	1301.3	554.9	251.2
<i>WP</i>	2.82	2.67	2.64	2.36	2.52	2.31	2.26	2.12	2.56	2.44	2.31	1.96
<i>V<sub>ini</sub></i>	92.7	85.2	82.5	57.0	118.1	86.8	77.1	70.2	120.5	112.7	100.1	77.0
<i>V<sub>end</sub></i>	91.7	84.4	81.7	55.3	117.2	86.1	75.4	69.3	118.8	111.5	99.2	76.6

<sup>a</sup> *N*: number of trees per hectare; *b* and *c*: parameters *b* and *c* of Weibull distribution; *V<sub>ini</sub>* and *V<sub>end</sub>*: stand volumes (m<sup>3</sup> ha<sup>-1</sup>) at beginning and end of cutting cycle, respectively; subscripts “s” and “n” refer to *P. sylvestris* and *P. nigra*, respectively.

**Table IV.** Land expectation value (*LEV*, euro ha<sup>-1</sup>), profit (*PRF*, euro ha<sup>-1</sup> a<sup>-1</sup>), mean annual harvest (*WP*, m<sup>3</sup> ha<sup>-1</sup> a<sup>-1</sup>), mean annual net income (*NET*, euro ha<sup>-1</sup> a<sup>-1</sup>), and managed forest value (*MFV*, euro ha<sup>-1</sup>) in the optimal management when *LEV*, *PRF*, *WP*, *NET*, or *MFV* was used as the objective function, for medium site characteristics, 20-year cutting cycle, and 2% discounting rate.

Objective variable	<i>LEV</i>	<i>PRF</i>	<i>WP</i>	<i>NET</i>	<i>MFV</i>
<i>LEV</i>	1331.2	25.0	2.31	45.1	2758.8
<i>PRF</i>	1318.2	25.1	2.28	43.9	2683.3
<i>WP</i>	861.5	15.8	2.56	49.6	3036.7
<i>NET</i>	–63.42	–2.68	2.59	49.2	3005.4
<i>MFV</i>	445.9	8.3	2.61	50.1	3068.6

### 3.3. Effect of site

#### 3.3.1. Elevation

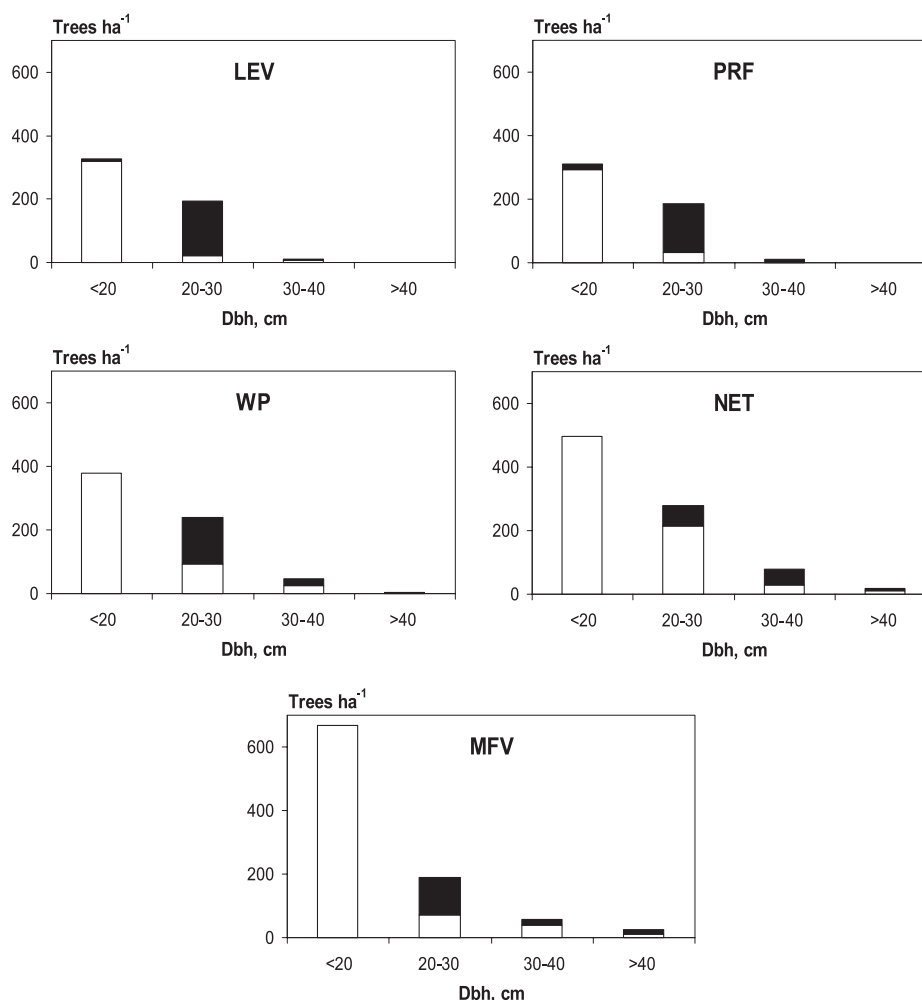
The effect of stand elevation was analyzed with the *LEV* goal, using low, medium and high elevation levels, while medium values were kept for other site characteristics. *LEV* was maximized for a 20-year cutting cycle and 2% discounting rate. The economic and yield values, as well as the initial stand volumes, were highest for an elevation of 400 m (Tab. V). The results for 900 and 1400 m elevations were similar. The proportion of *P. nigra* in the optimal number of trees per hectare prior to selection thinning increased when elevation decreased. The results were logical, bearing in mind the site requirements of the species [37].

#### 3.3.2. Plot factor (site fertility)

The effect of site fertility was analysed when *LEV* was maximized with a 2% discounting rate and 20-year cutting cycle,

using low, medium and high levels of plot factor (*PF*) in diameter growth models. High and low site fertility levels were defined using the standard deviation (SD) of the plot factor in the diameter growth modelling data (see [37]). A plot factor equal to SD represented high fertility, while *PF* = 0 was medium fertility and *PF* = –SD was low fertility. Medium site characteristics were used in models other than diameter growth.

The economic return and the yield values, as well as harvest percentages, increased with site fertility (Fig. 3). The land expectation value and mean annual harvest (in parenthesis) for low, medium and high fertility levels were 858.9 euro ha<sup>-1</sup> (2.01 m<sup>3</sup> ha<sup>-1</sup> a<sup>-1</sup>), 1450.8 euro ha<sup>-1</sup> (2.28 m<sup>3</sup> ha<sup>-1</sup> a<sup>-1</sup>), and 2164.8 euro ha<sup>-1</sup> (3.11 m<sup>3</sup> ha<sup>-1</sup> a<sup>-1</sup>), respectively. The optimal management was similar for medium and high site levels, the initial stand volume being slightly higher for high fertility levels. The optimal stand structure for low site fertility clearly included more small trees (*dbh* < 20 cm) than for the more fertile sites. The optimal number of trees per hectare prior to selection thinning was clearly higher for *P. sylvestris* than for *P. nigra*.



**Figure 2.** Harvested (black) and residual (white) number of trees per hectare in the four diameter classes used in the present study for optimal management when *LEV*, *PRF*, *WP*, *NET* or *MFV* was used as the objective function, medium site characteristics, 20-year cutting cycle, and 2% discounting rate.

### 3.4. Effect of timber price

The stumpage price per cubic meter was similar for all *dbh* classes (Tab. II), due to the importance of small-size poles in the region. To make the results comparable with the more common relationships between tree size and timber price, the effect of timber price was analysed by comparing the results obtained with the original *dbh*-price function with those obtained with a modified function (Fig. 4), as defined by Palahí and Pukkala [28]. The *LEV* was maximized using both of the price functions, for 20-year cutting cycle, medium site characteristics and different discounting rates. Use of the modified *dbh*-price function resulted in a more classical uneven-aged stand structure (inverse “J” shape) with more large trees (*dbh* > 30 cm) than was obtained with the original price function (Fig. 5).

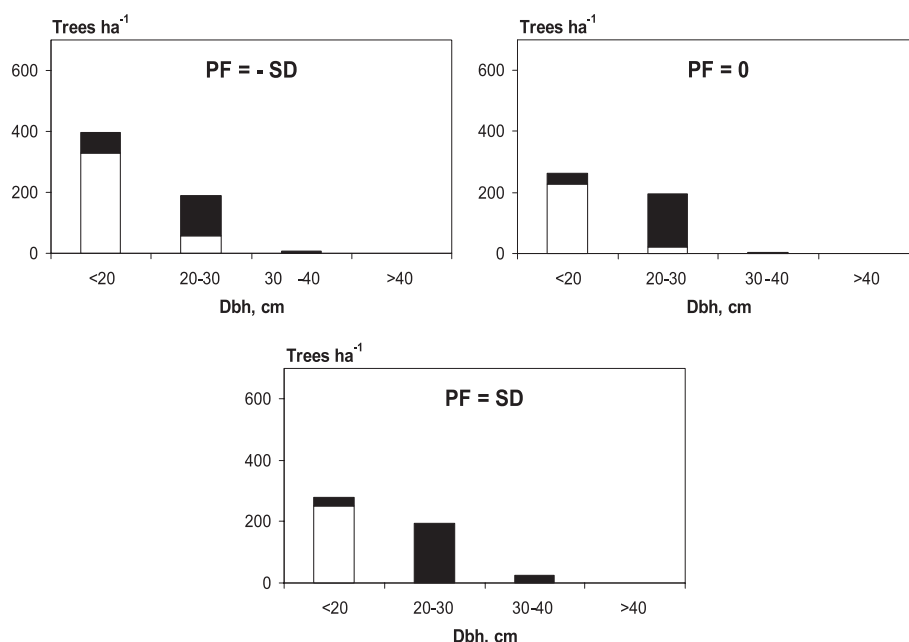
### 3.5. Effect of exponential diameter distribution

As decisions on the optimum management schedule for uneven-aged stands in Spain have traditionally been based on balanced diameter distributions characterized by de Liocourt’s “q” constant, the effect of drawing the initial tree list from a nega-

tive exponential diameter distribution (i.e., when the shape parameter *c* of the Weibull distribution is fixed to 1) was analysed. *LEV*, *PRF*, *WP*, *NET* and *MFV* were maximized using both an exponential (*c* = 1, no limits for *b*) and a uni-modal (free value for the shape parameter in the Weibull) diameter distribution (Fig. 6) for medium site characteristics, 20-year cutting cycle and 2% discounting rate. The use of an exponential diameter distribution led to lower maximal *LEV* and *PRF* than was obtained with uni-modal distribution; but the maximal *WP*, *NET*, and *MFV* were about the same for both diameter distribution shapes (Fig. 7). When the exponential distribution was used, the number of very small (*dbh* < 13 cm) and large trees (*dbh* > 30 cm) was greater, while with the uni-modal distribution, the number of small and medium trees (about 13–30 cm of *dbh*) was greater (Fig. 6).

### 3.6. Effect of biodiversity considerations

Non-timber outputs, such as the number of large trees, deadwood volume or presence of hardwoods in stands, have been proposed



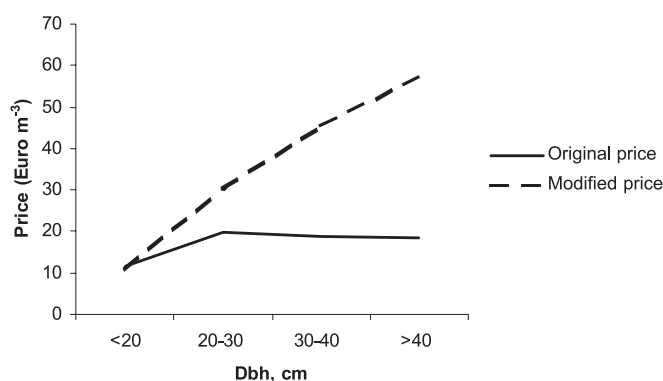
**Figure 3.** Harvested (black) and residual (white) number of trees per hectare in the four diameter classes used in the present study, for optimal management when *LEV* was used as the objective function, different plot factor (*PF*) standard deviation (*SD*) levels in diameter growth models (Eqs. (2) and (4)), medium site characteristics in models other than diameter growth, 20-year cutting cycle and 2% discounting rate.

**Table V.** Optimal combination of *DVs*, land expectation value (*LEV*, euro ha<sup>-1</sup>), mean annual harvest (*WP*, m<sup>3</sup> ha<sup>-1</sup> a<sup>-1</sup>) and stand volume with *LEV* goal, for different levels of elevation, medium slope, latitude and continentality, 20-year cutting cycle, and 2% discounting rate.

Variable <sup>a</sup>	Elevation (m a.s.l.)		
	400	900	1400
$N_s$	462	439	463
$N_n$	230	98	0
$b_s$	13.37	12.95	13.29
$b_n$	10.00	10.00	–
$c_s$	2.1	2.32	2.61
$c_n$	2.1	1.38	–
<i>LEV</i>	1746.7	1331.2	1360.7
<i>WP</i>	3.29	2.31	2.19
$V_{ini}$	121.8	86.8	76.6
$V_{end}$	120.0	86.1	76.0

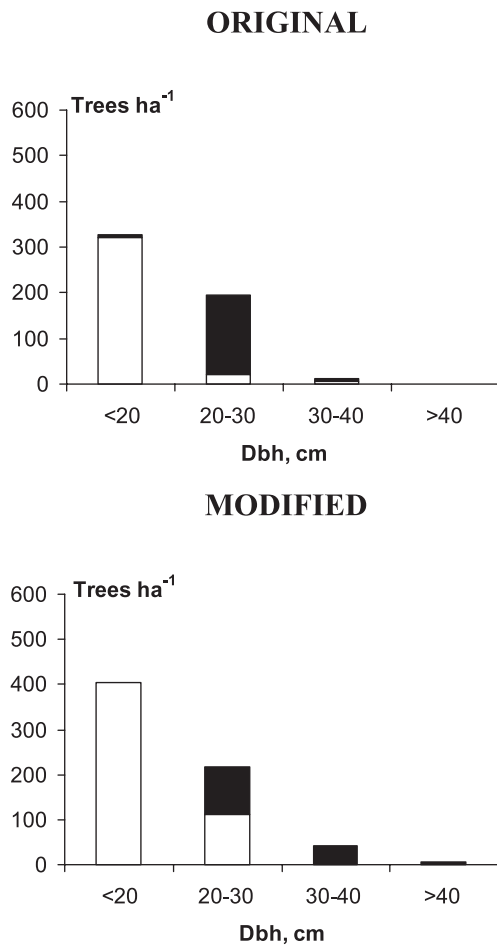
<sup>a</sup>  $N$ : number of trees per hectare;  $b$  and  $c$ : parameters  $b$  and  $c$  of Weibull distribution;  $V_{ini}$  and  $V_{end}$ : stand volumes (m<sup>3</sup> ha<sup>-1</sup>) at the beginning and end of the cutting cycle, respectively; subscripts “s” and “n” refer to *P. sylvestris* and *P. nigra*, respectively.

to be important for biodiversity in the forests of Catalonia [6]. With the available growth and yield model it was not possible to represent all those components of biodiversity. However, as an example of how this criterion could be included in the optimisation problem, the effect of retaining a minimum number of very large trees in the stand was evaluated. Camprodon [6]

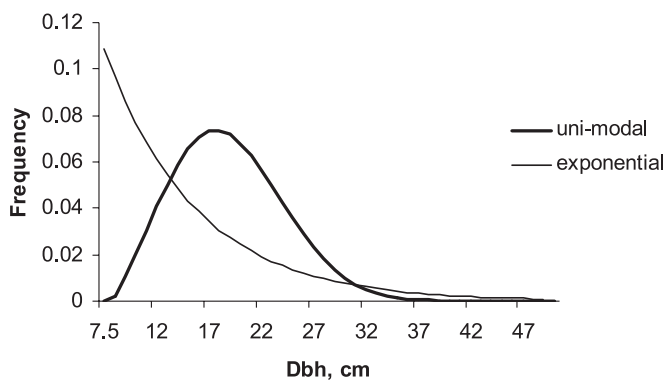


**Figure 4.** Original and modified *dbh*-price functions.

as a starting point for considering this aspect in Iberian forests, although he stressed retaining as many very large trees as possible while keeping the loss of profitability reasonable. We subjectively defined 30 large trees (*dbh* > 40 cm) per hectare as a good number for enhancing biodiversity in stands. Thus, a penalty function forcing the presence of a given number of large trees was added to the optimisation problem, and the management for a minimum of 10 and 30 large trees per hectare in a stand was optimised, (Fig. 8). The large-tree constraint significantly decreased profitability: the *LEV* without the biodiversity constraint was 1331 euro ha<sup>-1</sup>, with 10 large trees ha<sup>-1</sup> it was 758 euro ha<sup>-1</sup>, and with 30 large trees ha<sup>-1</sup> it was 194 euro ha<sup>-1</sup>. Wood production was the same as without the biodiversity constraints. The optimal standing volume increased from 86 to

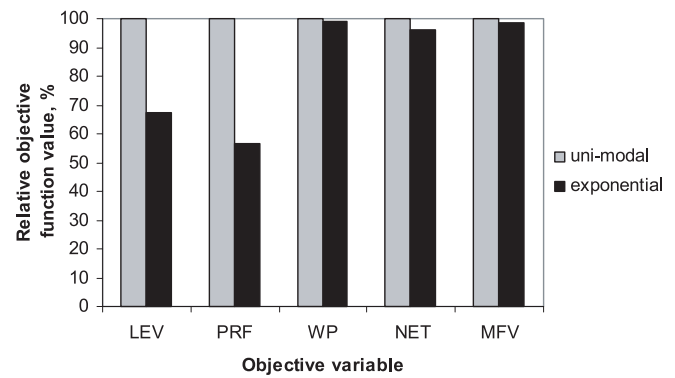


**Figure 5.** Harvested (black) and residual (white) number of trees per hectare in the four diameter classes used in the present study for optimal management with *LEV* objective function, the original and modified *dbh*-price functions and 2% discounting rate percentage.



**Figure 6.** Exponential and uni-modal *P. sylvestris* diameter distributions of the initial stand when *LEV* was maximized, for medium site characteristics, 20-year cutting cycle and 2% discounting rate.

90 m<sup>3</sup> ha<sup>-1</sup> when 10 large trees ha<sup>-1</sup> were required and to 120 m<sup>3</sup> ha<sup>-1</sup> with 30 large trees ha<sup>-1</sup>. Furthermore, the resulting



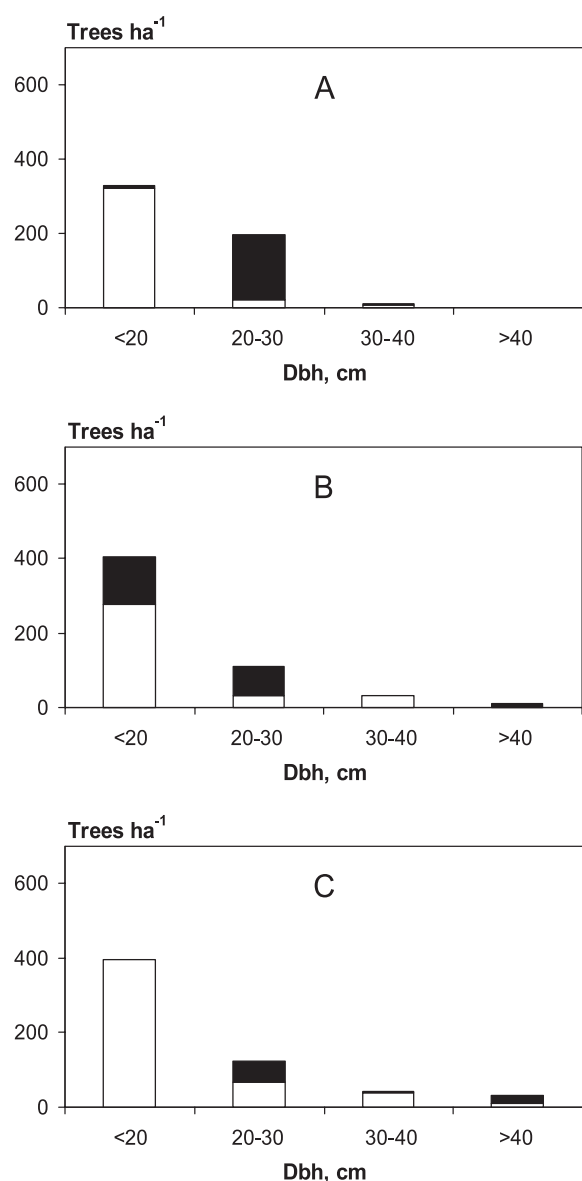
**Figure 7.** Comparison of efficiency between use of an exponential and a uni-modal diameter distribution to draw the initial stand when *LEV*, *PRF*, *WP*, *NET*, and *MFV* are maximized, for medium site characteristics, 20-year cutting cycle and 2% discounting rate.

optimal stand structures resembled the classical inverse exponential shape for uneven-aged stands more than the optimal structures without the biodiversity constraint did. When 10 large trees ha<sup>-1</sup> were required, the dominance of *P. sylvestris* in the optimal stand structure continued; but with a minimum of 30 large trees per hectare, the stand was dominated by *P. nigra*.

#### 4. DISCUSSION

The results obtained in this study were based on the growth and yield model of Trasobares et al. [37]. This model, developed by using data from systematically placed permanent sample plots of the Spanish National Forest Inventory in Catalonia, enables simulations of stand development for average site conditions, but accounts for only a small part of the site-specific variation in fertility among stands. The model may under-predict the growth of young fast-growing, rather even-aged stands [37]. However, it allows us to vary site fertility by modifying the value of the random plot factor in the diameter growth models (i.e., site-specific variation in growth not accounted for by the covariates in the fixed part of the model).

Different aspects had to be considered when the optimisation algorithm was chosen and used: (i) the single-tree simulator produced a discontinuous and non-convex response surface; (ii) the Hooke and Jeeves method is sensitive and inconsistent with respect to the number of *DVs*, starting points and feasible range of *DVs* [16, 17, 41]; (iii) the objective function values around the solution set tended to be flat, partly because the *dbh*-price function used was also very flat; (iv) to avoid a premature termination of the solution algorithm (i.e. a solution far from optimal), penalty functions had to be used, but without too large penalty parameters. To solve the above-mentioned problems, and reach solutions close to the global optimum, the response surface was carefully inspected. Furthermore, suitable values were found for the penalty parameters, first using trial and error and later using a sequential unconstrained minimization technique. The penalty parameters as well as the other optimisation



**Figure 8.** Harvested (black) and residual (white) number of trees per hectare in the four diameter classes used in the present study, for optimal management when *LEV* was used as the objective function, medium site characteristics, 20-year cutting cycle, and 2% discounting rate: without using biodiversity penalty function (*A*), using biodiversity penalty function of 10 large trees per hectare (*B*), using biodiversity penalty function of 30 large trees per hectare (*C*).

parameters were set in such a way that the 33 (direct) solutions obtained for each problem did not vary too much. As many as 5 000 random combinations of *DVs* were used to find the initial point of 30 out of the 33 direct search runs for each problem.

The effect of discounting rate and length of the cutting cycle on the optimal management of mixed stands of *P. sylvestris* and *P. nigra* mixed stands was clear. With a 10-year cutting cycle the optimal structures resembled young and rather even-aged stands. This means that the best way to reach sustainable equilibrium diameter distributions was with tree sizes close to the

maximum *dbh*-growth rate (up to 20 cm *dbh*). With stumpage prices independent of harvested volume and tree size, the 10-year cutting cycle was more profitable than the longer cycles because the opportunity cost of holding trees in the stand was lower. However, it should be remembered that the accuracy of the used growth and yield model [37] is lower for young fast-growing stands, given that this type of stand structure was not highly represented in the modelling data.

The use of profitability (*LEV*, *PRF*) as the objective function assigned interest charges depending on the growing stock. Maximisation of wood production, net income or managed forest value – equivalent to the classical interest-free forest rent criterion – permitted larger trees to occur in the residual distribution [1]. Nevertheless, the effect of the objective function, when these variables were all of the same type (with or without opportunity cost) was not significant, partly because of the flat price function. For example, the result of maximising *WP* or *NET* was practically the same.

An interesting result was that *P. sylvestris* dominated most of the calculated optimal diameter distributions. This was mainly due to the inherent characteristics (diameter growth, survival and ingrowth) of the growth and yield model used, because there are only slight differences in the stumpage prices of the two pine species (only the price of the largest diameter class was slightly higher for *P. sylvestris*). For the medium site characteristics used in the present study, *P. sylvestris* was growing faster than *P. nigra* at low and medium stand densities, while *P. nigra* was growing faster at high stand densities (e.g. when there many very large trees in the stand).

When there were no biodiversity constraints, *P. nigra* was never dominating in optimal stand structures. When *P. nigra* was forced to be the dominant species, the optimal management of the stands was very similar to stands dominated by *P. sylvestris*. Volume growth was slightly smaller, but stand densities and harvest percentages were practically the same; the stand volume was about 10 m<sup>3</sup> less for a *P. nigra* stand. Thus, the optimal management results presented in this study could also be used as a reference for managing *P. nigra* stands in Catalonia.

Since the flat *dbh*-price function used may be rather uncommon, the effect of using a more classical function with the unit price increasing with *dbh* was evaluated. The optimisations showed a clear relationship between optimal stand structures and stumpage prices. This is important because stumpage prices might vary considerably in practical situations, for example, depending on the region or on accessibility to the forest.

In the present study, the constraint of using a “balanced” (negative exponential) diameter distribution to draw the initial stand structures, instead of a more flexible uni-modal distribution, resulted in lower efficiency when profitability objectives were maximized [2] and similar efficiency when profitability was not considered in the objective function. There were two main reasons for this: the lack of flexibility of the shape-constrained Weibull distribution to produce high frequencies for medium *dbh*-classes (13–25 cm of *dbh*), which have the highest growth rates, and the greater number of large trees (*dbh* > 30 cm) in exponential diameter distributions, leading to higher opportunity costs. The results showed that if the aim is to define investment-efficient stand structures for uneven-aged stands in the region, a non-constrained diameter distribution should be

used to define the initial stand structures instead of the exponential diameter distribution approach.

We were aware of the possible limitations that the Weibull distribution may have in representing uneven-aged stands [2, 40]. The Weibull function used to generate the initial diameter distributions has an infinite tail and consequently always populates the larger diameter classes, which could have affected the determination of optimal initial stand structures. Other distributions such as the Johnson SB may provide more accurate descriptions of diameter distributions [33–35]. However, we decided to use the Weibull distribution because it requires only two parameters to be estimated (Johnson SB requires three parameters) and has proved to describe diameter distributions with similar accuracy as the Johnson SB [34]. Despite this, future studies should consider the use of other distributions to describe uneven-aged stand structures.

As an example of the use of biodiversity considerations in the optimisation problem, the effect of retaining a minimum number of very large trees in the stand was analysed. The lower profitability obtained when the large tree constraint was used was reasonable; keeping a certain number of very large trees in the stand led to higher standing volumes, and therefore the opportunity cost was higher. It was also logical that the stand was dominated by *P. nigra* when a minimum of 30 large trees per hectare was required, as *P. sylvestris* is more sensitive to stand density than *P. nigra* is (see [37]). The methods presented can be an important tool for quantifying the cost of favouring biodiversity in the *P. sylvestris* and *P. nigra* stands of the region. Nevertheless, further research should be conducted to assess the amount of a certain stand feature required to provide a suitable habitat for certain species. Furthermore, to predict the dynamics of other relevant non-timber attributes in stands, new models should be developed. It should also be remembered that for some species with large territories, biodiversity matters must be considered at the forest level rather than the stand level. Other non-timber forest values such as amenity or mushroom production could be also included in the optimization problem [29], but models expressing the relationships between these values and stand structures in the region should be developed first. Such applications would be an important contribution for helping managers analyse the ecological and social effects of the alternative management options of these forests, in addition to the classical economic effects.

Optimal management was sensitive to changes in site conditions. Changes in elevation affected optimal stand structures according to the requirements of the two pine species, while changes in site fertility affected harvest percentages and profitability. The simulation-optimisation system developed here was applied to different site fertility conditions by modifying the plot factor. In practical applications, however, a plot factor value describing site fertility will be difficult to calculate. Hence, if the aim is to assess site-specific growth potential carefully, a version of the growth and yield model that includes an easily measurable site descriptor corresponding to the site index of an even-aged stand would be better [36].

The simulation-optimisation system described in this article can be used to derive silvicultural instructions for reaching the optimal management regime for a given stand. These instructions can be used directly by forest managers. Nevertheless, in

practical forestry existing stands may differ considerably from optimal sustained diameter distributions. Due to this, given the existing and the optimal stand structure non-linear programming should be used in future studies to solve for the optimal conversion strategy [39].

The majority of forests cannot be managed by relying only on the stand-level approach because this often produces large fluctuations in annual harvests and revenues. In forest-wide applications, where the aim is to find an optimal schedule of treatments for all stands to best meet forest wide objectives and constraints, the system can help generate alternative treatment schedules for stands. These schedules can subsequently be analysed by a forest-level optimisation model (solved using linear programming or other procedures) to find an optimum forest-wide management regime.

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## REFERENCES

- [1] Bare B., Opalach D., Optimising species composition in uneven-aged forest stands, *For. Sci.* 33 (1987) 958–970.
- [2] Bare B., Opalach D., Determining investment-efficient diameter distributions for uneven-aged northern hardwoods, *For. Sci.* 34 (1988) 243–249.
- [3] Bazaraa M.S., Sherali H.D., Shetty C.M., *Nonlinear programming: theory and algorithms*, 2nd ed., John Wiley & Sons, New York, 1993.
- [4] Buongiorno J., Michie B., A matrix model of uneven-aged forest management, *For. Sci.* 26 (1980) 609–625.
- [5] Buongiorno J., Gilless J.K., *Forest management and economics: a primer in quantitative methods*, Macmillan, New York, 1987.
- [6] Camprodon J., *Tratamientos forestales y conservación de la fauna vertebrada*, in: Camprodon J., Plana E. (Eds.), *Conservación y gestión de la biodiversidad en ecosistemas forestales: Aplicación a la fauna vertebrada*, Universitat de Barcelona, Barcelona, 2001, pp. 135–179.
- [7] Chang S.J., Determination of the optimal growing stock and cutting cycle for an uneven-aged stand, *For. Sci.* 27 (1981) 739–744.
- [8] Consorci Forestal de Catalunya, *Taules de Preus de fustes i llenyes*, *Revista "Catalunya Forestal"* n° 56, 2002. Available from <http://www.forestal.net/> [updated 10 August 2002; cited 18 September 2002].
- [9] Díaz Balteiro L., Prieto Rodríguez A., Modelos de planificación forestal basados en la programación lineal: Aplicación al monte "Pinar de Navafria" (Segovia), *Invest. Agrar. Sist. Recur. For.* 8 (1999) 63–92.
- [10] Gonzalez J.M., Arrufat D., Meya D., Modelos de gestión silvícola para las masas irregulares de Pino laricio en el Prepireneo Catalán, *Rev. For. Esp.* 16 (1997) 14–20.
- [11] Gove J.H., Fairweather S.E., Optimising the management of uneven-aged forest stands: a stochastic approach, *For. Sci.* 38 (1992) 623–640.
- [12] Gracia C., Abril M., Barrantes O., Burriel J.A., Ibáñez J.J., Serrano M.M., Vayreda J., *Inventari Ecològic i Forestal de Catalunya: Mètodes*, Departament d'Agricultura, Ramaderia i Pesca, Generalitat de Catalunya, Barcelona, 1992.

- [13] Gracia C., Burriel J.A., Ibáñez J.J., Mata T., Vayreda J., *Inventari Ecològic i Forestal de Catalunya: Regió Forestal IV*, CREAF, Bellaterra, 2000.
- [14] Haight R.G., Evaluating the efficiency of even-aged and uneven-aged stand management, *For. Sci.* 33 (1987) 116–134.
- [15] Haight R.G., Getz W.M., Fixed and equilibrium endpoint problems in uneven-aged stand management, *For. Sci.* 33 (1987) 908–931.
- [16] Haight R.G., Monserud R.A., Optimising any-aged management of mixed-species stands. I. Performance of a coordinate-search process, *Can. J. For. Res.* 20 (1990) 15–25.
- [17] Haight R.G., Monserud R.A., Chew J.D., Optimal harvesting with stand density targets: managing Rocky Mountain conifer stands for multiple forest outputs, *For. Sci.* 38 (1992) 554–574.
- [18] Hall D.O., Financial maturity for even-aged and all-aged stands, *For. Sci.* 29 (1983) 833–836.
- [19] Hooke R., Jeeves T.A., “Direct search” solution of numerical and statistical problems, *J. Assoc. Comput. Mach.* 8 (1961) 212–229.
- [20] ICONA, Segundo Inventario Forestal Nacional (1986–1995) Cataluña: Lleida, MAPA, Madrid, 1993.
- [21] Leak W.B., An expression of diameter distribution for unbalanced uneven-aged stands and forests, *For. Sci.* 10 (1964) 39–50.
- [22] De Liocourt F., De l’aménagement de Sapinières, *Bull. Soc. For. Franche-Comté et Belfort*, Besançon, 1898.
- [23] Madrigal Collazo A., Ordenación de montes arbolados, ICONA, Madrid, 1994.
- [24] Maltamo M., Puumalainen J., Päivinen R., Comparison of Beta and Weibull functions for modelling basal area diameter distribution in stands of *Pinus sylvestris* and *Picea abies*, *Scand. J. For. Res.* 3 (1995) 333–342.
- [25] Meyer H.A., Structure, growth, and drain in balanced uneven-aged forests, *J. For.* 50 (1952) 85–92.
- [26] Miina J., Optimising thinning and rotation in a stand of *P. sylvestris* on a drained peatland site, *Scand. J. For. Res.* 11 (1996) 182–192.
- [27] Muchiri M., Pukkala T., Miina J., Optimising the management of maize-*Grevillea Robusta* fields in Kenya, *Agrofor. Syst.* 56 (2002) 13–25.
- [28] Palahí M., Pukkala T., Optimising the management of Scots pine (*P. sylvestris* L.) stands in Spain based on individual-tree models, *Ann. For. Sci.* 60 (2003) 105–114.
- [29] Pukkala T., Miina J., A method for stochastic multiobjective optimisation of stand management, *For. Ecol. Manage.* 98 (1997) 189–203.
- [30] Rautiainen O., Pukkala T., Miina J., Optimising the management of even-aged *Shorea Robusta* stands in southern Nepal using individual tree growth models, *For. Ecol. Manage.* 126 (1999) 417–429.
- [31] Rideout D., Managerial finance for silvicultural systems, *Can. J. For. Res.* 15 (1985) 163–166.
- [32] Roise J.P., A nonlinear programming approach to stand optimisation, *For. Sci.* 32 (1986) 735–748.
- [33] Schreuder H.T., Hafley W.L., A useful bivariate distribution for describing stand structure of tree heights and diameters, *Biometrics* 33 (1977) 471–478.
- [34] Siipilehto J., Improving the accuracy of predicted basal-area diameter distribution in advanced stands by determining stem number, *Silva Fenn.* 33 (1999) 281–301.
- [35] Tham A., Structure of mixed *Picea abies* (L.) Karst. and *Betula pendula* Roth and *Betula pubescens* Ehrh. stands in south and middle Sweden, *Scand. J. For. Res.* 3 (1988) 355–370.
- [36] Trasobares A., Pukkala T., Using past growth to improve individual-tree diameter growth models for uneven-aged mixtures of *Pinus sylvestris* L. and *Pinus nigra* Arn. in Catalonia, north-east Spain, *Ann. For. Sci.* 61 (2004) 409–417.
- [37] Trasobares A., Pukkala T., Miina J., Growth and yield model for uneven-aged mixtures of *Pinus sylvestris* L. and *Pinus nigra* Arn. in Catalonia, north-east Spain, *Ann. For. Sci.* 61 (2004) 9–24.
- [38] Valsta L., A comparison of numerical methods for optimising even-aged stand management, *Can. J. For. Res.* 20 (1990) 961–969.
- [39] Valsta L., Economic evaluation of uneven-aged management, in: Gadaw K. von, Nagel J., Saborowski J. (Eds.), *Continuous cover forestry: assessment, analysis, scenarios*, Kluwer Academic Publishers, pp. 281–293.
- [40] Vanclay J.K., *Modelling forest growth and yield: applications to mixed tropical forests*, CAB International, Wallingford, UK, 1994.
- [41] Wikström P., Eriksson L.O., Solving the stand management problem under biodiversity-related considerations, *For. Ecol. Manage.* 126 (2000) 361–376.