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► **To cite this version:**

Daniel Cintra, Pierre Argoul. Argumentary oscillation phenomenon (working document). 2013. <hal-00876552v1>

HAL Id: hal-00876552

<https://hal.science/hal-00876552v1>

Preprint submitted on 24 Oct 2013 (v1), last revised 5 Feb 2018 (v2)

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Argumentary oscillation phenomenon

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When a Duffing oscillator with one degree of freedom is submitted to an external harmonic force whose frequency is much higher than that of the oscillator, it is possible, under certain conditions, that the motion of the oscillator, once launched, is maintained at a stable energy level. This may occur when the force exerts its action only when the oscillator is close to its center position. The equation of motion can be written in canonical form :

$$\frac{d^2 x}{d\tau^2} + 2\beta \frac{dx}{d\tau} + x + \mu x^3 = A H(x) \sin\left(\frac{\nu}{\omega_0} \tau\right) \quad (1)$$

where :

x is the space coordinate,

β is the coefficient of viscous dissipation,

ω_0 is the natural angular frequency of the non-dissipative oscillator for low amplitudes in x ,

$\tau = \omega_0 t$ is the dimensionless time,

μ is a coefficient representing the non-linearity of the restoring force,

A is a parameter representing the magnitude of the external force,

H is a function of x representing the proportion of the external force acting on the oscillator as a function of the position x ($|H|$ varies between 0 and 1),

and ν is the angular frequency of the external force, independent of the oscillator.

To experiment with a pendulum, we put $\mu = -\frac{1}{6}$ to get a model of the restoring force close to the sine for $|x| \leq \frac{\pi}{2}$. We can take for the function $H(x)$ a gate function, which is 1 for $|x| \leq h$ and 0

otherwise. Writing that the solution $x(t)$ of equation (1) could be expressed as

$x(\tau) = a(\tau) \sin(\omega \tau + \varphi(\tau))$, where ω is a parameter to be determined, and $a(t)$ and $\varphi(t)$ respectively represent the amplitude and phase of x , which we assume being slowly varying with respect to the variations of the external force, we can write (1) in standard form and average the resulting system with respect to time, to get, if there is an odd integer n such that $\omega = \frac{\nu}{n}$ is close to

ω_0 , i.e. $\rho_n = \frac{\omega}{\omega_0}$ is close to 1, the following system of equations :

$$\frac{da}{d\tau} = -\frac{A}{4\rho_n} S(a) \sin(n\varphi) - \beta a, \quad \frac{d\varphi}{d\tau} = \frac{\rho_a^2 - \rho_n^2}{2\rho_n} - \frac{A}{4a\rho_n} D(a) \cos(n\varphi)$$

where :

a is the amplitude of x , averaged with respect to time,

φ is the phase of x , averaged with respect to time,

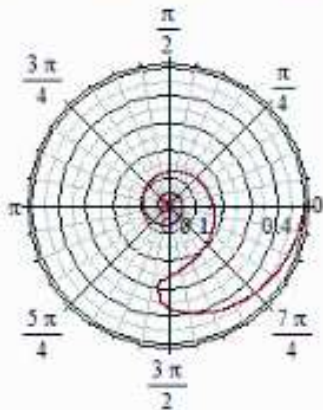
$$\rho_a^2 = 1 + \frac{3\mu}{4} a^2, \quad \rho_n^2 = \left(\frac{1}{n} \frac{\nu}{\omega_0}\right)^2,$$

and $S(a)$ and $D(a)$ are functions depending on the shape of the function H .

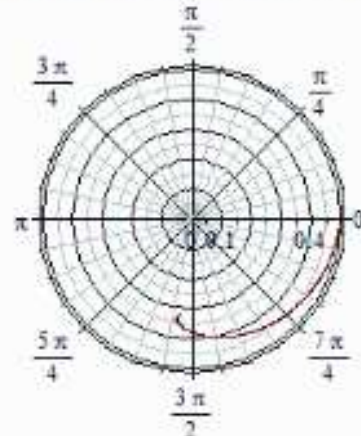
The study of the stationary solution and its stability shows that, under certain conditions, the system, once launched by an external pulse, can maintain its amplitude of motion due to the external force, although the latter has a frequency much higher than the natural frequency of the oscillator. In addition, when the system is non-linear ($\mu \neq 0$), there are several possible stable amplitudes. In van der Pol representation in polar coordinates, we obtain the following type of

chart:

$$\rho = 0.9936305732, n = 101, A = 0.0969, \\ \&gamma = 10000, \beta = 0.001, v = v, \varphi_0 = \frac{3}{202} \pi$$



$$\rho = 0.9936305732, n = 101, A = 0.0969, \\ \&gamma = 10000, \beta = 0.001, v = v, \varphi_0 = 0$$



In the graph on the left, we can see that the movement vanishes, after being nearly captured in the ($a=0.32$, $\varphi \approx 3\pi/2$) area, while in the right graph, we see that the movement is stabilized at ($a = 0.31$, $\varphi \approx 3\pi/2$). The distinction between these two cases comes solely from the initial conditions.