# From the input impedance to the synthesized pressure signal : application to the saxophone

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# FROM THE MEASURED INPUT IMPEDANCE TO THE SYNTHESIZED PRESSURE SIGNAL : APPLICATION TO THE SAXOPHONE

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## ABSTRACT

A method for time domain simulation of single reed wind instruments is presented, based on measured input impedance. The results of internal pressure simulations are compared with earlier experimental results obtained from a saxophone. The G fingering (first register) is studied and various oscillation regimes are presented, according to geometrical modifications on the saxophone neck. Sound examples illustrating the method are presented.

## 1. INTRODUCTION

For many years, the input impedance of single reed wind instruments has been a common tool used to evaluate the playing frequency of such instruments. During the past decade, the development of numerical simulations using physical modelling enabled doing synthesis from the knowledge of the resonator input impedance or with the help of waveguide filters techniques. Today, it seems to be very interesting to develop synthesis, in order to build a system to be used as a design aid tool for instrument makers. For such an application, the results obtained from the model should be compared with the experimental results in order to verify the validity of the method. In this paper we present simulation results obtained with the help of measured input impedance of an alto saxophone. These results can be compared with experimental results presented by Dalmont and Gilbert (1993).

Following this introduction, we give briefly in section two, the basic physical model used for the numerical simulation and the general technique developed to calculate the internal pressure. In the third section, we present the results applied to the saxophone.



## 2. FROM THE MEASURED INPUT IMPEDANCE TO THE SYNTHESISED PRESSURE SIGNAL

### 2.1. An elementary physical model of woodwinds.

The numerical simulation of a musical instrument needs a physical model to represent its functioning. This physical model is described below. It has been chosen to be very simple in order to describe the basic behaviour of the auto oscillating system without creating a complicated numerical algorithm (see for example Gazengel et al., 1994, Dalmont et al., 1994). We give here a set of non-linear and linear classical equations governing the functioning of a single reed instrument.

The resonator is considered to be a linear system, whose properties can be described by its input impedance $Z(j\omega)$ :

$$p(j\omega) = Z(j\omega)U_{in}(j\omega), \qquad (1)$$

where $p(j\omega)$ is the acoustic pressure and $U_{in}(j\omega)$ the volume velocity at the input of the tube in the frequency domain.

The modelling of the air flow entering from the player's mouth to the instrument is a delicate subject which has been dealt carefully by Hirsberg et al. (1990) for a static position of the reed. We give here only a simple model. If $P_m$ is the static pressure in the player's mouth, the acoustic flow entering the instrument $U(t)$, is related to the pressure jump $P_m$-$p(t)$ by Bernoulli's law :

$$P_m = p(t) + \frac{1}{2}\rho\left\{\frac{U(t)}{w.[y(t)+H]}\right\}^2, \qquad (2)$$

where $\rho$ is the air density, w the width of the reed, H the height of the air gap when the instrument does not play and $y(t)$ the position of the extremity of the reed. Moreover $U(t)$ is set equal to zero when the reed is completely closed. This condition is fulfilled if the position of the tip of the reed verifies $y \le -H$.

$$U(t) = \sqrt{\frac{2}{\rho}}.w.\sqrt{|P_m - p(t)|}.[y(t)+H].Sgn,$$

$Sgn = 1$ if $P_m - p(t) \ge 0$, (3).

$Sgn = -1$ if $P_m - p(t) \le 0$.

$U(t) = 0$ if $y(t) \le -H$.

For simplicity, we write $U(t) = NL[p(t), y(t)]$, where NL is the non linear function expressed by equation (3). The total acoustic flow $U_{in}(t)$ entering the bore is also due to the acoustic flow $U_r$ produced by the reed movement :

$$U_r(t) = S_r.\frac{dy(t)}{dt}, \qquad (4)$$

where $S_r$ is an effective area of the reed.

The total acoustic flow $U_{in}(t)$ can then be written :

$$U_{in}(t) = U(t) - U_r(t). \qquad (5)$$

Moreover, we suppose the reed does not bend against the curved lay of the mouthpiece but simply displaces, therefore obeying a simple harmonic oscillator equation without non linearity. In fact, even if it is not beating against the mouthpiece, the reed may not behave as a linear oscillator, but, at least far from its resonance frequency, the reed acts like a linear oscillator driven below resonance. The collision that occurs when the oscillator strikes the mouthpiece is made inelastic by setting the velocity to zero and the position $y(t)$ to -H :

$$\frac{d^2y(t)}{dt^2} + g_r\frac{dy(t)}{dt} + \omega_r^2 y(t) = \frac{p(t) - P_m}{\mu_r} \quad (6a)$$

$$\begin{cases} y(t) = -H \\ \dfrac{dy(t)}{dt} = 0 \end{cases} \quad \text{if} \quad y(t) \le -H \qquad (6b)$$

where $g_r$ is the reed damping factor, $\omega_r$ its natural resonance frequency and $\mu_r$ its mass per area.

## 2.2. Solving the problem in the discrete time domain

In the time domain, the behaviour of the resonator can be expressed with the help of h(t), the inverse Fourier Transform of Z(jω), but the use of h(t) consumes much calculating time because of the temporal length of this function. An alternative method consists of calculating the equivalent plane wave reflection function $r_p(t)$ = FT -1 [ $R_p(f)$ ], where $R_p(f)$ is the plane wave reflection coefficient defined by Schumacher (1981) as :

$$R_p(f) = \frac{Z(f) - Z_c}{Z(f) + Z_c}, \qquad (8)$$

$Z_c$ being the characteristic impedance of the wave guide, defined by $Z_c = \dfrac{\rho C}{S}$.

Supposing that $p_+(t)$ and $p_-(t)$ represent respectively the outgoing and incoming waves in the input of the resonator, equation (1) can be written in the time domain :

$$\begin{aligned} &p(t) = p_{past} + Z_c U_{in}(t), \\ &p_{past} = 2p_-(t) = r_p(t) * 2p_+(t), \\ &\text{where } 2p_+(t) = p(t) + Z_c . U_{in}(t)' \end{aligned} \qquad (9)$$

where * represents the convolution operation.

The set of equations (3), (4), (5), (6) and (9) is solved in the discrete time domain using the compensated bilinear transformation (for more details see Gazengel et al., 1994) for which y[n] and $U_r[n]$ can be written :

$$\begin{cases} y[n] = y_0 + y_1 p[n] \\ U_r[n] = U_{r_0} + U_{r_1} p[n] \end{cases} \qquad (10)$$

This set of equation leads to the non linear equation :

$$p[n] = p_{past} + Z_c NL(p[n]; y[n]) - Z_c U_r[n] \qquad (11)$$

which is solved using the Newton-Raphson method (see Table I. below).

Calculation of an approximation of the discrete reflection function using an FFT algorithm on $2^{NFFT}$ points from the measured input impedance (Dalmont and Bruneau, 1991).

⇓

Calculation of the "past" of the resonator:

$$p_{past}[n] = \sum_{k=1}^{n} r_d[k].\{p[n-k] + Z_c U_{in}[n-k]\}$$

⇓

Evaluation of the discrete response of the reed using the compensated bilinear transformation.

⇓

Calculation of the discrete solutions of the system: p[n], $U_{in}[n]$, y[n] using the Newton Raphson technique applied on equation (11).

**Table 1:** General method of simulation (see Gazengel et al., 1994)

## 3. APPLICATION TO THE SAXOPHONE

We use the simulation process described in section 2 and apply it to the low G fingering of an alto saxophone. This saxophone is modified in order that it can be played a quarter tone lower than usual. This was done with two tubes branched on the neck. Dalmont and Gilbert (1993) show that these two tubes can realise two major functions. On one hand, they can modify the first resonance frequency value without changing the second resonance frequency (modification 1). On the other hand, they can shift the two first resonance frequencies without modifying their relative inharmonicity (modification 2). For the low G fingering, the amplitude of the two first impedance peaks are almost equal. When playing the G fingering, the first modification creates a quasi periodic tone while the second one changes only the fundamental frequency and the spectrum of the mouthpiece internal pressure. Moreover these results depend on the musician's embouchure. For a tight embouchure, modification 1 creates a quasi periodic oscillation, for a relaxed one, this modification changes the fundamental frequency and the spectrum of the pressure signal.

We wish to know if the simulation process we developed can reproduce these effects when using measured input impedance of the G fingering modifieded with the modifications mentioned above.

For the simulation, we use the measured input impedance of the G fingering (no modification, modification 1 and 2) and the following reed parameters :

$\mu_a = 0,0231 \text{ kg/m}^2$,

$\omega_a = 20000 \text{ rad/s}$, $g_a = 3000 \text{ s}^{-1}$.

(parameters obtained from Worman, 1971).

We use three different values for the reed area $S_r$ in order to show the effects of the musician's embouchure on the calculated pressure signal :

$S_r = 4.10^{-5}$, $6.10^{-5}$, $8.10^{-5}$ $m^2$,

which correspond respectively to the three following length correction :

$\Delta l = 3,5$ mm ; 5,2 mm ; 7 mm (see Dalmont et al., 1994 for the estimation of these three values).

We present in Figure (1a) the simulation results obtained for the G fingering without modification and the G fingering modified with modification 2. Figure (1b) shows the pressure calculated for the G fingering modified with modification 1. Figure (2) shows the embouchure effects on the resulting discrete pressure signal calculated with the G fingering modified with modification 1 (quasi periodic oscillation). These results show that the simulation process enables obtaining oscillation regimes whose characteristics are very near those observed during the experiment (Dalmont et Gilbert, 1993).

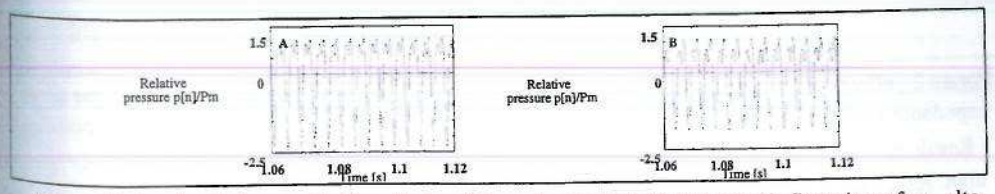


Figure (1a) : simulation results (mouthpiece internal pressure) for the low G fingering of an alto saxophone. The mouth pressure is $P_m = 1500$ Pa, the reed area $S_r = 4.10^{-5} m^2$. Result A : the input

impedance is not modified, the fundamental frequency is f = 229,9 Hz. Result B : the input impedance is modified such that the harmonicity does not change (modification 2), the fundamental frequency is f = 212,9 Hz.

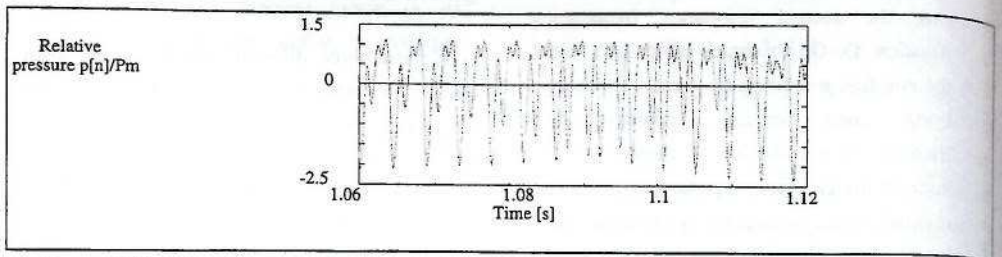


Figure (1b) : simulation results (mouthpiece internal pressure) for the low G fingering of an alto saxophone. The mouth pressure is $P_m = 1500$ Pa, the reed area $S_r = 4.10^{-5} m^2$. The input impedance is modified such that the harmonicity has changed (modification 1) : there is a quasi periodic oscillation

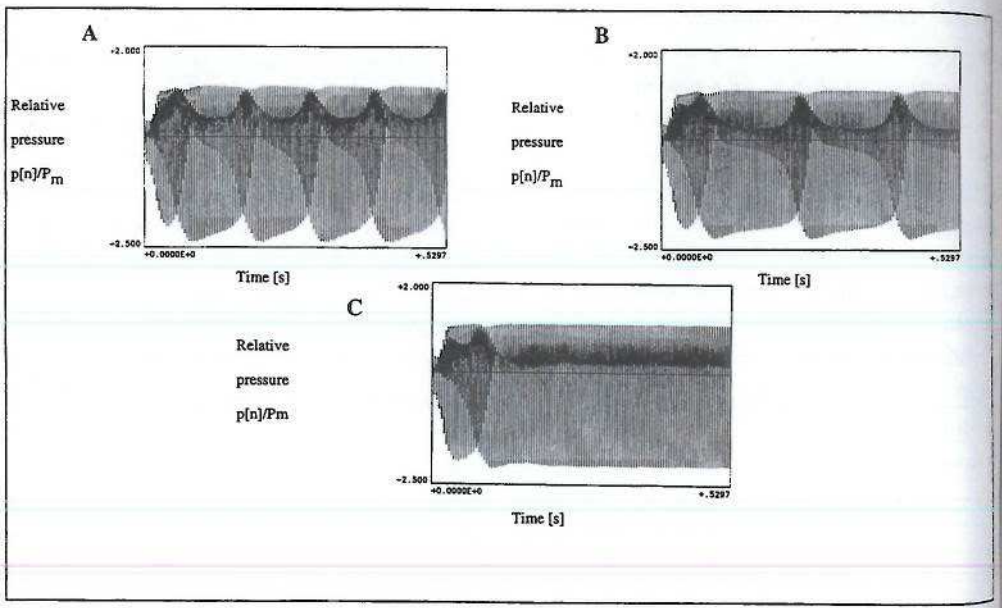


Figure 2 : effects of the musician's embouchure on the resulting pressure signal calculated with the input impedance modified with modification 1 (change of harmonicity). The mouth pressure is $P_m = 1500$ Pa. Result A : $S_r = 4.10^{-5} m^2$. Existence of a quasi periodic regime for which the "beating frequency" is high. Result B : $S_r = 6.10^{-5} m^2$. Existence of a quasi periodic regime for which the "beating frequency" is low. Result C : $S_r = 8.10^{-5} m^2$. There is not any quasi periodic regime any more.



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